Words and Pictures: New Light on Plimpton 322

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1. INTRODUCTION. In this paper I shall discuss Plimpton 322, one of the world's most famous ancient mathematical artefacts [Figure 1]. But I also want to explore the ways in which studying ancient mathematics is, or should be, different from researching modern mathematics. One of the most cited analyses of Plimpton 322, published some twenty years ago, was called "Sherlock Holmes in Babylon" [**4**]. This enticing title gave out the message that deciphering historical documents was rather like solving a fictional murder mystery: the amateur detective-historian need only pit his razor-sharp intellect against the clues provided by the self-contained story that is the piece of mathematics he is studying. Not only will he solve the puzzle, but he will outwit the well-meaning but incompetent professional history-police every time. In real life, the past isn't like an old-fashioned whodunnit: historical documents can *only* be understood in their historical context.

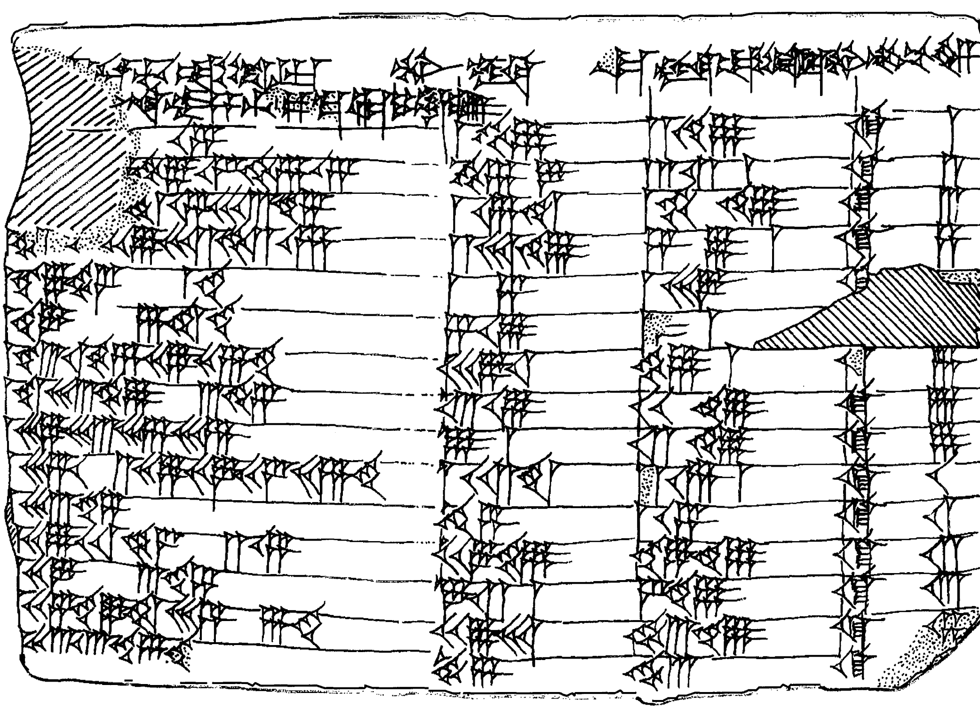


Figure 1. Plimpton 322 (obverse). Drawing by the author.

Let's start with a small experiment: ask a friend or colleague to draw a triangle. The chances are that he or she will draw an equilateral triangle with a horizontal base. That is our culturally determined concept of an archetypal, perfect triangle. However, if we look at triangles drawn on ancient cuneiform tablets like Plimpton 322, we see that they all point right and are much longer than they are tall: very like a cuneiform wedge in fact. A typical example is UM 29-15-709, a scribal student's exercise, from ancient Nippur, in calculating the area of a triangle [Figure 2]. The scale drawing next to it shows how elongated the sketch is.

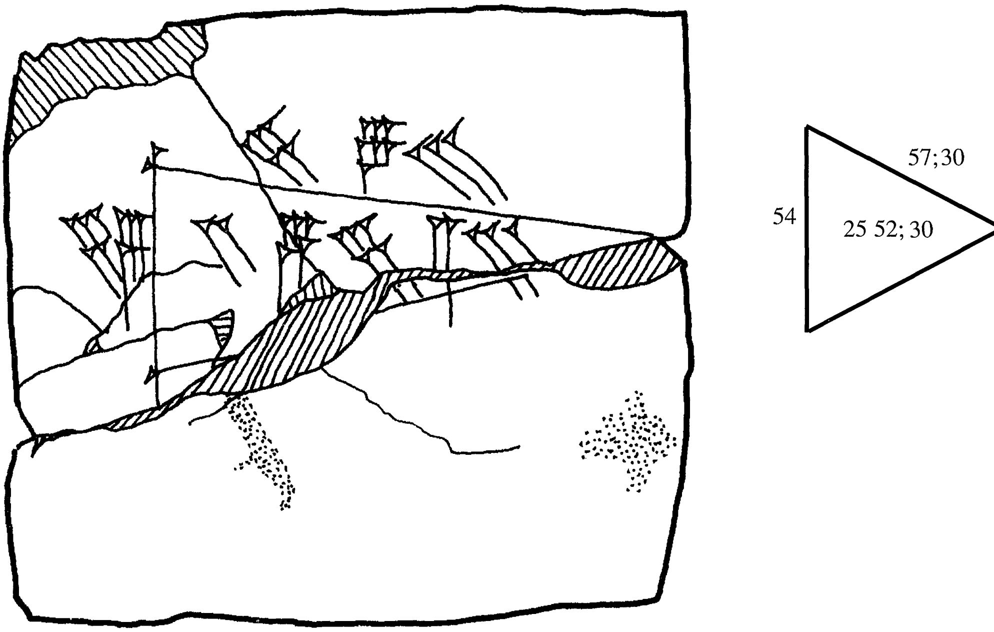


Figure 2. UM 29-15-709 (obverse). Drawing by the author [**26**, p. 29].

