Lost in a Forest

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1. INTRODUCTION. Fifty years ago, R. Bellman asked a remarkable minimization question (see Bellman [**7**], [**6**, p. 133], [**8**]) that can be phrased as follows:

A hiker is lost in a forest whose shape and dimensions are precisely known to him. What is the best path for him to follow to escape from the forest?

Call a path an *escape path* if it eventually leads out of the forest no matter what the initial starting point or the relative orientations of the path and forest. To solve the "lost in a forest" problem we must find the "best" escape path. Bellman proposed two different interpretations of "best," one in which the *maximum time* to escape is minimized, and one in which the *expected time* to escape is minimized. A third interpretation (see Croft, Falconer, and Guy [**12**, p. 40]) involves maximizing the probability of escape within a specified time period.

Bellman asked about two situations in particular: (1) the case in which the region is the infinite strip between two parallel lines a known distance apart, and (2) the case in which the region is a half-plane and the hiker's distance from the boundary is known. If "best" is taken to mean that the maximum time to escape is minimized, both of Bellman's particular situations have been investigated: for the first the minimax (i.e., shortest) escape path was found by Zalgaller [**45**] in 1961, and for the second the minimax escape path was described by Isbell [**21**] in 1957. In each of these two specific situations the shortest escape path is unique up to congruence. Little is known in either situation for other interpretations of "best."

Our objective in this survey is to focus narrowly on the case in which the "best" escape path is the shortest one. We establish a fundamental connection between the "lost in a forest" problem and L. Moser's well-known "worm problem" (see Moser [**31**], Wetzel [**41**]), and we utilize a partial result in the worm problem due to Poole and Gerriets [**35**] to define a large class of regions for which the best escape path is a line segment. We give two examples for which the shortest escape path is not unique. And finally we summarize the little that is known for a variety of regions having various elementary geometric shapes.

These problems can be phrased as "search" problems, in which one seeks the shortest search path to find the boundary of the region. For example, S. Burr (see Ogilvy [**33**, pp. 23–24, 149]) asked: *A swimmer is lost in a dense fog at sea, and she knows the shape of the shore and her distance from it. What is the best path for her to follow to search for the shore?*

Williams [**43**] has included the "lost in a forest" problems in his recent list "Million Buck Problems" of unsolved problems of high potential impact on mathematics.

2. A FEW GENERAL RESULTS. We begin by setting some language. A *path* γ is a continuous and rectifiable mapping of [0, 1] into $\mathbb{R}^2$. The path γ is oriented by increasing argument, from its initial point $\gamma(0)$ to its final point $\gamma(1)$. We write $\ell(\gamma)$ for the length of the path γ and $\{\gamma\}$ for its *trace*, i.e., its range $\{\gamma(t) : 0 \le t \le 1\}$.

We assume that a *forest* is a closed, convex region in the plane with nonempty interior. A path γ is an *escape path* for a forest F if it meets the boundary ∂F of F no matter how it is placed with its initial point in F, that is to say, if for each point P in F and each Euclidean motion μ for which $P = \mu(\gamma(0))$ the intersection $\{\mu \circ \gamma\} \cap \partial F$ is not empty. Among all escape paths for a forest F there is one whose length is as short as possible. The *escape length* β of a forest F is the length of this shortest escape path for F. If the forest F is bounded, a line segment whose length is the diameter of F is surely an escape path, so for a bounded forest F of diameter δ we have the inequality $\beta \leq \delta$.

We say that a path γ *fits* in a set S (or the set S *covers* the path γ) if there is a Euclidean motion μ so that $\mu(\{\gamma\})$ lies in S, or, equivalently, if S has a subset that is congruent to the trace $\{\gamma\}$ of γ. One fundamental mechanism for identifying a shortest escape path, employed frequently in the examples that we discuss, is the following:

Theorem 1. *Let F be a closed, convex region with nonempty interior O, and let γ be a path in F. If* (a) *γ does not fit in O, but* (b) *every path shorter than γ fits in O, then γ is a shortest escape path for F.*

Proof. Condition (a) means that γ is an escape path for F, because if the initial endpoint of γ is in O, then some other point of $\{\gamma\}$ must lie outside of O. To show that γ is a shortest escape path, we need only show that no shorter path is an escape path. But (b) says that every shorter path fits in O. ∎

In other words, the shortest escape path from a closed, convex region F is the shortest path that does not fit in the interior of F. This result is less helpful than one might think, because the shape of a shortest escape path must be known in advance and because the fitting condition (b) is typically very difficult to verify.

