notes

# MATHEMATICAL THEMES IN A FIRST-YEAR SEMINAR


Jennifer Schaefer
Jennifer Bowen
Mark Kozek
Pamela Pierce
**Editors**


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# in a First-Year Seminar

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# Mathematical Themes in a First-Year Seminar

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# Introduction

Jennifer Bowen, Mark Kozek, Pamela Pierce, and Jennifer Schaefer

The purpose of this volume is to share with undergraduate mathematics faculty a variety of ideas for teaching a mathematically-oriented first-year seminar (FYS). Our vision is that this *Notes* volume, containing 36 unique FYSs taught by a variety of authors from small liberal arts colleges to large research universities,[1] will serve as a handbook for faculty members interested in finding new topics, course structures, activities, or assignments to incorporate into any first-year experience course containing mathematical content or mathematical or quantitative themes.

## The History of First-Year Seminar in Higher Education

The modern FYS program can be traced back to the early 1970s at the University of South Carolina. In response to student riots on campus, then-president Thomas Jones established a seminar, University 101, taught by trained faculty that he hoped would "increase positive attitudes and behaviors towards the institution, enhance retention, communicate the value of education, and improve teaching in undergraduate programs" [7, p. 156]. Decades later, FYS courses have been built into the curriculum of colleges and universities in hopes of increasing student retention and student engagement and building a strong foundation for academic success.

The structure of such a course varies by institution. One FYS program may provide an extended orientation, while another may focus on developing students' skills in critical thinking, reading, and writing. Some seminars might contain a great deal of discipline-specific content and serve primarily as an introduction to a discipline. In other cases, an FYS may be a hybrid course that encompasses all of these characteristics. Regardless of structure, extensive research has shown that "the highest-quality first-year experiences place a strong emphasis on critical inquiry, frequent writing, information literacy, collaborative learning, and other skills that develop students' intellectual and practical competencies" [13, p. 9].

There is no doubt that the adaptable format of FYS is "among the reasons the seminar has found an enduring and extensive foothold in higher education" [11, p. 94]. However, it is the numerous benefits of FYS—demonstrated through extensive assessment and research from its inception—that have solidified its place in the undergraduate curriculum and as a high-impact educational practice by the Association of American Colleges and Universities [1]. In addition to increased rates of retention, "students who participate in FYS have demonstrated gains in academic achievement and grades; civic engagement; intercultural competence and multicultural awareness; positive relationships with faculty, staff, and peers; involvement on campus; and development of academic interpersonal and life skills" [11, p. 94]. These deeper connections with FYS instructional faculty facilitate the institution's goal of supporting first-year students in developing academic capacities that will lead to academic success and personal growth [7, p. 163].

## Opportunities and Challenges for Mathematics Faculty Teaching a First-Year Seminar

Given the inherent benefits of FYSs to our students, it is important that faculty from all disciplines take up their role as FYS instructors. While mathematics faculty may find teaching FYSs intimidating because of their heavy emphasis on writing, we can not leave this valuable opportunity to other disciplines. Luckily for mathematics faculty, this generation of students is primed for an FYS with a mathematical theme.

[1] Half of the contributions are from small liberal arts colleges and half are from public or private universities and colleges.

In recent years, we have witnessed an increase in the number of films and television shows portraying mathematics and mathematicians (*A Beautiful Mind*, *Hidden Figures*, *The Imitation Game*, *Moneyball*, *Numb3rs*, *21*). These films and television shows are often based on popular works of narrative non-fiction that are able to engage a broader audience than ever before by highlighting the *story* behind a mathematical achievement. In this way, film and television have given rise to a generation of students who have grown up watching stories "about mathematics." As a consequence, our students are either aware of or are applying mathematics in more ways than previous generations (sports analytics, hacking/hacktivism, data science, privacy ethics, Occupy Wall Street, online poker/gaming) and thus are keen to hear what we, as mathematicians, have to say.

Complementing this more mathematically-aware generation, FYSs provide mathematics faculty an opportunity to reach open-minded first-year students by engaging them with a story "about mathematics." This opportunity is often not available in our standard curriculum given the sequential nature of mathematics courses and the extensive set of required topics in most classes, especially at the undergraduate level. However, FYSs make it possible to involve students in mathematics, or in mathematics-related activities, even when there are no prerequisites. They also allow mathematics faculty to introduce students to the field of mathematics by teaching mathematical content in ways we would not regularly teach in a mathematics course. The question is *how*?

One of the editors took part in a conversation at the 2017 Joint Mathematics Meetings with a group of mathematicians from small colleges. They began sharing ways they had incorporated an array of mathematical readings into their FYSs. The editor volunteered that she spent a fair bit of time on the prisoner's dilemma in her seminar, which was about how people make decisions. Another faculty member contributed that they incorporated a good deal of probability into their seminar about how people evaluate risk. A third person shared that they were looking at common core standards as their seminar was evaluating the mathematical preparation of today's college students. Someone else had used the book *Innumeracy: Mathematical Illiteracy and Its Consequences* in their class.

Two general themes emerged from this conversation. First, mathematics faculty are teaching a variety of interesting FYSs with mathematical themes. Secondly, they are very interested in learning how to do this successfully. As mathematicians, we are typically not trained to teach academic writing or to lead discussion-based classes. In addition, our mathematics courses generally have one course text that drives the course schedule. We are not used to developing a syllabus based on multiple sources with various perspectives. In standard mathematics courses, we usually can expect that our students have a certain level of prerequisite knowledge. However in an FYS, this is not necessarily the case. Given these challenges, it would be valuable if the sharing of FYS themes, course structures, readings, and assignments through informal conversations could be disseminated more broadly with the mathematics community.

When we developed our own FYSs, we found many resources regarding mathematics and popular culture, yet we found very few resources describing how to organize an FYS on such a topic (or any math-related topic), especially in terms of reading and writing assignments. In James R. Hughes's December 2000 *PRIMUS* article "Mathematics and the First-Year Seminar," he stated that he could only find one article specifically dealing with a mathematical FYS [10, p. 360], namely, David T. Burkam's 1989 article "Using Writing to Teach Mathematics" in the MAA *Notes* volume *Teaching Mathematics within the Writing Curriculum*. We found that this number improved only slightly through 2017, as one can see from the following list of topics found in articles describing FYSs with mathematical themes.

- Chaos and Fractals [9]
- Critical Thinking and Scientific Reasoning [4]
- Cryptography/Cryptology [6, 12]
- Culture [2]
- Introduction to College Life and Mathematical Vignettes [1]
- Mathematical Transitions course [3]
- Quantitative Literacy [5, 10, 14, 16]
- Statistics and Data Science [17]

Hence there seems to be a need for a single, easily accessible resource describing the experiences of faculty who have successfully taught FYSs with mathematical themes. This is where our volume comes in.

## Structure of the Volume

The book begins with a chapter written by the editors highlighting important things to keep in mind when teaching an FYS. The remainder of the volume is arranged by mathematical theme: Cryptography; Gambling, Game Shows, and Game Theory; Mathematical Modeling and Data; Mathematics in Politics, Equity, and Social Justice; Mathematics in Popular Culture and History; Mathematics, Art, and the Natural World; Proofs and Problem Solving; and Quantitative Literacy. In each chapter, you will find a collection of articles describing the experiences of faculty who have successfully taught mathematically-oriented FYSs related to the chapter's featured theme.

Each article has the same overall structure, with the following four sections:

1. Background and Context: describes the author's institution and the FYS program at their college or university;

2. Mathematical Theme: provides a summary of their course's mathematical theme and a description of how this theme fit into the course structure;

3. Course Structure: provides an overview of the course structure while focusing on descriptions of effective resources, activities, assignments and/or modes of assessment; and

4. Reflections: describes what went well in past offerings of the course with any adaptations that they would suggest to readers who hope to implement these ideas in their own classrooms.

Some articles focus on a mathematical theme that will hold student and faculty interest for an entire semester. Other pieces describe a specific collection of resources and activities that made for a cohesive unit on a mathematical theme. Most articles focus on a few in-class activities or assignments that worked well in their FYS while incorporating learning objectives, the specifics of carrying out the activities or assignments, and/or the assessment tools utilized. In many cases, authors have included entire assignments or assessment tools, such as rubrics, either in the body or the appendix of their article. In fact, you will find the following course features described in the articles that follow: Assessment Tools; Class Activities; Course Schedule; Field Trips and Guest Lectures; Linked Course; Projects; Readings and Films; Student Helpers; and Writing Assignments. In the Quick Reference Guide, you can see which of these course features are highlighted in the articles of this volume. Because each author has included as many details as possible, our hope is that a reader thinks of each piece as a "how-to" article, and feels confident adapting the ideas for their own FYS.

This volume is not the type of text that needs to be read in a linear fashion. Feel free to skim the Table of Contents and start with themes or topics that interest you. You will find that many of the ideas are not theme specific. Readings, class activities, writing assignments or projects detailed in one FYS may work well with a different FYS theme. We encourage you to pick and choose information from a variety of articles as it will help you develop an FYS unique to you and your institution's learning objectives. Any content in the volume can be freely adapted for instructor use.

## Acknowledgements

Two of the editors organized the MAA Contributed Paper Session *Mathematical Themes in a First-Year Seminar* at the 2018 Joint Mathematics Meetings. The session had approximately two dozen speakers with at least 20 people in attendance for each presentation. Because of the success of this session, the editors co-organized two additional MAA sessions on *Mathematical Themes in a First-Year Seminar* at MathFest 2018 and the 2019 Joint Mathematics Meetings. We want to thank all of the speakers in these sessions for eagerly sharing information about their FYSs with the mathematical community.

The editors would also like to thank Stephen Kennedy of the AMS Acquisitions team and Mike Axtell and Liz McMahon, editors of the MAA *Notes* series, for their useful guidance in creating this volume. We appreciate the thoughtful feedback from the MAA *Notes* Board. We are particularly grateful to all of the contributing authors for their creativity and enthusiasm for their FYSs. Especially important are our own FYS students and their experiences in our own classrooms.

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# Advice for First-Year Seminar Instructors

Jennifer Bowen, Mark Kozek, Pamela Pierce, and Jennifer Schaefer

Perhaps you are reading this volume because you have been asked to teach a first-year seminar (FYS) for the first time. Creating your FYS might seem like a daunting task, especially if writing assignments are central to the course. At least it seemed that way to us when we started out. But with this chapter, and the rest of this volume, we hope to share with you what we wish we would have known when we taught an FYS for the first time—the things we learned "the hard way." We have compiled these suggestions from our experiences teaching FYSs, from the three sessions on *Mathematical Themes in a First-Year Seminar* that we led at the Joint Mathematics Meetings and at MathFest, and from the shared wisdom of our volume's authors.

In order to give you a head start on preparing, we describe some of the steps involved in pre-course planning: picking a theme, preparing your course, attending workshops, seeking out campus resources, selecting student helpers, and naming your course, and we offer various tips about teaching the course: course structure and pacing, content, reading assignments, class discussions, writing assignments, student presentations, and grading.

The generic FYS we are helping you plan meets for three contact-hours per week, for a 14 to 16-week semester. We assume the first-year students come from the broad undergraduate student population and that the seminar involves a non-trivial amount of writing. While many of the articles in this volume address the recommendations we describe below, we highlight some in which you can find useful examples of these course tools and techniques.

### Pre-course Planning (begins six months to a year before teaching the course)

- Theme: First and foremost, you want to pick a theme that you are excited about. After all, you are going to be spending a good deal of time on this theme, so it should be something that you enjoy. It could be mathematical or not, but if you are excited about the theme, then there will be a better chance that your students will engage with the theme as well.
- Preparation: You want to start planning your course as far in advance as possible—one year in advance is not too soon! Once you know that you are scheduled to teach an FYS, begin talking to other people. Read over articles, such as those found in this volume, to help you begin thinking about a potential theme. Tell your colleagues, friends, and family about ideas that you are tossing around in your head. Very often, someone will suggest a reading, a film, or an idea that proves to be a wonderful resource. Keep an electronic file on your computer with ideas for resources, so that you can go back and investigate them when you have time. Ask faculty colleagues who have taught an FYS before to sit down with you and discuss the following: how did they arrive at the topic for their seminar? how many books did they use? what worked well and what did not work well? what would they change the next time they teach the class? Your colleagues are fantastic resources, so use them! You may also ask to sit in on a colleague's FYS course (or regular course that overlaps pedagogically with FYSs) to observe how to lead discussions, administer writing or peer-review exercises, etc.
- Attend Workshops: At many institutions, FYSs fall under the umbrella of an established campus office, such as the Writing Program or First-Year Experiences. Often, these offices sponsor mini-courses or seminars for first-time instructors. If so, definitely attend the mini-course and try to get a sense of what the FYS program is all about at your institution. These workshops should cover the learning goals of the FYS program, specific requirements for writing assignments, ideas and advice for working with first-year students, and other key topics that are common to all seminars across campus. A course design workshop can help you flesh out specific ideas for your seminar, also.

- Resources and Specialists on Campus: There are probably a lot of resources on campus that can support you and your students.[2] Hopefully these will be shared with you through the umbrella office that works with FYSs or during the workshops for new faculty teaching in the program. Learn about these resources, meet the people involved early on, and use them! For example, your Writing Program might have a specialist who could come to your class for a day or two to teach your students successful strategies for peer review. Or, your library might have a research librarian who could visit your class to provide instruction on using library resources effectively. Many institutions have speaking/presenting coaches. These people are usually happy to stop by your class and share their expertise. In effect, this acts as an advertisement for their services, and students will (hopefully) continue to use the resources that they learn about in your class throughout their four years of college, laying the groundwork for a strong educational experience for your students. We suggest reaching out to these colleagues early, maybe one to three months before the start of the course, to account for the fact that their time might be in high demand by other FYS instructors once the academic term begins.
- Student Helpers: If your institution embeds student helpers,[3] such as undergraduate teaching assistants or peer mentors, in FYSs, choose these student helpers wisely, provided that falls under your purview as the course instructor. The right student helper can make teaching FYSs go much more smoothly for you. It works best when the student helper is someone you know and trust, is an excellent writer, and has taken courses with you, so they are familiar with your general teaching habits. Maybe use one of your academic advisees or course graders or someone you know from one of your mathematics courses, your interdisciplinary courses, or a previous version of your FYS. If you have never worked with a student helper before, ask your colleagues how they chose their student helpers and how they incorporated their helpers into their course. In particular, be aware of what a student helper is allowed and is not allowed to do (based on your institution's policy on the role of student helpers).
- Naming Your Course: Choose the name of your course carefully. You want to find the happy medium between having a flashy name and accurately representing to the students your course's content—mathematical or otherwise. The registrar, dean for academic advising, or colleagues who administer the FYS program may have some excellent tips on naming your course. They often have first-hand knowledge of what themes and buzzwords tend to resonate with entering students and whether or not to include the word *mathematics* in the title (assuming that your seminar deals with mathematical themes). At some institutions, if you put this word in the title, it may act as a deterrent for students choosing your course. However, it is entirely possible that those same students might feel deceived if your course has a mathematical component that was not indicated in the course title.

### Teaching Tips

- Course Structure and Pacing: At most institutions, FYSs have prescribed or recommended content, usually related to college-life or writing requirements, in addition to your thematic goals. When you plan your semester, please make sure you have allotted enough time for these elements of the course, even if it means that you do not get to cover as much of your theme as you originally intended. If your course meets for three hours per week, we recommend the following: on average, spend one hour per week discussing the reading or viewing for that week (so, your thematic content), spend one hour per week on writing exercises or college-life material (required content), and use your discretion to determine what is the best use of the remaining hour that week. This third hour could be used as extra time for thematic or required content, for in-class activities, ongoing class discussions, group work, guest lectures, student presentations, etc. (Nordstrom [Chapter 5] and Pierce [Chapter 6] provide daily course schedules, and Berger/Meyer [Chapter 9] provide a contact-hour breakdown of their major course components.)
- Content: Typically, an instructor teaching in the FYS program for the first time will try to cover too much content. Remember that this is not a Calculus 1 course with fixed set of topics that the students should learn prior to enrolling in Calculus 2. The thematic material is not nearly as important as giving your students a good first-semester experience. Helping them read more effectively, write more clearly, challenge assumptions, and think

[2]These resources can also be thematic resources, for example, Beecher's sustainability-themed FYS [Chapter 8] took advantage of various campus-wide sustainability offices or resources.

[3]We will not describe the duties of these student helpers, as at different institutions these titles, teaching assistant or peer mentor, might mean the same thing, or they might not. Also, at some institutions the student helper is only a writing resource, only a campus life resource, or both.

critically will help them throughout college and beyond. While the theme should be engaging, it is not necessary to inundate the students with new content.

- Reading Assignments: It is important to pace the readings appropriately. Many students will complete a long reading assignment for homework or a long writing assignment for homework, but not both. Avoid assigning long readings, even if a long reading is spread out over several weeks. We would suggest capping the reading at about 150 pages per week (for a three-credit hour seminar), and when possible, the reading assignments should be stand-alone reading assignments. (Whipple [Chapter 16] includes a schedule of course readings.) Although most of the readings will be chosen to support the theme, try for these readings to also be representative examples of academic writing (Kozek [Chapter 19] provides examples of such resources). To encourage students to keep their reading up-to-date, we suggest giving reading comprehension quizzes or requiring journal response submissions, especially early in the semester (Brown [Chapter 34] and Szydlik [Chapter 29] include reading assessments).

- Class Discussions: If you do not have a lot of experience leading class discussions, you may want to start by having small group discussions. Break the class into groups of three or four and ask them to discuss (a given topic) amongst themselves. Then walk around the classroom to get a sense of what the students are saying and who is doing all the speaking. To further give the students material to talk about, ask the students to write a journal response, either for homework or at the start of the class, on the discussion topic. Start by asking the students to share what they wrote with their group-mates. Once the students have had enough time to do this, then ask the students a follow-up question to their journal entry conversation. Finally, call all the groups back together and ask each group to present their conversation's outcome to the class. At this point, sometimes a full class discussion emerges organically. If so, let the students go with it. If not, move on to the next item on your agenda. Other resources and innovative ideas for classroom discussions can be found online at Cult of Pedagogy's "The Big List of Class Discussion Strategies" [2].

- Writing Assignments: It could be that your institution has a required list of writing assignments that you must include in your course. If not, many authors in this volume find assigning more frequent, shorter writing assignments to be effective for their courses. They describe writing assignments that are: one to two, two to three, three to five, five to seven, or eight to ten pages at the longest. Also, we suggest that you include a recommended word count (approximately 250 words per double-spaced page or 500 words per single-spaced page) with each page length requirement to discourage students from using formatting shenanigans to reach the required length. Unless it is explicitly required by your institution, we do not suggest assigning papers that are longer than ten pages (2500 words). Of course, the quantity of writing that you assign will vary depending on the credit hours awarded for your course. A three-credit hour course may require substantially more writing than a one-credit hour course.

  In addition, we recommend varying the type of writing projects you assign. An initial paper may ask students to write from personal experience. Another assignment could require students to analyze a reading. You could follow this with a paper that asks students to compare and contrast two ideas, themes, or resources. Varying the length and type of assignment will help your students develop a stronger and more diverse set of writing skills. In this volume, you will find numerous versions of common FYS writing assignments, such as those we just mentioned, journal entries, summary essays, persuasive essays, and research papers; and innovative writing assignments such as point-counter point essays (Montgomery [Chapter 33]), resumés and cover letters (McAllister/Overmann [Chapter 27]), blog entries (Treviño [Chapter 30]), creative writing (Saclolo [Chapter 13]), or grant proposals (Shank [Chapter 31]).

  While most of the writing assignments will deal with content associated with your theme, you will need to remember to incorporate writing skills into your assignments. For example, early in the semester it is a good idea to run an in-class session on how to write a strong thesis statement. Your student helper or a specialist from your institution's Writing Center can help you lead this session. Once the students have gained sufficient proficiency at developing strong thesis statements, you can then embed this skill into each subsequent writing assignment. Analyzing evidence, comparing and contrasting sources, effective strategies for peer-review, and revising and properly citing references are other valuable skills that you may want to address (Bowen [Chapter 35] includes a template for a peer-review exercise; Wiglesworth [Chapter 3] includes a template for peer-reviewing a presentation, and Schaefer [Chapter 17] includes a revision memo). By introducing a limited number of skills throughout the course, you will ensure you are not assessing an overabundance of competencies in any given assignment.

It is essential to give students focused prompts and detailed directions for writing assignments.[4] Remember that these are first-year students and some, if not all, have limited experience with academic writing. For example, if you expect students to use direct quotes from a course reading to support their thesis or avoid outside resources altogether, you probably need to tell them this (at least at first). If you want students to choose their own topic, you should approve the topic before the students start the writing process. The more specific your prompt is the more closely your students' papers will align with your expectations (Anderson [Chapter 12] and Parsley [Chapter 10] include very detailed writing prompts).

For each writing assignment, allow sufficient time—perhaps multiple weeks—for the students to go through various "check-in" deadlines along the way (outlines, annotated bibliographies, drafts, peer-reviews). In addition, try not to have the important assignments due during "peak" times of the semester. For example, during the last week of the term, students are likely to have a major assignment due in all of their classes. They may choose to prioritize a course in their prospective major over their FYS, or they may choose to prioritize the most valuable assignment due that week.

- Student Presentations: At some institutions, student presentations are a required part of an FYS. Your students may, for example, write a final paper, have a group project, or create a poster that they present to the class or a broader audience. If your course has a presentation component, then remember to allow ample time to discuss with your students the skills needed to create and deliver their presentation (Lakins [Chapter 25] includes speaking prompts). Please explore available campus resources that train students to give slide presentations or to create posters. If you have not used presentations or posters in any of your previous courses, then definitely consult with faculty colleagues who have. Many articles in this volume describe projects that have a presentation component.
- Assessment Tools: One of the most challenging parts of teaching FYSs is grading student writing. Start by asking your colleagues how they grade papers in their FYSs. If your institution offers seminars for first-time instructors, these will often include samples of effective grading rubrics or guidelines. Start with these, but also survey your close colleagues, especially if you know someone in the Writing Program, the Humanities, or the Social Sciences Division who has to grade papers in all of their other courses. Do not try to reinvent the wheel. Ask people who have been grading papers longer than you have what works for them and use that as your starting point. Another helpful resource is the American Association of Colleges & Universities VALUE rubrics [1]. These assessment tools include rubrics concerning written communication, critical thinking, oral communication, and others (Greenwald [Chapter 20] and Hummel [Chapter 2] also include detailed rubrics).

In this section, we have provided broad guidelines or initial suggestions on how to undertake various components of an FYS. Now that you are aware of things to keep in mind while developing your FYS, we suggest you read our volume's articles. You will find how the volume's authors have put all of these pieces together in light of their chosen theme, their institution, and their vision for their course. Remember to glance though the Table of Contents and the Quick Reference Guide where you can find which of these course features the authors have highlighted in their articles. Good luck with your course, and happy planning!

## Bibliography

[1] Association of American Colleges and Universities. "VALUE Rubrics". 2009. `www.aacu.org/value-rubrics`

[2] Jennifer Gonzalez, "The Big List of Class Discussion Strategies", Cult of Pedagogy, October 15, 2015. `www.cultofpedagogy.com/speaking-listening-techniques/?`

[4]When possible, try to make your assignments unique and specific, so that students are not tempted to look for—and find—pre-existing essays on the web.

# Part I

# Cryptography

Cryptography is a popular mathematical theme for first-year seminars among both faculty and students. It can be taught with minimal mathematical prerequisites and is interdisciplinary by nature, making it appeal to a broad audience. We include two articles in this section, but other articles can be found in the literature and two have been cited in the introduction.

# 1

# Developing Research, Writing, and Speaking Skills Through a Seminar in Cryptology

Randall D. Helmstutler and Keith E. Mellinger

**Abstract** In this article we describe the evolution of our first-year seminar in cryptology as we adapted it to meet university-wide learning outcomes regarding communication and research skills. As the course is offered with no prerequisites, we outline how we deliver this course in modern cryptographic techniques without assuming anything beyond high-school-level algebra. We also give an overview of how speaking, writing, and research skills—components required of all seminars regardless of discipline—have been integrated into the course content.

## 1.1 Background and Context

The University of Mary Washington (UMW) is a public liberal arts university of around 4,000 undergraduates, located about 50 miles south of Washington, DC. The first-year seminar course (FSEM) is a university-wide general education requirement, delivered by all colleges and departments as a central part of the first-year experience. Each section of the FSEM course is grounded in a discipline—with contributions from all academic departments—but with a common set of learning outcomes to be employed across all sections. As part of its reaffirmation of accreditation by the Southern Association of Colleges and Schools, in 2013 the university made the redevelopment of its FSEM courses the focus of its required five-year Quality Enhancement Plan (QEP). The goal of the QEP was to provide all first-year students the necessary skills in research and communication in order to promote success in all four years of their studies, integrating these skills into all FSEM courses regardless of discipline or topic.

The university first offered first-year seminars in 2006. The program took root quickly and by 2008 the FSEM was a requirement for all incoming first-year students as part of the general education curriculum. With a cap of 15 students per section, the faculty at large have been responsible for delivering roughly 70 sections of FSEM courses per year. From the beginning, the mathematics department at UMW saw this new program as an opportunity to attract majors and teach exciting new topics outside of the current curriculum. This was also a chance to promote mathematics to new students as more than rote solving of quadratic equations and memorization of the unit circle. Over the years, the mathematics department created six new FSEM courses on topics such as chaos theory, mathematics and art, and cryptology.

One of the groundrules for our FSEM courses is that there must be no prerequisites or barriers to entry: all courses should be open to all students. For mathematicians wishing to do interesting mathematics, this can present a challenge. Unlike some disciplines, mathematics is often very linear-building: students cannot reasonably learn integration by parts without first understanding some basics of derivatives. But the challenge does not stop there. University guidelines set by the faculty in the QEP required that each course contain significant research, writing, and speaking components, no matter if the FSEM was new or already-existing. The second author developed the FSEM *Cryptology* in the fall of 2009, soon after drawing the attention of the first author who refined and expanded many of the materials. The approval of the QEP in 2013 and its pending implementation in 2014 required a significant overhaul and re-thinking of the course. Our end result is a successful mathematics course for first-year students requiring no prerequisite knowledge that develops the basic research, writing, and speaking skills necessary for success in college.

## 1.2 Mathematical Theme

Our FSEM in cryptology gives a broad view of cryptographic techniques, starting with the simple shift cipher of Julius Caesar and ending in the modern era with the public-key protocol of Rivest, Shamir, and Adleman (RSA). In addition to developing the mathematics necessary for creating these algorithms, we also consider the cryptanalysis of each encryption algorithm we encounter. While all the encryption algorithms are based on modular arithmetic, the cryptanalysis techniques in cipher-breaking often involve parts of mathematics unrelated to this, such as probability and statistics. Pure ingenuity also plays a large part in cryptanalysis, so this is the perfect setting to showcase the creative aspects of doing mathematics. (Indeed, the cryptanalysis units are the students' favorites.) The cryptanalysis of ciphers often involves not only modular arithmetic and probability but also linguistics, showing how seemingly disparate fields can relate to and complement each other in powerful ways.

Even though the mathematical foundations of an algorithm like RSA are highly sophisticated, we have found it possible to teach this material without prerequisites to a first-year audience. Certainly not many rigorous mathematical proofs are given, but the students do leave the course with an honest working knowledge of the algebra of $\mathbb{Z}_n$ and the applications to cryptography. The structure of the course and the ordering of the topics has been carefully designed to make the development of modular arithmetic a natural evolution of ideas and not something pulled out of thin air. Bearing in mind the goals of the QEP, we have taken the course topics and designed assignments around them to develop the students' skills in research, writing, and speaking. The topics in the course naturally lend themselves to group discussions and formal presentations ("here's how we broke this cipher..."), while the historical aspects of cryptology provide a wide range of resources and opportunities for research and writing projects.