We tend to think of mathematics as relatively culture-free; i.e., as something that is *out there*, waiting to be discovered, rather than a set of socially agreed conventions. If a simple triangle can vary so much from culture to culture, though, what hope have we in relying on our modern mathematical sensibilities to interpret more complex ancient mathematics? Unlike Sherlock Holmes we cannot depend solely on our own intuitions and deductive powers, and we cannot interrogate the ancient authors or scrutinise their other writings for clues. We therefore have to exploit all possible available resources: language, history and archaeology, social context, as well as the network of mathematical concepts within which the artefact was created. In the case of Plimpton 322, for instance, there are three competing interpretations, all equally valid mathematically. As I shall show, it is these contextualising tools that enable us to choose between them.

Plimpton 322 is just one of several thousand mathematical documents surviving from ancient Iraq (also called Mesopotamia). In its current state, it comprises a four-column, fifteen-row table of Pythagorean triples, written in cuneiform (wedge-shaped) script on a clay tablet measuring about 13 by 9 by 2 cm [**20**, Text A, pp. 38–41]. The handwriting of the headings is typical of documents from southern Iraq of 4000–3500 years ago. Its second and third columns list the smallest and largest member of each triple—we can think of them as the shortest side s and the hypotenuse d of a right-angled triangle—while the final column contains a line-count from 1 to 15. Part of the tablet has broken away at the beginning of the first column but, depending on whether you believe the column has fully survived or not, it holds the square of *either* the hypotenuse *or* the shortest side of the triangle divided by the square of the longer side l. Whether it lists d^2/l^2 or s^2/l^2, this column is in descending numerical order. The numbers are written in the base 60, or sexagesimal, place value system. I shall transliterate them with a semicolon marking the boundary between integers and fractions, and spaces in between the other sexagesimal places [Figure 3].

[*ta*]-*ki-il-ti ṣi-li-ip-tim* [*ša* 1 *in*]-*na-as-sà-ḫu-ma* SAG *i-il-lu-ú*	ÍB.SI$_8$ SAG	ÍB.SI$_8$ *ṣi-li-ip-tim*	MU.BI.IM
[(1) 59] 00 15	1 59	2 49	KI.1
[(1) 56 56] 58 14 50 06 15	56 07	1 20 25	KI.2
[(1) 55 07] 41 15 33 45	1 16 41	1 50 49	KI.3
(1) 53 10 29 32 52 16	3 31 49	5 09 01	KI.4
(1) 48 54 01 40	1 05	1 37	KI.[5]
(1) 47 06 41 40	5 19	8 01	[KI.6]
(1) 43 11 56 28 26 40	38 11	59 01	KI.7
(1) 41 33 45 14 3 45	13 19	20 49	KI.8
(1) 38 33 36 36	8 01	12 49	KI.9
(1) 35 10 02 28 27 24 26 40	1 22 41	2 16 01	KI.10
(1) 33 45	45	1 15	KI.11
(1) 29 21 54 2 15	27 59	48 49	KI.12
(1) 27 00 03 45	2 41	4 49	KI.13
(1) 25 48 51 35 6 40	29 31	53 49	KI.14
(1) 23 13 46 40	28	53	KI.15

Figure 3. Transliteration of Plimpton 322.

There have been three major interpretations of the tablet's function since it was first published [Figure 4]:[1]

1. Some have seen Plimpton 322 as a form of trigonometric table (e.g., [**15**]): if Columns II and III contain the short sides and diagonals of right-angled triangles, then the values in the first column are $\tan^2$ or $1/\cos^2$—and the table is arranged so that the acute angles of the triangles decrease by approximately 1° from line to line.
2. Neugebauer [**19**], and Aaboe following him, argued that the table was generated like this:

> If p and q take on all whole values subject only to the conditions
>
> (1) $p > q > 0$,
>
> (2) p and q have no common divisor (save 1),
>
> (3) p and q are not both odd,
>
> then the expressions
>
> $$x = p^2 - q^2 \quad \text{[our } s\text{]},$$

[1] Incidentally, we can dismiss immediately any suspicion that Plimpton 322 might be connected with observational astronomy. Although some simple records of the movements of the moon and Venus *may* have been made for divination in the early second millennium BCE, the accurate and detailed programme of astronomical observations for which Mesopotamia is rightly famous began a thousand years later, at the court of the Assyrian kings in the eighth century BCE [**3**].

line	α	p	q	x	$1/x$
1	44.76°	12	5	2 24	25
2	44.25°	1 04	27	2 22 13 20	25 18 45
3	43.79°	1 15	32	2 20 37 30	25 36
4	43.27°	2 05	54	2 18 53 20	25 55 12
5	42.08°	9	4	2 15	26 40
6	41.54°	20	9	2 13 20	27
7	40.32°	54	25	2 09 36	27 46 40
8	39.77°	32	15	2 08	28 07 30
9	38.72°	25	12	2 05	28 48
10	37.44°	1 21	40	2 01 30	29 37 46 40
11	36.87°	2	1	2	30
12	34.98°	48	25	1 55 12	31 15
13	33.86°	15	8	1 52 30	32
14	33.26°	50	27	1 51 06 40	32 24
15	31.89°	9	5	1 48	33 20

Figure 4. The proposed restorations at the beginning of the tablet according to the trigonometric, generating function, and reciprocal pair theories.

$$y = 2pq \quad \text{[our } l\text{]},$$
$$z = p^2 + q^2 \quad \text{[our } d\text{]},$$

will produce all reduced Pythagorean number triples, and each triple only once [**1**, pp. 30–31].

The quest has then been to find how p and q were chosen.

3. Finally, the interpretation first put forward by Bruins [**5**], [**6**] and repeated in a cluster of independent publications about twenty years ago [**4**], [**9**], [**30**] is that the entries in the table are derived from reciprocal pairs x and $1/x$, running in descending numerical order from 2;24 $\sim$ 0;25 to 1;48 $\sim$ 0;33 20 (where $\sim$ marks sexagesimal reciprocity). From these pairs the following "reduced triples" can be derived:

$$s' = s/l = (x - 1/x)/2,$$
$$l' = l/l = 1,$$
$$d' = d/l = (x + 1/x)/2.$$

The values given on the tablet, according to this theory, are all scaled up or down by common factors 2, 3, and 5 until the coprime values s and d are reached.

How are we to choose between the three theories? Internal mathematical evidence alone clearly isn't enough. We need to develop some criteria for assessing their historical merit. In general, we can say that the successful theory should not only be mathematically valid but historically, archaeologically, and linguistically sensitive too.

