

THOMAS HARRIOT: FATHER OF MODERN NOTATION

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TO DR. SLOAN DESPEAUX

DEPARTMENT OF MATHEMATICS

WESTERN CAROLINA UNIVERSITY

BY

LAYLA BIDDIX

Students learn mathematical symbolism early in their education. Any third grader can convey with ease the meanings of “+”, “-”, “=”, “<”, and “>” when asked about these symbols. However, these same students are rarely ever taught the origins of the symbols that they utilize with nearly every mathematical equation that they encounter. Most of these students will never be taught that several of these symbols were created and used by mathematician and astronomer Thomas Harriot.

Although little is known of Thomas Harriot, his influence on mathematics is prevalent even at the most basic levels of mathematical manipulations. There exist many suggestions for why Harriot created symbols for his mathematical works, but no concrete reasoning for why Harriot began using symbolism in his mathematics is known. The mixed reaction from his mathematic colleagues and the fact that Harriot did the majority of his work in the early 1600s without the use of a printing press, make it extraordinary that many of the symbols Harriot utilized and the notation in which he wrote in his mathematical manuscripts are still in use today.

Harriot always had a knack for symbolism. Harriot’s work with symbolism began with his creation of his own alphabet based on the Algonquian language.¹ After spending time and forming a friendship with an Algonquian Indian that some of Harriot’s friends had kidnapped from what is now known as North Carolina, he began writing a language based on what he learned from his new Indian friend.² This new language included a branch of mathematics, in which he represented powers and roots using cosmic or algebraic numbers.

1. Jacqueline Stedall, “Symbolism, combinations, and visual imagery in the mathematics of Thomas Harriot,” *Historia Mathematica* 34, no. 4 (November 2007): 382.

2. Ibid.

Harriot's use of symbolism indicated his recognition of symbols used by some of his mathematics colleagues.³ Harriot incorporated some symbols from other mathematicians in his own mathematical work, while others were his own creation.

Even though, mathematically speaking, Harriot was well before his time, he was definitely not the first mathematician who utilized symbolism in mathematical computations. Several of Harriot's mathematical predecessors, including Bombelli, Stifel, Clavius and Viète, also used symbolism in their mathematical works.⁴ Not all of the symbols created and used by these mathematicians stood the test of time; while most of Harriot's symbolism or variations of Harriot's symbols have remained important components of mathematical computation and have continued to be in the everyday use of modern mathematicians.

Mathematical symbolism first appeared in Harriot's work on falling materials.⁵ Using approximately the same weight of wax and lead, Harriot hypothesized that bodies of matter fall at different rates of change.⁶ In this manuscript on falling material, Harriot represented the volume of lead by a , the volume of wax by $8a$, the volume of air by b , the distance traveled by lead by f , and the distance traveled by wax by g .⁷ Although his physics were incorrect, Harriot arrived at the correct answer using the equation $b - a, b - 8, a: f \bullet g$.⁸ This was only the beginning of symbolism in Harriot's works.

Harriot had the best opportunity to expand his use of symbolism when he dealt with common operations like addition, subtraction, multiplication, and division. Because these

3. John W. Shirley, *Thomas Harriot: A Biography* (New York: Oxford Press, 1983), 110.

4. Stedall, "Symbolism", 397; Shirley, *Thomas*, 110.

5. Stedall, "Symbolism", 384.

6. Ibid.

7. Ibid.

8. Ibid.

operations were so common, Harriot created his own symbols to use as shorthand. During Harriot's time, the modern addition and subtraction symbols had already become common practice, so it was not surprising to find these symbols in Harriot's manuscripts.⁹ In addition to those symbols, Harriot created and used two more addition symbols, \pm and \mp , so that he could work with multiple equations simultaneously.¹⁰ To denote the parentheses required in multiplication, Harriot used single commas.¹¹ Also, in a rather confusing manner, Harriot occasionally used commas to denote multiplication involving numbers and variables.¹² To multiply two or more quantities with variables, Harriot wrote:

$$\begin{array}{|l} a+b \\ b+c \end{array} = ab + bb + ac + bc.$$

¹³ It was also common for Harriot to use “•” located between two numbers when indicating the desire to multiply these two numbers together. When he dealt with division, because the modern notation of dividend over divisor (a/b) had become prevalent, Harriot used this notation.¹⁴

Harriot had an extensive list of miscellaneous symbols he used when writing his mathematical manuscripts. Because Harriot used symbolism as a means of shorthand mathematical expression, this is not surprising. Several of his symbols seem to be entirely too obvious, including using “|” for lines, “□” for squares, and a cube formation to represent

9. Jacqueline Stedall, , *The Greate Invention of Algebra: Thomas Harriot's Treatise on Equations* (New York: Oxford University Press, 2003), 8.