## 1.3 Course Structure

There are two over-arching components of the course structure: the cryptology techniques and the QEP content. The first is of course particular to our FSEM, but the enhancements required by the QEP are universal to all FSEMs regardless of discipline. This presents a unique set of challenges, especially for mathematics faculty who may not be intimately acquainted with communication theory, writing styles in other disciplines, or research tools outside mathematics.

At a base level, the course is delivered in an inquiry-based learning approach. Roughly, a typical content unit might progress in the following stages:

- Introduction to a new cipher; experimenting in using it
- Development of the mathematics behind the cipher; proving that it "works"
- Exploring vulnerabilities in the encryption; developing the cryptanalysis
- Completing a project utilizing and extending these concepts.

The second step involving the mathematics is often the hardest and demands the greatest amount of time and guidance. The final step is where the QEP components are integrated into the course material.

### 1.3.1 Cryptographic Content

In our course we examine a handful of encryption systems in content units, all in a very intentional order. The ordering of the topics is one of the keys to the success of the course: this makes the development of modular arithmetic seem natural and understandable. The "correct" ordering also gives a controlled increase in sophistication along the way, both in the mathematics and the resulting cryptanalysis. In this way, no giant leaps are made. Currently, most instructors teach the following topics in the given order, where we have indicated the mathematical or cryptologic "point" of each topic:

- **The Caesar shift cipher.** Encryption by shifting the alphabet by a fixed amount, represented by maps of the form $f(x) = x + a$ mod 26. Naturally motivates addition modulo 26. Cryptanalysis by single-letter frequency of the most common character.
- **Affine ciphers.** Encryption by a substitution cipher of the form $f(x) = mx + a$ mod 26. Introduces the multiplicative aspects of modular arithmetic, including invertibility issues. Cryptanalysis by single-letter frequency of the two most common characters. Requires solving a $2 \times 2$ linear system modulo 26.
- **The Vigenère cipher.** Our first block cipher. Acts by a series of disjoint shifts. The cipher typically destroys letter frequencies, demanding a new approach to cryptanalysis. Requires a deep statistical analysis of the text and interpretation of some statistics (Kasiski test, index of coincidence, Friedman's formula).
- **The Hill cipher.** Our second block cipher. Acts by matrix-vector multiplication. Motivates matrix algebra and determinants. Cryptanalysis by two-letter ("digraph") frequencies.
- **Diffie-Hellman key exchange.** Our first public-key protocol, entering the modern era. Motivates the need for primitive elements in a finite field. Our first example with no feasible attack.
- **RSA.** Our first and only public-key encryption scheme. Motivates greatest common divisors and Euler's $\varphi$ function. Explains the importance of the search for large primes. Applications to digital signatures and authentication. No feasible attacks.

In any given unit, students discover the relevant mathematics and cryptography within guided group-work assignments. Class and small-group discussions pinpoint the major discoveries and iron out the difficulties; these discussions reflect the speaking component of the QEP goals. Units on the first four encryption methods end with a cryptanalysis project, where students get to apply what they've learned to break some codes. This is by far the students' favorite part of the course, as it allows them to use their creativity in a math class (of all places). An example of such an assignment is given in Appendix 1.4.

In these code-breaking assignments, each working group receives a segment of "stolen" ciphertext through Canvas, our course management platform. Usually the ciphertexts are designed to have some added layer of difficulty not discussed in class, so that some honest ingenuity (and perserverance) is required of the students. In making such assignments, it is critical that the students have access to user-friendly software with some cryptographic capabilities. This may present a challenge: the lack of the right piece of technology can force some of these topics off-limits in a course like this. We have been lucky enough to discover ECrypt, an intuitive and easy-to-use Java applet written by Tim McDevitt at Elizabethtown College.[1] Though no longer in development, we have been using this as our sole cryptanalysis tool with few issues since the course was first delivered in 2009.

### 1.3.2 QEP Content

The QEP components are woven throughout the entire course, so that they do not seem isolated or separated from the mathematical content. In order to equip faculty from all disciplines with resources in teaching these components, the office of the director of the QEP provides online learning modules on research, writing, and speaking skills. Each module is embedded in every FSEM instructor's Canvas shell, making the modules easy to use and creating consistency across sections regardless of discipline. Currently there are 12 modules, four in each area; a complete list of all online learning modules is given in Appendix 1.4. Modules end with a short content quiz, the scores being accessible to the QEP office as well as the instructor, giving instant access to data for assessment of the QEP goals.

We have found that the design of the *Cryptology* FSEM lends itself to a natural inclusion of the QEP online learning modules. Most assignments have some relation to at least one of the modules, so we assign the relevant module several days in advance. This way we can discuss something concrete about these skills as we launch the assignment. For instance, getting students used to speaking up in front of everyone is a challenge and a source of anxiety among the students. Before we have our first group discussion of the semester, students complete the module on communication

[1] Downloadable from `www2.etown.edu/ECrypt/ECrypt.htm`.

apprehension. While they are preparing their first formal presentation on their cryptanalysis of an assigned cipher, they will simultaneously complete a module on the effective use of visual aids in presentations. As we are nearing the research paper assignment (outlined in Appendix 1.4), students participate in modules on finding a good research topic and searching for and vetting good, reliable resources. Throughout, we use the context of these assignments as a reason to introduce students to the various support centers on campus—our research library staff, the Writing Center, the Speaking Center—thereby connecting our first-semester students with these valuable resources aimed at student success and skills development in these areas.

## 1.4 Reflections

The FSEM *Cryptology* has been taught almost every year since its development in 2009, sometimes running two sections in a given semester. At 15 students per section, we have reached close to 200 students in this small seminar. Although hard to measure exactly, it is clear to us that most of our students in the seminar actually did not become mathematics majors; we know of only a handful of that did. While we certainly love the idea of the course attracting majors, there is also satisfaction that comes from so many non-majors being in the course. These non-majors, many of whom obtain their degrees in other liberal arts disciplines, leave our course equipped with a better understanding of the utility of mathematics. Many non-majors end their mathematics studies with algebra, trigonometry, or precalculus, areas that sometimes leave students scratching theirs heads and asking "why did we study this?" Students from our FSEM understand not just basic mathematics behind a modern application, they learn a completely new area of mathematics and develop an increased level of comfort in communicating quantitative information, something that is severely lacking in our society today. The communication aspects of the course—both written and oral—are essential to its success.

As mentioned above, the course would not work without the right kind of software. ECrypt has worked well for us, but not without some bugs. For one, it cannot correctly invert all invertible matrices in the execution of the Hill cipher, and sometimes a round of encryption/decryption does not reveal the original plaintext as it should. This seems to affect only the Hill cipher and not all key choices, so the instructor must choose wisely in writing problems. As it is no longer developed, there is the constant concern that at some point ECrypt will simply not work anymore, voiding out all of our assignments. To be proactive, we should work on writing Mathematica routines to achieve the same functionality, but this requires a skill-set that current instructors may not have.

We have also found that there is no suitable textbook for such a course. Most textbooks have significant prerequisites, assuming something on the order of an introductory discrete mathematics or proof-writing/foundations course. The one possible exception is *Cryptology* by Beutelspacher [1], which meets the goal of assuming no advanced mathematics. It is fairly easy to read, but the content does not exactly align with what we currently teach in the course. We've used this text in the course before but have gone text-less the last few years, using this as a reason to amplify the IBL-approach to the seminar.

Modulo these two small issues, at this point in its evolution the course runs very smoothly. It would admittedly be difficult to integrate all of the QEP skills into the class without centralized administrative support and coordination. The founding and operation of our QEP office required significant support from the university, financial and otherwise (as it was tied to successful accreditation this was not a hard case to make). There must be someone dedicated to rallying, supporting, and coordinating the faculty, department chairs, student support centers, and administrators to make all the pieces come together coherently, not to mention creating all of the online modules. It is a campus-wide effort.

Because the QEP was tied to accreditation, there is a vast amount of data assessing the goals and desired outcomes of the QEP and the FSEM learning goals. In addition to the QEP module quiz scores, student presentations are videotaped and sample research papers are collected across all sections and are then evaluated relative to the learning outcomes. While all of this data preliminarily shows a positive effect (including on the rate of retention), we cannot parse out the effect in our particular FSEM. The FSEM program as a whole is successful, and our section is one piece of that.

In the early years of offering this FSEM, we often struggled with teaching the parts of the course that may be foreign to a mathematician whose last non-math writing course was in college 20 years before. Certainly we know how to write mathematics well, but that is not the sort of writing instruction that a first-year student needs. We certainly know what makes a good and reliable journal in mathematics, but we do not necessarily know how to recognize contemporary and well-respected sources in, say, historiography. The implementation of the QEP essentially addressed all these issues. The online modules were prepared with input and content from experts in writing and speaking, but formulated in a way that a first-year student could understand. The engaging content of the modules supplied instructors with talking points for the next class meeting. We found that the QEP modules gave just as much guidance to the instructors as it did the students.

In all, we have found the experience of introducing first-year students to the mathematics of cryptology a rewarding pursuit. We consider the topics and approaches discussed in our cryptology FSEM as worthy of serious study in their own right, but they are also a good source of material to develop our students' skills in research, writing, and speaking. If we want them to learn how to research/write/speak well, they should have something interesting to research/write/speak about. That we can do this in an area of mathematics that is contemporary, relevant, and completely new to them is a compelling reason to offer a cryptology seminar for first-year students.

## 1.5 Bibliography

[1] Albrecht Beutelspacher, *Cryptology*, MAA Spectrum, Washington, DC, 1994.

## Appendix

### The QEP Modules

The university's QEP office produced 12 online learning modules to develop students' skills in college-level research, writing, and speaking. Each module contains reading material, a video (many that are original productions by the QEP office), a practice quiz, and a final skills quiz used for assessment of the learning outcomes. Below we list the topics of the modules, grouped by skill-set.

- **Writing Skills**
  - *The Writing Process*: created by our Writing Center, this module focuses on the general writing process and outlines steps to follow for successful writing.
  - *Introductory and Concluding Paragraphs*: introduces students to the purpose of introductory and concluding paragraphs and gives students tips for writing each.
  - *Punctuation: Commas and Semicolons*: this module targets students who struggle with comma and semicolon usage, addressing the bulk of common mistakes made with these punctuation marks.
  - *The Editing and Revision Process*: teaches students about the process of editing and revising a paper and where they should focus their attention throughout each process.
- **Speaking Skills**
  - *Communication Apprehension*: outlines strategies for dealing with communication apprehension and helps students prepare for discussions and presentations.
  - *Effective Visual Aids*: helps students enhance their presentations by explaining the purpose of visual aids and outlining effective design and use of these aids, showing negative examples for emphasis.
  - *Class Discussions*: introduces students to students' and professors' perceptions of class discussion, giving students a guide to help them become successful and engaged participants.
  - *Communication Theory*: introduces students to basic ideas of communication theory, including miscommunication and modes of persuasion.
- **Research Skills**
  - *Finding a Research Topic*: walks students through the process of finding a topic for their research papers.
  - *What to Type in Search Boxes*: teaches students how to effectively search databases for peer-reviewed academic sources.

  - *The CRAAP Test*: helps teach students how to differentiate between appropriate and inappropriate sources for their research by assessing five categories: currency, relevance, authority, accuracy, and purpose.
  - *Deconstructing Citations*: teaches students how to trace back the information found in citations to find sources for their own research.

## A Group Project

This assignment is typically the first group project assigned to the class. Since this is early in the semester, they know only two basic ciphers: affine and Vigenère encryption. They should also understand the essential advantages, deficiencies, and vulnerabilities of each. This assignment assesses their understanding of the standard cryptanalysis of these ciphers, including applying basic statistics to determine which of these two ciphers was used for encryption of the "stolen" ciphertext. Students complete an online learning module on the effective use of visual aids, the assignment culminating with a group presentation on their methodology for breaking their cipher.

GROUP PROJECT #1

**Parameters:** This assignment is to be done in your working teams. For this assignment, you may *only* use ECrypt, your Vigenère square, materials from our Canvas site, and each other; you may not cheat by searching the web for automated code-breaking software. Two parts of the project make up 10% of your semester grade, scored out of 25 points. The middle phase counts under your Modules grade.

This project will be completed in three phases. As a baseline assumption, everyone in the group receives the same grade on the group-components (Phases 1 and 3). Upon examining your group-evaluation sheets, "problematic contribution" will result in an individual grade deduction after a conference in my office. All three parts of the project must be completed in order to receive a non-zero grade.

PHASE 1: BREAKING A CIPHER UNDER TIME CONSTRAINT (10 points)

- This will take place on Monday 9/26.
- Elect one member of your group as scribe. This person is in charge of taking detailed notes on your tactics of cryptanalysis (both the successes and the failures). Because we are operating under a time constraint, it is certainly fine if this log sheet is handwritten.
- Each group will receive a unique piece of ciphertext via Canvas. You may assume that the ciphertext was encrypted using an affine or Vigenère cipher. If the latter was used, the keyword could be found in a standard English dictionary.
- Your mission is to recover the plaintext by the end of class.
- To receive full credit for this portion of the project, your group must do two things:
  1. Email me the original plaintext as soon as you have recovered it.
  2. Have the scribe hand in the log sheet of your cryptanalysis at the end of class.

PHASE 2: THE VISUAL AIDS MODULE

- This is to be completed before class on Wednesday 9/28, so that we may discuss it then.
- Log on to Canvas and go to the "Modules" tab. You should see the "Effective Visual Aids" module with seven subparts underneath.
- Start at the obvious place: the link that says "Start Here." Read the instructions.
- Finally, take the associated quiz. Your quiz grade will automatically be sent to me.

PHASE 3: GROUP PRESENTATION ON YOUR CRYPTANALYSIS (15 points)

- This will take place on Monday 10/3. This may take more than one day, so you may have to wait until Wednesday to present. Be prepared to go on Monday.
- Prepare a presentation of length *at most* 15 minutes describing your group's attack on the given ciphertext. Some tips follow.

- I cannot imagine how you could do this without using your preferred presentation software package (PowerPoint or not). For instance, including screen captures of relevant ECrypt statistics would greatly help us to understand what you did, and more importantly, *why* you did it. In Windows, using the *snipping tool* is the easiest way to do this.
- Your presentation should be professional, informative, and easy to follow. Learn to go slowly, and practice this.
- I want to *learn something* from your presentation. Your colleagues do, too.
- There is a fine line between making a presentation interesting and eye-grabbing versus making it cheesy. Ponder this as you prepare.
- We will dedicate class time to the preparation of your presentation, where you are free to consult me and your peers for advice.
- Practice.
- All of the things I said above—and much more—were also in the presentation we had from the Speaking Center consultants. Make use of the Speaking Center!

## The Research Project

This is the culminating assignment for the term, one in which students must pull together all the research, writing, and speaking skills they've learned. It is also the most writing-intensive assignment of the semester. Students are assigned a research paper on the topic of their choice in cryptology, ending the semester with a summary presentation to the class. The students receive plenty of support throughout the process, including specialized sessions with a university research librarian, together with online learning modules on finding appropriate references and selecting a good, focused topic.

RESEARCH PROJECT

**Parameters:** This assignment is to be completed individually and is worth 15% of your semester grade. Your papers will be graded according to the following rubric, scored out of 50 possible points.

| | |
|---|---|
| Quality of claims/thesis | 10 points |
| Quality of arguments | 15 points |
| Quality of writing | 15 points |
| Oral presentation | 10 points |

Papers will be due Friday, December 9th in electronic form. Please email a PDF version of your finished product to me by 9:00 a.m. on the due date, saved under the file name

`LastName_final.pdf`

On the paper itself:

- The hardest parts are (a) settling on a topic and (b) finding legitimate, useful sources. We have two online modules to learn how to do this, along with a scheduled visit with the FSEM librarian. All of this should help greatly.
- Make sure that you actually have something interesting to say about your topic. You can't just regurgitate facts from your readings. You need to assert something (your "thesis") and then back it up with evidence from your readings.
- Papers should be double-spaced with 1-inch margins. There is no page requirement, though I can't see how anything less than four pages could get the job done.
- Regarding references, I'm not very particular about the style of your bibliography. When in doubt, MLA is a fine choice. The standard in mathematics is set by the American Mathematical Society (AMS), and is commonly known as "AMS plain style" but I don't care if you use this or not.

The process of writing your research paper and presenting your findings will occur in several helpful stages. They are:

- Students complete the *CRAAP Test* online learning module by 9:00 a.m., Monday, October 24th.

- The class meets the FSEM research librarian in Simpson 225B on Monday the 24th for a session on how to find good sources for a research paper. If at all possible, bring a laptop.
- Students complete the *Finding a Research Topic* module by 9:00 a.m., Friday, October 28th.
- At this point, you should have enough tools at your disposal to (a) find a research topic that interests you and (b) find at least two good sources on that subject. On Wednesday, November 2nd each student will bring to class a hard copy of their research proposal. We will trade proposals and perform the CRAAP test on your colleagues' references to see if they qualify as legitimate. More details on the proposal stage are below.
- Students will complete a preliminary draft of their research paper by Wednesday, November 16th and each will bring a hard copy to class. We will then have an in-house editing and revision workshop, given by the Writing Center staff.
- Students will electronically submit their final papers by the deadline of Friday, December 9th. There is no final exam in this course.
- Each student will prepare a 10-minute maximum presentation on their research topic and findings. We will use all three days of the last week of class to deliver these presentations, having five presentations per day. The presentation order will be determined by a random drawing held the week before. Volunteering for a Monday presentation slot is allowed.

On the proposal process:

- The first step is to settle on a topic. Our FSEM research librarian and the online modules will show you how to do this.
- Next, find *at least two* decent sources on this topic. Make sure they pass the CRAAP test.
- After browsing your sources and learning something, formulate an intelligent question or claim about your subject. Either is fine. At this early stage, a question may be all that you can form. (Hopefully, in the course of your research, you'll be able to answer it.)
- I will provide a common form requesting your topic, your research question, and your sources. Fill this out and bring it class on Wednesday, November 2nd. We will trade them and a colleague and I will check your sources.

A final helpful hint...

If you visit the website of the National Cryptologic Museum you will find this link

`www.nsa.gov/about/cryptologic-heritage/museum/exhibits/`

to a list of their exhibits. You could use this as a starting point to think about a topic. The bonus is, of course, if you find an interesting topic here *you can actually visit the museum and see real artifacts related to your topic* on our field trip.

# 2

# *Secrecy and Security*

Livia Hummel

**Abstract** *Secrecy and Security* is a first-year seminar course exploring the history and mathematics of encryption with an eye towards modern applications and information security issues. A specifications grading approach is used to encourage student engagement and exploration while accommodating the wide differences in mathematical abilities of students. This course also utilizes a Designated Tutors program where two writing lab tutors are assigned to the course and work closely with the students and instructor on the research paper and poster components.

## 2.1 Background and Context

The University of Indianapolis is a liberal arts institution with about 4,100 undergraduates. The first-year seminar (FYS) is part of the university-wide general education curriculum and courses feature a variety of disciplinary topics influenced by faculty interests. Every FYS is shaped around the following learning goals [5, p. 7]:

- using a sequence of readings to develop the intellectual focus of the seminar and provide material for discussion.
- providing instructional support for research by asking students to engage in some form of structured investigation beyond the common sequence of readings.
- providing instructional support for writing, using an array of different writing assignments. (In particular, students should write regularly, though not all writing need be graded.)
- providing structured occasions for students to take an active part in shaping discussion.

Each course achieves these learning goals by engaging students in disciplinary or interdisciplinary issues and cutting-edge topics. Faculty are encouraged to choose topics that are both engaging and interesting to students, as well as to the faculty member teaching the course. The only prerequisite is that students are prepared to enter freshman composition the semester they enroll in the FYS. Students can take an FYS either semester; however a majority of the courses are offered in the fall semester. FYS courses are capped at 20 students and the semester is 15 weeks (excluding finals week). *Secrecy and Security* is a three-credit hour course that meets for three 50-minute sessions each week.

*Secrecy and Security* is one of two FYS courses offered by the Department of Mathematical Sciences. Both courses in the department require a math placement score indicating a student is ready to enroll in a course satisfying the mathematics competency general education requirement. In order to add this math competency requirement, a request for an exception was made to the General Education Core Committee during the course approval process. FYS courses in our department and other natural sciences departments were granted this exception when it was demonstrated as necessary to access introductory material for the course. Unlike every other FYS course offered by other departments, the courses in the Department of Mathematical Sciences do not satisfy the requirements for another area in the general education core, as the department is uncomfortable with either of these courses being a terminal mathematics course for a student based on learning goals and scope of covered content. Despite not satisfying a second general education requirement, these courses have been fully enrolled. Strong enrollment is certainly influenced by these courses thus far only being offered in the fall semester when the number of students exceed the number of open seats, combined with strong encouragement during summer registration to enroll in an available fall semester FYS. The two courses are now offered on an alternating schedule as course loads permit; *Secrecy and Security* will be offered for the first time during a spring semester in 2020.

## 2.2 Mathematical Theme

*Secrecy and Security* explores the history and mathematics of encryption with an eye towards modern applications and information security issues. The challenge and reward of this course has been weaving together historical context, mathematics, and discussions of relevant current events into a meaningful narrative applicable to students' daily lives. Unlike a standard mathematics course, *Secrecy and Security* allows me to engage students in a wide variety of discussions by having students actively seek out examples of information security issues developing around them.

About half the course is devoted to teaching mathematical content. In the first half of the course, students are exposed to a variety of mathematical concepts necessary to understand classical ciphers. Students learn modular arithmetic in order to study the Caesar shift, affine, and Vigenère ciphers. The affine cipher also requires students to review some number theory notation as well as functions and their inverses. Studying the permutation cipher requires an introduction to permutations. Using $2 \times 2$ matrices, students learn matrix-vector and matrix-matrix multiplication modulo 26 in order to understand the Hill cipher. Some elementary combinatorics is introduced to allow students to understand the complexity of the Enigma machine, and appreciate the difficulty the Allies faced when trying to break this cipher. Towards the end of the semester, binary numbers and the conversion between binary and base 10 numbers are presented to give students additional context as we touch on modern computer-based ciphers. However, we don't go into great mathematical detail about how the modern ciphers work.

## 2.3 Course Structure

*The Mathematics of Encryption: An Elementary Introduction* [1] is the required textbook for the course. This book provides

- a basic overview of the history of cryptography (Chapters 1, 3 and 10)
- an introduction to several classical ciphers and their strengths and weaknesses (Chapters 2, 4, 5)
- an introduction to modern cryptography. (Chapters 6 and beyond, although I only focus on section 6.1 and Chapter 10 to avoid covering more technical content.)

The mathematics needed to understand and work with (or attack) discussed ciphers is introduced within the context of each cipher. As the semester progresses, the class works through the textbook in chronological order. The textbook is supplemented by news articles, journal articles, and other sources uncovered by student research. The typical class day usually consists of either a class discussion over an assigned reading, an interactive lecture, or an activity related to or in support of their investigative project, a final project that includes a paper and a poster presentation.

### 2.3.1 Class Discussions

The first week of class students are asked to look for a news article about cryptography or other information security issues (such as data breaches). Students write summaries of these news items and present their findings to the class as part of their first graded homework assignment. This activity helps students identify the myriad of areas where these issues arise and start thinking about potential topics for their investigative project.

Discussion days can focus on topics in the textbook or a supplemental reading assignment. When we are using the textbook as a basis of our discussion, students are assigned a short reading (5-10 pages) to prepare before class. Normally these readings center on a historical use of cryptography. During class students are asked to share information that interested or surprised them in the reading and raise questions they had about the material. Using a list of prepared questions to help sustain or restart conversation, I often also add historical details or context to what appeared in the reading.

The readings from the textbook are supplemented by current news articles related to cryptography or data security. Each student is required to lead an in-class discussion and can choose a news article, or a historical section of our text, to form the basis of the discussion. Students are required to prepare a set of questions to facilitate conversation. If appropriate to the topic, I encourage students to take a side (for instance, is this an ethical reason to access private data?). I try to also play devil's advocate when students' opinions are fairly uniform.

### 2.3.2 Interactive Lecture

At least half the semester involves lectures related to the mathematics needed to use particular ciphers and how encryption and decryption works for each covered cipher. Once each cipher has been presented (Chapters 2 and 3), the authors introduce techniques malicious third parties use to intercept and decrypt ciphertexts (Chapter 4 and 5). For all sections, I assign students reading in advance; the lecture elaborates upon and allows students to practice the mathematics or ciphers covered in the reading. Throughout the semester I assign seven to nine homework assignments from the textbook exercises.

New mathematical concepts are almost always immediately applied to a cipher. During class we walk through examples demonstrating the mathematics used to encrypt messages. For most ciphers, the decryption process is fairly straightforward, so during the next class period I often start with a ciphertext written on the board, provide students with the decryption key, and ask them to discover the message with their classmates. As we work through the classical ciphers, students are assigned problem sets of about five to six problems to practice mathematical skills and to perform encryption and decryption themselves with the ciphers we have recently studied.

After discussing the ciphers, we turn to discussing ways malicious actors can reverse engineer messages from ciphertexts. I begin each class with a ciphertext; as a class students attempt to decrypt the message using the techniques discussed in the section. Since students take the lead on this activity, decryptions may take two class periods if incorrect assumptions are initially made. While I want students to appreciate the methods used to break ciphers, there are fewer homework exercises assigned during this section. Most attack methods require a frequency analysis of letter occurrence, necessitating large ciphertexts that are more time consuming to decrypt. Some decryptions are only demonstrated in class due to the computational requirements of the process.

### 2.3.3 Investigative Project and Designated Tutor Program

Running alongside daily activities and assignments is an investigative project that includes a paper and a poster presentation and allows students to relate encryption and data security to their field of study, future career aspirations, daily life, or other interests. While I provide a suggested list of topics that expand upon information provided by the textbook, students are allowed to select their own topic. In previous semesters, students have explored topics such as the role of women in the history of cryptography, the history of the Navajo Code Talkers, election security, security of critical infrastructure, government surveillance programs, and instances of espionage. Only once has a student (an advanced mathematics major) explored modern ciphers, which required learning mathematics beyond the scope of the course.

The project is broken into a series of assignments to help keep students on track, as well as to give me an opportunity to periodically check in on their progress. These intermediate assignments include selecting a project topic, an annotated bibliography, preparing a first draft for an in-class peer-review workshop, a final draft of their paper, and a poster presentation. The poster presentation exposes students to the wide variety of topics related to the course theme that we wouldn't otherwise have time to discuss; students are often surprised by the applications discovered by their classmates. Timing of these assignments can be found in the Appendix (pp. 16–17).

The type of paper students write depends on the topic and range from informative to argumentative. I ask all students to use the Mathematical Association of America Journal Reference Guide [7], given the motivating theme of this course. As this citation style is unfamiliar to the students, I build in several opportunities for them to practice citing resources properly and receive feedback on their attempts. For instance, when students present a summary of a news item related to cryptography or information security during the first week of class, I do not initially provide a required citation style. To prepare students for the annotated bibliography, they later revise the citation of this first news item to follow the Reference Guide.