A great deal of emphasis has been laid on the uniqueness of Plimpton 322; how nothing remotely like it has been found in the corpus of Mesopotamian mathematics. Indeed, this has been an implicit argument for treating Plimpton 322 in historical isolation. Admittedly we know of no other ancient *table* of Pythagorean triples, but

Pythagorean triangles were a common subject for school mathematics problems in ancient Mesopotamia. This point has been made before (e.g., by Friberg [**9**]) but hasn't yet proved particularly helpful in deciding between the three interpretations of Plimpton 322. What we shall do instead is to make some *new* comparisons. None of the comparative material itself is new though: all but one of the documents I have chosen were published at the same time as Plimpton 322 or decades earlier.

First, Plimpton 322 is a table. Hundreds of other tables, both mathematical and nonmathematical, have been excavated from Mesopotamian archaeological sites. What can we learn from them?

Second, if Plimpton 322 is a *trigonometry* table, then there should be other evidence of measured angle from Mesopotamia. We shall go in search of this.

Third, Plimpton 322 contains words as well as numbers: the headings at the top of each column should tell us what the tablet is about. Some of the more difficult words also appear on other mathematical documents from Mesopotamia. Can they help us to understand their function on Plimpton 322?

Finally, Plimpton 322 was written by an *individual*. What, if anything, can we say about him or her, and why the tablet was made?

2. TURNING THE TABLES ON GENERATING FUNCTIONS. Let's start with some very general contextualisation. We can learn a lot about any tablet simply from its size, shape, and handwriting.

Plimpton 322 is named after its first Western owner, the New York publisher George A. Plimpton (see Donoghue [**8**]). He bequeathed his whole collection of historical mathematical books and artefacts to Columbia University in the mid-1930s along with a large number of personal effects. Surviving correspondence shows that he bought the tablet for $10 from a well-known dealer called Edgar J. Banks in about 1922 [**2**]. Banks told him it came from an archaeological site called Senkereh in southern Iraq, whose ancient name was Larsa [Figure 5].

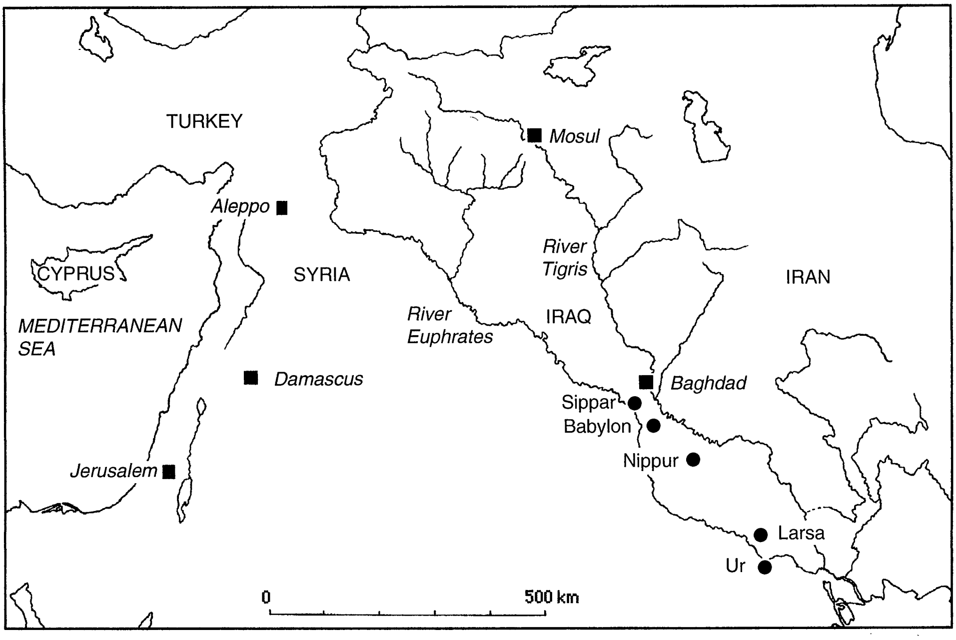
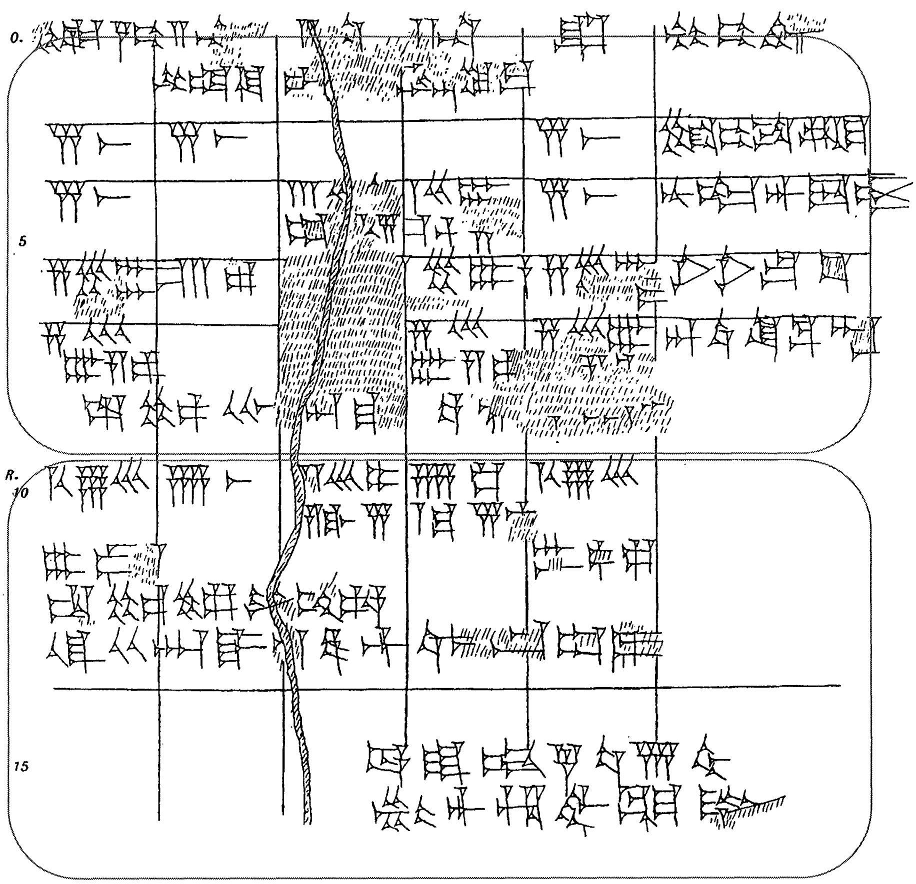


Figure 5. Map of the archaeological sites mentioned in the text. Drawing by the author.