10. Ibid.

11. Ibid.

12. Ibid.

13. Ibid.

14. Jacqueline Stedall, *A Discourse Concerning Algebra* (New York: Oxford University Press, 2002), 92.

cubes of numbers.¹⁵ He sometimes used the letter “s” to indicate when to conduct the “sum of” a combination.¹⁶ Also, Harriot commonly wrote aa, aaa, and aaaa for a^2 , a^3 , a^4 and so on.¹⁷ Most of these symbols are no longer recognized in mathematics most likely due to a lack of need. Another reason for not using these symbols is that modern mathematics has more effective notation for the symbols that Harriot used.

On the other hand, some of Harriot’s symbols were not so straightforward. The origin of thought for these symbols is not only unknown, but also highly questioned. Based on the connotations Harriot associated with certain symbols and because he used some of the same symbols found in modern mathematics in a different manner, modern mathematicians would likely experience difficulty when following Harriot’s manuscripts. The best example of this is Harriot’s use of $^2\sqrt{}$ and $^3\sqrt{}$. In Harriot’s mathematics, these symbols represented roots of two or three terms, not the square or cube root, as in modern day mathematics.¹⁸ Other symbols Harriot used would be completely foreign to the modern mathematician. For example, Harriot typically denoted “equivalent to” using a ladder-like symbol consisting of two vertical lines with additional horizontal lines connecting the vertical lines depending on the number of equivalences.¹⁹ Additionally, Harriot sometimes used an elongated “=” sign centered between two numbers or variables to denote subtraction of two numbers. For example, Harriot wrote $a==b$ to denote $a - b$.²⁰ A majority of these symbols served as the precursor to other modern symbols. It is likely

15. Stedall, *Symbolism*, 388.

16. Ibid.

17. Stedall, *Discourse*, 90.

18. Stedall, *Symbolism*, 388.

19. Ibid.

20. Ibid.

that these symbols were never adopted into modern mathematics with their original meaning intact due to their unpopularity among Harriot's mathematical peers.

It was very common for Harriot to take symbols from other mathematicians and alter them. For example, Harriot's equality symbol was much like the "=" sign used by Robert Recorde in his 1557 *The Whetstone of Witte*, except Harriot added two vertical lines connecting the two horizontal lines.²¹ Sometimes, however, Harriot used "||" to denote equality. Additionally, Harriot changed the common vowels used by Viète, including *A* and *E*, used to denote variables in algebra, to lowercase vowels like *a* and *e*.²² Like Viète, Harriot used letters to denote known and unknown numbers. For example, Harriot used lowercase vowels when referring to an unknown number in an equation and lowercase consonants for numbers that were known constants.²³

Two of Harriot's most well known accomplishments are the less than and greater than symbols, which Harriot adapted from previous mathematicians, as well. The original symbols for less than and greater than were \lrcorner and \llcorner , respectively. Harriot's greatest accomplishments became staples in modern mathematical notation. He began using two elongated, curved lines coming together at a point, almost like that of a horn or cornucopia, to represent less than or greater than.²⁴ Like the less than and greater than symbols modern mathematicians use, the symbol pointing to the left represented less than and the

21. Stedall, *Discourse*, 90.

22. Stedall, *Greates*, 5.

23. Florian Cajori, "A Revaluation of Harriot's *Artis Analyticae Praxis*", *Isis* 11, no. 2 (December 1928): 316.

24. Stedall, *Greates*, 8.

symbol pointing to the right represented greater than. These symbols evolved into the modern notation for less than and greater than, $<$ and $>$, respectively.²⁵

Finally, the greatest of Harriot's symbolic notations was his juxtaposition of two variables written side-by-side to represent multiplication, like ab .²⁶ The use of this notation has undoubtedly remained unwaveringly prevalent in mathematics.

Many researchers question Harriot's reasoning for integrating symbolism into his mathematical manuscripts, though there is no widely accepted notion for why he began the movement to symbolic mathematics. Several theories exist, however, for why Harriot began this movement. It is commonly believed that Harriot most likely began using symbols to make equations easier to understand by creating a universal mathematical language. This eliminated the possibility of misinterpretation of mathematical definitions of words in equations.²⁷ Because the equations Harriot worked with were lengthy and composed of many words, Harriot intended to include symbols to shorten the length of the equations. Also, Harriot's mathematical peers at the time were discovering links between algebra and geometry.²⁸ Harriot's symbols only further advanced the connection of these two branches of mathematics. Using symbols in mathematical equations would not only shorten the length of equations, it would shorten the length of solutions, saving the mathematician solving the equation valuable time.

During the time of Harriot's work, symbolism in mathematics was not completely foreign but remained minimal. To have a mathematician like Harriot begin utilizing symbolism at such a great frequency illustrated a vast divergence from the norm. Most

25. Stedall, *Discourse*, 90.

26. *Ibid.*

27. Stedall, "Symbolism", 389.

28. *Ibid.*

mathematicians who utilized symbols used them sparingly. Consequently, there were mixed reactions from the mathematical community in relation to Harriot's recurrent use of symbols. There were those who supported and those who opposed this movement to symbolism.