The Designated Tutor program provides an additional layer of support to students through the research and writing process for their investigative project. The Designated Tutor Program assigns two student Writing Lab tutors to an FYS course. These tutors are undergraduates themselves and are writing experts, not course content experts. As the instructor I design and indicate activities where the expertise of a Writing Lab tutor would be particularly helpful and work with the tutors so they understand the requirements of writing assignments. Students are split into groups across the tutors and have three required individualized meetings with their assigned tutor. The required meetings include one regarding their proposal for a project idea, one about their annotated bibliography, and one discussing the second draft of their paper. The tutors also provide me feedback on common issues that may need to be addressed as a class as well as feedback on student progress and readiness for tutor meetings.

In-class activities are also built into the semester to prepare students for their Designated Tutor meetings. Before their first meeting, students participate in a worksheet-guided discussion/brainstorming session of their peer's potential project ideas. Before the second meeting we have an in-class discussion about vetting sources for credibility and authority, as well as the expectations of the annotated bibliography and the reference style. A peer-review workshop leads to the second draft of their paper which they discuss with their tutors. In addition to introducing the purpose of the peer-review workshop to students in advance, I also provide the class with a list of guided reading questions to make their peer-review workshop more productive.

### 2.3.4 Assessment

I use specifications grading in *Secrecy and Security*, as I want student grades to reflect their engagement with the material and their work on their investigative project. Specifications grading attempts to better align course grades with learning outcomes; a student's grade is based on their level of performance in several important mastery areas. (See Linda Nilson's *Specifications Grading: Restoring Rigor, Motivating Students, and Saving Faculty Time* [4] for more details.) One of the challenges of applying this method to the mathematics classroom is creating specific rubrics for each mastery area, which requires a significant amount of course set-up work by the instructor. However for my FYS, this grading system is applied by focusing on five areas that affect a student's grade. See Appendix 2.4.4 for details on these categories.

## 2.4 Reflections

I have taught *Secrecy and Security* four times and the course is constantly evolving, sometimes influenced by news events that occur mid-semester. Since the first iteration of this course, changes have been made to include non-graded writing in a more meaningful way, encourage students to shape and lead in-class discussions, better support students as they complete their investigative projects, and ensure homework and project assessments encourage rather than discourage student learning and curiosity throughout the semester.

### 2.4.1 Regular Writing Opportunities

In the original proposal for the course, I included weekly journal entries to satisfy the FYS learning goal that "students should write regularly" [5]. Each journal entry included writing prompts asking students to find a news article related to the course and summarize it, to reflect on difficulties or items of interest that arose during class the previous week, or to report on current progress of their project.

However, in practice weekly journals did not integrate well into the course. Students would forget to submit them or ignore the prompts. As the semester continued prompts became more open-ended, and I tended to fall behind on reading and grading journal entries. Overall, the entries would slowly became less meaningful writing experiences and turned simply into a task to complete every week.

During the most recent iteration I removed weekly journals and increased writing within homework and intermediate assignments related to the investigative project. Most of these assignments are often graded based upon completion, rather than quality of student writing.

### 2.4.2 Encouraging Student Discussion

Engaging students in class discussions continues to be a challenge. I always hope these discussions will emerge organically, however many of my students don't follow national news on a regular basis. This has led me to create more structured discussions where I assign a particular news item and help instigate the discussion through prepared questions. While this encourages student reflection and participation, it does not provide opportunities for students to lead these discussions

In the most recent iteration, I required each student to lead an in-class discussion of a news item, including preparing a list of questions to guide discussion. This provided the opportunity for students to take ownership of the content and learn how to lead these activities. This assignment needs further modification, as most students wait until the last three weeks of the semester to lead these in-class discussions. Even though it ensures I'm not adding new mathematical content at the end of the semester, it might be helpful to distribute these discussions across the semester.

### 2.4.3 Writing Support for Investigative Project

While the intermediate assignments were in place from the first iteration of this course, the Designated Tutor Program was first included in the second iteration. The Designated Tutors ensure each student receives a certain level and quality of support throughout the investigative project. Working with the tutors has also aided me in improving rubrics, developing in-class activities, modifying intermediate assignments, as well as changing the purpose of certain required tutor meetings.

When this course was offered during the 2016 election year, a student asked if they could discuss election fraud. Although I approved this topic, it became readily apparent that students in general were not closely considering the quality of their sources, nor their implicit bias. I now lead a discussion on identifying bias in and the authority of a particular resource, emphasizing the need to balance conflicting views within their final paper, and acknowledge the potential for bias in the sources supporting their thesis. The annotated bibliography now requires students to address bias and credibility of their listed resources.

The annotated bibliography and second draft meetings have been a regular part of the course since implementation of the Designated Tutor program. However, the third meeting continues to evolve to address student writing needs. Initially students discussed a draft of their final poster with their tutor; the brainstorming meeting replaced this to strengthen the framework of the entire research and writing process. In the next iteration, this third meeting will change again: students will create and discuss an outline for their paper before beginning their first draft. The goal is to give the tutors and myself an opportunity to address structural issues in the paper before a full draft has been written; students have been reluctant to make significant changes to their paper after the second draft meeting.

One drawback to the support provided by the Designated Tutors is that students are less likely to seek assistance from me on their papers. Even though this saves me grading time, it isolates me from the research and writing process. In an attempt to address this, I recently added a required office hour attendance to encourage students to visit me at least once during the writing phase. It would also be ideal to adapt some of the required Designated Tutor meetings to in-class group activities guided by the tutors and myself. This change may make it more clear that I am an active participant in the process. However, I have been unable to make this modification due to tutor schedule conflicts.

### 2.4.4 Supporting Student Learning through Assessment

In the first iteration of the course I graded homework based upon correctness, with an eye for detail. Students were very frustrated with their homework grades. Students would either spend significant amounts of time on the problems, ending sometimes in frustration, or just give up completely and not do their homework. Thus I moved homework assessment away from a points-based system to a Pass/Almost/Try Again assessment. Each homework problem is given either a Pass (correct, except for perhaps some minors errors), Almost (a significant mathematical error was made while taking the correct approach), or Try Again (the problem is entirely incorrect or missing). Each homework assignment receives a Pass if at least 80% of the problems receive a Pass, an Almost if at least 60% of the homework problems receive a Pass, and a Try Again otherwise. The percentages for the Pass and Almost grades are set intentionally low to encourage struggling students to improve on their previous work. Students are given three opportunities to resubmit three homework assignments throughout the semester to receive an improved grade, or submit project components late for full credit.

With this grading scheme students are more likely to ask questions on homework based on where they are in the problem-solving process. Students are also able to explore the material without being penalized for deficiencies in their mathematical background that the FYS is not designed to handle.

Changing homework assessment led me to move towards a specification grade scheme for determining course grades as well (see Appendix 2.4.4). I am still modifying the grading categories used to ensure they reflect the intended learning outcomes for the course. The categories for homework, project components, and Designated Tutor meetings have stayed consistent each semester. In the past I also included categories for attendance/class participation and weekly journal entries; these were replaced recently by leading an in-class discussion and required office hour attendance. Attendance/class participation was removed as I would often forget to take attendance and found class participation difficult to track across the different types of class sessions we had throughout the semester. I'm hoping that having students lead an in-class discussion better encourages attendance and participation, and that requiring an office hour helps support homework and project grades.

While I feel specifications grading lends itself better to assessing the course based on FYS course goals, this method does have its downsides. It takes time to lay the groundwork for why things are graded this way. It is also difficult for a student to scan their grades in the learning management system and know where they stand at any point in the semester. Hence students need to request status updates on their grade as it is a cumbersome process to compute current grades, as well as mid-term and final grades.

## 2.5 Bibliography

[1] Margaret Cozzens and Steven J. Miller, *The Mathematics of Encryption: An Elementary Introduction,* American Mathematical Society, Providence, Rhode Island, 2013.

[2] Denise Kreiger, (2012), "Research Paper Rubric," Instructional Design and Technology Services, SC&I, Rutgers University, Accessed January 14, 2019, `www.dropbox.com/s/6gosl0s0wecoxaa/ResearchPaper_rubric.docx`.

[3] Mathematical Association of America, (n.d.), " Mathematical Association of America Journal Reference Guide," Accessed January 14, 2019, `maa.org/sites/default/files/pdf/pubs/MAA_ReferenceGuide.pdf`.

[4] Linda B. Nilson, *Specifications Grading: Restoring Rigor, Motivating Students, and Saving Faculty Time,* Stylus Publishing, Sterling, Virginia, 2014.

[5] University of Indianapolis General Education Core Committee, (2014), "Gen Ed Core Curriculum Omnibus," Accessed August 14, 2019, `my.uindy.edu`.

## Appendix

### Requirements and Timeline for Investigative Project

*This document is provided to students within the syllabus as a guide to the intermediate assignments building to the final investigative project. The project consists of both a written research paper and a poster presentation.*

Your investigative project will delve into a particular topic related to cryptography and information security not covered in class. These topics can range from the mathematical topics related to certain encryption algorithms, the perception of cryptography in the media, cryptography in art, or a historical event where cryptography was a key player. Your first task will be to explore topics and find one that fits your interests. In the first part of the project, you will be asked to write a paper describing what you've learned after your investigations. The second part of the project involves sharing what you've learned with your classmates, in the form of a poster presentation. Your paper must have a thesis which you will defend or support with your research. Some examples of what a thesis could include would be

- an argument persuading your reader that a particular side in a debate is most correct, most ethical, most practical, etc.
- a stance on a historical situation or context
- an informed opinion regarding your topic of choice supported by research.

Your poster may either have the same thesis, or may be more informational in nature in presenting a part or all of your topic. You may choose to form your poster around a subset of the research in your topic area (as your entire topic is too large to be adequately encompassed in a poster).

Your audience for the paper is your classmates; you may assume your reader has the same content knowledge as you have by virtue of being a student in the class. Your audience for the poster is a more general audience (the poster session will be open to all members of the campus community). Thus, you should be prepared to (either verbally or on your poster, based on space constraints) define any terms or ideas that an attendee might ask you, even if it is something we covered in class.

## Summary of Deadlines and In-Class Activities

- In-Class Activity: Project Topic Brainstorming: Week 2
- Project Ideas Meeting with Designated Tutors: Weeks 3 and 4
- Project Idea Due: end of Week 4
- In-Class Discussion: Vetting Sources and Using the MAA Citation Guidelines: Week 4
- Annotated Bibliography and Source Vetting Meeting with Designated Tutors: Weeks 5 and 6
- Finalized Project Idea and Annotated Bibliography Due: end of Week 6
- Draft 1 Due to Peer-Review Partner: Week 11
- Peer Review Workshop: Week 11
- Draft 2 Meeting with Designated Tutor: Week 11 until Week 14
- Final Draft of Paper Due: Week 14
- Poster Presentation: Final Examination day and time; usually only one hour is used and a different room may be used for space considerations.

**A Note on Citation Format:** For this class you will be using the citation format used by the professional mathematical community (specifically the Mathematical Association of America's Journal Guidelines [7]). A reference guide is provided on the learning management system, and examples will be provided on the learning management system as well. It is important you follow the guides and examples (while it may look like the Chicago Manual of Style method, there are differences, so be careful!). Any questions that you have can be directed to Dr. Hummel or your Designated Tutor.

## Grading

Rubrics for the Project Literature Review, Paper and Poster will be posted on the learning management system. The point breakdown for each component of the project is below.

| | |
|---|---|
| Project Topic Submission | 5 points |
| Annotated Bibliography | 10 points |
| First Draft and Peer Review | 10 points |
| Final Paper | 100 points |
| Poster and Presentation | 50 points |
| Total | 175 points |

## Investigative Project Rubrics

*These are the grading rubrics for the written paper and the poster presentation. These are provided to the students within the syllabus along with the Requirements and Timeline document provided in the Appendix (pp. 16–17).*

### *Secrecy and Security* Paper Rubric

*This rubric is an adaption of the rubric created by Denise Kreiger [2].*

| Criteria | Beginning | Developing | Accomplished | Exemplary | Total |
|---|---|---|---|---|---|
| **Paper Focus: Position (Thesis)** | **0–15 points** Research paper's central purpose or position statement is generally unclear and paper lacks focus overall. | **16–17 points** Research paper's central purpose or position statement is somewhat unclear and needs to be developed further; focal point is not consistently clear. | **18–19 points** Research paper's central purpose or position statement is apparent and is the focal point of the paper for the most part, but may digress from it on occasion. | **20–23 points** Research paper's purpose or position statement is well-developed, readily apparent, and clearly stated. Consistently maintains the focal point throughout the paper. | /23 |
| **Depth and Evidence (Sources)** | **0–15 points** Central purpose or position is not supported with sufficient research sources and/or are generally not relevant, accurate, or reliable and/or sources are generally not integrated well in the paper. | **16–17 points** Central purpose or position is largely supported but with only a few research sources and/or some sources may not be relevant, accurate, and reliable and/or some sources are not integrated well in the paper. | **18–19 points** Central purpose or position is supported with at least five research sources that are mostly relevant, accurate, and reliable. Sources are integrated well in the paper for the most part. | **20–23 points** Central purpose or position is supported in-depth with at least five research sources that are highly relevant, accurate, and reliable and add to the strength of the paper. Sources are skillfully referenced throughout the paper. | /23 |
| **Organization** | **0–15 points** Paper lacks logical organization and impedes readers' comprehension of ideas. Central purpose or position statement is rarely evident from paragraph to paragraph, and/or is missing required components. | **16–17 points** Paper is organized in general, although occasionally ideas from paragraph to paragraph may not make sense to the central purpose or position statement and/or be clear as a whole and/or may be lacking a required component. | **18–19 points** Paper is organized for the most part. Ideas are arranged logically and usually linked clearly from paragraph to paragraph connecting to the central purpose or position statement. Includes required components for the most part. | **20–23 points** Paper is well-organized. Ideas are arranged logically, flow smoothly, with a strong progression of thought from paragraph to paragraph connecting to the central purpose or position statement. Includes all required components. | /23 |
| **Writing Quality & Adherence to Format Guidelines** | **0–15 points** Paper shows a below average/poor writing style using MAA guidelines and appropriate standard English. Frequent errors in spelling, grammar, punctuation, spelling, usage, and/or formatting errors. Language and style poorly suited to intended audience. | **16–17 points** Paper shows an average and/or casual writing style using MAA guidelines and standard English with some errors in spelling, grammar, punctuation, usage, and/or formatting errors. Language and style only sometimes suited to the intended audience. | **18–19 points** Paper shows above average writing style and clarity in writing using MAA guidelines and standard English with few errors in grammar, punctuation, spelling, usage, and/or formatting errors. Language and style appropriate for intended audience. | **20–23 points** Paper is well written and clear using MAA guidelines and standard English characterized by elements of a strong writing style and basically free from grammar, punctuation, spelling, usage, or formatting errors. Language and style appropriate for intended audience. | /23 |

| | | | | | |
|---|---|---|---|---|---|
| **Timeliness and Length of Paper** | **0–3 points** Paper is submitted 2–3 days (49–72 hours) after the deadline and/or is lacking in length. | **4–5 points** Paper is submitted 1–2 days (25–48 hours) after the deadline and/or is somewhat lacking in length. | **6 points** Paper is submitted within 1 day (24 hours) after the deadline; meets the required length (at least ten pages for the body). | **8 points** Paper is submitted by the deadline; meets the required length (at least ten pages for the body). | /8 |
| **TOTAL POINTS (sum of 5 Criteria)** | | | | | /100 |

## *Secrecy and Security* Poster Rubric

| | | |
|---|---|---|
| 1) | Does the poster demonstrate a topic in secrecy and security? | /5 |
| 2) | Does the poster give some background on the topic? | /5 |
| 3) | Does the poster have enough prose, but not too much? | /5 |
| 4) | Does the poster have a good organization that promotes understanding? | /5 |
| 5) | From the poster alone, can someone learn about this topic? | /5 |
| 6) | Does the presenter demonstrate a good understanding of the content of the poster during the poster session? | /5 |
| 7) | Is the poster visually appealing? | /5 |
| 8) | Is the information on the poster correct? | /5 |
| 9) | Did the presenter demonstrate poise and professionalism during the poster session? | /5 |
| 10) | Did the student fill out an evaluation for two other students? | /5 |
| | **Total** | /50 |

## Rubric for Final Course Grade in Syllabus

*This section includes the portion of the class syllabus describing the method used to determine the final course grade.*

Rather than assigning points to each individual assignment and computing your grade based on total points earned, your grade will be based upon your level of accomplishment in various grade categories. (Points may be used to determine your level accomplishment on certain assignments.)

The goals of this grading method are to

- help ensure your final grade better reflects your learning and accomplishments in this class
- move the focus away from point accumulation to learning
- empower you (the student) to take control over what grade you want to earn, work towards that goal, and have a clearer picture as where you stand on that goal

The five categories I will be basing your grade on are as follows:

- Homework and Other Assignments (graded based on level of accomplishment and not points)
- Leading an in-class discussion (graded based on level of accomplishment and not points)
- Investigative Project (graded based on points)
- Meeting with Dr. Hummel at least once throughout the semester (during office hours, or outside of office hours) (graded based on level of accomplishment and not points).
- Meeting with your Designated Tutor (graded based on level of accomplishment and not points).

The minimum requirements for earning each letter grade are as follows. Note, if minimum requirements in a grade are are not met, your grade drops to the next lowest applicable grade.

| **Homework and Other Assignments: Submitted/ Pass Rate** | **Leading In-Class Discussion Grade** | **Grade on Project Components** | **Required Office Hour Attended** | **Number of Designated Tutor Meetings** | **Course Grade** |
|---|---|---|---|---|---|
| 90% sub / 80% Pass | Pass | $\geq 158$ | Yes | 3 | A |
| 4 requirements meet A requirements, 1 requirement meets B+ requirements | | | | | A− |
| 80% sub / 70% Pass | Pass | $\geq 140$ | Yes | 3 | B+ |
| 4 requirements meet B+ requirements, 1 requirement meets C+ requirements | | | | | B |
| 3 requirements meet B+ requirements, 2 requirements meet C+ requirements | | | | | B− |
| 70% sub / 60% Pass | Almost | $\geq 123$ | Yes | 3 | C+ |
| 4 requirements meet C+ requirements, 1 requirement meets D+ requirements | | | | | C |
| 3 requirements meet C+ requirements, 2 requirements meets D+ requirements | | | | | C− |
| 60% sub / 50% Pass | Almost | $\geq 105$ | No | 1 | D+ |
| 4 requirements meet D+ requirements, 1 requirement meets F requirements | | | | | D |
| 3 requirements meet D+ requirements, 2 requirements meet F requirements | | | | | D− |
| If you do not fulfill the requirements for a D− | | | | | F |

**Virtual Tokens:** Life happens. To ease the impact of circumstances on your grade, you will be provided with three virtual tokens. You may "cash in" a token in one of the following ways:

- Re-submit a homework assignment on which you received a Try Again or No Pass. (Original assignment must be attached to the resubmit, along with a signed honor pledge, provided on the learning management system.) All homework resubmits must occur by noon on the last day of classes.
  - The annotated bibliography is eligible for this token usage; other components of the project are not. This submission must happen before the peer review of your first draft.
- Receive permission to turn in a homework assignment late without penalty. This must be redeemed within a week of the original due date.
  - The project ideas, annotated bibliography, and final paper are eligible for this token usage. Other project components are not.

I will have a place in the online Gradebook to indicate the number of remaining tokens you have at any time (including a note regarding when any tokens were cashed in). Unredeemed tokens have no point value.

# Part II

# Gambling, Game Shows, and Game Theory

A first-year seminar grounded in mathematics lends itself quite well to a discussion of basic probability theory, risk, and statistics for an educated citizen. The following seminars include examples from casino games, game shows, two-player matrix games, and voting theory while also focusing on cooperation, defection, and the social and economic impact of these choices.

# 3

# The Mathematics and Impacts of Gambling

Lesley W. Wiglesworth

**Abstract** This article discusses the course *The Mathematics and Impacts of Gambling*, a First-Year Seminar course taught at Centre College in which students explore the probability and odds of casino games as well as the social and economic impact of the gaming industry. Effective course assignments and experiences are detailed, including their objectives and assessments. Adaptations and reflections on the various implementations of the course developments are also discussed.

## 3.1 Background and Context

Centre College is a selective liberal arts college of approximately 1,500 students located in rural Kentucky. Centre attracts highly motivated students from around the country, and increasingly from around the world, although a majority of students come from the South and Midwest regions. The college consistently ranks in the top 50 liberal arts colleges in the *US News and World Report* rankings. The General Education curriculum houses the First-Year Seminar courses, all of which are taught during the CentreTerm, an intensive three-week term in January in which students only take one course. All First-Year Seminar courses have three goals: 1) to provide a small-group learning situation that will engage students and faculty in an intensive intellectual experience; 2) to introduce students to a discipline's basic concepts, modes of thought, or procedures in an innovative way; and 3) to foster basic educational skills such as how to read critically, think logically, and communicate effectively. In order to meet these goals, the enrollment in FYS courses is capped at 15 students.

Professors choose any theme or topic they desire to center the course around, but the course cannot have any prerequisites. In addition, the course must address the following competencies: 1) awareness of social context; 2) written communication; 3) oral communication; 4) critical inquiry; and 5) creative exploration. The courses offered vary greatly. Some recent courses include *Mindfulness in the Digital Age* (taught by a sociology professor), *Rainmaking: The Study of and Preparation for Leadership* (taught by the College's president), and *History of Food in America* (taught by a history professor). Recent courses with mathematical themes include *Statistics in Sports*, *Renewable Energy*, and *Mathematical Impossibilities*.

## 3.2 Mathematical Theme

Examples of glorified gambling fill popular culture and film, causing a false perception of the gaming industry in the students' minds. Movies such as *21* [1], *Casino Royale* [2], and the *Ocean's Eleven* franchise [9] all depict well-dressed adults winning big hands at the tables, making the fantasy of "beating the house" seem attainable. The course *The Mathematics and Impact of Gambling* studies areas of probability and discrete mathematics, using them to examine lottery games, keno, and several casino table games such as roulette, poker, blackjack, and craps. After exploring the games and understanding both the mathematical and casino odds, we examine the social and economic impacts of the gaming industry, as well as ethical controversies centered on gambling. Beginning the course with the mathematics of casino table games and lotteries allows the students to immediately understand how hard it is to "beat the house."

## 3.3 Course Structure

The course meets for 16 three-hour class periods during the January term. In addition to regular classroom learning and experiences, the course provides the students with several special learning opportunities such as field trips and guest speakers. A tentative schedule of the course topics is provided in Appendix 3.4.

Since no FYS courses have prerequisites, knowledge of computing probability or odds cannot be assumed. On the first day, we begin with the basics: defining sample space, events, and discussing different counting principles. Then the students move to computing basic probabilities, answering questions such as what is the probability of drawing a king or queen from a standard deck of cards, the probability of drawing a king or a spade, the probability of drawing two consecutive aces from a deck, etc. These questions gradually build to a discussion of Antoine Gombaud's (the Chevalier de Méré) unprofitable game [7] and fairness in the game of Penny Ante [5]. After working through these computations and examples, students are better prepared to discuss the casino games. We begin with poker where the students not only play the game but also compute the probabilities of certain five-card hands in poker, and probabilities of obtaining certain hands on the flop, turn, and river in Texas Hold'em. We then continue with other casino games such as roulette, craps, and blackjack, concluding the mathematics portion of the course with lottery games and Keno.

Once the students have completed their study of the casino games, we move to exploring the impacts of the gaming industry. Because of Centre College's location in Kentucky, the students examine the horse racing industry and its impact on the state. The horse racing industry has a large economic impact on Kentucky, which provides a segue into the economic impact casinos have on the cities and states in which they are located. Though many view the economic impact as largely positive, there are many impacts that are negative. Therefore, we begin discussing the negative impacts, focusing largely on poverty and addiction. To better understand the social impacts of the gaming industry, the students take part in several special learning opportunities. These are detailed below:

- **Game Days**: In order to truly understand casino games, students must play them. On designated Game Days, students decide which casino games they would like to play though most prefer craps, blackjack, and poker.

- **Volunteering at the Local Bingo Hall**: Students volunteer two nights at Title Town Bingo, the local bingo hall, for five hours each night. Since the students are typically under 21 years old, they cannot enter a casino. Volunteering at the bingo hall enables them to watch people gamble, make betting mistakes, and dream of winning. The students rotate between selling concessions and selling pull-tabs. Pull-tabs are similar to an instant lottery ticket, and winning tickets can be turned in for a monetary prize. In their reflection papers after volunteering, every student commented on the pull-tabs and reflected on the odds of winning. Some even attempted to calculate the expected value of the pull-tabs. For example, one student wrote, "I cannot imagine how poor the odds are on those pull-tabs, so I felt guilty allowing the players to throw away their money so easily. The games were designed for them to lose, but they would play anyways in the hopes that it would give them a win. Selling the pull-tabs made me feel responsible that they were losing so much money." These two nights allowed students, many from more urban areas, to interact with the rural community surrounding the college. Another student commented, "The gambling night also opened me to a lot of the suffering that many in rural areas feel."

- **Book**: The class reads *Confessions of a Slot Machine Queen: A Memoir* by Dr. Sandra Adell [1]. In this book, Dr. Adell details her experiences growing up in Detroit as an uneducated African-American teenage mother who ran away from home. After overcoming the struggles of her youth, Dr. Adell earned a PhD, and became a university professor. She then risked everything playing the slots in casinos. This book allows students to realize that gambling is not usually glamorous and helps them understand the power of gambling addiction. It also allows students to see that gambling addiction can happen to people they respect, or even people they themselves could be.

- **Guest Speakers**: Throughout the term, I arrange for several guest speakers to meet with students either in person or through video conferences. In past years, these have included Sandra Adell, author of *Confessions of a Slot Machine Queen*; Allen Godfrey, Executive Director of the Mississippi Gaming Commission, the department responsible for regulating the gaming industry in the State of Mississippi; Dan Davis, President and founder of Southern Research Group (a full-service marketing research firm) that has served as a global consultant for the gaming industry; and Daniel Elliot, Representative for Kentucky House District 54. In addition to engaging with the speakers, the students also write reflections on the content of the presentations.

- **Field Trips**: Because of its relatively close proximity to Centre College, one of the class days is spent traveling to Churchill Downs racetrack (home of the Kentucky Derby). There students engage with professional horse racing handicappers and employees of the track to learn about the odds of horse racing as well as the business side of the track. After learning about the racing industry, different types of horse racing bets, and how to compute the cost of the exotic bets (exacta, trifecta, boxes, wheels, pick-3s, etc.), students can try their own luck at the race track. While no horses are running at Churchill Downs in January, the track allows simulcast wagering. This field trip provides students with an opportunity to place real bets if they choose, and observe other people gambling at the race track.

  The class also travels to the Kentucky Lottery headquarters. There, the students take part in a discussion with one of the Senior Vice Presidents to learn about the security of scratch-off lottery tickets and live drawings. They also learn about past lottery scandals and the odds of winning the Kentucky games such as Keno, Powerball, Mega Millions, and Lucky for Life. In addition, they watch a live lottery drawing, helping them to appreciate all of the time and security that take place behind the scene to ensure fairness.

A variety of assignments and assessments are used throughout the course. During the first portion of the course that focuses on probability, students complete assigned nightly exercises, take daily quizzes to assess their understanding of material, and end the unit on probability and games with an exam to assess their comprehension of the mathematics. As the focus shifts to the impacts of the gaming industry, students give a 20-minute presentation, write a paper, and write numerous responses and reflections to the course material. More detailed descriptions of class assignments are below:

- **Homework**: Students have daily assignments to complete before each class. Moreover, since a good portion of this class is hands-on, students also complete and turn in several worksheets for credit. For observed probability in the mathematics of gambling portion of the course, these typically give students a specific way to keep track of the events they observe. Calculations using their data from these worksheets are a form of homework. Typically, these will accompany or follow the activities outlined in the course schedule (see Appendix 3.4).