Vast numbers of cuneiform tablets were being illicitly excavated from Larsa at that time. Several big museums, such as the Louvre in Paris, Oxford's Ashmolean Museum, and the Yale Babylonian Collection bought thousands of them. Although Plimpton 322 doesn't look much like other *mathematical* tablets from Larsa, its format is strikingly similar to administrative tables from the area, first attested from the late 1820s BCE. The tablet YBC 4721 [**12**, no. 103], for example, is an account of grain destined for various cities within the kingdom of Larsa [Figure 6]. It was written in the city of Ur, then under Larsa's political control, in 1822 BCE and is now housed at Yale.

Like Plimpton 322 it is written on a "landscape" format tablet (that is, the writing runs along the longer axis) with a heading at the top of each column. Entries in the



Grain debit	*For Ur*	*For Mar-...*	*For......*	*Total*	*Its name*
301 <gur>	301 <gur>			301 <gur>	Lipit-Suen
301 <gur>		214 gur, 285 sila	86 gur 15 sila	301 <gur>	Nur-Dagan
296 <gur>	180 gur	[60 gur]	56 <gur>	296 gur	Ili-eriba
277 gur 200 sila		[ ]	277 <gur> 200 sila	277 <gur> 200 sila	Šamaš-kima-ilišu
From 23 gur 40 sila of [Šamaš-...-aplu's] troops/workers.					
1176 gur 20 sila	481 <gur>	274 <gur> 485 sila	420 gur 95 sila	1176 gur 20 sila	

From the grain of Lu-am-...
And from 23 gur 40 sila of Šamaš-...-aplu's troops/workers.
Month 1, day 7,
Year that Rim-Sin became king (1822 BCE).

Figure 6. YBC 4721, after Grice [**12**, pl. XL]. 1 gur = 300 sila ≈ 300 litres.

first column are sorted into descending numerical order. Calculations run from left to right across the table, while the final column lists the names of the officials responsible for each transaction. Although the scribes of Larsa mostly used the cuneiform script to write a Semitic language called Akkadian, they often used monosyllabic words from a much older language, Sumerian, as a kind of shorthand. Like the final column of Plimpton 322, the last heading on YBC 4721 carries the Sumerian writing MU.BI.IM for Akkadian *šumšu* ("its name"). Unlike Plimpton 322 though, the text is dated in the final line. There are about half a dozen published tables from the Larsa area with these same characteristics: all of them are dated to the short period 1822–1784 BCE and so, therefore, is Plimpton 322.

So we can already say that Plimpton 322 was written by someone familiar with the temple administration in the city of Larsa in around 1800 BCE, at least twenty years before its conquest by Babylon in 1762. Sherlock Holmes, if he ever made it to Babylon, would have been over 100 miles away from the action: no ancient mathematics has ever been found there.

And the fact that Plimpton 322 follows the same formatting rules as all other tables from ancient Larsa leads us to dismiss Neugebauer's theory of generating functions. If the missing columns at the left of the tablet had listed p and q, they would not have been in descending numerical order and would thus have violated those formatting rules. Nor, under this theory, has anyone satisfactorily explained the presence of Column I in the table. There are other good reasons to eliminate the generating function theory; I deal with them in [**29**]. The trigonometry table and the reciprocal pairs remain.

3. CIRCLING ROUND TRIGONOMETRY. We saw at the beginning of this article how differently from us the people of ancient Mesopotamia thought about triangles; that contrast with modern concepts runs right through their plane geometry.

For instance, YBC 7302 [**20**, p. 44] is roughly contemporary with Plimpton 322 [Figure 7]. From its circular shape and size (about 8 cm across) we know the tablet was used by a trainee scribe for school rough work. It shows a picture of a circle with three numbers inscribed in and around it in cuneiform writing: 3 on the top of the circle, 9 to the right of it, and 45 in the centre. Now 9 is clearly the square of 3, but what is the relationship of these numbers to 45? The answer lies in the spatial arrangement of the diagram. Looking closely, we can see that the 3 lies directly on the circumference of the circle, while the 45 is contained within it. The 9, on the other hand, has no physical connection to the rest of the picture. If on this basis we guess that 3 represents the length of the circumference and 45 the area of the circle, we should be looking for the relationship $A = c^2/4\pi$. We have the c^2—that's the 9—and if we use the usual Mesopotamian school approximation $\pi \approx 3$, we get $A = 9/12$. This translates in base 60 to $45/60$, namely, the 45 written in the circle.

When we teach geometry in school we have our students use the relationship $A = \pi r^2$; none of us, I would guess, would use $A = c^2/4\pi$ as a formula for the area of a circle. In modern mathematics the circle is conceptualised as the area generated by a rotating line, the radius. In ancient Mesopotamia, by contrast, a circle was the shape contained within an equidistant circumference: note that there is no radius drawn on YBC 7302. There are many more examples of circle calculations from the early second millennium, and none of them involves a radius. Even when the diameter of a circle was known, its area was calculated by means of the circumference. We also see this conceptualisation in the language used: the word *kippatum*, literally "thing that curves," means both the two-dimensional disc and the one-dimensional circumference that defines it. The conceptual and linguistic identification of a plane figure and one