For bounded regions we have a fundamental compactness result:

Theorem 2. *If every path of length less than L fits in a compact, convex set F, then every path of length L also fits in F.*

Proof. Let $\gamma : [0, 1] \to \mathbb{R}^2$ be a given path of length L. For each n in $\mathbb{N}$, let

$$I_n = \left[\frac{1}{n+2}, \frac{n+1}{n+2}\right],$$

and let $\gamma_n : I_n \to \mathbb{R}^2$ be the restriction of γ to the interval I_n. For each n there is a set F_n congruent to F that contains $\{\gamma_n\}$. According to the Blaschke selection theorem (see Benson [**9**, pp. 134–35] or Eggleston [**13**, pp. 64–67]), the sequence $\langle F_n \rangle_{n \in \mathbb{N}}$ contains a subsequence $\langle F_{n_k} \rangle_{k \in \mathbb{N}}$ converging to a compact, convex set F_0 that contains $\{\gamma\}$ and is congruent to F. (A careful proof of this last assertion was given by Radunović [**36**].) ∎

Finally, we examine briefly the connection with Moser's well-known but still unsolved "worm problem": *Find the convex region of least area that covers each path of length one.* (See L. Moser [**30**], W. Moser [**31**], and Wetzel [**41**].) Let $\mathcal{C}_x$ be the family of all paths in the plane of length at most x.

Theorem 3. *The escape length β of a bounded forest F is the largest x for which F is a cover of $\mathcal{C}_x$.*

Proof. Let $s_0 = \max\{s : F \text{ is a cover for } \mathcal{C}_s\}$. If $x < s_0$, then every path of length x fits in the interior O of F and consequently is not an escape path for F; and it follows that

$x < \beta$. Hence $s_0 \leq \beta$. But since no path of length x smaller than β can be an escape path, every such path must fit into O, which is to say that F is a cover for $\mathcal{C}_x$; and it follows that $x \leq s_0$. Hence $s_0 \geq \beta$. ■

Poole and Gerriets [**35**] showed in 1973 that a 60° rhombus $R(L)$ (shaded in Figure 1) with longer diagonal of length L is a cover for every path of length at most L. Call a compact, convex set X *fat* if it contains points P and Q so that (a) PQ is the diameter of X and (b) the 60° rhombus *R(PQ)* with longer diagonal PQ fits in X. We call such a rhombus *diametral.*

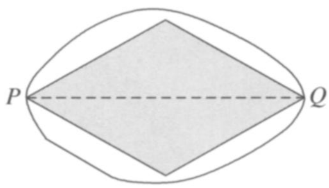


Figure 1. A fat convex set.

Theorem 4. *The escape length of a fat forest is its diameter.*

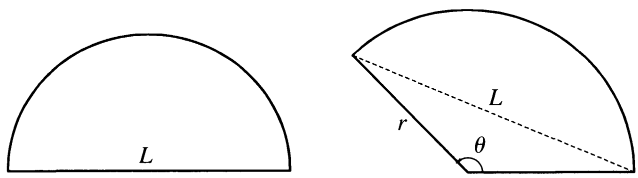
Proof. Suppose that the fat forest F has diameter δ and that $R(\delta)$ is a diametral 60° rhombus. A line segment of length δ does not fit in the interior of F. According to the theorem of Poole and Gerriets, any path of length less than δ fits in a 60° rhombus strictly smaller than $R(\delta)$ and consequently in the interior of F. So the claim follows from Theorem 1. ■

One can establish a slightly stronger similar result by replacing the 60° rhombus with the somewhat smaller cover found in 1992 by Norwood, Poole, and Laidacker [**32**].

3. SPECIAL SHAPES. In this section we collect what is known for regions of various special shapes. Many of these special shapes are fat, so the matter is settled by Theorem 4. Regions that are elongated in some sense are more difficult, and results are known for only a few situations.