- **Reflections**: Throughout the term, students submit reflection papers on various topics related to the course material. These include assigned reflections on mathematical questions, articles, documentaries, responses to guest speakers, or insights the class generated. For the first reflection, students are required to read the article "Poker Science: Math, Game Theory Can Help A Gambler's Strategy, Study Says" [6] and make an argument as to whether they believe poker is really gambling. The prompt requires students to reflect on both their mathematical knowledge and class experiences with the game, relate them to the article, and then draw a conclusion.

- **Chip Count**: At the beginning of the first class period, students are given $500 in chips to use for roulette, craps, poker, blackjack, and occasional side bets. Students earn a grade based on their performance with their chips and their betting throughout the term. Tying the students' chips to their grade has caused students to think more about betting strategies and behaviors. It also better simulates the behavior of betting with something important to them, as if they were using their own money in a casino.

- **Presentations**: The final two class periods are reserved for student presentations on a choice of assigned topics. There are many topics under the category of gambling that are not covered as part of the standard course material, but these other topics are no less important. Students give a 20-minute presentation to the class on their topic. Some topics are: sports gambling, pai gow poker, other variations in poker, online gambling, variations in blackjack, side bets in table games, cheating the casino, and the psychology of casinos. During the presentations on topics that involve betting, the presenter is expected to not just teach the class how to play the game but to also calculate the odds and expected values of the bets. Because one of the competencies of our First-Year Seminar courses is oral communication, students are given immense feedback on their presentations. They turn in an outline of their presentation and must rehearse their presentation with an assigned peer. This peer provides feedback, both positive and negative. Then both the student and peer meet with me to discuss the presentations. Finally, during the actual presentation, all students evaluate the presentations using the peer evaluation form in Appendix 3.4. In addition to returning the grade on the presentation, I summarize the comments from the peer evaluation forms as additional feedback.

- **Formal Paper**: Students write an argumentative research paper on whether they think Kentucky should continue to have legalized gambling and whether Kentucky should expand gaming. In addition to using reputable sources, many students that argue against the expansion of gaming in Kentucky using the mathematics learned in the course. They cite the difference in the casino odds and true odds of the table games. Appendix 3.4 contains the paper prompt distributed to the students for this assignment. To better meet the competency of written communication, students have two peer conferences on their paper prior to turning in the final draft.

## 3.4 Reflections

Over the past eight years, I have taught four different iterations of this course. Every iteration has been different, and each time, changes are made to the course assignments and experiences to improve perceived weaknesses in the previous iteration. These iterations have led to a fairly stable version of the course which I look forward to teaching again soon with little change. The rest of this section is devoted to how the course evolved into its current form and what seems to work extremely well.

The first time I was asked to teach a First-Year Seminar, I searched for a topic that would be rather atypical in a college math curriculum. I toyed with teaching cryptography and coding or a variation of graph theory but soon realized that these courses would not meet the criteria of Centre College's First-Year Seminar program. Each would have been more of a math course than an FYS course. I then explored subjects for which mathematics could be incorporated, such as mathematics in music, the movies, architecture, and gambling, ultimately settling on gambling.

During my second year at Centre College, I taught my first FYS course. The course was called *Mathematics in Games and Gambling*. I considered two textbooks, *The Mathematics of Games and Gambling* by Packel [8] and *Mathematics in Games, Sports, and Gambling* by Gould [3], ultimately settling on the former. As a second-year faculty member, I still lacked some confidence in front of a classroom and was hesitant to simply use notes and external readings, thus using a text was very helpful. The text provided me with general organization of the material that was invaluable, particularly for my first time teaching the course.

In addition to learning about the mathematics involved in casino games and the lottery, several days of this first iteration were spent discussing the probabilities of various board games and the mathematics behind card tricks. The course was essentially a lower-level probability course, despite trying to avoid the course being a "math" course. The feedback I received from the College's administration was that the course was too math heavy and did not meet the FYS goal of *awareness of social context*. I agree with this assessment, particularly since the assignments mostly consisted of nightly homework, two tests, and a final exam, all of which focused on evaluating the students on their understanding of the mathematics of the games.

To address social issues better in the second iteration, the course was given a new course number, renamed *Understanding the Games We Play*, and focused exclusively on gambling. Students were also introduced to addiction and to the economics related to gambling. It is this iteration where students were first required to read Dr. Sandra Adell's memoir and engage with her through a video conference. The exclusion of board games and card tricks and the inclusion of addiction and economics made the course much stronger. The course shifted from a focus on mathematics and probability to the various aspects of gaming.

Though the second iteration of the course was more unified and better met the college's goals, two main issues still needed to be addressed. First, as students were able to perceive, I was not nearly as comfortable teaching the social portions of the course as I was the mathematics. Second, the course name was misleading since it lacked the word mathematics. Because First-Year Seminar courses are not allowed to have prerequisites, the lack of any mention of mathematics in the title was deceptive to students. As a result, many students without an interest in math registered for the course (despite the course description). Our three-week term is intense due in part to its compressed nature, and there are several course days where students are spending a majority of the three-hour time block discussing probabilities. If a student lacks an interest in mathematics, then the course can be painful at times.

To overcome these difficulties, I spent a lot of time reflecting on my teaching of this course and the students' behaviors, leading to the following observations and resolutions:

- **I was not nearly as comfortable with the social context portion of the course as the mathematics portion of the course and this needed to change.** I realized that the source of this discomfort was leading open-ended discussions in class. At the time, my typical classroom was more structured and offered little open-ended discussion. To improve my ability and confidence, I emailed several professors from other disciplines in the college with more experience and expertise to ask for guidance. Together, we had an excellent conversation about engaging students in discussions. We talked about approaches for preparing, facilitating, and evaluating discussions that greatly enhanced the future incarnations of the course.

- **The students seemed to enjoy playing the casino games in class, but they were not taking the games seriously.** The purpose in playing the games and using casino chips was to simulate what it would be like for them to play in a real casino. However, they had no attachment to the chips they were given and thus were extremely careless. For example, students would put all their money on a single number in roulette. I struggled to help the students establish a connection between their chips and the class because they were not gambling with their own money. To remedy this, I decided to tie the students' chips to their grade. A small percentage of a student's final grade is now dependent on how well they manage their chips during class. This resulted in the students focus shifting from "having fun" to thinking about betting strategies and behaviors.

- **Students struggled to comprehend gambling addiction.** Students had difficulty understanding why a person would enter into a casino knowing they were using the last dollars in their bank account. To them, it just seemed easy to make the choice to stay at home. Even after reading Dr. Sandra Adell's book and hearing her story, gambling addiction seemed distant. To aid in making the material more real, students are now required to volunteer at the local bingo hall, allowing them to actually see individuals gamble. During the most recent teaching of the course, I also asked students to not use their cell phones for 48 hours in order to parallel the behavioral addiction of gambling with behavioral cell phone addiction. While this assignment did make students more aware of their own behaviors, the assignment was difficult because of how much students rely on their phones for day-to-day tasks. In the future, I will instead ask students to give up social media or texting for 48 hours.

- **The student reflections are extremely important.** They offer opportunities for students to think about the material at a deeper level and critically engage with course concepts. This is particularly true for the special learning opportunities. The act of reflecting often gives more meaning to the experience. Students make connections between their observations, experiences, and opinions, and must explain and analyze the course concepts from their own perspective.

Incorporating more social impacts of gambling into the course initially caused me to be concerned that the mathematics would be forgotten by students at the end of the term. Upon reading my most recent course evaluations, my fears were unfounded. Many students commented on the mathematics and their appreciation of it. For example, on the course evaluations, one student wrote, "This course forced me to focus on concepts of math behind an everyday activity, creating a more exciting and fun environment of learning."

The changes to my teaching of the course and to the course assignments have made this First-Year Seminar course much stronger. In addition, the students are able to articulate arguments about gambling topics and use mathematics to support these arguments. The course helps them develop their skills as independent learners, their ability to think critically, and their ability to communicate orally and in writing. Moreover, the course now achieves an appropriate balance between mathematics and the social aspects of gaming, allowing the students to connect their academic learning to wider personal and social concern.

## 3.5 Bibliography

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# Appendix

## Tentative Course Schedule

| Class | Topic |
|---|---|
| 1 | Probability Introduction |
| 2 | Poker |
| 3 | Odds and Roulette |
| 4 | Craps |
| 5 | Blackjack |
| 6 | Blackjack and Game Day |
| 7 | Horseracing |
| 8 | Churchill Downs Field Trip |
| 9 | Economics of Gaming |
| | Guest Speaker: Daniel Elliott (R), Kentucky House District 54 |
| 10 | Lottery and Keno |
| 11 | Addiction |
| | Guest Speaker: Dr. Sandra Adell |
| 12 | Kentucky Lottery Field Trip |
| 13 | Paper Conferences |
| 14 | Game Day and Presentation Evaluations |
| 15 | Presentations |
| 16 | Presentations |

## Argumentative Research Paper Prompt

### Research Paper

This course is designed to teach you about the mathematical odds of some casino games and expose you to the social and economic factors of gaming. For this assignment, you will write an argumentative research paper.

Two bills will be discussed at this session of the Kentucky legislature this month. The first bill, HB41, would remove the current prohibition on casino gambling in the state. In doing so, Kentucky would authorize up to four new casinos in the state. The second bill, SB22, would authorize sports betting in Kentucky, on the condition that the current federal law prohibiting sports betting is changed.

For this paper, you will

- Determine your opinion on whether casinos should be legalized in Kentucky and make an argument supporting your opinion.
- Research sources appropriate to your topic and effectively use those sources as evidence to support and prove your thesis.
- Cite all sources using proper MLA or APA format, including proper parenthetical citations.

**Expectations**: Your research paper must be 5-7 pages, typed, double-spaced, in 12-point Times New Roman font. All body paragraphs must relate back to your thesis. They must also be organized, clear, and focused. All sources must be cited using a proper format as stated above, and you should have at least five required sources.

**About Required Sources**: A strong and well-researched paper includes a variety of sources. All of your sources must be reputable. Good sources include scholarly journals, newspapers and reliable magazines, some websites, our guest speakers, etc.

## Presentation Peer Evaluation Form

FYS Presentation Peer Evaluation

Presenter:

Topic:

**Preparation:** Did the presenter seem well-prepared for the talk?

1 2 3 4 5 6 7 8 9 10
no somewhat absolutely

**Content**: Was the talk intellectually stimulating?

1 2 3 4 5 6 7 8 9 10
no somewhat absolutely

**Presentation**: Were visuals used effectively? Did the speaker speak clearly?

1 2 3 4 5 6 7 8 9 10
no somewhat absolutely

**Delivery**: Did the speaker communicate well with the audience and explain concepts clearly?

1 2 3 4 5 6 7 8 9 10
no somewhat absolutely

State a question you would ask the speaker about their topic.
What was your favorite aspect about the presentation?
What would most improve this presentation?

# 4

# *Wheels and Deals*: A First-Year Seminar Based on Game Shows

Alison Marr

**Abstract** *Wheels and Deals: A Survey of Television Game Shows* is a first-year seminar that explores television game shows through a liberal arts lens. This article describes how mathematics is used in a two-week unit as well as how the rest of the course is structured. Relevant sources and rubrics for the major papers and projects are given.

## 4.1 Background and Context

Southwestern University is a private liberal arts college of about 1,500 students. Every student who enrolls at Southwestern takes either a first-year seminar (FYS) or (for transfer students) an advanced-entry seminar (AES) in their first semester. The FYS/AES program is a campus-wide program and about one faculty member from each department participates. Students select their FYS/AES from a list of topics and descriptions before they arrive on campus. Instructors for the seminars and their home departments are not given so that students do not always choose a seminar in their intended major and/or avoid seminars in disciplines they think they do not like. The hope is that each seminar has a variety of student interests and backgrounds. All students who are enrolled in the same seminar live together on the same floor of their first-year residence hall to create living learning communities. FYS courses are typically between 12 and 18 students. The seminars begin a week before the actual semester starts with a 12-hour week of classes. Then, the seminars meet twice a week for 75-minute periods for ten more weeks finishing a few weeks before the rest of the semester. The course counts as a full four-credit course and is graded in the same way as other courses (on an A-F plus/minus scale). As stated in the annual call for proposals, the mission of the seminars is to help the new student begin to practice an education that arcs over the whole course of the student's experience and across the curriculum. Seminars introduce and reflect upon intellectual skills common to the liberal arts: formulating cogent questions, forging connections between methods of inquiry, recognizing and challenging assumptions, seeking out and listening to multiple perspectives, and rethinking the role of reading, writing, and discussion in inquiry and student-centered learning. Faculty can choose any topic for their seminar as long as the course objectives align with this mission.

## 4.2 Mathematical Theme

Imagine you are on the television game show, *Let's Make a Deal*. The host, Monty Hall, shows you three doors. Behind one door is a car. Behind the other two are goats. Monty asks you to pick one of these three doors. Let's say you pick door number 2. Monty (knowing the location of the goats and car) opens door number 3 and shows you a goat. Monty then asks you, "would you like to stay with door number 2 or switch to door number 1?" This question, known as The Monty Hall problem, has sparked debate amongst mathematicians, studies by psychologists, and thoughts by philosophers, and was the inspiration for my FYS course, *Wheels and Deals: A Survey of Television Game Shows*. While I was inspired by a classic math problem, the course looks at game shows through a liberal arts lens.

The mathematical theme of *Wheels and Deals* is related to the mathematics found in television game shows and we discuss these ideas in a two-week unit. We explore mathematical topics including basic probability, conditional probability, Pascal's triangle, expected value, the multiplication principle, and the prisoner's dilemma. These topics were chosen because of the ease with which students from all backgrounds can engage with the material. While this was the only mathematical content I choose to cover in this seminar, one could certainly extend this and study other interesting mathematics involved in game shows. Some potential additional areas of exploration could be: bidding in Final Jeopardy! or on *The Price is Right*, the randomness (or lack there of) of the *Press Your Luck* screen, or discussing Instant Insanity as seen on *Survivor*. In addition, game shows are a rich source of all kinds of potential statistics projects. One of my former FYS students ended up doing her senior capstone project with me on the differences between the types of trivia questions asked on *The Chase* in the UK versus *The Chase* in the US, basing her work on similar analyses performed using data from *Who Wants to be a Millionaire* by Hestroni [10, 9]. This type of project could certainly be incorporated into an FYS course if mathematics and statistics was the central focus of the course.

## 4.3 Course Structure

The seminar has four main units: defining a game show, analyzing the components, history of game shows, and the mathematics of game shows. A majority of the course is taught in a discussion based style with some interactive lecture in the mathematics unit. We begin with some discussion of what is a television game show. Is *Survivor* a game show? Is *The Voice* a game show? To prepare for our discussions, students read various articles on this exact question to help formulate their position. In particular, over the summer students have read the first few chapters of our main course text, *The Quiz Show* by Su Holmes [12, pp. 1–31]. Once on campus, students read excerpts from Mittell's text [14, pp. 29–55] on genre and television and the conclusion from Hoerschelmann's game show text [11, pp. 149–155]. We make a list of characteristics that a show in this genre might have. In the past, these lists have included things like a host, contestants, timed play, rules, a stage, sound effects, and prizes. Then, we host a debate in class on the statement: *The Voice* is a game show. Students are randomly selected to either be on the pro side, con side, or be a judge. This debate helps students formulate their thoughts for their first writing assignment in the class where they take a position on this same topic and argue their position. The instructions and rubric for this assignment can be found in Appendix 4.4.

After this unit, we return to the Holmes [12, pp. 58–165] book and spend a day thinking deeply about the various components of a game show. Students are split into groups and each group is assigned one chapter. Those groups present to the class what they found interesting about their chapter. Depending on the content of those presentations, we may talk about knowledge and power and how that relates to gender and who is typically seen in the roles of host, models, and contestants. On the other hand, we might discuss the design of the set, placement of the hosts and contestants, and sound effects used. Sometimes we discuss the differences between game shows in the US and abroad and what those differences might say about our societies, especially given the Holmes text is based mainly in the UK. Student presentations can lead to discussions about race, gender, socioeconomic status, and/or sexual orientation in the context of who's playing the game and who's watching the show. Some of these conversations and topics return when students present their scholarly articles or when students write their final paper (see Section 4.3.2).

Next, we move to the history of television game shows. We watch the the Game Show episode of *PBS Pioneers of Television* [16], some clips from the television mini-series *Game Show Moments Gone Bananas!* [7], and students read the history chapter from Holmes [12, pp. 32–57]. In addition, over the course of the semester students have to watch three different game shows and reflect on what they saw. All of this helps them get an idea of the type of shows out there and provides some of the history of the genre. As part of this unit, students bring in what they believe to be the five most critical moments in game show history and we form a timeline of events and discuss each of the items before deciding on our collective ten most important moments in game show history. During this time, students gather in the dorm one evening to watch the film *Quiz Show* [18] which gives them an idea about the quiz show scandals of the fifties. Another important part of understanding the history of television game shows is getting a chance to talk to former contestants. Each time I have taught the seminar I have invited one or two former contestants to come to class and discuss their experiences from the contestant perspective. Moreover, students read *Prisoner of Trebekistan* by Bob

Harris [8] over the summer which gives them a chance to hear about how one popular *Jeopardy!* contestant related his multiple appearances to his own life (and how students can relate his journey to their own lives and new college experiences).

The course also contains the unit on mathematics that is described in Section 4.3.1. The seminar ends with a collection of cumulative assessment projects including a final paper, game show creation project, an exam, and short three minute presentations where students connect the ideas they've learned in the course related to game shows to some other course or their lives. More details on the final paper and game show creation project are given in Section 4.3.2.

### 4.3.1 Mathematical Unit

The first day of the unit is spent exploring the Monty Hall problem. The problem is introduced via a brief, interactive lecture after which I take an informal poll of what students think we should do: stay, switch, or it doesn't matter. Then, students begin running their own simulations of staying versus switching with dice and car/goat index cards. We collect this data and students immediately start to see that switching is the optimal strategy. We then work on crafting a definition for probability and focus on the long term aspect of the definition. After this, we go through varied ways of reasoning through the Monty Hall problem that don't involve a simulation including listing the sample space and drawing a probability tree. We watch clips from the TV show *NUMB3RS* [15] and the film *21* [1] allowing students to see other explanations of the problem. If time allows, we discuss other interesting conditional probability puzzles such as "I have two children, one of which is a boy born on a Tuesday. What is the probability I have two boys?" Many times we have had a discussion about the difference between the Monty Hall Problem and the final situation in *Deal or No Deal* when contestants are asked if they want to switch the case they have in front of them with the final one remaining on stage. This question arises naturally in class and is a great opportunity to talk about what conditional probability really is and how the information the host does (or does not) have can really change how we approach a problem.

The second day is spent on expected value in the context of *Deal or No Deal* and *Who Wants to Be a Millionaire*. We start by discussing what expected value is and how to calculate it. Then, we play an online simulation of the *Deal or No Deal* game. After each round of the game, we calculate our expected value and compare that to the banker's offer to decide if we should continue playing or take the offer. We then spend a brief amount of time discussing the ideas of being risk neutral, risk averse, or risk loving as described on Harvey Mudd College's Math Fun Facts website [20]. The students enjoy playing the game and picking their cases and if time allows, we play a second round of the game at the end of class. After this example, we move to a discussion of expected value in the context of *Who Wants to Be a Millionaire*. We use the model outlined by Quinn [17, pp. 82–83] to discuss the possible scenarios one can face moving from the $16,000 question to the $32,000 question in the original US version of the show. After this, we spend a brief amount of time discussing the Multiplication Principle in the context of the fastest finger round on *Who Wants to be a Millionaire*, again drawing from Quinn [17, p. 83].

The third day is used to discuss two mathematical examples from *The Price is Right*: Plinko and Master Key. To introduce the mathematical ideas found in Plinko, students are given the ABRACADABRA exercise described in Butterworth and Coe [2, p. 20] as homework. In this exercise, they are given the word ABRACADABRA spelled out in a diamond shape where the first row has a single A, the next row has two Bs, the third row three Rs, up to the 6th row with 6 As and then the number of letters per row decreases down to one final A. They are asked to find the number of ways to spell ABRACADABRA starting with the first A and moving to an adjacent letter in the next row. They arrive in class ready to give their guesses. Then, we discuss the strategies they used to come to their count and arrive at a "best" strategy to count these sorts of things that involves Pascal's triangle. I then usually show a clip or two of people playing Plinko in case students are unfamiliar with the game. Then, they get to work on figuring out a strategy for the real Plinko board and work in groups to figure out the number of ways to reach each of the slots based on where they drop their Plinko chip using ideas from Pascal's triangle. Using an Excel spreadsheet and the counts they come up with, we find the expected value for each of the slots and discuss what a best strategy might be (again using guidance from Butterworth and Coe [2, pp. 21–23]). We usually have enough time after this to talk about one other pricing game and I have usually used Master Key. Again, I show them a clip of how the game works. Then, using guidance from Carlton and Mortlock [4] we analyze the game and all the possible probabilities that take place at different scenarios. We then rewatch the same clip and I pause it at points asking questions about probabilities of winning at each stage.

This is a great chance to remind the students about conditional probability in the context of a different game.

The final day explores game theory and the prisoner's dilemma based on the game show *Friend or Foe*. First, I show the class a clip of this Game Show Network show as it is not one that many students know. Using the activity outlined in Coe, et al. [3, pp. 30–32], we then play an in-class version of *Friend or Foe*. This in-class version uses trivia questions based on class material and is usually placed right when we are ready to review for the upcoming exam. After we play the game, we discuss the prisoner's dilemma in the context of this game as well as the more classical scenario. Students learn about finding a Nash equilibrium, how to draw payoff matrices, and the difference between strongly dominant and weakly dominant strategies. We also discuss that prisoner's dilemma scenarios were seen in the first few seasons of *Bachelor Pad* (for any fans of *The Bachelor* series).

At the end of the unit, students are given a writing assignment. After learning about the Monty Hall problem, students read Keith Devlin's article [5] that describes the Monty Hall problem and the mathematics behind it for a general audience. Their task is to take one of the other mathematical ideas from a game show that we just learned (Master Key from *The Price is Right*, Plinko from *The Price is Right*, *WWTBAM* and fastest finger, *Deal or No Deal* and expected value, *WWTBM* and expected value, or *Friend or Foe* and the prisoner's dilemma) and write a similar type of article describing the mathematics from that show for a general audience. They are asked to write two to three pages that includes a clear description of the game and the mathematical ideas present in that game. A rubric and full description of the assignment is given in Appendix 4.4.

We spend one additional day returning to the Monty Hall problem and discussing chapter 6 (Cognitive Monty) from Rosenhouse's *The Monty Hall Problem* [19, pp. 133–153]. This chapter looks at some of the psychology behind what makes the Monty Hall problem so tricky for humans to comprehend. It allows us the chance to connect mathematics to another discipline and in particular gets students to think more deeply about what makes this problem so interesting and why there is a whole book dedicated to this one problem.

### 4.3.2 Major Assignments

One of the earliest major assignments relates to getting students to learn how to use the library to find scholarly sources. As part of this assignment, students have to find three scholarly articles on television game shows that might interest them. This is not an easy task as game shows have largely been ignored as a genre by scholars. Once they have found three articles and learned to correctly give their full citations, students are assigned an article to present to the class in groups of two. They have ten minutes to summarize the article for the class. These presentations give students a chance to practice both their ability to accurately summarize information and their oral communication skills. Furthermore, this project is a great chance to see how game shows apply to many other fields such as communication studies, economics, television studies, or feminist studies. These articles are also listed in the course management system, so that when students start writing their final papers they have a bibliography of possible scholarly sources.

The final writing assignment is a five to seven page paper where each student compares and contrasts two television game shows and what those shows say about us as a society. Students sign up for their pair of game shows and no two students can have the same pair. Students have to use at least three sources in this paper including one scholarly source. This gets students to engage with the library once more and work on properly citing sources. Furthermore, this assignment gets students to think deeply and in an academic way about a seemingly unacademic topic. The paper by Laist [13] comparing *Jeopardy!* and *Wheel of Fortune* serves as a good model. Instructions for the assignment and how it is graded are given in Appendix 4.4.

One of the assignments that occurs throughout the semester is the creation of the students' game shows. Early in the semester, students were put into groups of three to four and told to create their own television game shows. They submit progress reports along the way and we work on getting all the logistics of their shows worked out. Then, students prepare a game show proposal with all the details of their show. Finally, we select two different times and locations to present their shows to the campus community. We do a lot of advertising and students recruit peers from their dorm and friends to join us in the the audience for the shows. To complete their game show project, students individually write a reflection on their experience. This project introduces them to group work that they will see in many of their college classes. Additionally, students get to show their creative side in the creation of various components of the show. Students have been very creative in the artwork for their logos, creating music for the show, coming up with interesting concepts for the shows, and even some of the "acting" done by the hosts. The two writing components of this assignment also expose students to technical writing (in the proposal itself) and reflective writing. Additional details about the various components of this assignment and how it is graded can be found in Appendix 4.4.

## 4.4 Reflections

I have taught this course a total of six times. As a mathematician, teaching this course was certainly a challenge. I had never really crafted a reading list, created (or provided feedback on) major writing assignments, or led class discussions. However, through professional development and the support of my colleagues, I was able to create a successful course and have used these new pedagogical tools to inform the way I teach my mathematics classes.

One area that I have worked on quite a bit is the writing component. I am now happy with the amount and various types of writing the students are assigned. Students get a chance to argue a position, write mathematics for a general audience, write a research paper, write a technical game show proposal, and write a reflection. These are all types of writing they may see in their future classes and this exposes students to each of these types of writing and allows them to get feedback early in their career. In terms of written feedback, I have found that giving each student three global things to work on at the end of each draft that I read helps students improve their papers. This saves me the time of doing line edits and correcting grammatical mistakes. If grammar is one of their weaknesses, I generally mark the mistakes in the first paragraph and make a global comment (as one of their three things to work on) that they should seek help from the campus writing center on ways to improve their grammar.

In terms of critical reading, students often struggle with the Holmes text, but this is a struggle I want them to have. Students often mention they have not read anything like the Holmes book before and it really gives them a chance to see what college-level reading might be like. I am always happy to push them to read challenging material. However, in some iterations of the seminar, I have assigned chapter 7 (Philosophical Monty) from *The Monty Hall* book by Rosenhouse [19, pp. 155–174], but first-year students struggle reading this chapter much more than chapter 6 (that I do assign), so I have not assigned chapter 7 in recent versions of the course.

The game show project has consistently worked well. In recent years, I added an in-class day close to game show presentation day where I spend 10-15 minutes with each group going over the minute-by-minute details of the show and solving any last minute problems (the need for an outlet, confirming the layout of the stage, making sure they have contestants, etc.). This has helped the shows run a lot smoother on presentation day. A few years ago, Southwestern started a First-Year Symposium where various first-year seminars presented work from their class to the campus community and Board of Trustees. I now have two of the game shows present during this time. The other two are usually scheduled for one evening that same week. This schedule has worked well for us and I will likely continue that setup moving forward. The reflective writing piece that students do as the conclusion to this project is often the best writing I see from the students throughout the course of the semester.