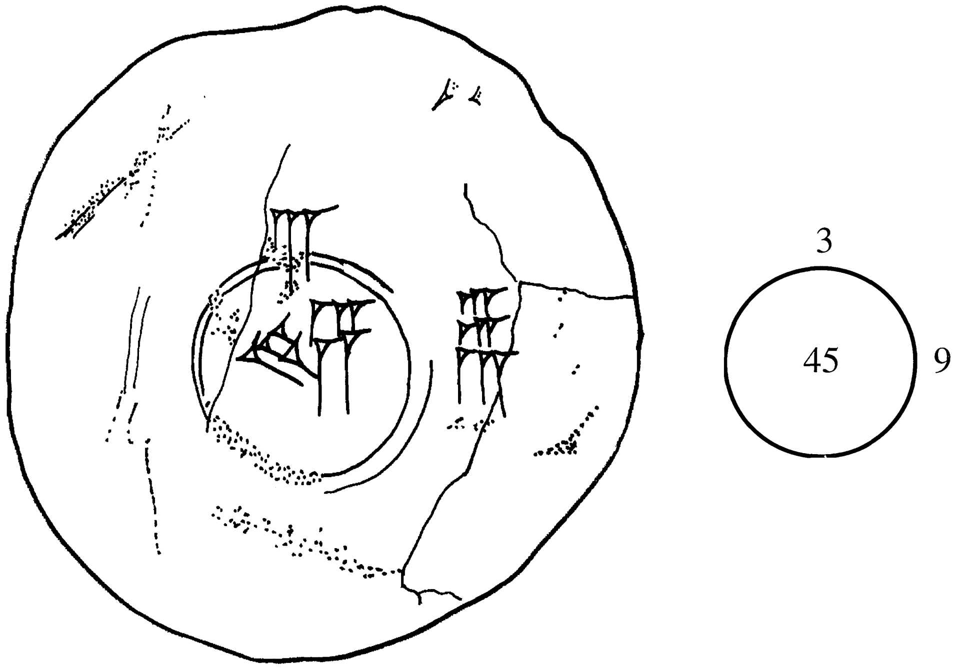


Figure 7. YBC 7302 (obverse). Drawing by the author.

of its external lines is a key feature of Mesopotamian mathematics. For instance, the word *mitḫartum* ("thing that is equal and opposite to itself") means both "square" and "side of square." We run into big interpretational problems if we ignore these crucial terminological differences between ancient Mesopotamian and our own mathematics.

What does this tell us about Plimpton 322? That if plane figures were conceptualised, named, and defined from the inside out, then the centre of the circle and the idea of the rotating radius could not have played an important part in Mesopotamian mathematics. And if the rotating radius did not feature in the mathematical idea of the circle, then there was no conceptual framework for measured angle or trigonometry. In short, Plimpton 322 cannot have been a trigonometric table.

This should have been our intuition on later historical grounds anyway. Nearly two millennia after Plimpton 322 was written, Ptolemy conceptualised the circle as a diameter rotating about its centre in order to simplify his calculations of chords of arc—but those chords were functions of arc, not of angle (Toomer [**31**, p. 47]). (Ptolemy, working in Roman Egypt in the second century CE, was heavily reliant on Mesopotamian traditions: he used astronomical data from first millennium Assyria and Babylonia, adapted Mesopotamian mathematical methods, and even calculated in base 60.) Over the following millennium several generations of Indian and Iraqi scholars compiled tables of half-chords, but the conceptual transition from arc to angle was slow and halting.

Returning to the early second millennium BCE, I should emphasise two points. First, I do not mean that the ancient Mesopotamians *did not know* that circles could be generated by rotating radii. There is a great deal of visual evidence to show that they did. For example, BM 15285, a compilation of plane geometry problems from Larsa, depicts several circles whose deeply impressed centres reveal that they were drawn by means of rotating compasses [Figure 8]. But Mesopotamian mathematical concepts were as

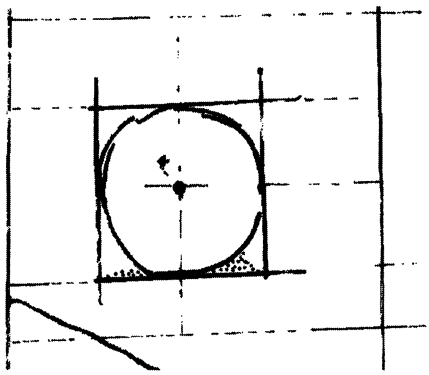


Figure 8. BM 15285 (detail). Drawing by the author [**25**, p. 214].

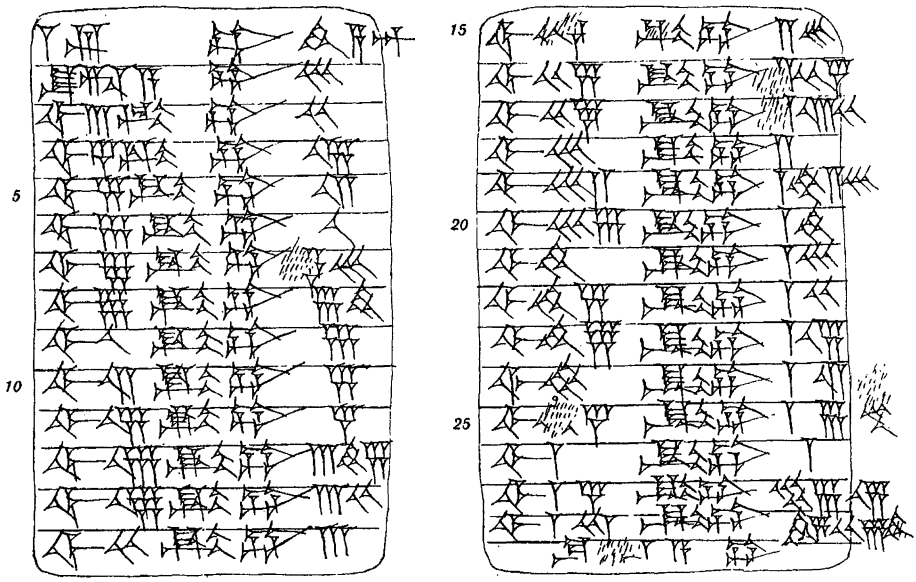
socially bounded as ours are: although we often draw circles free-hand without radii, even in mathematics classes, it would rarely cross our minds to teach our students $A = c^2/4\pi$. Equally, radii were known and used in ancient Mesopotamia, but played little part in the dominant outside-in conceptualisation of plane geometry. Second, neither do I mean that there was no concept of angle *at all* in ancient Mesopotamia. Gradients were used to measure the external slope of walls and ramps in formulations like "for every 1 cubit depth the slope (of the canal) is 1/2 cubit" (YBC 4666, rev. 25; [**20**, text K]). There was also a rough distinction made between right angles and what we might call "wrong angles," namely, those configurations for which the Pythagorean rule held true or not, with probably a 10° to 15° leeway (see Robson [**24**]).