Supporters of Harriot's symbolism saw the mathematical benefits. Supporters understood that Harriot was attempting to make mathematics, a difficult field of investigation, much easier by eliminating the added confusion of wordy equations.²⁹ Harriot's symbolism received support from fellow mathematicians because they recognized that the use of symbols could be used as short hand, making solving an equation a less time consuming task.³⁰ Lastly, mathematicians believed that symbolism helped them to understand the problems that they were completing by serving as a means of clarifying any information that could have been misinterpreted in the wordy equations that were typical of the late 16th and early 17th centuries.³¹

Those who opposed saw Harriot's symbolism as an unnecessary nuisance. The opposition felt that Harriot's symbols created more work for mathematicians. They believed that utilizing symbolism meant that mathematicians would have to learn symbols, the meanings for these symbols, and what methods of solving were appropriate.³² They believed that integrating symbols into the mathematics that they were accustomed to was too drastic of a change. Others felt that the use of symbols would allow some of the mathematical meanings to slip through the cracks.³³

29. Ibid., 390-391.

30. Ibid.

31. Ibid.

32. Ibid.

33. Ibid.

Because Harriot never published any works during his life time, he was not able to see his movement towards symbolism in mathematics begin to take shape. However, Harriot included in his will that his works should go into publication. Therefore, his first published work was *Artis Analyticae Praxis* in 1631, following his death in 1621 from what is believed to have been skin cancer.³⁴ *Artis Analyticae Praxis* is the only true example of Harriot's symbolism in print. It included not only his use of symbolism in mathematics, but also Harriot's methods of solving mathematic equations and his mathematical notation were well beyond his time. In this work, Harriot toyed with the idea of imaginary numbers, a thought unheard of during the early to mid-1600s.³⁵

Recently, there has been much debate over whether or not Harriot's untimely notation acknowledged both positive and negative roots. However, when solving many problems in *Artis Analyticae Praxis*, Harriot did not use negative roots as solutions. For example, employing many symbols during a problem involving cubics, Harriot solved down to $ee=c cc+\sqrt{-d dddd}$, at which point he wrote that this is impossible.³⁶ Later, Harriot clarified that, although he toyed with the idea, it was impossible to have any solutions that are negative roots.

Another groundbreaking mathematical concept covered by Harriot in *Artis Analyticae Praxis* is his use of setting mathematical statements equal to zero to determine solutions.³⁷ Harriot took statements like $(a - b)$ and set it equal to zero, arriving at $a = b$.

34. Ibid., 470.

35. Cajori, "Revaluation", 317.

36. Ibid.

37. Muriel Seltman and Eddie Mizzi, "Thomas Harriot: Father of English Algebra?" *Mathematical Intelligencer* 19, no. 1 (January 1997): 48.

Harriot also set statements like this equal to negative numbers, if needed.³⁸ This was the first time that a mathematician used zero as a number of value.³⁹

Although a few of Harriot's symbols have remained untouched, like his less than and greater than signs and his use of juxtaposition for any two variables multiplied together, some of his symbols have been altered in the movement toward modern mathematics. The reasons behind these alterations to Harriot's symbolism and notation are not known. However, some of these changes were carried out by the editors of Harriot's work, as in the case of Nathaniel Torporley, Harriot's first editor in 1620.⁴⁰ During the editing of Harriot's work, Torporley began abbreviating $aaaa$ to a^{IV} to shorten his work.⁴¹ However, Harriot's second editor, Walter Warner, wanting to withhold Harriot's notation, did not publish this abbreviation in Harriot's *Artis Analyticae Praxis*.⁴² Fellow mathematician Kersey followed suit when he published in 1673, giving his interpretation of Euclid's *Elements*, explaining that he believed the notation of repeated letters for powers to be "plainest for learners".⁴³ Also, Harriot's use of the letter a remained the most commonly used variable in mathematical manipulations until Descartes introduced the variable x .⁴⁴

Thomas Harriot will remain one of the most influential mathematicians. The symbolism of Harriot's mathematics has shaped modern notation in a great way. His work with symbolism changed mathematics, and influenced the way that every student in a mathematics classroom learns notation and carries out mathematical manipulations.

38. Ibid., 47-48.

39. Ibid.

40. Cajori, "Revaluation", 397.

41. Ibid.

42. Ibid.

43. Ibid.

44. Ibid.

Because his methods were unorthodox for the time, Harriot's work stood out, making it easy for him to remain such an important part of the history of mathematics. Harriot began doing things mathematicians had never thought of, especially with his work on entirely symbol-based mathematics. Even with mixed reactions, vastly changing methods of solving mathematical equations and a lack of resources due to the time period several of Harriot's symbols have withstood the test of time.

Although his motivations for this movement towards symbolism in mathematics have not yet been fully discovered, Harriot's work towards a fully symbolic mathematics was successful. While most mathematics students may not know Harriot's name, they reap the benefits of his notation every time they solve an equation.

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