Circular disk. The first progress on Bellman's problem of which we are aware was made by O. Gross [**19**] in an unpublished 1955 Rand technical report in which he settled the case of the circular forest and examined a few other shapes. Assuming the disk has unit radius, the fact that the shortest escape path is a straight line segment of length two follows immediately from Theorem 1. Indeed, a segment of length two is an escape path, and no shorter path can be an escape path, because as Gross noted, if any path of length less than two is placed so that the point midway between its endpoints lies at the center of the disk, then that path cannot meet the periphery of the disk (because the median of a triangle is shorter than the average of the two adjacent sides). It should be noted that the disk is fat, so the claim also follows from Theorem 4. Gross [**19**] also included some inchoate observations about other shapes, including the convex hull of two externally tangent disks, the equilateral triangle, and the infinite strip. (The circular disk was discussed a few years later in volume 2 of *The USSR Olympiad Problem Book* (Shklarsky, Chentzov, and Yaglom [**37**, pp. 22–23, 136–137, 367]), a book that unfortunately has never been translated into English, and still later by Tóth [**40**].)

Circular sectors. The shortest escape paths for a semidisk and for certain large-angle circular sectors are also line segments of length the diameter of the region. In the 1960s Meir showed that every path of length L lies in a closed semidisk of diameter L (see Wetzel [**42**]), and it follows from Theorem 1 that the shortest escape path from the semidisk of diameter 1 (Figure 2a) is a straight line segment of length 1. A few years later, Wetzel [**42**] showed more generally that the circular sector $S(r, \theta)$ of radius $r = (L/2)\csc(\theta/2)$ (Figure 2b) contains a congruent copy of every path of length L. It follows from Theorem 1 that when $\theta \geq 60°$, the shortest escape path from a circular sector with angle θ and radius $(1/2)\csc(\theta/2)$ (so that it has diameter $L = 1$) is a straight line segment of length 1. The shortest escape path when $\theta < 60°$ is not known.



(a) The semidisk. (b) The circular sector.

Figure 2. Circular sectors.

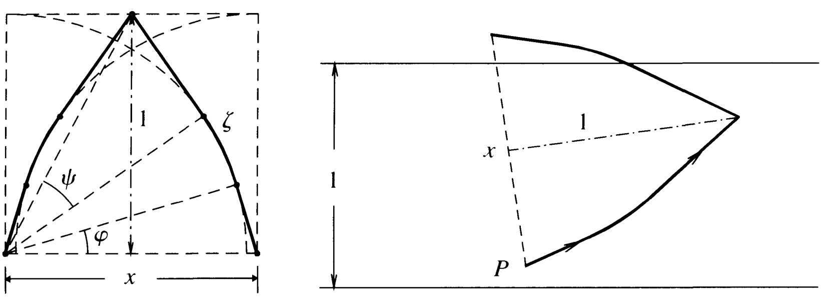
Infinite strip. Bellman asked for the shortest escape path for an infinite strip, the region between two parallel lines a known distance apart. The solution was described in 1961 by V. A. Zalgaller [**45**]. Suppose the strip has unit width. Zalgaller showed that the shortest escape path ζ is the symmetric path formed by four line segments and two circular arcs arranged as in Figure 3a, with

$$\varphi = \arcsin\left(\frac{1}{6} + \frac{4}{3}\sin\left(\frac{1}{3}\arcsin\frac{17}{64}\right)\right),$$
$$\psi = \arctan\left(\frac{1}{2}\sec\varphi\right).$$

It has length

$$\ell_0 = 2\left(\frac{\pi}{2} - \varphi - 2\psi + \tan\varphi + \tan\psi\right) \approx 2.278292.$$

Figure 3b shows this path as an escape path from a forest in the shape of an infinite strip of unit width. The hiker, initially at point P, follows this path and eventually



(a) The broadworm. (b) The shortest escape path.