To sum up so far: the theory of generating functions is organisationally implausible, while the trigonometric theory is conceptually anachronistic. We are left with the theory of reciprocal pairs—how does it measure up to our historical expectations?

4. WORDS COUNT TOO: RECIPROCAL PAIRS. We can start by recognising that reciprocal pairs—unlike generating functions or trigonometry—played a key role in ancient Mesopotamian mathematics. Our best evidence is from the scribal schools of nineteenth and eighteenth century Larsa, Ur, and Nippur, where thousands of surviving practice copies show that scribal students had to learn their sexagesimal multiplication tables in the correct order and by heart. The first part of the series was the set of thirty standard reciprocal pairs encompassing all the sexagesimally regular integers from 2 to 81 (thereby including the squares of the integers 1 to 9) [Figure 9]. The trainees also learned how to calculate the reciprocals of regular numbers that were not in the standard list and practised division by means of finding reciprocals, as this was how all Mesopotamian divisions were carried out (see Robson [**26**, pp. 19–23]).

Looking at the reciprocals proposed as the starting point for Plimpton 322 [Figure 4], it turns out that although only five pairs occur in the standard list, the other ten are widely in evidence elsewhere in Mesopotamian mathematics (as constants, for instance), or could be calculated trivially using methods known to have been taught in scribal schools. None of them is more than four sexagesimal places long, and they are listed in decreasing numerical order, thereby fulfilling our tabular expectations. But we haven't yet explained the purpose of the first surviving column: what are all those s^2/l^2 (or d^2/l^2) doing there?

The headings at the top of the table ought to tell us: that, after all, is their function. We have already seen that the last column, which contains only a line-count, is headed like the other tables from Larsa with the signs MU.BI.IM meaning *šumšu* ("its name").



Two thirds of 1 is 0;40.
Its half is 0;30.
The reciprocal of 2 is 0;30.
The reciprocal of 3 is 0;20.
The reciprocal of 4 is 0;15.
The reciprocal of 5 is 0;12.
The reciprocal of 6 is 0;10.
The reciprocal of 8 is 0;07 30.
The reciprocal of 9 is 0;06 40.
The reciprocal of 10 is 0;06.
The reciprocal of 12 is 0;05.
The reciprocal of 15 is 0;04.
The reciprocal of 16 is 0;03 45.
The reciprocal of 18 is 0;03 20.
The reciprocal of 20 is 0;03.
The reciprocal of 24 is 0;02 30.
The reciprocal of 25 is 0;02 24.
The reciprocal of 27 is 0;02 13 20.
The reciprocal of 30 is 0;02.
The reciprocal of 32 is 0;01 52 30.
The reciprocal of 36 is 0;01 40.
The reciprocal of 40 is 0;01 30.
The reciprocal of 45 is 0;01 20.
The reciprocal of 48 is 0;01 15.
The reciprocal of 50 is 0;01 12.
The reciprocal of 54 is 0;01 06 40.
The reciprocal of 1 00 is 0;01.
The reciprocal of 1 04 is 0;00 56 15.
The reciprocal of 1 21 is 0;00 44 26 40.
<Its half>

Figure 9. MLC 1670, after Clay [**7**, no. 37].

The two columns immediately preceding that, we remember, contain what we can conveniently think of as the shortest sides and diagonals of right-angled triangles. They are headed ÍB.SI$_8$ SAG and ÍB.SI$_8$ *ṣi-li-ip-tim* for *mitḫarti pūtim* and *mitḫarti ṣiliptim* ("square of the short side" and "square of the diagonal," respectively). This contradiction disappears when we recall that Mesopotamian plane figures are defined and named for their key external lines. We can thus adjust our translations to read "square-side" of the short side and diagonal, respectively. (I am using the translation "diagonal" here rather than "hypotenuse" to indicate that this is a general word for the transversal of a figure, not restricted to triangles.)

Last and by far the most difficult, the heading of the first surviving column reads

[*ta*]*-ki-il-ti ṣi-li-ip-tim*
[*ša* 1 *in*]*-na-as-sà-ḫu-ú-ma* SAG *i-il-lu-ú*

(for Akkadian *takilti siliptim ša ištēn innassaḫûma pūtum illû*), where square brackets mark missing cuneiform signs that I have restored. Surprisingly, no one has been able to improve convincingly on the translation made by Neugebauer and Sachs when they first published Plimpton 322 [**20**, p. 40]. They were uncertain about the first word and the last word, as well as what was missing at the beginning of the second line. In fact the last word is legible, if a little squashed. The breaks at the beginnings of the lines can be filled in, and the whole understood, through comparison with other mathematical documents that use the same terminology. We end up with something like this:

> The *takiltum*-square of the diagonal from which 1 is torn out, so that the short side comes up.

To understand what exactly that means, and how it relates to the reciprocal pairs, we need to look at one more mathematical tablet, YBC 6967 [**20**, text Ua]. This tablet is almost certainly from late nineteenth to early eighteenth century Larsa, like Plimpton 322. It contains instructions for solving a school problem about reciprocal pairs. As Jens Høyrup has shown, we can best understand this sort of mathematics not as algebra but as a very concrete cut-and-paste geometry [**13**, pp. 262–266]. Once again square brackets show restorations of missing text.

> [A reciprocal] exceeds its reciprocal by 7. What are [the reciprocal] and its reciprocal?

The product of the mystery reciprocals is by definition 1 (or any power of 60). The fact that their difference is an integer suggests that we should think of them as integers too. We can thus conceptualise them as the unknown lengths of a rectangle with area 60 [Figure 10].

> You: break in half the 7 by which the reciprocal exceeds its reciprocal, and 3;30 (will come up). Multiply 3;30 by 3;30 and 12;15 (will come up).

Following the instructions, we can move the broken piece of the rectangle to form an L-shaped figure, still of area 60, around an imaginary square of area 12 1/4.