I'm constantly thinking of ways to improve the seminar. Between the game show project, writing assignments, and exam, students often feel that too many assignments are due near each other, so the pacing of the course could still be improved. This likely means removing content or assignments, but that is hard for me given the work I have put into developing many of these assignments and units. As a mathematician, I always want to add new and engaging mathematical ideas to the seminar. Students do not seem to struggle with the mathematics I have used thus far in the class, making me question if I could push the level of mathematics to even more advanced topics. As a discrete mathematician, I would love to add in the math behind Instant Insanity (as seen on *Survivor*). To make space for this topic, I would likely remove the Fastest Finger discussion and/or the Master Key probability example as those are fairly basic. I am also always updating the documentary students watch related to the history of television game shows. I recently saw *Game Changers* [6] hosted by Alex Trebek and am considering replacing the PBS documentary [16] with this one.

Engaging with first-year students, teaching outside of my discipline, and exploring a topic I am passionate about have been some of the most rewarding parts of teaching in the first-year seminar program. The liberal arts approach to the class allows students to really think deeply about a topic that normally may not have felt very academic and this to me is the ultimate goal of an FYS course.

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# Appendix

## Rubric for Paper 1: Is *The Voice* a Game Show?

**FORMAT:** Papers should be typed, double-spaced, margined, stapled, and proofread. Feel free to print your papers double-sided. Always document your use of sources appropriately and with a consistent style. MLA style might be most familiar to you. Aim for around three pages.

**INSTRUCTIONS:** Discuss the following statement: "*The Voice* is a game show." Feel free to agree, disagree, or simultaneously agree and disagree. Make your position known by creating a clear thesis statement. Your goal is to convince me of your position.

Your paper will be graded using the following rubric:

**10–9 = Advanced:** writer demonstrates strong control and skill in this trait; many strengths present
**8–7 = Proficient:** writer demonstrates effective control and skill; strengths outweigh weaknesses
**6–5 = Developing:** writer demonstrates equal number of strengths and weaknesses in this trait
**4 and below = Needs Improvement:** writer is not yet showing control or skill in this trait

| **Paper Criteria** | **Score** |
|---|---|
| **Quality of Ideas:** Range and depth of argument; logic of argument; quality of research or original thought; appropriate sense of complexity of the topic; appropriate awareness of opposing views. | |
| **Organization:** Includes introduction, conclusion, and well developed body paragraphs; logical and clear arrangement of ideas; effective use of transitions; unity and coherence of paragraphs. | |
| **Introduction:** Grabs the reader's attention; contains a developed thesis statement. | |
| **Clarity and Style:** Ease of readability; appropriate voice, tone and style for assignment; clarity of sentence structure; gracefulness of sentence structure; appropriate variety and maturity of sentence structure. | |
| **Sentence Structure and Mechanics:** Grammatically correct sentences; absence of comma splices, run-ons, fragments; absence of usage and grammatical errors; accurate spelling; careful proofreading; attractive and appropriate manuscript form. | |

## Rubric for Paper 2: Mathematics for a General Audience

**FORMAT:** Papers should be typed, double-spaced, margined, 2-3 pages, stapled, and proofread. Feel free to print your papers double-sided. Always document your use of sources appropriately and with a consistent style. MLA style might be most familiar to you.

**INSTRUCTIONS:** Recall the article "Monty Hall" by Keith Devlin. Your task is to write a similar essay describing the mathematics in one of the other game shows we mentioned in class. You can choose from these topics: Master Key, Plinko, *WWTBM* Fastest Finger, *Deal or No Deal* and Expected Value, *WWTBM* and Expected Value, *Friend or Foe* and the Prisoner's Dilemma.

Your paper will be graded using the following rubric:

**10–9 = Advanced:** writer demonstrates strong control and skill in this trait; many strengths present
**8–7 = Proficient:** writer demonstrates effective control and skill; strengths outweigh weaknesses
**6–5 = Developing:** writer demonstrates equal number of strengths and weaknesses in this trait
**4 and below = Needs Improvement:** writer is not yet showing control or skill in this trait

| Paper Criteria | Score |
|---|---|
| **Description of Problem:** Clear statement of problem to be described. | |
| **Mathematics:** Clarity and correctness of mathematics. | |
| **Organization:** Includes introduction, conclusion, and well developed body paragraphs; logical and clear arrangement of ideas; effective use of transitions; unity and coherence of paragraphs. | |
| **Clarity and Style:** Ease of readability; appropriate voice, tone and style for assignment; clarity of sentence structure; gracefulness of sentence structure; appropriate variety and maturity of sentence structure. | |
| **Sentence Structure and Mechanics:** Grammatically correct sentences; absence of comma splices, run-ons, fragments; absence of usage and grammatical errors; accurate spelling; careful proofreading; attractive and appropriate manuscript form. | |

## Rubric for Final Paper: Game Shows and Society

**FORMAT:** Papers should be typed, double-spaced with 1 in. margins, 11 pt. font, 5-7 pages, stapled, and proofread. Feel free to print your papers double-sided. Always document your use of sources appropriately and with a consistent style. MLA style might be most familiar to you. Your paper must use at least three sources including one scholarly journal article.

**INSTRUCTIONS:** Choose two game shows. Be sure to watch a few episodes of each show you choose. Then, compare and contrast what these shows say about us as a society. Think about set design, host choice, music choice, etc. Anything that you think is interesting! Sign up for your pair of game shows on Moodle. No two papers should have the same pair of game shows, so sign up early. ("Us" can be humans in general or Americans or another culture (or cultures) appropriate to the game shows you select.)

Your paper will be graded using the following rubric:

**10–9=Advanced:** writer demonstrates strong control and skill in this trait; many strengths present
**8–7=Proficient:** writer demonstrates effective control and skill; strengths outweigh weaknesses
**6–5=Developing:** writer demonstrates equal number of strengths and weaknesses in this trait
**4 and below=Needs Improvement:** writer is not yet showing control or skill in this trait

| Paper Criteria | Score |
|---|---|
| **Quality of Ideas:** Quality of research and original thought; appropriate sense of complexity of the topic; range and depth of arguments. | |
| **Development:** Clarity of thesis statement; good development of ideas through supporting details and evidence; interesting title | |
| **Organization:** Includes introduction, conclusion, and well developed body paragraphs; logical and clear arrangement of ideas; effective use of transitions; unity and coherence of paragraphs | |
| **Clarity and Style:** Ease of readability; appropriate voice, tone and style for assignment; clarity of sentence structure; gracefulness of sentence structure; appropriate variety and maturity of sentence structure | |
| **Sentence Structure and Mechanics:** Grammatically correct sentences; absence of comma splices, run-ons, fragments; absence of usage and grammatical errors; accurate spelling; careful proofreading; attractive and appropriate manuscript form | |

## Game Show Project

Your game show project consists of a presentation, proposal, and reflection. First, you will have one written group proposal consisting of the items listed below. Second, each group will premiere their game shows at a designated time and day. Finally, each group member will submit a personal reflection as described below.

1. Your group game show proposal should be a formal document that you would present to a television network in an attempt to get your show on TV. After reading this proposal a network executive should have a clear idea of how your game show would work. It should contain the following headings (and possibly others depending on your show):

a. Program title
b. Target audience
c. Suggested time-slot
d. Length (mins)
e. Brief outline (2-3 sentences)
f. Outline running order
g. Merchandising opportunities (i.e., what could you sell?)
h. Detailed synopsis
i. Sample questions
j. Show logo
k. Suggested host
l. Outline budget (Max campus: $30) Campus and real world versions
m. Set design (including illustration)
n. Contestant selection

2. Game Show Presentation Rubric
Each area scored out of 10:

**Concept:** Originality of idea, game show characteristics, well-defined rules
**Preparation:** Effort put into premiere, flow of show
**Entertainment Value:** Audience reaction, correct show length
**Artistic Elements:** Music, sound effects, set design, costumes
**Real World Value:** People interested in being contestants, worthwhile to a network

3. You will each need to write a two-page reflection essay on the game show creation process. This essay can include a discussion of group dynamics, how you felt during the process, what you liked/disliked about the project, etc. I will be the only one reading these reflections, so feel free to write about anything related to the process.

# 5

# Game Theory in Popular Culture: A First-Year Writing Seminar

Jennifer Firkins Nordstrom

**Abstract** This chapter describes a first-year writing course based on game theory in popular culture. The students watch films such as *Dr. Strangelove*, *Return to Paradise*, and *Rebel without a Cause*, and read a novel called *The Mind Game*. Using the mathematical background of two-player matrix games, the course focuses on themes from game theory such as cooperation and defection, assumptions of game theory such as rationality and perfect knowledge, and models such as the prisoner's dilemma and chicken.

## 5.1 Background and Context

Linfield College is a small liberal arts college in northwest Oregon with approximately 1,500 undergraduate students. Each first-year student is required to take an Inquiry Seminar, which is a four-credit course meeting for approximately 200 minutes per week during our fourteen-week semester.[1] This course functions primarily as a first-year writing course. Inquiry seminars are taught by faculty from across the institution who choose their own topic and use that as a framework for teaching students how to craft argumentative essays and a research paper. Each course enrolls 16-18 students who themselves represent a wide variety of academic interests and backgrounds. Since the focus is on writing, each course requires students to write two to three argumentative essays and a research paper. To encourage students to think critically about their own writing, students are required to demonstrate a process of revision as well as reflection on the evolution of their own work. However, the real driving force of the inquiry seminar is the intentional focus on the development of inquiry. Students spend significant class time engaging with questions related to the course topic. They then use writing and discussion to learn how to think through multiple perspectives and support a position. The topics should offer complex questions that have no clear answers. Faculty have significant freedom and encouragement to design courses that step away from traditional academic themes. Faculty often teach courses that speak to a particular passion and choose themes that are relatable to the experiences of new students. Successful courses allow students to quickly develop some expertise in a topic and form connections to their own experience. This article will focus on incorporating the mathematical theme of game theory into a first-year writing course, but a more detailed discussion on incorporating inquiry into this course is given in "It is All About Inquiry: A Cross-Disciplinary Conversation about the Shared Foundations for Teaching" [11].

## 5.2 Mathematical Theme

I have designed a first-year writing seminar based on game theoretic themes found in popular culture. I have a passion for game theory and enjoy seeing it as a theme in popular movies. The course provides an investigation of game

[1] Most inquiry seminars meet two or three times per week. My course met for three 65-minute blocks each week.

theory through film, television, and fiction. It explores ideas such as perfect information, the prisoner's dilemma, and the volunteer's dilemma. It examines the assumptions, mathematical models, and historical development of game theory. Students learn the basic mathematical underpinnings of game theory and then apply these concepts to themes of conflict, cooperation, and trust. In particular, it covers mathematical ideas and techniques in the topics of zero-sum games, matrix games, equilibrium points, dominated strategies, strategies for finding equilibrium points, finding mixed strategy equilibria, von Neumann's minimax theorem, non-zero-sum games, and Nash equilibria. If we have time, we sometimes are able to cover coalitions and bargaining games. Overall, about 15% of the class periods are devoted to primarily mathematical content. The amount of mathematics that students incorporate in their papers varies, some include payoff matrices and analysis, while others focus more on ideas of assumptions, dilemmas, or the decisions of a particular character. Using popular culture as a vehicle to explore game theory gives students a familiar context to explore the more mathematical content. Students can quickly become the "experts" on the topic since they can apply the mathematical framework to their favorite films or even to bigger societal questions. One challenge is getting the students to ask interesting questions-at-issue that do not have a "correct" answer; this is especially true when they think the content and justification are mathematical. In game theory, students can use a mathematical frame to explain a situation, say, a conflict between two characters, but how they apply the frame and what assumptions they make about a character's motivations are easily debatable.

Since the intention of an inquiry seminar is to delve deeply into a topic, we spend the entire semester engaging in questions about rationality, value, mathematical models for conflict, and how cooperation and defection affect group dynamics. Although the models are mathematical, students can discuss the application of the model to a wide array of disciplines such as psychology, economics, political science, and biology. Using popular culture as the context for the material gives students an easy entry into the themes, while the wide variety of applications helps students connect the material to their own academic interests and provides relevance for their writing.

## 5.3 Course Structure

My inquiry seminar meets for three 65-minute classes each week. The long class time allows for significant discussion time. We also use class time for students to read and critique each other's writing in small groups of three to four students. I do very little lecturing, and only when I need to introduce some mathematical material. In general, students have a reading, viewing, or writing assignment due each class. We then use the class time to discuss the reading or viewing. To facilitate discussion, half the students are required to bring a notecard with a question-at-issue and half bring a one-page response paper to such a question. For example, the day after a movie viewing, students in the first half of the class alphabetically will bring a notecard with a discussion question that they develop for the film. The other half of the students bring a one-page response paper based on their own thesis statement relating to the film. The following class, the half that brought a card bring a paper and vice-versa. After the next film, we switch which group brings cards and which brings papers to the first class. But for each film, every student is responsible for a card and a paper. They work in small groups to improve their questions, develop thesis statements in response to their questions, and then highlight possible evidence to support the thesis. Students who bring a paper to the first discussion do need to have a question in mind, but may choose to write an improved question for the second class. Similarly, students who bring a card first often choose to answer a different question in their response paper. The cards provide a framework for beginning the discussions, and the papers provide practice developing an argumentative essay. Students can then use these discussions and initial drafts to build their ideas for the larger writing assignments: two five-page argumentative essays and one ten-page research paper. Before each major paper is due we have both a thesis workshop and a day for peer review of the current draft. The thesis workshop has students work in groups of three to four students to make sure each thesis is an answer to a question-at-issue, is debatable, and can be supported with evidence. The peer-review class breaks students into pairs to read and critique each other's drafts. We generally have time for each student to work in two separate pairs.

Teaching a discussion-based writing course can be daunting for mathematics faculty. But math faculty are well-trained in how to structure logical arguments to support our ideas. This training is a real asset when teaching first-year students to write a thesis and support it with logical structure and evidence. I give the students significant freedom to choose what to write about and guide them on how to support their arguments. For each of the five-page argumentative essays, the students are assessed on their ability to state a clear thesis, one that a peer might reasonably disagree with, and their ability to provide contextual evidence from the film or reading that supports their thesis. Their argument must connect to the thesis, and any conclusions drawn must connect to the evidence.

The course is organized around three main themes: the assumptions of game theory, some classic models of two player games, and group dynamics in multiplayer games. Each unit is organized around relevant films and readings with corresponding writing assignments. Although the course focuses on these three main themes, we also cover some of the historical development of game theory. The students read *Prisoner's Dilemma* [13], which provides a nice framework for the course with history, an introduction to game theory, and an exploration of some of the common dilemmas. Additionally, "Game Theory in Popular Culture: Battles of Wits and Matters of Trust" [9] makes a good introduction to the course and was the original motivation for the development of this inquiry seminar. An outline of the entire course is given in the Appendix (p. 51).

### 5.3.1 The Assumptions: Perfect Knowledge, Rationality, and Value

The first theme explores the basic assumptions of game theory. In particular, we discuss ideas of perfect knowledge, rationality, and value. The main film we watch as a class is *Dr. Strangelove or: How I Learned to Stop Worrying and Love the Bomb* [5]. This film is particularly relevant to the course as it references and parodies some of the early founders of game theory. It explores concepts of preventative war, deterrence, and perfect information. It provides interesting characters for the discussion of rationality in that there is a blurring of the lines between behavior we often societally characterize as irrational with the more mathematical definition of rational behavior.

The main paper in this unit has the following essay prompt:

> Consider a game-theoretic scenario in a novel or film or your choice. How is the situation affected by the assumptions? Do all of the players seem to be using the same assumptions? How do the assumptions differ from standard game theory? What is the director or writer trying to convey about the characters through game theory?

Students are not required to address all of the questions, but may focus on one or two. Although *Dr. Strangelove* provides a common foundation for discussing rationality, perfect knowledge and value, I encourage students to write about films they are more familiar with where they notice these themes. Some films that are also relevant are *The Dark Knight* [8] in which students can successfully argue the rationality of the Joker, *The Princess Bride* [15] as most students are familiar with the classic poison scene, and *Mystery Men* [18] where there is another example of a "you know that I know that you know" scene. For more information on the mathematics in these films, see "Game Theory in Popular Culture: Battles of Wits and Matters of Trust" [9, pp. 87–91].

### 5.3.2 The Models: Chicken and the Prisoner's Dilemma

The second theme is a focus on common two-player non-zero-sum games such as battle of the sexes, chicken, and the prisoner's dilemma. These models provide a strong framework for discussing conflict and cooperation among characters. For this unit we watch *Return to Paradise* [17], *Rebel Without a Cause* [14], and the 1984 version of *Footloose* [16]. The film *Return to Paradise* is centered around a classic prisoner's dilemma in that the two main characters need to decide if they will cooperate to save a third friend, resulting in a three-year prison sentence for each cooperator, or defect leaving any cooperator to serve a six-year sentence. If both defect, then their friend will die in prison. Although framed as a standard prisoner's dilemma, the film adds nuance to the game in that although players can communicate, the communication is only as strong as their trust in each other. The other two films, *Rebel Without a Cause* and *Footloose* contain iconic chicken scenes. Even though viewing only the scene, rather than the entire film, may be sufficient for demonstrating the game of chicken, a more substantial question is how the game is used in a film, rather than what are the details of the actual game. To really understand why a writer or director would include a game of chicken in a pivotal scene, the scene must be viewed in the context of the entire film. Here the mathematical context of chicken as compared to prisoner's dilemma can be enlightening. In particular, a prisoner's dilemma has a clear equilibrium point (both players defect), and the dilemma arises when you assume players are equally rational, and thus must make the same strategy choice. If they know this, then they are both better off choosing to cooperate. However, in a chicken game, the two equilibrium points occur when the players choose the opposite strategies (one swerves while the other goes straight). Thus, the dilemma is in how to ensure the two equally rational players choose opposite strategies. Students can then extrapolate the themes of cooperation and conflict from these films to broader societal commentary. More information about chicken and the prisoner's dilemma in film can be found in [9, pp. 91-95].

For this unit, I also have students read *The Mind Game* by Hector MacDonald [7], a novel with explicit themes of game theory. The novel is an important part of the course, as I like to include a broader representation of popular culture than just films. Since the references to game theory are explicit, this novel is a good vehicle to discuss how popular culture uses and misuses mathematics more broadly. The general essay prompt for this section focuses on *The Mind Game*, but can easily be adapted to the other films:

> *The Mind Game* claims to have broad themes of game theory running throughout the story. Pick two characters in the story and relate their experience to game theory. For example, what game are they playing? What is the author trying to convey about the characters through the game? Is the author truly applying game theory? Is there a winner of the game? Did players choose optimal strategies? How would alternate strategies have changed the outcome of the game? You are encouraged to develop your own questions connecting the novel and game theory.

Again, students can focus their papers on one or two of these questions. Since all of the students are writing about the same material, this is a good opportunity to emphasize the peer-review process. The students have equal access to the possible evidence, thus they can be more critical of whether that evidence is sufficient to support the thesis. They can provide valuable feedback to each other and suggest additional evidence from the book to support a claim.

### 5.3.3 Group Dynamics: a Multiperson Prisoner's Dilemma and the Volunteer's Dilemma

The last section of the course covers games with more than two players, in particular a class-wide prisoner's dilemma and the volunteer's dilemma. These games provide robust models for group dynamics and often lead to some of the most interesting discussions in the course. I introduce each of the dilemmas with class experiments. The prisoner's dilemma experiment is adapted from an experiment conducted by Douglas Hofstadter, as described in *Metamagical Themas: Questing for the Essence of Mind and Pattern* [4, pp. 739–755]. In this experiment each student secretly submits a choice to cooperate or defect. The students receive points as if they played a prisoner's dilemma game with each classmate. The total points received (always real points towards the course grade) depends on how many cooperators and defectors the class has. For example, if a class has 3 cooperators and 21 defectors, each cooperator would get 3 points from the other two cooperators, but 0 points from each other defectors, for a total of 6 points. Each defector would get 5 points from the three cooperators, and 1 point from each of the 20 other defectors, for a total of 35 points. See the Appendix (p. 47) for the specific class activity. The defectors always do better than the cooperators, but everyone in the class does better if there are more cooperators. Generally there are very few cooperators, who often then feel offended by the defectors. The class discussions can lead to impassioned arguments about the "right" strategy in a prisoner's dilemma.

For the volunteer's dilemma experiment, I use a classroom response system where students can record answers in real time, anonymously to peers, but not anonymously to me (since part of their grade will be determined by their answers). I present a list of increasingly difficult choices in which students need to decide to volunteer or not. For example, "The lights go out in your neighborhood, do you call the power company or let someone else do it?" or "The class has been captured by an evil villain, do you volunteer to get eaten by sharks to save everyone else? If no one volunteers, everyone gets eaten by the sharks." After each question we talk about why someone might choose to volunteer or not. In all the classes where I have used this game (in addition, to the writing seminar, I regularly teach a quantitative reasoning course using game theory), I have *always* had at least one volunteer for each scenario, until I get to the last questions where they count for actual points: "Choose 1 point or 5 points. As long as at least one person chooses 1 point, everyone will get the number of points they chose. If no one chooses 1 point, everyone gets 0 points." This is followed by a question with the same setup, but at least five people must choose 1 point. Most of the time I do not end up awarding any points. This leads to great discussions about the difference in theoretically volunteering and actually volunteering. For more detailed discussion along with follow-up questions for these experiments, see Appendices 5.4 and 5.4 and *Introduction to Game Theory: a Discovery Approach* [10, Sections 4.5, 4.6].

After the volunteer's dilemma experiment the class reads "The Tale of Happiton" in *Metamagical Themas* [4, pp. 767–780]. This essay is a parable about a volunteer's dilemma in which residents of Happiton need to write postcards to a demon to reduce the chance of all the residents dying from a poisonous gas. Originally written as an analogy for nuclear war, the Tale can also be applied to issues such as voting and climate change in which one individual's actions seem inconsequential, but collectively can make an impact. If there is time at the end of the course we discuss coalition games with more than two players. Gillman and Housman have a nice treatment of these games in *Models of Conflict and Cooperation* [3, pp. 287–333].

The main writing assignment is to tie together popular culture themes with academic writing in the form of a research paper. For an example of an academic paper incorporating the humanities and game theory, I have students read "Game Theory and Literature" [1]. This paper provides a model for discussing conventions of academic writing and how it relates to the themes of the course. I give students a very broad prompt:

> Choose a game-theoretic dilemma and examine how it has been applied in literature, fiction, television, or film. What does this dilemma convey about the characters, story, or society? Your paper should cite at least three academic sources and at least three novels, films, or television shows. Be sure to provide evidence to support your claim.

For this paper, students often look at academic work done in biology, psychology, or political science, but since those works rarely refer to popular culture, it is incumbent on the student to make a compelling connection. In this paper I am really looking for students to pick one theme that has been covered in the course and pull together several examples from popular culture to frame the theme. Here students might choose a large idea such as whether it is a better strategy to cooperate or defect, or whether the concept of rationality of the characters can be used to distinguish horror films from suspense films, or how game theory and psychology can explain the bystander effect. Often students' thesis statements build on arguments they have made in one of their previous papers. I allow a fairly broad definition of "research" as there is not much academic writing connecting popular culture to game theory. Students are excited to be entering into an academic conversation where there are few experts before them. Every time I teach this course, I am impressed with the nuanced connections students make between popular culture and the academic field of game theory.

## 5.4 Reflections

I have taught an inquiry seminar on game theory in popular culture four times in seven years. I feel, overall, the course has been extremely successful. The accessibility of popular culture allows my students to bring their own expertise to the classroom. The focus on how mathematical models can be used to frame human behavior provides a rich context for having productive classroom discussions. Although my course need not have any quantitative aspect, and the level of engagement with rigorous mathematical ideas has depended on the students in the course, I have also used game theory to teach a quantitative reasoning course several times. I have developed an open access textbook for my quantitative reasoning course *Introduction to Game Theory: a Discovery Approach* [10]. A blend of the more quantitative material with the popular culture applications could satisfy a variety of first-year seminar structures. The text includes two sections on applications to popular culture with a series of possible essay questions.

One challenge as a professor is to let go of the idea that I need to be the expert. I may be completely unfamiliar with a particular popular culture reference. Instead I need to view my role as more of a guide. I suggest connections between math and popular culture as a model for the students to find their own examples. Since one of the main goals of an inquiry seminar is to develop an ability to write an argumentative essay, and, in particular, to back up a claim with evidence, students learn they need to convince me of their argument just as they do their peers.

I must admit that I was unsure at first about teaching a writing course. But I realize, after teaching it, that mathematicians are, in fact, well-trained to teach students to write argumentative essays. In our math classes we teach students to carefully connect ideas together and make sure their arguments follow logically from one idea to the next. This is exactly the skill students need to develop in their writing. First-year students often think that writing college essays requires complex "academic" language. But when they focus on writing clear arguments, their papers improve. Although I correct grammar and sentence structure in their papers, I focus on the content, connections, and flow of the argument. Thus, I do not feel I am teaching an English class, but rather helping them apply good logical structure to written work, something I do regularly in the math classes I teach. I also think it is important for students to see and integrate mathematical ideas outside of a "math" class. Too often students compartmentalize mathematics as something that only occurs in a math classroom. In a course such as this, they can see how mathematical arguments and ideas can integrate with areas such as social science and humanities. Additionally, they can see how, once they have access to some simple mathematical ideas, they can bring a richer analysis to their own writing.

When I first designed this course, I was warned not to make my essay prompts too broad as first-year students struggle to find a clear voice. I did not heed that warning and have allowed my students considerable freedom to find their own topic and popular culture references. Yes, I can get students who struggle to find a good question and back it up with strong evidence, but more often that not, I am truly amazed at the connections my students make between their favorite films, television series, and novels with the deep themes of the course. Sometimes my students are concerned about using a reference that I have not seen or read. I approach this by using the same conventions we would use in academic writing: they should be able to provide just enough context that I can see the connection. I should not have to see the entire film to see how they are using it as context or evidence for their claims. This takes practice for most students but is a broad principle that they can apply to writing throughout their college career. This also helps encourage the transition from the professor being the authority to the student as the authority. Once the student feels ownership over his or her argument, he or she can really begin to be part of an academic conversation though writing.

Given how many of the course themes relate to the film *The Dark Knight*, students often ask why I don't include it instead of, say, *Rebel Without a Cause*, especially since *The Dark Knight* has an iconic prisoner's dilemma in the ferryboat scene. While I agree that *The Dark Knight* provides ample relevant material, most of my students have seen it, and I have consciously chosen to require less familiar films since this provides a more equitable student experience, as all of the students are seeing, reacting to, and processing the film for the first time. With a more popular film, some students may have seen it dozens of times, while others not at all. Additionally, since I cannot show all the relevant films, students now have a wider range of films to apply to the theme. They are, of course, encouraged to include the material from *The Dark Knight* themselves, and many do. The least successful film is often *Rebel Without a Cause* as students have a hard time relating to it, as they find it to be dark, humorless, and rather dated. The relationships between the characters are somewhat subtle, so they have a hard time understanding the motivations of the characters. The last time I taught the course, I added *Footloose* and shifted the focus to a comparison of the two films. This seemed to work much better as the contrast in the films was easier for the students to discuss.

One of the goals of an inquiry seminar is to get students to find questions-at-issue in whatever they are studying. These are questions that the academic community (which might be the class itself) does not view as settled. They illuminate areas of disagreement. An academic essay is relevant if it can further the discussion on areas of disagreement. As a mathematics professor, I am often seen as trying to illuminate mathematical ideas that have long been settled by the academic community, but of course, mathematical research attempts to settle areas of disagreement (though we usually characterize these areas as "unknown"). The context of game theory, especially in how it relates to behavior, has easily approachable areas of disagreement, such as whether a character is rational or whether it is better to cooperate or defect. The biggest drawback is that since students are comfortable with these issues at the beginning of the course, some of them stay with the same general issue (such as rationality) throughout their writing assignments. This can work if they apply successively deeper arguments, but in one course I did have to "forbid" the topic of rationality for some of their later essays, short response papers, and discussion questions. Although students are often able to relate the concepts to their own lives, I would like to foster more discussions relating game theory to current events and societal issues. For example, discussions of cooperation and defection strategies in the US Congress, the volunteer's dilemma and climate change, and "the mad man" strategy in chicken as it relates to negotiations with North Korea. Although the class has been able to discuss these topics, the discussions would be much richer if the students had more familiarity with current events.