Figure 3. Lost in an infinite strip.

escapes from the forest, no matter how the forest is positioned with respect to the path; and there is no shorter path that guarantees this outcome. The distance $x = \sec\varphi \approx 1.043590$ is algebraic of degree six:

$$3x^6 + 36x^4 + 16x^2 - 64 = 0.$$

Recall that the *minimum width* of a bounded set in the plane is the width of the narrowest infinite strip that can cover the set. Zalgaller's path ζ is the shortest path in the plane whose minimum width is one. Dually, ζ is the unique path of length ℓ_0 whose minimum width is as large as possible. It was rediscovered in 1968 by Schaer [**38**], who called it the *broadworm* and provided a careful proof of its uniqueness; it was rediscovered again in 1986-87 by Klötzler [**24**] and Klötzler and Pickenhain [**25**], who called it the *universal escape path*; and it was rediscovered yet again in 1989 by Adhikari and Pitman [**1**], who called it the *caliper*. The path plays a significant role in the worm problem (see Finch [**15**], Wetzel [**41**]).

Rectangle. Suppose that a rectangular region is $1 \times r$, with $r \geq 1$. The diameter δ of the rectangle is the length of the diagonal, $\sqrt{1+r^2}$. As earlier, let ℓ_0 be the length of Zalgaller's ζ path, whose minimum width is 1.

If $r < (\ell_0^2 - 1)^{1/2} \approx 2.047099$, then every path shorter than δ lies in a $\ell_0/\delta \times \ell_0 r/\delta$ rectangle that, in turn, fits properly inside the given rectangle. It follows that no path shorter than δ can be an escape path. Thus the straight line segment of length δ is the shortest escape path in this case. (If $r \leq \sqrt{3}$ the $1 \times r$ rectangle is fat, but this is not true when $\sqrt{3} < r \leq (\ell_0^2 - 1)^{1/2}$.)

If $r = (\ell_0^2 - 1)^{1/2}$, then the diagonal and the Zalgaller path ζ have the same length, and both are shortest escape paths. This observation (noted by Tóth [**40**] at a time when, however, he was unaware of Zalgaller's result) points up the fact that shortest escape paths need not be unique.

If $r > (\ell_0^2 - 1)^{1/2}$, then the shortest escape path is Zalgaller's path ζ. Indeed, ζ is an escape path, and any shorter escape path would be a better path for the infinite strip, contrary to Zalgaller's result.

The assertion involved here that paths of a certain length fit in a rectangle of a certain size follows from a result of Jones and Schaer [**22**, pp. 5–6], who in an unpublished 1970 University of Calgary research report used Minkowski's inequality to show that an $a \times b$ rectangular region contains a congruent copy of *every* path of length L if and only if

$$\begin{cases} a^2 + b^2 \geq L^2, \\ \min\{a, b\} \geq L/\ell_0. \end{cases} \tag{1}$$

The earliest published source for this fact appears to be Schaer and Wetzel [**39**] (where the result for squares is explicit in Theorem 4, and conditions (1), while not explicitly stated, are implicit in the results of sections 2 and 4). Makeev [**28**] has also given a short, direct argument. As a high school student in 1982, Tóth [**40**] gave a direct argument for the case of a square and considered the rectangle case as well.

Regular polygons. For each $n > 3$ the region F_n bounded by a regular n-gon is fat (Figure 4), so the escape length is the diameter $\delta(F_n)$, namely,

$$\delta(F_n) = \begin{cases} \csc \frac{\pi}{n} & \text{when } n \text{ is even}, \\ \frac{1}{2}\csc \frac{\pi}{2n} & \text{when } n \text{ is odd}, \end{cases} \tag{2}$$

where the edge length is taken to be 1. (Tóth [**40**] included some remarks about this problem for even n.)

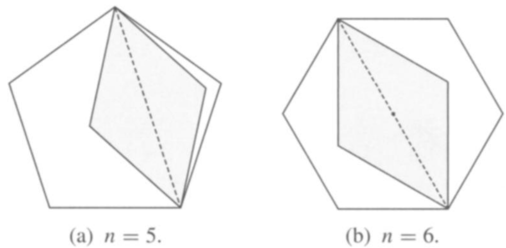
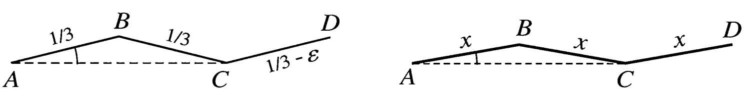


Figure 4. Regular polygons.