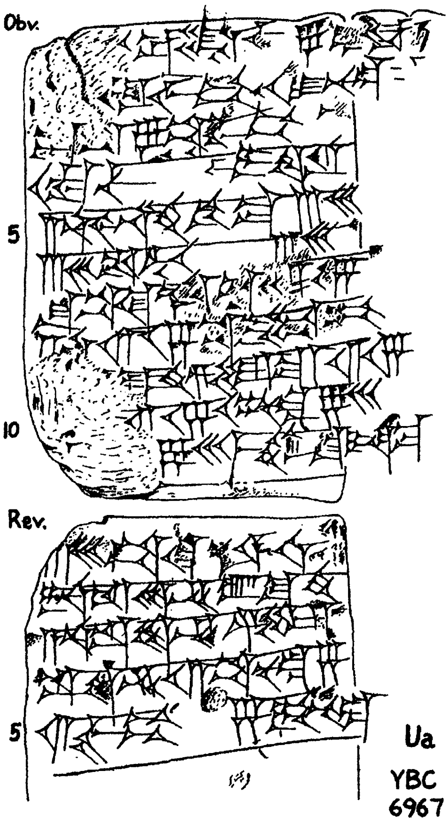
> Append [1 00, the area,] to the 12;15 which came up for you and 1 12;15 (will come up). What is [the square-side of 1] 12;15? 8;30.

Together, therefore, they comprise a large square of area 72 1/4 and side 8 1/2.

> Put down [8;30 and] 8;30, its equivalent, and subtract 3;30, the *takiltum*-square, from one (of them); append (3;30) to one (of them). One is 12, the other is 5. The reciprocal is 12, its reciprocal 5.

We remove the vertical side of the imaginary small square from that of the large composite square, reverting to the smaller side of the original rectangle, a side whose length is 5. We find the longer side of the rectangle by adding the horizontal side of the imaginary square onto that of the large composite square and arrive at the answer 12.

If instead we choose a reciprocal pair whose product is not 60 but 1, their product can be imagined as a much longer, narrower rectangle than in Figure 10. But the semidifference of the reciprocals, $(x - 1/x)/2$, can still be found and the rectangle rearranged to form an L-shaped gnomon, still of area 1. Its outer edges will still be the lengths of a large square, and its inner edges the lengths of a small square. That is, we will have a composite large square that is the sum of 1 (itself a square) and an imaginary small square. This set of three squares, all generated by a pair of recip-



[A reciprocal] exceeds its reciprocal by 7.
What are [the reciprocal] and its reciprocal?
You: break in half the 7 by which the reciprocal exceeds its reciprocal, and 3;30 (will come up).
Multiply 3;30 by 3;30 and 12;15 (will come up).
Append [1 00, the area,] to the 12;15 which came up for you and 1 12;15 (will come up).
What is [the square-side of 1] 12;15? 8;30.
Put down [8;30 and] 8;30, its equivalent, and subtract 3;30, the *takiltum*-square, from one (of them); append (3;30) to one (of them).
One is 12, the other is 5.
The reciprocal is 12, its reciprocal 5.

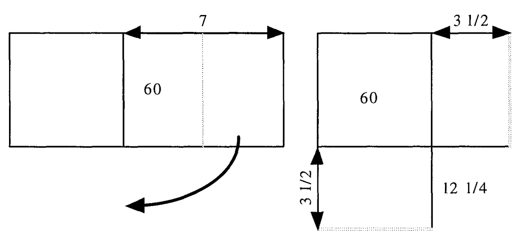


Figure 10. YBC 6967, after Neugebauer and Sachs [**20**, pl. 17].

rocals, obeys the Pythagorean rule $d^2 = s^2 + l^2$. Their sides, in other words, are the Pythagorean triple we having been looking for.

Let us look again at the heading of Column I:

> The *takiltum*-square of the diagonal from which 1 is torn out, so that the short side comes up.

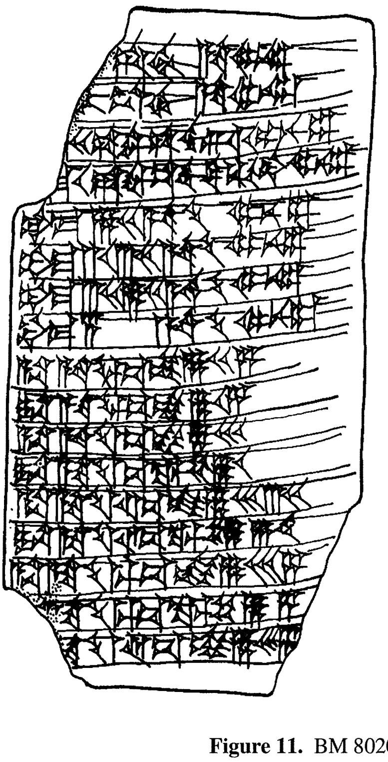
It describes the area of the large square, composed of 1 plus the small square—the verb *ilûm* ("come up"), we have seen, is the standard term for "to result." Our restoration dilemma is now solved: we should put 1s at the beginning of every entry. There is one small terminological discrepancy left to deal with: in Plimpton 322 *takiltum* refers to the area of the large composite square, while in YBC 6967 it means the side of the small imaginary square. We know by now to expect squares and their sides to be named identically so that is not a problem. The word itself, a technical derivative of the verb *kullum* ("to multiply lengths together into areas") does not suggest that its meaning should be restricted to either the little square or the big square but that its pattern of attestation is restricted to exactly these cut-and-paste geometrical scenarios.

We have found, then, the most historically, culturally, and linguistically convincing of our three interpretations of Plimpton 322: a list of regular reciprocal pairs, each four places long or shorter, was drawn up in the usual decreasing numerical order on the missing part of the tablet. They were used to find the short sides s and diagonals d of triangles with long sides of length $l = 1$ by the method of completing the square. One of the intermediate results was recorded in the first extant column. Then common factors were eliminated from the triples produced to give the coprime short sides and diagonals listed in Columns II and III.

All we need to know now is who wrote Plimpton 322 and for what purpose—but that is easier said than done!