Overall, I have really enjoyed using game theory to teach writing. Since each student comes to the course with their own applications to popular culture, the material and their writing never seem stale. I am often excited by the connections that students see between the course and their own lives. I've had students decide to major in math because they see an application to a field of interest. An inquiry seminar is designed to teach students about engaging in the academic conversation, but this idea can be intimidating to first-year students, as they do not see themselves in a position to contribute to the conversation. Since so little academic work has been done at the boundary of mathematics (or biology, or psychology, or economics) and popular culture, students can feel that their voices and ideas contribute to the conversation in a substantial way.

## 5.5 Bibliography

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## Appendix

### Sample Activity: a Class-wide Prisoner's Dilemma

*This activity is used after the prisoner's dilemma game is introduced in class. I have students submit their responses by email before class, but this experiment can also work with clickers, or even notecards. Students then answer the questions about the results in small discussion groups.*

Each member of the class secretly chooses either C (cooperate) or D (defect). This letter then represents the student's strategy against each member of the class with payoffs as follows for each pair of players: if they both cooperate, they each get 3 points. If they both defect, they each get 1 point. If one cooperates and one defects, the cooperator gets nothing, but the defector gets 5 points. Each player then gets the sum of the points from each pair of games. For example, in a class of 20 students, if there are three cooperators and 17 defectors, each cooperator will get 6 points (3 from each of the other two cooperators), while each defector gets 31 points (3 from each of the three cooperators and

1 from the other 16 defectors). Note, if everyone cooperates, they each get 57 points; while if everyone defects, they each get 19 points. So everyone does better with more cooperators, but a single defector will always get more points than a single cooperator.

Once everyone has recorded their answers, the class can answer the following questions.

1. Summarize the class responses.
   (a) How many Cs were there?
   (b) How many Ds were there?
   (c) What was the payoff to each C?
   (d) What was the payoff to each D?
2. Determine the payoff matrix for the class-wide prisoner's dilemma. Note, although you played this game with each other person in the class, this is still a two-person game.
3. What are some reasons people chose C? What are some reasons people chose D?
4. Thinking about the idea of *rationality*, what appears to be the most rational choice, C or D? If everyone is *equally* rational, then what would everyone do? If everyone is equally rational, should everyone choose the same thing?
5. Now suppose everyone is equally (and perfectly) rational, *and* everyone knows that everyone else is equally (and perfectly) rational. What should everyone choose?
6. Consider the following game:

   | | C | D |
   |---|---|---|
   | C | (3, 3) | (0, 50) |
   | D | (50, 0) | (.01, .01) |

   What would you do? Why? What seems to be the most rational thing to do? Why?
7. Consider the following game:

   | | C | D |
   |---|---|---|
   | C | (1000, 1000) | (0, 100) |
   | D | (100, 0) | (100, 100) |

   What would you do? Why? What seems to be the most rational thing to do? Why?
8. Looking at all three of the above games, can you think of what sort of payoffs you would need in order to cooperate (C)? What about to defect (D)?

## Sample Activity: a Volunteer's Dilemma Experiment

*This activity is presented without any prior introduction to volunteer's dilemma. I have always used clickers or Poll Everywhere. Although the responses should be anonymous to the students, I give students actual course points for the last four questions.*

This activity needs to be played as a class. All players need to be able to respond without being able to see the responses of others. Answers may be revealed before moving on to the next question.

Without sharing your answers with others, select your answer to the following questions. Try to be as honest about your answer as possible.

1. The lights go out in the neighborhood. Someone needs to call the power company. If someone calls, everyone's lights go on.
   (a) Call
   (b) Don't call

2. The same as in (1), but now you have to wait on hold for 5 minutes.
   (a) Call
   (b) Don't call
3. The same as in (1), but now you have to wait on hold for 30 minutes.
   (a) Call
   (b) Don't call
4. The same as in (1), but now you have to pay a $.50 service fee.
   (a) Call
   (b) Don't call
5. The same as in (1), but now you have to pay a $2.50 service fee.
   (a) Call
   (b) Don't call
6. The phone lines go down in your small mountain community. You have to hike three miles in the snow to notify the power company.
   (a) Hike to notify the phone company
   (b) Stay home and let someone else do the hiking
7. Everyone in your class cheats on an exam. The professor says if someone confesses they receive an F. If no one confesses, everyone fails.
   (a) Confess
   (b) Don't confess
8. Evil Dr. No captures the class and puts you in all in a shark tank separated so you can't communicate. If one person volunteers to be eaten then the rest go free. If no one volunteers after 10 minutes all get eaten by sharks.
   (a) Volunteer
   (b) Don't volunteer
9. Evil Dr. No captures your family and puts you in all in a shark tank, separated so you can't communicate. If one person volunteers to be eaten then the rest go free. If no one volunteers after 10 minutes all get eaten by sharks.
   (a) Volunteer
   (b) Don't volunteer
10. For any *Big Brother* [2] fans: choose to eat all your favorite foods for a week or nasty "slop" for a week. If at least three people say slop, everyone gets what they asked for. Otherwise everyone is on slop.
    (a) Favorite foods
    (b) Slop
11. OK, now let's get serious about this. Answer 5 points or 1 point. If at least one person says 1 point, everyone gets the number of points they chose. Otherwise, everyone gets 0 points. (These are *real* homework points!)
    (a) 5 points
    (b) 1 point
12. Answer 20 points or 1 point. If at least one person says 1 point, everyone gets the number of points they chose. Otherwise, everyone gets 0 points.
    (a) 20 points
    (b) 1 point

13. Answer 6 points or 5 points. If at least one person says 5 points, everyone gets the number of points they chose. Otherwise, everyone gets 0 points.
    (a) 6 points
    (b) 5 points
14. Answer 20 points or 1 point. If at least five people say 1 point, everyone gets the number of points they chose. Otherwise, everyone gets 0 points.
    (a) 20 points
    (b) 1 point

## Sample Activity: Comparing the Class-wide Prisoner's Dilemma and the Volunteer's Dilemma

*This activity has students compare the games in the two previous class activities. It is presented after a discussion about what it means to cooperate or defect in prisoner's dilemma and volunteer's dilemma.*

In class we played a game called the volunteer's dilemma. One example was everyone choses "1 point" or "5 points." If at least one person writes down 1 point, then everyone gets the number of points they wrote down. If no one chooses 1 point, then everyone gets 0.

The purpose of this activity is to compare the class-wide prisoner's dilemma with the volunteer's dilemma.

Recall that in the class-wide prisoner's dilemma, defectors got 1 point for each other defector, and 5 points for each cooperator. Each cooperator got 0 points for each defector, and 3 points for each other cooperator.

1. In the class-wide prisoner's dilemma what effect does one defector have on the group? (You can answer this in terms of points.)
2. In the class-wide prisoner's dilemma what effect does everyone's defection have on the group?
3. In the class-wide prisoner's dilemma what effect could your own defection have on the group?
4. In the volunteer's dilemma what effect does one defector have on the group? (You can use the example above or one of the more extreme ones from class.)
5. In the volunteer's dilemma what effect does everyone's defection have on the group?
6. In the volunteer's dilemma what effect could your own defection have on the group?
7. Considering your answers above and to previous work with the prisoner's dilemma, in the class-wide prisoner's dilemma, which is more rational to be a cooperator or a defector? Why?
8. Whichever you chose in (7), explain what would happen if everyone was the most rational. Is it now more rational to do the opposite?
9. Considering your answers above, in the volunteer's dilemma, which is more *rational* to be a cooperator (volunteer) or a defector? Why?
10. Whichever you chose in (9), explain what would happen if everyone was the most rational. Is it now more rational to do the opposite?
11. The volunteer's dilemma can also be called "class-wide chicken." Try to describe this class-wide game in terms of "swerving" and "going straight." How do the payoffs for the volunteer's dilemma relate to the payoffs for chicken?

## Sample Schedule

*This is a schedule of topics for the course, including suggested readings and films.*

| Class | Reading | Discussion |
|---|---|---|
| Day 1 | | Intro |
| Day 2 | "Battles of Wits" | Assumptions of game theory |
| Day 3 | | Developing questions in game theory |
| Day 4 | *Prisoner's Dilemma*, Ch 1, 2 | Dilemmas |
| Day 5 | *Prisoner's Dilemma*, Ch 3 | Zero-sum games, minimax theorem |
| Day 6 | *Prisoner's Dilemma*, Ch 4, 5 | Game theory and war |
| Day 7 | | *Dr. Strangelove* |
| Day 8 | | *Dr. Strangelove* |
| Day 9 | | Examples of game theory in movies or TV |
| Day 10 | | Examples of game theory in movies or TV |
| Day 11 | *Prisoner's Dilemma*, Ch 6 | Prisoner's Dilemma |
| Day 12 | | Workshop thesis for Essay 1 |
| Day 13 | | Peer review of draft essay |
| Day 14 | *Prisoner's Dilemma*, Ch 8 | Do people really behave this way?<br>*Friend or Foe* [6], [12] |
| Day 15 | | *Return to Paradise* |
| Day 16 | | *Return to Paradise* |
| Day 17 | | Practicing the naysayer, how to introduce objections |
| Day 18 | | Why does game theory matter?<br>Applications to political and social situations |
| Day 19 | *The Mind Game* | *The Mind Game* |
| Day 20 | | *The Mind Game* |
| Day 21 | | Workshop thesis for Essay 2 |
| Day 22 | | Peer review of draft essay |
| Day 23 | *Prisoner's Dilemma*, Ch 10 | Chicken |
| Day 24 | | *Rebel Without a Cause, Footloose* |
| Day 25 | *Prisoner's Dilemma*, Ch 11 | *Rebel Without a Cause, Footloose* |
| Day 26 | "Game Theory and Literature" | Writing a research paper |
| Day 27 | *Prisoner's Dilemma*, Ch 12 | Volunteer's dilemma |
| Day 28 | "Tale of Happiton" | Applications and implications of "Happiton" |
| Day 29 | | Workshop thesis for final paper |
| Day 30 | | Mixed strategies, repeated games |
| Day 31 | | Peer review of draft paper |
| Day 32 | | Cooperative games with more than two players |
| Day 33 | | Final Presentations |
| Day 34 | | Final Presentations |
| Day 35 | | Final Presentations |

Note, all the films are viewed outside of class time. I reserve our library viewing room for one showing of each film, but students are welcome to watch the films on their own. The films are on reserve in the library, where the students can check them out for 2 hours or view them individually in the library.

# 6

# A First-Year Seminar on Decision Making

Pamela B. Pierce

**Abstract** The focus of my first-year seminar, titled *Decisions, Decisions*, is on the thought processes and mental tendencies related to various forms of decision making. Because the topic of the seminar is so broad, I can bring up almost any controversial topic and have the students debate and discuss the issues surrounding this topic in class. Wanting to incorporate some mathematical ideas into the course, I have created several activities focused on the prisoner's dilemma, basic game theory, elementary probability, and voting theory that are appropriate for first-year students and that tie in with the course theme. Here I share five in-class activities that I have found particularly effective for incorporating these mathematical themes into my first-year seminar.

## 6.1 Background and Context

The College of Wooster is a residential liberal arts college in northeast Ohio with approximately 2,000 undergraduate students. We enroll a diverse student population from 47 states and 45 different countries. Approximately 20% of the students are domestic students of color, while 13% are international and 55% are female.

Every year Wooster offers 37 to 39 sections of First-Year Seminar (FYS)—enough to accommodate the incoming class of students in sections of 15 students each. The seminars, organized through the office of the Academic Dean, do not have a home in any particular academic department and do not count toward any other graduation requirements other than the FYS requirement. The instructor for each FYS class serves as the academic advisor for the students enrolled in the course, which allows for frequent contact between student and advisor. These seminars serve as the foundation in a scaffolded curriculum that prepares each student at Wooster to complete an Independent Study thesis or project in their senior year [9].

The FYS program runs in the fall semester, which lasts for 14 weeks. Wooster does not count credit hours, but each FYS counts for one credit in a curriculum requiring 32 credits for graduation. Instructors have some flexibility when choosing the time slot for the class, with some choosing to meet twice a week in 80-minute sessions, and others choosing to meet three times a week in 50-minute sessions. My class meets on Monday, Wednesday, and Friday for 50 minutes at a time.

The goals for the FYS program include introducing the students to critical reading, writing, and thinking and developing skills in effective communication, information literacy, and argumentation. Instructors often devote some class time to introducing the students to various campus resources and to discussing specific elements of transitioning to college-level academic work. Overall, the class is supposed to be discussion-based, so that students can share ideas, learn from one another, and find ways of making strong arguments to support a point. With this broad agenda, instructors are given the freedom to design their own course, choosing any accessible topic, provided that there are no prerequisites attached to it. While the instructors also choose their own readings, films, assignments and activities, there are guidelines for the quantity of writing required in each FYS section and the number of assignments that must incorporate a revision process.

## 6.2 Mathematical Theme

The broad topic of my seminar concerns how people go about making decisions, and this theme allows me to guide the course in a variety of directions. One important advantage of this topic for me is the opportunity to incorporate some mathematical themes and ideas into the course. While my class is not a math-focused seminar, I am able to engage the students in several fun mathematical activities that deal with decision-making. I would estimate that 20% of class time is spent introducing mathematical concepts to the class. The mathematical topics that I incorporate are specifically chosen to fit into the theme of making decisions. For example, as we discuss how individuals make decisions, it is natural to bring in a unit on game theory. We can then ask: is there a systematic or logical way that a player can decide what strategy to choose when playing a certain game? As we move on to discuss how groups can arrive at a collective decision, various voting systems naturally come up and we do a short unit on the mathematics of voting theory.

I intentionally do not put any hints that the seminar will deal with mathematics in the title of the course. Once I have a group of interested students, I can incorporate these math-themed units into the class in a natural way. There is no mathematical background assumed or required. Throughout the course, students begin to realize that there can be some mathematics behind our individual and collective decision making, and that mathematics is more a part of our lives than we often think it is. One of my goals for the seminar is for students to leave my classroom with an awareness of the ways in which the tools found in mathematics can help with decision making, critical thinking, and logical reasoning.

## 6.3 Course Structure

This course aims to address the following questions: “How do individuals make decisions?” and “How do groups make decisions?” This natural breakdown works well and provides a bit of structure to the semester. The central textbook, *Thinking Fast and Slow* [8], is written by behavioral economist Daniel Kahneman. Other readings include excerpts from *How We Decide* by Jonah Lehrer [11], *The Paradox of Choice* by Barry Schwartz [11], *The Power of Habit* by Charles Duhigg [2], *The Other Wes Moore: One Name, Two Fates* by Wes Moore [12], and several short articles (e.g. [1], [3]).

Most of the class meetings are spent engaging students in a class discussion of the readings or films. Because writing is an important part of the course, I typically spend one day per week discussing the writing process and having the students participate in exercises and workshops designed to improve their writing skills. Further information on the writing assignments is provided in Appendix 6.4. In the subsections below I describe some of the math-themed activities that I incorporate into the class. The first few subsections deal with individual decision-making, while the last deals with group decision-making. An abbreviated course schedule is provided in Appendix 6.4 and presents the timing of these activities in relation to other course activities and assignments.

### 6.3.1 The Prisoner’s Dilemma: Dice Battle and “Split or Steal”

The prisoner’s dilemma is a well-known game in which two individual players must choose to either “cooperate” or “defect,’” and the outcome of the game depends on the selections made by both players. It is heavily studied in game theory as well as in many other fields because it can be used to model numerous situations involving cooperation. Since the dilemma can be expressed in several different ways, the options for the individual players may be expressed as “cooperate” vs. “defect,” “remain silent” vs. “betray,” or, as in the following example, “split” or “steal.”

Early in the semester, as the students are still getting to know one another, I schedule a day for all of us to play “dice battle.”[1] The idea is simple: students roll a die and compare their roll to their opponent’s roll. The higher roll is declared the winner, while a tie is considered a “do-over.” After nine rolls, the player with more wins moves on to the next round of the battle, and we continue this elimination strategy until we have a single class winner, who wins a small gift card to a local coffee shop. The dice battle itself is very simple, but it is not the *real* game.

[1]This idea was shared with me by Dr. Dean Fraga, Professor of Biology at The College of Wooster, who developed the game for his FYS.

Just as we are about to begin the dice battle, I interrupt and tell them that there is another game that we will play first: the "Split" or "Steal" game. This provides an opportunity for each student to earn some bonus points that can be added to each die roll during the dice battle, and this is where the prisoner's dilemma comes into play. Each student will pair up with another student who is not their opponent in the first round of the dice battle. Each team of students is awarded two bonus points, and by playing the "Split or Steal" game, each pair will decide how the bonus points will be distributed. Each student writes down on a hidden piece of paper whether they want to "split" or "steal" the two points that are up for distribution. If both players agree to split the points, then they each get one point to use in all upcoming dice battles in the competition. If both students write "steal" on the paper, then neither one gets any points. And, if one person writes "split" while the other writes "steal", then the stealer walks away with both of the points and the other person gets nothing.

Once the "Split" or "Steal" game is finished, we go back and play the dice battle as planned, except that some students now have one or two bonus points that are added to the number on their die roll on every turn in the battle. Students immediately see how having additional bonus points is a tremendous advantage in the dice battle. If the activity works as planned, then the ultimate winner of the dice battle game is someone who chose to "steal" while their partner chose to "split," and consequently had two bonus points to help them throughout the competition.

We play the dice battle game several times, changing the way that the "Split or Steal" pre-game is played each time. The second time, for example, every person makes a play—either "split" or "steal"—but they do not know who their opponent will be. Each student writes their name and their play on a folded slip of paper, and I randomly choose two slips of paper simultaneously for each "contest."

This activity really brings the prisoner's dilemma to life for the students. When we have our follow-up discussion, there are a number of interesting questions that can lead to a lively classroom discussion. Are some people more predisposed toward cooperation? Does greediness always pay off? Does it help to know your opponent? What is your strategy when you don't know your opponent? Why did some of you choose to "steal" when your opponent was unknown, and "split" when your opponent was sitting next to you? If your choice were to remain unknown to others, would that affect your play? What are some of the applications of this dilemma to other fields of study? While there are numerous games that can be found online to illustrate the prisoner's dilemma, this one is relatively quick, simple, and enjoyable for the students [6].

### 6.3.2 Game Theory: Nim and The Dollar Auction

After our unit on the prisoner's dilemma, I want to get students thinking more broadly about two-player games and developing a winning strategy whenever possible. The classic game of Nim provides a nice introduction to strategic games. I have the students pair up and play a particular version of Nim. We begin with a pile of 21 beads, and a "turn" consists of a player removing either one, two, or three beads from the pile. The person who takes the last bead loses the game. Students take turns acting as Player One and Player Two. Given enough time and some guidance, students can often discover that there is a winning strategy for Player Two. As a concluding exercise related to Nim, I have the students do an in-class writing exercise where they must write a clear explanation of the winning strategy for Player Two.

Shortly after the exploration of Nim, I have a day where the students take turns pairing up with different opponents to play tic-tac-toe, dots and boxes, and rock-paper-scissors. Students must determine which games have a winning strategy for one or both players. If one can be found, they must write it out and one team presents that strategy to the class. If there is no winning strategy for a specific game, one of the groups presents to the class their explanation as to why no such strategy exists. In a follow-up class session, I provide some of the basics of game theory, introducing a definition of a two-person, zero-sum game [4, pp. 467-487]. I then have the students work in teams to identify which games played in class fall into this category. As before, one team presents their findings and explains why a given game fits the definition of a two-person zero-sum game.

A frequent assumption in the field of game theory is that players always act as "rational agents"—that is, they will always choose a strategy that maximizes their own personal gain, or utility. As we saw in the prisoner's dilemma, however, when both players attempt to maximize their individual utility, a suboptimal outcome follows. To further explore whether a player should always act as a "rational agent," I use a non zero-sum sequential game called the Dollar Auction designed by economist Martin Shubik to illustrate a paradox in rational choice theory [16]. To begin, I offer up a dollar bill to the class. In round one, everyone places a bid (secretly, on a piece of paper) stating how much they would be willing to pay for the dollar (in five-cent increments only). The highest bidder wins the dollar, and the second highest bidder must pay the money that they bid, but gets nothing. This unusual game is confusing at first, but after playing several rounds, students typically learn to bid either very high or very low. We then discuss the various strategies adopted by each student, and we ask whether each person was acting as a rational agent or not.

Round two of the Dollar Auction is the more typical version envisioned by Shubik. In this round, we use the same rules, but we proceed as an auction. If one player bids 10 cents, then there is an incentive for another player to bid 15 cents, because there is a chance to get a dollar for 15 cents. Of course, if this happens, then the player who bid 10 cents has the incentive to bid 20 cents, so as not to lose 10 cents by being the second highest bidder. If players continue to act as "rational agents," this will continue until the bidding reaches more than one dollar! Of course, this quickly becomes ridiculous, and therein lies the paradox. It is a good way to illustrate what it means to act as a rational agent, and it can lead to lively classroom conversations.

### 6.3.3 Some Basic Probability: The Linda Problem

In his book, Kahneman describes research that he and his colleagues have conducted in order to learn more about the human brain and its thought processes. I have successfully turned some of these research descriptions into in-class activities that I use before the students have been assigned to read that part of the text. A classic example is "The Linda Problem," developed by Amos Tversky and Daniel Kahneman to illustrate how our notions of "representativeness" can often contradict the laws of probability. I read the following (rather outdated) description of Linda to the class:

> Linda is thirty-one years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations [8, p. 156].

After this background information about Linda is presented, the students are then asked to rank the following scenarios for Linda based on their likelihood, or probability.

1. Linda is a teacher in an elementary school.
2. Linda works in a bookstore and takes yoga classes.
3. Linda is active in the feminist movement.
4. Linda is a psychiatric social worker.
5. Linda is a bank teller.
6. Linda is a bank teller and is active in the feminist movement.

What happened in Tversky and Kahneman's research, and what is repeated in my classroom, is the following: students rank scenario 6 as more likely than scenario 5, even though most mathematicians can see that the probability of scenario 6 is less than the probability of scenario 5. Students are making a representativeness heuristic error, and helping them to see this can aid with their logical thinking. When I ask students to describe how they ranked the above scenarios for Linda, they typically reply with comments such as, "When I pictured Linda in my head, she was more of a bookstore/yoga kind of person," and "I envisioned Linda as doing something meaningful—for a cause that she believed in." When I probe further, and restrict students' attention to scenarios 5 and 6, students note that a Linda who works as a bank teller and remains active in the feminist movement is more consistent with their vision of Linda than a Linda who simply worked as a bank teller.

While all of their comments make sense, I want students to see that scenario 5 has to have a higher probability than scenario 6. To help the students see this, I ask the following question: Suppose that there are 100 people who fit the description of Linda above. If you had to guess the fate of those 100 people, about how many would you place into each of the 6 categories above? Now the students begin to see that the sets are not mutually exclusive. I send the teaching assistant to the board to draw a Venn Diagram. He asks the students which sets could intersect with other sets, and this helps him to draw the diagram accurately. Once a decent Venn Diagram is on the board, I ask students to fill it in with some hypothetical numbers that describe the fates of the 100 people fitting Linda's description. I write the terms "Venn Diagram," "subset," "intersecting," and "mutually exclusive" on the board, and students give interpretations (but not formal mathematical definitions) of all of these terms. I inform the students that these terms are fundamental when one studies set theory or elementary probability. If all goes well, students begin to see that if A is a subset of B, then the probability of being an element of set A is less than the probability of being an element of set B. They discover the flaws in their thinking and learn a little bit about probability.

### 6.3.4 Voting Theory: A Fake Election

Voting is one way that a large population of people can make a decision together. We begin this unit by discussing how an individual chooses to vote. Since the students tend to be around 18, we discuss what factors might influence their voting choices: parents, friends, social media, etc. I then ask the students how we might find the best method for aggregating all of those individual preferences into a population-wide decision. To investigate the different ways of arriving at a group decision, I incorporate the following activity on voting theory into the class.

I pull some material from *For All Practical Purposes: Mathematical Literacy in Today's World* [4], a publication of the Consortium for Mathematics and its Applications (COMAP). To engage the students in the lesson, I have them "act out" many of the voting scenarios. For a hypothetical election, I give goofy names to the candidates, and I give each student a paper with a set of candidate rankings that represent their individual voting preferences for the day. Each student gets an opportunity to share their voting preferences with the group, and this often becomes an enjoyable experience, with students creating fake reasons for supporting their fake candidates in the fake election.

I ask students to brainstorm on the type of voting system we should use, and I ask them to think about some of the desirable properties of a voting system. Ultimately, I introduce them to multiple voting systems, including plurality voting, Condorcet's method, sequential run-off, sequential pairwise voting, the Hare system, and Borda count methods [4, pp. 285–306]. The preference sheets given to each student are rigged so that the different voting systems yield different winners. Students begin to see how it might be advantageous, under certain voting scenarios, for someone to make an insincere vote for a candidate who is not their first choice. They are also able to identify the numerous problems that exist within certain voting systems. Arrow's Impossibility Theorem [4, p. 301], which states that with three or more alternatives, there does not exist a voting system that always produces a winner and satisfies some desirable properties of a voting system, is a highlight of this section.

## 6.4 Reflections

This FYS, *Decisions, Decisions*, has been the most successful of the five different versions of FYS that I have taught while at Wooster. The title served to intrigue students and draw them into the course. The students were very eager to discuss their personal experiences with decision-making, so the subject allowed for some good "get to know each other" conversations.

While it was not a mathematics-based seminar, the course allowed for the introduction to several interesting mathematical concepts. The students were engaged with the mathematics that was incorporated into the class, and they enjoyed the notion of a paradox that appeared several times throughout the semester. They genuinely seemed to learn some of the basic concepts in probability and game theory, and the seminar might have inspired some students to pursue these topics further.

I liked the interdisciplinary feel of the course. We sometimes discussed topics that could be categorized as psychology or behavioral economics, which allowed me to explore outside my area of expertise a bit, and learn about fields that I don't often have the chance to study myself. There was also a great deal of variety in the class. We read a personal story, a behavioral economics book, and some popular books about decisions. We watched a movie, *Owning Mahowny* [13], we watched a podcast [16], and we read recent articles about teenagers and decision-making [1], [3]. A lot of the content felt very relevant to the students' lives, which was also an advantage.

There are several improvements that I hope to make the next time that I teach the course. First, I would like to invite one or more guest speakers into the class to discuss the fields of behavioral economics and psychology, fields from which a lot of the course content is drawn. Secondly, while the mathematics topics are engaging and fun, I did not have enough assessment tied to these activities. Most of the grades related to these activities were points for participating actively in the class sessions. Next time, I plan on creating some graded homework assignments that are related to the mathematical topics discussed in class. Further, at the end of the semester I would like to ask students to write an essay describing how mathematics relates to critical thinking and decision making.

The students seemed fairly pleased with the seminar–even those who were surprised by the incorporation of mathematical content. While I don't think that I was able to convince any students to major in mathematics, they certainly seemed intrigued by aspects of the course–particularly the prisoner's dilemma. I look forward to teaching this course again.