The shortest escape path is not known in the case of an equilateral triangular region. Assuming the triangle has unit side, one might expect the shortest path to be a straight line segment of length 1. Gross [**19**], however, observed that for sufficiently small ε the path pictured in Figure 5a, with $\angle CAB = 15°$ and $CD = 1/3 - \varepsilon$, is an escape path for the equilateral triangle of unit side, and its length is less than 1. (It is easy to see that any ε with $0 < \varepsilon < 0.013$ works.) Prompted by an equivalent question posed by Graham [**18**] in 1963, Besicovitch [**11**] found the escape path of length $3\sqrt{21}/14 \approx 0.981981$ pictured in Figure 5b, where $\angle CAB = \arcsin(1/\sqrt{28}) \approx 10.9°$ and $x = \sqrt{3/28}$. (He obtained his result by solving an optimizing equation numerically; the radical expressions were found by Steven Knox in 1994.) Besicovitch conjectured that this path is the shortest. Although this conjecture is likely to be correct, little progress has been made toward its proof.



(a) The Gross path. (b) The Besicovitch path.

Figure 5. Two zigzag paths, both with $CD \parallel AB$.

Nothing seems to be known concerning the escape problem for nonequilateral triangles.

Line. The last two special cases are of a somewhat different character, and it is a little more natural to phrase them in plain geometric language as searches in the plane.

Bellman assumed in his second situation that the region is a half-plane and the initial point is a known distance from the edge. So, given a point P, we seek the shortest search path to find a line m, given its distance d but not its direction from P. The shortest search path, shown in Figure 6, was described in 1957 by Isbell [**21**]. It has length $(\sqrt{3} + 7\pi/6 + 1)d \approx (6.397242)d$. In an important 1980 article Joris [**23**] supplied a complete and detailed proof. Melzak [**29**, pp. 150–153] also considered Isbell's problem.

Circle. In 1961 Gluss [**17**] considered the problem of searching for a circle of known radius in the plane, given its distance from the starting point but not its direction. As Gluss noted in his abstract (p. 357), "The problem appears to be of some practical

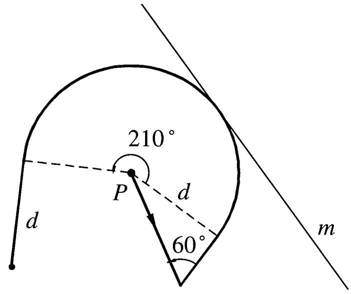


Figure 6. Search for a line m.

significance, since it is equivalent to that of searching for an object a given distance away which will be spotted when we get sufficiently close—that is, within a specified radius." There are two different situations: *exterior*, in which the starting point P is *outside* the target circle C, and *interior*, in which the starting point P is *inside* the target circle C. Gluss considered the case in which the starting point P is outside the target circle. We consider the two situations separately.

The exterior case. This is a natural situation in which the region is not convex. Suppose that the target circle C has radius s, and that the initial point P lies outside C at distance $r + s$ from its center. Let Γ be the circle of radius r and center P, and let C' be an auxiliary circle of radius s whose center O' is chosen arbitrarily at the given distance $r + s$ from the starting point P (Figure 7). Let R be the point on Γ and Q the point on C' so that QR is tangent to Γ at R and the ray $O'Q$ bisects $\angle PQR$. (Locating Q requires the numerical solution of an intractable trigonometric equation.) Gluss's search path follows the segment PQ to C', then the segment QR back to Γ, then along Γ to S, and finally to T on C' along the tangent line SO'. His persuasive heuristic reasoning that the search path $PQRST$ so defined is the shortest has not, to our knowledge, been put on a more rigorous footing. Gluss observed further that for $s \to \infty$ with $r = 1$, this path approaches Isbell's path (Figure 6).

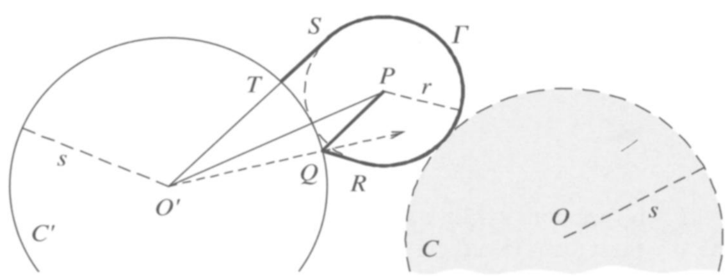


Figure 7. The exterior case with $r/s = 0.50$.