5. IN SEARCH OF AN AUTHOR. Ancient Mesopotamia was a culture that prized anonymised tradition over individual creativity. Even the greatest works of literature were attributed to deities or to long-dead historical figures (see Michalowski [**17**]). It is very unlikely that we will ever be able to put a name to our author, let alone outline his or her personality or life history. We can find out a great deal of more general information though. For instance, it is virtually certain that our author was male: all the known female scribes from ancient Mesopotamia lived and worked much further north, in central and northern Iraq. We can also rule out the possibility that our author was a mathematician in either of the senses we normally mean. He cannot have been a *professional* mathematician—the professionalisation of academic disciplines is a phenomenon of the very recent past. Nor was he likely to have been an *amateur* mathematician like those of Classical Antiquity and the Middle Ages, i.e., an educated member of the merchant classes or ruling elite for whom wealth, high status, or royal patronage provided enough leisure time to indulge his mathematical inclinations [**18**, ch. 7]. There is not one example of this type of individual in the whole of Mesopotamia's three-thousand year history. Rather, he must have been someone who used literacy, arithmetic, and mathematical skills in the course of his working life.

We can say something more positive about the author's identity by recalling some of our earlier conclusions. First, the methods used to construct Plimpton 322—reciprocal pairs, cut-and-paste geometry, completing the square, dividing by regular common factors—were all simple techniques taught in scribal schools. Our author could have been a trainee scribe or a teacher. Second, he was familiar with the format of docu-



[If each] square side is [. . .], what is the area?
[If each] square side is [. . .], what is the area?
[If] each square side is 20, what is the diagonal?
[If] each square side is 10, what is the border?
If the area is 8 20, what is the circumference?
If the area is 2 13 20, what is the circumference?
If the area is 3 28 20, what is the circumference?
If the area is 5, what is the circumference?
To the area of the circle add 1/2 a length: 8 25.
From the area of the circle take 1/2 a length: 8 15.
To the area of the circle add 1 length: 8 30.
From the area of the circle take 1 length: 8 10.
To the area of the circle add 1 1/3 lengths: 8 33 20.
From the area of the circle take 1 1/3 lengths: 8 06 40.
To the area of the circle add 1 1/2 lengths: 8 35.
[From] the area of the circle take 1 1/2 lengths: 8 05.
[To] the area of the circle add 1 2/3 lengths: 8 36 [40].

Figure 11. BM 80209 (obverse). Drawing by the author.

ments used by the temple and palace administrators of Larsa. That rules out the option that he was a student, but indicates instead that he was a professional bureaucratic scribe. In that case he would have been highly numerate, for the vast majority of ancient administrative documents related to quantity surveying or accountancy. If the author of Plimpton 322 was a teacher, then he was almost certainly a bureaucrat too: we know the names and primary professions of about half a dozen ancient Mesopotamian teachers, and all of them had careers in temple administration.

It is highly unlikely, however, that Plimpton 322 was written for the temple bureaucracy: its organisational structure most closely resembles a class of school mathematics documents that we might call "teachers' problem lists." A good example is BM 80209, originally from ancient Sippar near modern Baghdad but now housed in the British Museum [**10**]. It repeats a few school mathematics problems over and over, each time giving a different set of numerical data that will yield a tidy integer answer [Figure 11]. Plimpton 322 is also a repetition of the same mathematical set-up fifteen times, each with a different group of well-behaved regular numbers. It would have enabled a teacher to set his students repeated exercises on the same mathematical problem, and to check their intermediate and final answers without repeating the calculations himself.

6. CONCLUSIONS. I stated at the beginning that this paper would be both about Plimpton 322 and about historical method more generally. A great deal of the history of mathematics concerns periods, languages, and settings that we know a lot about and share common ground with: we are already more or less familiar with Galois's cultural background, for instance, or Newton's. We are also helped enormously by knowing their identities, their life histories, other writings by them and their contemporaries. This allows us to contextualise the mathematical content of their work, helping us to understand it as they did. But when we start to study mathematics from cultures whose languages, social practices, and common knowledge we do not share, we have to work considerably harder at positioning it within a historically and mathematically plausible framework.[2]

Plimpton 322, analysed solely as a piece of mathematics, looked very modern, although it was impossible to say which branch of modern mathematics it most closely resembled: trigonometry, number theory, or algebra. It seemed millennia ahead of its time, incomparably more sophisticated than other ancient mathematical documents. But if we treat Plimpton 322 as a cuneiform tablet that just happens to have mathematics on it, a very different picture emerges. We see that it is a product of a very particular place and time, heavily dependent on the ancient scribal environment for its physical layout as a table, its mathematical content, and its function as a teacher's aid. All the techniques it uses are widely attested elsewhere in the corpus of ancient Mesopotamian school mathematics. In this light we can admire the organisational and arithmetical skills of its ancient author but can no longer treat him as a far-sighted genius. Any resemblance Plimpton 322 might bear to modern mathematics is in our minds, not his.

ACKNOWLEDGMENTS. This is the text of an invited address presented to the Joint Mathematics Meetings in New Orleans on 10 January 2001. It is a pleasure to thank Jim Tattersall for inviting me to the conference

[2]For the social history of Mesopotamian mathematics, see Nissen et al. [**21**], Høyrup [**14**], and Robson [**25**, pp. 138–183], [**27**], [**28**], all with further bibliography. Friberg [**11**] is a comprehensive survey of Mesopotamian mathematical techniques. For the social history of Mesopotamia in general, see Walker [**33**], Roaf [**23**], Postgate [**22**], Kuhrt [**16**], and Van De Mieroop [**32**].

and Victor Katz for first asking me to write this up, many years ago, and then pursuing me persuasively but persistently until I did. I am extremely grateful too to Jane Siegel of the Rare Books and Manuscripts Collection of Columbia University Libraries, who provided invaluable help in tracking down original documentation and permitted me to collate the original tablet. The drawing in Figure 10 is reproduced with permission from the American Oriental Society. A longer and more technical version of this article appears in *Historia Mathematica* under the title "Neither Sherlock Holmes nor Babylon: a reassessment of Plimpton 322" [**29**].

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I had a feeling once about Mathematics, that I saw it all—Depth beyond depth was revealed to me—the Byss and the Abyss. I saw, as one might see the transit of Venus—or even the Lord Mayor's Show, a quantity passing through infinity and changing its sign from plus to minus. I saw exactly how it happened and why the tergiversation was inevitable: and how the one step involved all the others. It was like politics. But it was after dinner and I let it go!

——Winston S. Churchill
My Early Life: A Roving Commission
Charles Scribners Sons, New York, 1930