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# Appendix

## Writing Assignments

*There were five major writing assignments in this course, each ranging from three to six pages in length. All of these papers went through a drafting process. For essays 1 and 4, the initial draft was read by the professor; for essay 2, the initial draft was read by the student teaching assistant; and for essays 3 and 5, the students participated in a peer-review process, receiving feedback from two other members of the class before making revisions (and the option of meeting with the teaching assistant for further feedback). These writing assignments directly address the college requirement that students learn about critical reading, writing, and the drafting process. What follows are abbreviated versions of the paper assignments.*

- Essay # 1: *The Other Wes Moore.*
  Prompt: As you reflect on *The Other Wes Moore* [12], think about the reasons why these two young men, who grew up in similar situations, ended up following two completely different life paths. There were some very important decisions that were made by different characters in the book. Describe which decision or decisions were the most influential and explain why these decisions were pivotal in the lives of the two young men.

- Essay # 2: A Personal Decision.
  Prompt: Identify an important, nontrivial decision that you have faced in your life, and discuss why the decision was important. What were the consequences of the decision? What made the decision a difficult one? What process did you go through in order to help you come to a decision? Did you make a list of pros and cons? Did you talk with other people? Was there one particular insight that allowed you to make the decision clearly? Do you think that you made the decision that was right for you at the time?

- Essay # 3: Ways of Describing the Brain.
  Prompt: We have read about multiple ways of describing how the brain operates. In *Thinking Fast and Slow*, Kahneman [8] describes the brain as having a "System 1" and a "System 2." Lehrer, in his book *How We Decide* [11] describes how we have a "rational brain" and an "emotional brain." In some ways, however, Lehrer believes that the instantaneous decisions made by a talented quarterback, for example, are still mysterious. Plato envisioned the brain as a "chariot being pulled by two horses," one of which represents "negative, destructive emotions" [11, pp. 9–10]. Choose one of these ways of describing how the brain operates and describe it in detail. Then thinking back to the decision described in essay # 2, describe your decision-making progress in terms of this way of thinking about the brain.

- Essay # 4: The Teenage Brain.
  Prompt: In our readings from last week (see [1], [3]), we found that teenagers seek out more thrills, take more risks, and value rewards more than at any other point in their lives. These traits can lead teens to make unwise decisions, such as driving while distracted, trying addictive drugs, and engaging in unprotected sex, among others. What is the best way to ensure that children get through these teenage brain years safely, developing into mature adults with strong decision-making skills? Is this the responsibility of the parents? the school? someone else?

- Essay # 5: Are Decisions at the Root of All of Our Actions?
  Prompt: In this course we have seen examples of the following:

  - Wes Moore, who got involved in drugs and violence, and allegedly committed murder [12]
  - The young woman who was a meth addict, in *The Narrative of Privilege* [10]
  - Ann Klinestiver, who developed a gambling addiction while on Requip, a drug for Parkinson's disease in the book *How We Decide* [11]
  - Gambling addict Dan Mahowny, in the movie *Owning Mahowny* [13]
  - Gambling addict Angie Bachmann, in *The Power of Habit* [2]
  - Brian Thomas, who killed his wife in the middle of the night, reportedly while sleepwalking in *The Power of Habit* [2]

  There seem to be times when people take actions that are not rooted in a conscious decision-making process, but instead on some other brain-based process such as biologically-based addiction, ingrained habits, or brain damage. Where do we draw the line between actions that follow from decisions and actions that are not the result of decisions? You should discuss some of the examples above, and you may wish to bring in your own examples. Please discuss at least three examples, and be sure to address addictive behavior in at least one of your examples.

## Brief Course Outline

*Below is a general course outline for Decisions, Decisions:*

| Week | What is due? | In-Class |
|---|---|---|
| 1 | Moore [12], whole book (summer reading assignment)<br>First version of Essay #1 | M: Group Discussion (GD): Why Wooster? Rules for group discussion.<br>W: Bring draft of Essay #1 to class. Discuss thesis statements.<br>F: (GD) What decisions are responsible for the fates of the two Wes Moores? |
| 2 | Kahneman [8], Chapter 1<br>Final version of Essay # 1 | M: Dice battle/Split or Steal—illustration of the prisoner's dilemma.<br>W: Bibliographies, citations, and formatting a paper.<br>F: Description of prisoner's dilemma. Watch Radiolab: *Golden Balls* [7]. |
| 3 | Lehrer [11], Intro and "The Quarterback in the Pocket" | M: (GD) How does a good quarterback make a decision? Introduce Essay # 2.<br>W: Writing a good intro paragraph. Begin drafting Essay #2.<br>F: Group activity on applications of the prisoner's dilemma. |
| 4 | "The Narrative of Privilege" [10]<br>First version of Essay # 2 | M: Brief presentation on iterated prisoner's dilemma. (GD) Discuss article.<br>W: Time management workshop with staff from the Learning Center.<br>F: (GD) Further article discussion, decisions made early in life, choosing crime. |
| 5 | Kahneman [8], Chapter 2<br>Final version of Essay # 2 | M: (GD) Discuss *System 1* and *System 2*. Compare Kahneman and Lehman.<br>W: Avoiding common grammar mistakes and eliminating excess words.<br>F: Play Nim in teams. Clearly articulate winning strategy for one player. |
| 6 | Kahneman [8], Chapter 3<br>First version of Essay # 3 | M: Reading quiz. (GD) Discuss the reading. Kapernick's decision.<br>W: Peer review of Essay # 3.<br>F: Tic-tac-toe; dots and boxes; rock, paper, scissors. Winning strategies? |
| 7 | Kahneman [8], Chapter 4<br>Final version of Essay # 3 | M: Two-person, zero sum games: definition, examples; (GD) discuss reading.<br>W: Mental health resources. Meet with peer mentor to discuss spring courses.<br>F: The Dollar Auction. (GD) Do people always act as rational agents? |
| 8 | Kahneman [8], Chapter 5<br>Read 2 articles on the teenage brain [1], [3] | M: The Linda Problem.<br>W: Thinking about your major, planning ahead, using summers effectively.<br>F: (GD) Discuss teenagers and decisions. What is your experience? |
| 9 | Lehrer [11], "Story of Anne K."<br>First version of Essay # 4 | M: (GD) Teenagers and decision-making. Are the authors too harsh on teens?<br>W: Workshop with our librarian partner: scholarly articles and how to find them.<br>F: (GD) Discuss the reading from Lehrer book (Requip, Parkinson's, addiction). |
| 10 | Research assignment on gambling addiction.<br>Final version of Essay # 4 | M: Watch *Owning Mahowny*.<br>W: Watch *Owning Mahowny*.<br>F: (GD) How do decisions factor into this movie? Similarities with Anne K.? |
| 11 | Duhigg [2], Chapters 3, 4, 5 | M: (GD) Continued discussion of gambling addiction.<br>W: Director of Off-Campus Study visits to discuss spending a semester abroad.<br>F: (GD) Discuss the sleepwalker who murdered his wife. Is this a defense? |
| 12 | Read handouts on voting theory | M: (GD) How did we develop rules for our class discussions?<br>W: Fake election. Support your candidates!<br>F: Presentation on voting theory—different methods. |
| 13 | Kahneman [8], Chapter 6<br>First version of Essay # 5 | M: Finish voting theory. Discuss Arrow's Impossibility Theorem.<br>W: Peer review of Essay # 5.<br>F: Review math topics. Does math help with logical thinking? |
| 14 | Schwartz [11], Chapters 1, 2, 3<br>Final version of Essay # 5 | M: How much is too much? Listen to Schwartz on Radiolab. Do you agree?<br>W: (GD) How does abundance affect our decision-making in the US? elsewhere?<br>F: Pizza and course wrap-up. |

# 7

# “I Signed Up For A First-Year Seminar, But I Got A Math Class”: The Challenges of *Chance*

Mark Bollman

**Abstract** *Chance* is a first-year seminar taught eight times at Albion College that focuses on applications of probability and statistics for the educated citizen. The course combines in-class activities with Minitab exercises and culminates with a field trip to a local casino, where students experience the interplay between theory and practice. Because the seminar includes mathematics, it has been challenging to attract students to *Chance*. In this article, I describe several in-class activities and laboratory exercises that help to incorporate some basic probability and a discussion of risk into the course.

## 7.1 Background and Context

The First-Year Seminar (FYS) program at Albion College, a four-year Michigan liberal arts college currently enrolling about 1,550 students, has been in place for over 15 years. One seminar is required of all entering freshmen. FYSs at Albion are 1-unit courses, equivalent to four credit hours. While the original intention was to provide an introduction to an academic discipline in a small (16-student maximum), discussion-based, writing-intensive class, this has changed over time to increase the focus on making the transition from high school to college academic life. This is facilitated in part through a student mentor for each seminar, who is assigned to work with the students on transition issues in one 50-minute group meeting per week separate from the class. First-year seminar instructors serve as their students' academic advisors through at least their first semester at Albion. FYS instructors at Albion are free to design courses around the content of their choice; a faculty committee provides guidance to new instructors and must approve courses before they are taught.

These seminars are drawn from all academic departments, with topics chosen by the faculty member teaching the course. *Chance* is a seminar I have taught eight times that focuses on applications of probability and statistics for the educated citizen, and can trace its origin to a similar course developed at Dartmouth College. Applications include sports, medicine, law, surveys and sampling, gambling, and pattern detection. *Chance* meets four days per week, for 65 minutes each day, over a 15-week semester. Most weeks, one class session meets in a computer lab for experiments using Minitab statistical software, where students generate and analyze data and prepare formal lab reports. Since the aspects of the first-year experience more properly called “adjustment to college life” are largely covered with the student mentor, approximately 95% of our classroom work focuses on mathematics. The remaining 5% of class time is devoted to general discussion related to academic advising prior to individual advising meetings with each student.

## 7.2 Mathematical Theme

*Chance* covers elementary probability and statistics at a college algebra level. A fundamental idea that we develop early in *Chance* is the notion of *randomness*. The course begins right after the College's matriculation ceremony on the Friday before the semester starts, when the students are presented with the "Based on the toss of a fair coin, either flip a coin 200 times or imagine 200 coin tosses" challenge. The students' results are examined as the centerpiece of the first class day's events. Course readings and activities return to the question of randomness over the course of the semester; one early in-class demonstration uses uranium glass, a Geiger counter, and a computer microphone to illustrate a genuinely random, rather than pseudorandom, process.

Albion provides considerable funding for FYS travel; other seminars have traveled to such distant places as Cameroon, France, Niagara Falls, and Yosemite National Park. The capstone of *Chance* is a day trip to the Soaring Eagle Casino in Mount Pleasant, Michigan, where students get a hands-on opportunity to compare randomness in theory and practice as it presents in games of chance. This trip is scheduled for November, once all of the students have turned 18—the minimum gambling age at this tribal casino. The College funds vans for the 2-hour drive to Mount Pleasant as well as dinner at the casino. Students are not required to gamble, though nearly everyone does to at least a small extent.

## 7.3 Course Structure

In *Chance*, most non-lab days carry a reading assignment; these classes begin with a short three-point reading quiz which serves as an attendance check and emphasizes the importance of doing the reading. Students are invited to submit questions for these quizzes; a question used on the quiz gains its submitters a bonus point and increases the likelihood that they'll get that question right. Readings include both academic and popular books dealing with uses of probability and statistics. The reading list for *Chance* has changed several times over the eight times I've taught the course; the most recent list follows:

- *Basic Gambling Mathematics: The Numbers Behind The Neon*, Bollman [3].
- *Taking Chances: Winning With Probability*, Haigh [6].

  These two books form the mathematical backbone for the course. One challenge I faced in teaching *Chance* was finding the "right" probability textbook, with the topics I cover in class but without too much extra material. After trying several books that didn't have the desired mix of topics, I have written my own textbook, *Basic Gambling Mathematics*. *Taking Chances* has been in the rotation every year and includes some fairly involved mathematics, including elements of game theory, among its discussion of games, though it is not primarily a mathematics textbook. Mathematical homework assignments are drawn from these two books and from locally-written problems.

- *The Numbers Game*, Blastland and Dilmot [2].

  One point I try to stress in teaching *Chance* and talking about the course with students and colleagues is "It's about more than just gambling." In the promotional material for the course, the casino field trip is not mentioned and gambling is merely listed as one among several applications of probability and statistics to be discussed. Another way I expand the course beyond gambling is to include a non-textbook on the reading list that describes other applications of probability and statistics, usually without a lot of equations. The exact book has varied over time as new books are published; this self-described "commonsense guide to understanding numbers in the news, in politics, and in life" currently fills that slot.

- *Fortune's Formula*, Poundstone [9].

  This book describes Edward O. Thorp's work with Claude Shannon on strategies for counting cards at blackjack and developing wearable computers to predict the outcome of roulette spins, and continues—as Thorp did—to consider investments as a more serious subject where randomness is present but can be overcome by careful thinking. An extended short essay question on the similarities between investments and gambling is released to the students in advance and comprises 50% of the third hour exam.

- *How To Win More: Strategies for Increasing a Lottery Win*, Henze and Riedwyl [7].

  One week of *Chance* is designated "Lottery Week", and includes close study of lotteries such as Powerball and daily number drawings. This book includes a discussion of the mathematics behind techniques to choose

Powerball numbers that decrease one's chances of having to share the jackpot in the unlikely event that one matches all of the drawn numbers.

- *Slot Machine Strategy: Winning Methods for Hitting the Jackpot*, Symms [11].

  Some students prefer to play slot machines rather than table games while on the casino trip; most students look over the wide selection of machines on offer at least briefly. This book goes into some detail about slot machines and money management strategies useful for those who choose to play them. While nothing can increase a player's chance of winning on a given machine, there are ways to select a machine and choose paylines or denominations that can improve the likelihood of walking away with some money. The College has purchased a slot machine for my use in teaching this course, which we use in an in-class activity. We also cover video poker when completing this unit.

Albion includes a Common Reading Experience as part of the FYS program. However, the chosen book has never been a good fit for *Chance*. Students discuss the book with their student mentor and attend a convocation based on it, but I don't work with it in class.

A few *Chance* sessions are necessary lectures on the more technical matters under consideration, but most non-lab classes include an in-class activity connected to randomness. Examples of these activities follow.

- After the initial assignment on coin flipping, we consider other ways to randomize pennies: by spinning and by standing them on edge and pounding the table to knock them over. Physics and geometry are used to explain why these methods do not produce heads and tails equally often.
- The coupon-collector's problem asks the question "Given $n$ different coupons, equally distributed among a very large number of packages with one per package, how many packages must be purchased to collect at least one of every different coupon?". This problem is examined through simulation with cards or dice, and the results are compared to the theoretical predictions.
- Since *Chance* is taught in the fall, I have always used the class date closest to September 11 as a day of discussion of personal probability as people interpret risk—rationally or irrationally. Several articles that appeared in 2002 and 2006 that advocate for a rational response to terrorism form the day's assigned reading [1, 5, 8]. This worked a lot better from 2002–2006 than it did in 2016, as one might expect.
- One class session recreates Buffon's needle problem, where short straws are tossed at random onto a grid of horizontal lines and the number of straws crossing a line is used in computing an estimate of $\pi$.
- In "Sampling the Senate", students look at the difference between biased and random samples chosen from the United States Senate. Typically, the challenge to "Name five members of the current US Senate whom you have definitely heard of" produces a larger average length of service than a random sample does.
- During Lottery Week, we conduct several lotteries in class. In the first, students choose a two-digit number, and we look at the distribution of the digits in the numbers chosen. Subsequent lotteries include "pick the number closest to 2/3 of the mean of all choices", and an experiment where students choose five hypothetical tickets for the Michigan Lottery's weekly Lotto–47 drawing. Before the drawing, we consider the expected distribution of student tickets matching zero to six numbers and then compare that to the actual results.
- "To Begin Or Not To Begin?" presents a game of chance in which two players take turns drawing chips from an urn, and asks if going first is an advantage or a disadvantage. Students play the game several times to develop an experimental answer.
- We play the game show *Deal Or No Deal*, and explore the connection between the banker's offer and the mean value of the remaining cases.
- Expected value is explored through an experiment using sports parlay cards from Las Vegas casinos. Students compute which wagering option gives the best (least negative) expectation.
- Two days are devoted to an extended genetics experiment with dice that simulates the related effects of inherited traits and random mutation.
- An in-class experiment asks students to construct 90% confidence intervals for ten known quantities, such as the number of countries in OPEC [7]. Most students turn out to be very overconfident in their estimation skills. In some years, this has been followed by a lab on confidence intervals.

- We watch and discuss the History Channel documentary on one of the MIT blackjack teams [4].
- In the runup to the casino trip, several class activities focus on practice with popular casino table games, including blackjack, roulette, and craps. These practice rounds include both the rules of the various games and how one conducts oneself at the tables. While it is not possible to simulate the casino experience completely in a classroom—all of the chips come back to me at the end of the hour, and there's no real risk—advance familiarity with the game rules and casino etiquette allow students to make the most of our limited time on the casino floor.

  The 21st-century poker boom has brought several students to *Chance* with an interest in Texas Hold'Em poker. While that game cannot be avoided in the readings and discussion, I try to discourage students from playing poker on the casino trip; there are experienced local players who stake out the poker room looking to take advantage of new poker players. (Soaring Eagle sits three miles from 18,000-student Central Michigan University, so inexperienced poker players are not rare.) There's a big difference between casual Hold'Em games played in dorm rooms and games played for serious money in a casino, and students need to be aware of the extra risks involved. That said, while some students have lost money playing poker despite my warnings, the biggest winner in eight years of *Chance* won $400 in the poker room.

All lab assignments are locally written; they have evolved over my years teaching *Chance*, with specific topics flowing in and out of the lab curriculum.

1. **Introduction to Minitab** gets students acclimated to working with Minitab through some simple experiments in generating and graphing data.
2. **Simulation** builds on the ideas from the first lab and from some in-class activities by guiding students through simulations of coin tosses, die rolls, and other random phenomena.
3. **Binomial Distributions** explores the simulation of a variety of binomial experiments and compares theoretical and experimental probabilities, including the use of the binomial distribution to estimate sports-related probabilities.
4. **Looking at Variance** uses real and simulated data on rainfall in Lakeland, Florida to examine the influence of the standard deviation (SD) of a data set on the shape of the data and the influence of the data set on the SD. The Empirical Rule is introduced in this lab.
5. **Law of Large Numbers** explores expected value in several settings, including the challenge of airline overbooking, and examines such effects as how the experimental probability that passengers will have to be bumped from an oversold flight changes as the size of the airplane increases.
6. **Drinking With Dice** functions as a mid-semester lab practical exam. Students are presented with a hypothetical game where players in a bar roll dice to determine who pays for successive rounds of drinks, and are tasked to devise and execute a simulation, and then to evaluate the data, to determine whether or not the game is fair.
7. **415,000 Dice!** calls for students to simulate that many die rolls in an exploration of variations on the casino carnival game of chuck-a-luck and the underground dice game Twenty-Six.
8. **Casino Innovations** looks at new games that have been proposed for or introduced into casinos, and invites students to compare experimental and theoretical probabilities for these games.
9. **Cryptography** is a Web-based lab exploring how pattern detection can be useful in encrypting and decrypting information. This non-gambling activity explores how known frequencies of English-language letters can be used in a variety of encryption schemes. Students work to find the patterns in encrypted strings that "look random."
10. **Linear Regression** examines the related topics of correlation and regression, and explores how they are used for detecting relationships among variables. This includes a look at random variation, in keeping with the theme of the course. Examples are drawn from auto maintenance and geology, as well as from data collected from students at the start of the semester.

## 7.4 Reflections

One of the biggest challenges I have faced in teaching *Chance* is in the title of this paper: From the beginning, many students have come to the class expecting a "fun and easy" class—certainly not a course where they were expected to know mathematics. That FYSs be fun and easy appears to be a campus-wide expectation, which works against courses that are not FYSs as well as *Chance*. I addressed this in the most recent versions of the course by stating some quantitative expectations up front in the official FYS brochure from which students select their preferred seminars:

> **Prerequisites**: Students enrolling in *Chance* must have completed at least a precalculus-level mathematics course in high school, something where exponential, trigonometric, and logarithmic functions were studied. In addition to being a First-Year Seminar, it should be noted that *Chance* is a serious mathematics course. Students who dislike mathematics should select a different seminar. Additionally, *Chance* students must be 18 years old no later than November 1 [of the year the class runs].

Nonetheless, student reluctance to engage with the introductory-level mathematics used in *Chance* continues to be a sticking point.

Courses evolve, and faculty interests shift, over time. These shifts are healthy. At Albion, this has taken the form of an intentional effort to make FYSs more of an extended orientation to college life than a traditional content-focused college course in an effort to recruit faculty to teach in the program. For my part, as I saw seminars drifting away from content, I felt a very strong desire to spend more time teaching mathematics in my College-wide courses.

In 2011, I reworked *Chance* into a course that I now teach in the Albion College Honors Program as Great Issues in Humanities: Perspectives on Gambling. (Albion's Honors Program requires students to complete four interdisciplinary, discussion-based, and writing-intensive Great Issues courses: one each in natural science, social science, humanities, and fine arts. As with FYSs, the specific content of a Great Issues course is left to the instructor to determine. I had previously taught a Great Issues in Natural Science course.) This course preserves some of the readings, computer work, and mathematics from *Chance* while adding new readings from history, philosophy, and literature and sharpening the focus from applied probability and statistics in general to gambling. Teaching *Chance* in the Honors program allows me to teach and discuss mathematical content rather than focusing on orientation issues, and allows me to bring mathematics to Albion's more academically curious students.

With the shift away from academic content in seminars at the College level and an opportunity for me to teach much of the same material in the Honors Program without most of the non-mathematical expectations, the future of *Chance* as a first-year seminar at Albion is uncertain. While *Chance* may have run its course, its benefits continue. My new involvement with the mathematics in *Chance* has led to other opportunities to work with students in similar areas and directly impacted my own scholarly activities. I have had the opportunity to direct student research in gaming mathematics several times since I began teaching *Chance*. Albion provides considerable funding for summer research through its Foundation for Undergraduate Research, Scholarship, and Creative Activity (FURSCA). Several students have received FURSCA funding to work with me on a variety of projects including Spider Craps (craps using 8-sided dice), card games with nonstandard decks including a version of euchre as a casino table game, and computer simulation of new keno games.

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## Appendix

In addition to daily reading quizzes, weekly lab reports, and occasional homework and exams, *Chance* includes three extended writing assignments, which are described below.

### Project #1: Native American Gaming

*What follows here is the students' first extended writing assignment, which is assigned in Week 3. This assignment is preceded by a class session with a College librarian to acquaint students with the library's resources and some of the expectations for college-level scholarship.*

The first paper assignment for this semester will be a six-page (minimum) essay on some aspect of Native American gaming.

As you are surely aware, we will be taking a field trip later this semester to the Soaring Eagle Casino in Mount Pleasant, MI (www.soaringeaglecasino.com) . The Soaring Eagle is operated by the Saginaw Chippewa tribe (www.sagchip.org), and is one of a vast collection of American casinos run by Native American tribes. These arose beginning in 1988, when the Indian Gaming Regulatory Act legalized casinos run by tribes on reservation lands. This act changed the landscape of legalized gambling in the US forever. Your assignment is to research and report on some aspect of Native American gaming.

I must approve your specific topic in advance. Possible issues that might interest you follow. Remember that your assignment is a six-page paper—be sure that your ideas are focused enough that you can cover them adequately in a work of approximately that length. People have written books about each of these topics—and your research should include reading some of them.

- How has the growth of Native American gaming affected other casinos' operations? (Example: Is there any connection between the rise of Native American gaming in Michigan and the three casinos in Detroit?)
- How have casinos affected the standard of living of tribal members?
- How have casinos affected the economy near the reservation land where they are located?
- What impact have Native American casinos had on the expansion of other forms of gambling?
- Conversely, what impact has the expansion of other forms of gambling had on established tribal casinos?

One option for choosing a manageable topic might be to pick one of these topics and then focus your interest on a specific tribe, casino, or geographical area.

### Project #2: Casino Games

*Project #2 is assigned in Week 7, after we've read and discussed most of* Basic Gambling Mathematics *[3]. Students at this point are more familiar with a wide range of casino games and are more proficient in the elementary probability necessary to analyze them.*

The second paper assignment for this semester will be a six-page (minimum) essay on a specific casino game. You should select one of the following games:

- Baccarat
- Bingo
- Blackjack
- Caribbean stud poker
- Craps
- Keno
- Let It Ride
- Roulette
- Slot machines
- Texas Hold'Em poker
- Video poker

There is no need to pre-register this topic. Your paper should consider at least some of the following questions (Considering them all would be difficult to do in six pages.):

- What is the history of your chosen game?
- How is the game played?
- How has the game changed over time?
- How has the rise of computers changed your game, or how is it likely to do so in the future?
- What variations on your game have been tried in casinos?
- What strategies for players have been proposed for your game?
- What is the house advantage for your game?

(Note: We will be studying some of these games in class. Your paper must go into more detail than we do in class.)

In addition, your paper must include some mathematical analysis of at least one component of your game. This could be the calculation of the expected value of one or more wagers, for example. Equations can be tough to type—you should either use Equation Editor to format them correctly or leave some space in your text and handwrite the equations in later in black ink. The equations will take more room than you think at first.

The following grading rubric is used for Project #2; similar rubrics are used for the other projects:

| Objective | Value | Description |
|---|---|---|
| Mathematics | 10 pts. | Is your mathematical work both appropriate for your game and correctly presented? |
| Content | 15 pts. | Have you adequately covered your topic as you have defined it? |
| Research | 10 pts. | Are your sources credible and appropriately scholarly? Have you documented your research correctly? |
| Organization | 10 pts. | Does the paper flow well? Is it properly structured as an essay rather than as a list of answers to questions? |
| Mechanics | 5 to $-15$ pts. | Is the essay cleanly written? Are there so many technical errors (spelling, grammar, etc.) that they interfere with your message?<br>**Scoring**: There will be a one-point deduction for each error up through 20. (Past that point, I stop counting, although your "Organization" score will be affected.) |

## Project #3: Casino Trip Reaction Paper

*The third project is assigned near the end of the semester and focuses on the casino field trip, which typically occurs in mid-November, after all of the students have turned 18 but before Thanksgiving. Usually, this is in Week 12 or 13, and the paper is then due after Thanksgiving in week 15. The passage that follows is taken from a comprehensive handout that outlines the logistics of the trip and what I expect of the students.*

You will write a three-page ($\geq$ four pieces of paper) paper about this trip. In this paper, you should describe the role that chance played in the casino's operations as you either observed or experienced it. You might consider the following questions (and others, of course) as you explore—a lot of this can be done without actually gambling:

1. How is the casino encouraging patrons to gamble? (Consider issues of mass psychology as well as the mathematical issues.)

2. Are the odds/probabilities on any game of chance available to patrons? How available? What information is readily available?
3. How easy is it to find information on the mathematics behind the games? (A formal but clandestine search for this information might be interesting.)
4. For those of you with an interest in the issues beyond the mathematics: how are the ethical dimensions of legalized gambling in view here?

If you are looking for more information, you may find it useful to consult the websites of some casinos and look at their descriptions of their gaming activities.

# Part III

# Mathematical Modeling and Data

Data is becoming more prevalent in our daily lives. Working with data connects mathematics with various other academic disciplines, and as such, lends itself nicely as a mathematical theme for a first-year seminar. The first-year seminars described in this section focus on teaching students how to model and analyze data and include topics such as sustainability, sports ranking, and network modeling using graph theory. In addition, this section offers an example of a first-year seminar that is team-taught with a biologist in which students observe and study a restored ecosystem.