The reasoning specializes correctly to the case in which the target is a point ($s = 0$) at known distance. More generally, if one knows only the distance to a target object but nothing about its shape, then, as Melzak [**29**, p. 150] remarked in 1973, the shortest search path is the path pictured in Figure 8: move the known distance in any direction and then describe a circular path about the starting point.

The interior case. If the starting point P lies inside the target circle of radius s but nothing further is known, then the best search path is the line segment of length $2s$ (as

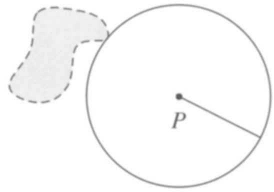
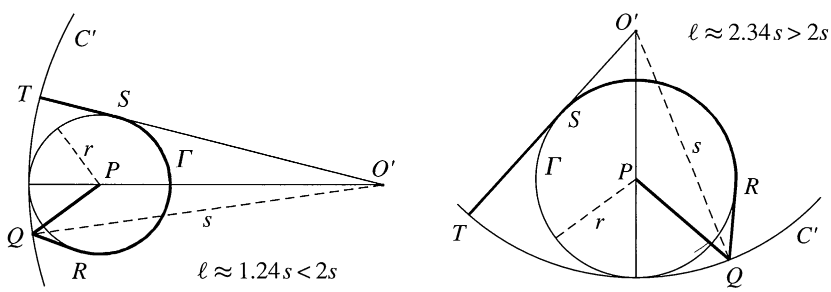


Figure 8. Search for an object at known distance.

Gross [**19**] noted in 1955). But if the distance r to the circle is also known, the interior problem seems to yield to the same heuristic considerations that Gluss used for the exterior case, at least when $r \le s/2$. The shape of the shortest search path depends only on the ratio $\rho = r/s$ of the two given distances. Figure 9a shows the Gluss search path *PQRST* for the case $\rho = 0.20$. Its length is about $1.24s < 2s$. Figure 9b shows the path for $\rho = 0.40$. Its length is about $2.34s > 2s$. Since for fixed s and $0 \le \rho \le 1/2$ the length $\ell(\rho, s)$ of the Gluss path is an increasing continuous function of ρ, there is a value ρ_0 for which the Gluss path has length $2s$. Numerical investigations give the approximate value $\rho_0 \approx 0.333454$. So we suggest that in the interior case, the length $\beta(\rho, s)$ of the shortest search path is as follows:

$$\beta(\rho, s) = \begin{cases} 0 & \text{when } \rho = 0, \\ \ell(\rho, s) & \text{when } 0 < \rho \le \rho_0, \\ 2s & \text{when } \rho_0 \le \rho \le 1/2, \\ (2-\rho)s & \text{when } 1/2 \le \rho \le 1. \end{cases}$$

In particular, when $\rho = \rho_0$ both the Gluss path and the straight line segment are search paths of length $2s$; and we have a second situation in which the shortest search path is not unique.



(a) $\rho = 0.20$. (b) $\rho = 0.40$

Figure 9. Interior Gluss paths.

4. RELATED QUESTIONS. There are many interesting variants and related questions in the literature. Baeza-Yates, Culberson, and Rawlins [**5**], [**4**] conjectured that a logarithmic spiral is the best path to follow to escape from a half-plane forest if one does not know how far one is from the edge, but this has apparently not yet been established rigorously. The "beam detection problem" asks for the shortest plane path that meets every line that meets a given compact, convex set (see Faber and Mycielski [**14**], Finch [**15**]). In insightful articles [**46**], [**47**], and [**44**], Zalgaller suggested, without rigorous proof, possible solution curves for a variety of interesting extremal problems in space, including the question of the shortest curve that has minimal width one. Other variants were considered by Anderson and Fekete [**2**], Baeza-Yates and Schott [**3**], Hassin and Tamir [**20**], López-Ortiz [**26**], and Papadimitriou and Yannakakis [**34**].

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