# 8

# Measuring Sustainability

Amanda Beecher

**Abstract** *Measuring Sustainability* is a First-Year Seminar given to expose students to the mathematical underpinnings of the interdisciplinary topic of sustainability. Ramapo College has a rich tradition of sustainability practices on campus, so I am able to utilize faculty and staff to inform students about these activities. The course includes several mathematical modeling projects to expose the quantitative side of sustainability evaluation and decision making. The course ends with students completing a campus sustainability research project. This article overviews the course, including examples of these projects, as well as additional sources and topics for sustainability-themed community engagement projects.

## 8.1 Background and context

Ramapo College of New Jersey is a public liberal arts institution enrolling about 6,000 undergraduate students. It is primarily an undergraduate institution, but does have some professional graduate programs. It is the youngest of the New Jersey State colleges, being founded in 1969 with its first graduating class in 1973. The mission of the College reflects these times in the four pillars of the school: interdisciplinary and experiential learning, and international and intercultural understanding. More recently, diversity, inclusiveness, sustainability, student engagement, and community involvement have been added formally to the list, although they have always been part of its identity. The First-Year Seminar (FYS) course reflects the College's mission by requiring each section to be built around a theme linked with a tenet of the College. In order to facilitate the transition from high school to college in and out-of-classroom, the theme stands as an academic reflection of the Ramapo College community and values. Most of the out of classroom transitions are handled in discussions and activities with the two peer facilitators each week. This four-credit course is given during a full 15-week semester with 200 minutes contact hours per week, which includes 50 minutes per week for the peer facilitator.

At Ramapo College, FYS has its own designation as interdisciplinary (INTD) in the general education program, and sits outside any one school. Most schools on campus require their students to select from the school's FYS topics, while others allow students to explore any topic. The only closed sections of FYS are for the honors students, who take their FYS as a cohort. Thus, my section of FYS is populated with up to 25 first-year students from the School of Theoretical and Applied Sciences (which includes mathematics), the Anisfield School of Business, and undecided students. Faculty and staff are encouraged to develop an FYS, but there is no requirement in the sciences or mathematics to do so. Because of this, the FYS director is always excited when faculty from these disciplines volunteer to teach a section.

In addition to having a theme of the College, the FYS course is also writing-intensive. The course must have several written assignments in which students receive feedback and are given the opportunity to incorporate that feedback in a resubmission. At least one of these written assignments must be based on the Summer Reading program required of all first-year students. The course includes information literacy instruction from a librarian, who also sets up a course page that includes several common databases used for journals in the discipline related to the theme. The course also has an oral communication outcome and for many years was the only general education course that required an oral presentation.

## 8.2 Mathematical Theme

The theme of my FYS section is sustainability. Ramapo College has a rich history of sustainability programs and initiatives since its founding during the environmental movement. We were one of the first US colleges to have a Director of Sustainability and have had a faculty member serve in this role under various names for several decades. It was a strategic choice to link the content of the course with a topic that our students have a natural inclination to study.

I chose *Measuring Sustainability* to highlight the quantitative viewpoint that this course takes on the subject. We study the metrics used to quantify sustainability and use quantitative decision-making techniques to answer sustainability-related questions or impacts. The sustainability FYS course has been taught once from this quantitative viewpoint, and is included as an example of a "Sustainability across Campus" course at Ramapo College. Other courses in this category, such as *World Sustainability*, offer students a qualitative or sociological viewpoint of the topic. The mathematical content of the course is typical of a general education or quantitative reasoning mathematics course, so no particular mathematical background is expected from the students entering the course.

My intention for this course is to give students an interdisciplinary modeling experience through open-ended real-life questions related to sustainability themes. This is consistent with the approach to mathematical modeling outlined in the GAIMME report [7]. We discuss and utilize the math modeling process as a formalism to our problem-solving approach. With the various backgrounds in mathematics of the students, I select project topics consistent with a standard quantitative reasoning course. This provides an inclusive environment that showcases the power of mathematics in problem solving, but not limiting the success of addressing real problems for those without a particular background. So, much of the direct mathematical instruction is to formalize and utilize the math modeling process in each part of the course. Any other math instruction is done on an ad hoc basis, as different teams utilize different mathematics for addressing the problems. All projects were completed in groups, which allows the groups to utilize their collective mathematical background. This course developed out of a Mathematical Association of America's Professional Enhancement Programs (MAA PREP) [9] workshop called Undergraduate Sustainability Experiences in Mathematics (USEMath) held at Shippensburg University in Shippensburg, PA.

## 8.3 Course Structure

The course is designed to be project-based, so that students work in groups to solve open-ended problems. I use a mathematical modeling approach consistent with the GAIMME report [7] as a problem-solving process. Answering the questions posed requires students to identify the specific problem to be solved, make assumptions, solve and analyze solutions, assess the reasonableness of their results, and make recommendations based on those results. My view of this course is to develop a mathematical modeling mindset by addressing quantitative sustainability problems, not on a specific set of mathematical tools. So, these problem solving activities are a catalyst to enhance their understanding of the sustainability-related concept, as well as see the quantitative underpinnings of these problems. The requirements of the course also naturally lend themselves to this model, as groups must regularly report on their results in written or oral communication. Generally, the course is divided into the topics of personal choice and campus sustainability. Much of the campus sustainability information is given by other faculty and students as speakers in class or guiding field trips to see sustainability efforts on campus. The course ends with a community engagement project requiring students to work on a campus sustainability initiative to measuring its impact as broadly defined in the College's strategic plan.

### 8.3.1 Personal Choice Topics

The topics that I consider personal choice are the open-ended problems that are sometimes not well defined, consistent with modeling problems. Questions that require students to answer "Is it worth it?", "Which is better?", "What should you/your parents/we do?" given some scenario. Many of these problems came from the summer MAA PREP workshop mentioned, either presented during the session or developed by the other participants. I only provide links to sources I used (potentially updated or adapted links) and recognize that there are likely multiple sources for similar problems. Examples of topics used in this course follow.

### Light Bulbs

There are three standard light bulb choices; incandescent/halogen, compact florescent light (CFL), and the light emitting diode (LED). I bring them to class and give students an overview of them. This is an opportunity to compare the shape, the light they emit, and the heat they give off. I tell them about the Bright Tomorrow Lighting Prize (L Prize), in which the US government offered $10 million to replace 60 watt incandescent light bulb with a high-efficiency lighting alternative. This contest encouraged the scientific community to develop energy saving technologies. The Philips company won by making an LED light bulb that costs more than $40 per bulb [12]. So, is it worth it to replace your existing light bulbs? This work is based on a presentation by Victor Donnay and adaptations by his students [1].

### Car Choice

My husband and I need to buy a new car. He just wants to replace our Honda Accord with a newer model, because our current one has lasted a long time and had minimal repair expenses. I want a Toyota Prius, because I commute to work and believe the savings in gas will offset the higher initial cost. What should we do? In other words, is a hybrid worth it? While this was an actual discussion between my husband and I, it is certainly not unique [3]. During the course, gas prices varied wildly, so students revisited this question to see how gas prices affects their model and recommendations.

### Solar Panels

I ask students, should your parents go solar? Students use their home electricity bill or my sample bill to compare with real data collected by the photovoltaic panels on the roof of the Sharp Sustainability Center at Ramapo College, see Appendix 8.4.3.

### Ecological and Carbon Footprints

There are many metrics to measure the impact our choices have on the Earth. In this course, we considered both the Carbon Footprint [2] and the Ecological Footprint [6]. We discussed why each is used and why some argue the Ecological Footprint is the better resource. We do not investigate the methodology of these computations, but rather analyze and discuss the results as an information tool to understanding our own impact on the world. A more thorough activity investigating the Earth's biocapacity from the Ecological Footprint written by Rikki Wagstrom [11] is available.

### Food and Foodprints

Students have two activities to work on food related issues. We consider the environmental cost of food by computing the impact of breakfast [8]. This activity demonstrates the impact of food sourcing on our environment. The worksheet may be too computationally difficult for FYS students, but the discussion of the results was insightful and effective.

The main project in the food section is based on students' own food consumption. Students keep a food journal for about two weeks. They write down everything they eat, including snacks and drinks. They select two sample days, one that represents an average food day and one representing their most sustainable food day. They use approximate carbon dioxide equivalent (CO2e) of the food categories (available online) to discover how sustainable their diet is on average and on best days. Students discover through their own use that vegetable-based diets had the lowest CO2e and meat the most. This is not surprising as many know of Meatless Monday as a sustainability initiative. Healthy students are surprised to know that their lean meats, while better than red meats, were still over triple the CO2e of vegetables.

## 8.3.2 Campus Sustainability Topics

The course culminates in a campus sustainability project requiring both a ten-page paper and a class presentation, see Appendix 8.4.3. Students are required to work in teams of three or four on a project that impacts Ramapo College's sustainability initiatives. We read the College's strategic plan, focusing on the sections about sustainability, to introduce this topic. Students may be surprised that such documents are openly available. Discussing the goals within the plan provides an excellent opportunity to talk about the values and expectations of the College, as well as the oversight and planning it takes to achieve them. This positions students to value their projects as part of a larger college process. For assessing student work, peer and instructor feedback were given using rubrics for written and oral presentations developed by the all-college general education curriculum committee.

There are many potential sustainability initiatives on campus [4, 5, 10], so this course serves as an opportunity to reach out to the campus community to inform the students and me on the status of this work. Through interactions with faculty and staff, students are brought into the Ramapo College community, as required for this course. Inviting speakers to class or taking the students on field trips to get sustainability perspectives on campus is reported to be a memorable component of the course.

### Faculty Colleagues

Ramapo College has undergraduate majors in Environmental Science, Environmental Studies, and Sustainability as well as a Master's in Sustainability program. I invite colleagues from these programs to speak with FYS students. I specifically prompt them to discuss campus activities that students can participate in or use for their campus sustainability project. Students love guest speakers (even if just another professor) and we all learn more about campus activities. At the end of the course, the final campus project papers are sent to the colleague, highlighting those that are projects they specifically mentioned in class.

### Sharp Sustainability Center

The president of the New Jersey Higher Education Partnership for Sustainability (NJHEPS) gave us a short presentation and tour of the Sharp Sustainability Center, a green building on campus, including the solar panels that produce the data students use in their solar decisions project. Other green elements include a geothermal heat pump, concrete floors, repurposed materials, prebuilt materials, and no water urinals. Groups of students research some of these features to report on the pros and cons of the selected building practice, and discuss why the practices may not be standard in all new construction.

### Student Clubs

The president of 1Step spoke with the class about existing projects of the club. The student club 1Step is focused on sustainability and environmental issues, with a strong emphasis on making impactful campus changes. She expressed how college administrators are making data-driven decisions, so gave specific examples of where the use of data collection or approximation can move current projects forward. Thus, inviting students to use club projects for their course project.

### Dining Services

Dining services at Ramapo College are contracted through an off-campus food service company. The dining hall manager must work with the college community to ensure a variety of quality foods that meets student (and parent) expectations, while also maintaining the standards of the company. This company has its own sustainability goals. Thus, the manager regularly partners on projects for campus sustainability, such as made-to-order food, no trays in the dining halls, Meatless Monday, and acquiring an industrial-sized composter. He is open to new ideas and excited to work with students on any project that could move the food services towards its own sustainability goals.

### Residence Life

The Director of Residence Life works on activities in and around the residence halls that advance the sustainability initiatives of the College associated with student housing, such as reducing energy, water, and single-use plastic usage and minimizing waste. She welcomes students to be part of this process.

### Project Topics

The project ideas originated from the sources above or student interest. They may not be applicable to all institutions. Students select a project topic and present it to the class. Additionally, all students then rank their top three choices and are assigned to groups. Topic options have included:

- How do we get water stations installed in residence halls and ensure students use them?

- How can we reduce the use of single-use plastic water bottles?
- How can we reduce the paper waste on campus?
- How do we limit lights left on?
- How do we decrease food waste in the cafeterias?
- How do we increase healthy eating and/or sustainable eating on campus?
- How much do people recycle and how do we get them to recycle more?

## 8.4 Reflections

### 8.4.1 Course Goals

The main goal of FYS is to help transition students to campus life in and out of the classroom. The course is structured to mirror this transition by starting with activities aimed at personal choices by them or their parents, moving into things happening on campus, and ending with them making a contribution through their campus sustainability project. I believe this course meets the goal by bringing students into the college community.

This course is writing intensive, so must meet criteria designed to improve students' ability to write. This includes opportunities for students to rewrite some assignments incorporating feedback from the instructor. Ramapo College has a Center for Reading and Writing, where I can refer (and mandate) weak students to go for help to support their writing. One unexpected result of having so many group activities is that students would peer-review work from other members of the group. This resulted in better writing and a more cohesive submission. Also, the multiple presentations and reports required introduce students to the expectations of college-level work. The success in these areas certainly depends on their previous experiences, but setting the expectation and allowing for rewrites to achieve them made the overall writing activities better.

### 8.4.2 Campus Partners

I am surprised by the excitement and willingness of faculty and staff to share their time and knowledge with the FYS class. This is an opportunity for them to recruit students to work on a project that may advance their own initiatives, join a club, apply to a program, or just be more aware of sustainability at Ramapo College. From this perspective, I realize that my requests are not a burden on their time, but an opportunity for them to free up some.

The dining hall manager and director of residence life were unable to attend this course in the past, but will be invited to speak in future FYS sections. They both assisted students in their campus sustainability project and expressed interest in being more involved throughout the course.

Moreover, campus partners listened to student recommendations and made changes based on these projects. For example, the reduction of plastic water bottles group proposed to the Director of Residence Life that the gift to all incoming first-year students be a reusable water bottle. They compared the cost of the water bottle to the backpack students had been receiving and showed it was cheaper. The water station group emphasized that if water stations were installed in the residence halls, then the students can utilize their bottle instead of purchasing the single-use water bottles. As a result, all first-year students have received a reusable water bottle at orientation the following year and each year since. Additionally, the residence hall water stations were prioritized as the most necessary on campus, and installed in each residence halls two years later. First-year students were able to see that their work had a direct impact on decisions made on campus. This was a powerful outcome of this course.

### 8.4.3 Student Outcomes

Out of the classroom, 68% of my students participated in a sustainability related activity. The activities students reported were attendance at the Campus Sustainability day, attending a talk on sustainable gardening, attending a sustainability club meeting, joining the bee keeping club, applying to the sustainable housing community, or getting a group of friends together to buy their produce at the local farmers' market. Many students embraced the ideas that what they do matters and made behavioral changes as a consequence.

On the final survey, I ask students “In what ways (if any) has your participation in this class changed your behavior or the way you think?” A selection of the responses are below.

- “This class has changed my behavior in many ways. For example, I turn off all the lights in my residence hall when leaving, whereas before I would think “I paid $13,000 for this, I’m just going to leave it.””
- “This class has made me more aware of my energy use. I never realized the impact long showers, or keeping the heat/lights on had on the environment. I now keep the lights off, recycle more, take shorter showers, and print double sided because I am more aware of the issues.”
- “My participation in this class has changed my behavior and the way I think very much. For instance, I used to only recycle occasionally and I always loved eating chicken or red meat. However, now I recycle all the time and I am a vegetarian! This class has definitely changed me for the better.”
- “From my participation in this class, I feel as if my confidence to raise my hand in order to share my opinion and thoughts has increased. I also find that I am more likely to take a leadership position in my group projects. ”
- “This class has made me realize that my opinions and contributions to research matter. If I feel strong enough about something, I have the potential to make a difference if I put hard work into it.”

Most students speak about ways in which the content changed their day-to-day habits, like the first three responses. However, there are some that find more meaningful experience through group work and research. I am particularly surprised and encouraged by these responses.

I believe the campus sustainability project is more successful than I had imagined. I witness students taking on tasks, collecting data, and even groups working together to share data. It is very impressive. I thought students would need a lot of guidance, but they take over and quickly make the project their own. It is a true research experience by students in their first year of college.

As a final thought, I received an email from a student just before she graduated saying, “Since your FYS class, I have used our big Measuring Sustainability project as a talking point in a number of interviews for part-time job opportunities and internships throughout my college career.” The project provided evidence that she could work effectively in a group, under a deadline, to produce quality results. These are soft-skills employers are seeking. It is another unexpected and positive outcome that often are seen by others that use community engagement projects.

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# Appendix

## Photovoltaic Panel Analysis

*This appendix includes the prompt questions and writeup guidance for one project in the course. The questions are stated as one might expect in a quantitative reasoning course using some basic mathematics, but then the synthesis of their work into a contextualized report is the key feature of this project and regularly used in this course.*

Purpose: We took a tour of the sustainability center. We saw the photovoltaic panels and viewed the energy collected by those panels. This exercise is to contextualize the amount of energy that is collected as compared with normal household use. Recall that the president of NJHEPS suggested that the Sharp Sustainability Center produces about twice as much energy as a normal household would need.

Compare these figures with the historical data given on your home electric bill. This will require you to look at your electricity bill and compare the past kWh usage with the actual amounts from the panels. This was used in the light bulb exercise.

1. Do the photovoltaic panels on the sustainability center produce enough energy to cover the needs of your home in the past year?
   (a) What is the surplus or deficit?
   (b) Based on this figure, estimate the number of panels needed for your home. Recall that the Sharpe Sustainability Center has 81 panels.
2. Compare the energy generation of the panels to the home usage on a monthly basis over the past year.
   (a) In what months would the panels produce enough energy to fulfill the needs of the home? In what months do they fail to meet the needs?
   (b) Compute the total energy saved each month if these panels were on your home. Sum the monthly energy savings to give an annual savings based on monthly need.
   (c) Based on your estimated cost of kWh from your electric bill, what kind of savings would there be each month over a year for your home?
   (d) If these same savings were to continue, how much would you save over the 25 year life of the panels?

To be submitted: Write an analysis of your findings.
This should include:

- **an introductory paragraph** The introductory paragraph should give context to the problem and the purpose of the analysis.

- **a paragraph or two explaining your work** The explanation of your work should be in words, but you may include equations to support your work, tables to organize your calculations, or other mechanisms to make the work clear. Be sure to include a discussion of any choices (assumptions) you made in the process
- **and a conclusion** The conclusion should provide a result and a recommendation for how this information can be used.

Upload to Moodle by Oct 17. If you used any information outside of the provided website or electric bill, then you must use in-text citations and provide a list of references at the end.

## Campus Sustainability Project Description

*The following is the project guidance given to students to lay out my expectations for their final campus sustainability projects with the due dates for the individual portions. This project resulted in a presentation and paper given by each project team of 3–4 students.*

The course culminates in a campus sustainability project. We have discussed initiatives on campus and now it is your turn to measure and analyze the progress of sustainability initiatives. You will work in teams of three or four on a project that has an impact on our sustainability initiatives.

A successful project may require:

- you to collect your own data. Data is a hard commodity to find, so you may need to go out and collect actual data. Remember that surveys do not always produce valid and reliable data. The more you can collect it yourself, the more you can authenticate it. This will give great power to your project.
- you to analyze and extrapolate on your data. It is unlikely you will collect it for the entire campus. You may need to find a representative sample of the data and use it to estimate the value for the entire college community.
- you to speak with campus staff. Prepare for any interviews or meetings about your project and do not waste their time. They should be conducted in a professional manner and be aware that some information that you seek may not be available.
- an investigation of the problem in a broader context. Sustainability problems are not unique to Ramapo, so providing information about the scope and difficulty of the problem will add great value in why your contribution is so important. This requires a literature review of valid and reliable sources.
- graphics, which need to be properly documented if found on the internet.
- outside sources. You will need to be clear what information came from which source. If you use someone else's ideas, then you need to cite that person. Give credit where credit is due. Choose either Chicago, MLA, or APA style to format your citations and be consistent. In your presentation, the last slide(s) will be your works cited page.

Time line and Description: All of these events are scheduled in Moodle and grading rubrics are provided.

**Proposal** Due November 18th uploaded to Moodle, a template is given on Moodle.

**Proposal Presentation** Due November 18th in class. You will have five minutes to present your project to the class. The more information you are able to provide about your plan, the more useful feedback you are likely to receive. There is not need for slides, but each member of the group should present. We are preparing for the final group presentation.

**Draft Paper** Due December 4th uploaded to Moodle, a template is given on Moodle.

**Final Presentation** Due December 9th in class. You will have 15–20 minutes to present the results of your project to the class. It should be a comprehensive presentation and indicate what work was completed by the group. Every member of the group must speak or the grade of the entire group will suffer.

**Final Paper** Due December 12th uploaded to Moodle, a template is given on Moodle.

Grade Distribution: Proposal 10%, Proposal Presentation 10%, Draft Paper 10%, Final Presentation 30%, Final Paper 30%, Team Evaluation 10%

# 9

# An Interdisciplinary Course to Engage First-Year Students in Biomathematical Research

Clinton K. Meyer and Heidi Berger

**Abstract** We leveraged multiyear research collaboration and combined the expertise of two faculty members to team teach *If You Build It, They Will Come: A Biomathematical Approach to Restoration*. This course provided hands-on approaches to both biological field work and mathematical modeling. Our class was offered in the first-year seminar program at Simpson College, a small (about 1,300 students), private liberal arts institution in Indianola, Iowa. Class activities were centered on two projects. The first was a comparison of restored prairies involving field sampling, statistical analyses, and a research paper. The second was the creation of an agent-based model of another restored ecosystem culminating in presentations. Students found this course a rigorous introduction to college and other STEM courses through honing of writing, critical thinking, and quantitative skills. While the course approach was effective, we felt the amount of content was overly ambitious. We revised the course so that students gained exposure to both disciplines but focused on either statistics or mathematical modeling for their final project.

## 9.1 Background and Content

The Simpson Colloquium (SC101) first-year program is part of the campus-wide general education curriculum, and a faculty member from each academic department typically offers one section each fall. The learning goals of the SC101 courses include a focus on written communication and critical thinking. Additionally, the course introduces students to available resources and opportunities to help ease the transition to college. All incoming first-year students take an SC101 course and enrollment for each section is capped at 18 students. Each SC101 course is worth four credits, but seat time is decided by faculty members. We scheduled the course to include three one-hour lectures and one three-hour lab time per week. This was critical to ensure that we had ample time for field work, laboratory work, and using computers. The class met 40 times, with a total of 68 hours of classroom engagement over the 15-week term. Course instructors are free to choose topics with the stipulation that there are no prerequisite requirements. Peer leaders are assigned to the class to assist with instruction and assessment of writing and to build community. In particular, each class was assigned a Writing Fellow and a SC Leader.

## 9.2 Mathematical Theme

The course was entitled *If You Build It, They Will Come: A Biomathematical Approach to Restoration* and incorporated two disciplines represented by the authors, namely biology and mathematics. This idea stemmed from a multi-year research collaboration between the authors and several biology and math undergraduate students. The course offered an introduction to tallgrass prairie restoration, and the interdisciplinary theme was using mathematics to inform biological inquiry. Two mathematical techniques supported this approach: inferential statistics and agent-based modeling. This content comprised approximately 30% of the contact hours for the course. Statistics were utilized in the first half of the course, during which students conducted biological field sampling of plant and invertebrate communities. Sampling in prairies gave students firsthand experience in assessing restoration in one habitat type. Those skills were extended in the second half of the course. Students chose additional habitats (e.g., boreal forest) and modeled restoration strategies to mitigate the effects of invasive species.

## 9.3 Course Structure

We have conducted interdisciplinary research since we arrived at Simpson in 2008. One of our more fruitful collaborations was an assessment of recovery of ground-dwelling invertebrates following grassland restoration. Our collaboration involved eight students over two summers. We realized this topic and related methodology not only had broad appeal to undergraduates, but was also accessible to early career students. Thus, we began discussing how to incorporate similar work in the classroom.

Our goal for the semester was to create an authentic interdisciplinary experience combining both mathematics and biology. Of the 68 total hours of classroom engagement, we dedicated five in-class hours to available resources and had guests from the library, counseling services, academic resources, career development, and financial aid. We also required students to attend Study Abroad Fair, Majors and Minors Fair, and at least two campus-wide lectures. We discussed and workshopped statistics for a total of seven hours. We discussed and workshopped math modeling for 11–12 hours. Furthermore, students had 4–5 hours to work on group projects leading up to presentations. Students were engaged in field sampling and sample processing for approximately 15 hours. We designed the course to focus on field work early in the semester while the weather was conducive. Laboratory work, statistical analyses, and mathematical modeling were completed later in the semester. Our disparate disciplinary backgrounds provided students with distinct perspectives on approaching a complex issue through collaboration.

### 9.3.1 Service Component

All SC101 courses included an opportunity for students and faculty to team build while providing service to the campus or broader community. First-year students moved to campus several days before other students to help with adjustment, and during that time they completed their service component. We designed our opportunity to provide students with a short but meaningful experience in prairie restoration. We volunteered at Neal Smith National Wildlife Refuge for 3-4 hours. Neal Smith National Wildlife Refuge is an approximately 3,500 hectare tract of land in Jasper County, Iowa that includes a few small native prairie patches with the bulk of the refuge comprised of restored prairie and savanna. Led by refuge staff, students helped to clean, sort, and prepare seeds for sowing and removed invasive weeds on the refuge, both critical components to the restoration process.

### 9.3.2 Discussion of Prairie Restoration and Management

Initial in-class activities helped provide additional background for prairie ecology and restoration management. The class discussed excerpts from Chris Helzer's book *The Ecology and Management of Prairies in the Central United States* [6] to appreciate the structure and function of tallgrass prairie, as well as key considerations for restoration. Students also discussed primary and secondary research articles to gain broader context in restoration theory (e.g., [1]). Speakers with firsthand knowledge of prairie restoration and ecological research spoke to the class. Dr. Pete Eyheralde, Assistant Professor of Biology, William Penn University, discussed his work on the role of bison in seed dispersal [5]. The authors strongly feel that peer mentorship can be a powerful pedagogical tool. To provide this perspective, three students with whom the authors had previously collaborated presented the results of their research, "Estimating the influence of plant richness on ground-dwelling invertebrates in restored prairie of central Nebraska."

### 9.3.3 Biological Field Work and Analysis

The biological work addressed the following research question: what is the influence of plant diversity on invertebrate abundance and diversity in tallgrass prairie? To provide functional statistical analyses, we needed to consider replication of sites. Thus, we chose four restored prairies within a 40-minute radius of campus. To address our research question, we chose two sites with low plant diversity and two sites with high plant diversity. There was an inverse relationship between percentage of invasive plants (e.g., smooth brome, *Bromus inermis* Leyss., and reed canarygrass, *Phalaris arundinacea* L.) and plant diversity. During the lab time for four consecutive weeks, we visited a field site to sample plant and invertebrate communities. At each site, students worked in groups (3-4 each). Each group established a 30-meter representative transect within the vegetation. Insects were sampled first to minimize the impact of disturbance, and one student within each group performed the insect sample at each site to minimize bias. The sample consisted of 30 sweeps of a standard insect net along the transect. The contents of the sweep net were emptied into a labeled plastic bag and frozen (–10° C) until identification. Insects were identified to the lowest practical taxonomic group, typically family, using appropriate resources ([4], [8]). Then, students sampled plants using an interrupted line transect approach. At each meter, students identified the closest plant for a total of 30 plants per transect. Unidentified plants were brought back to the lab for further identification with relevant field guides and taxonomic resources ([2], [3]). We spent the next four lab periods on identification and calculation of ecological parameters including abundance, richness, and Shannon–Wiener diversity of both insect and plant communities.

The authors provided instruction on basic statistical analyses (t-tests, ANOVA) so that students could conduct hypothesis testing using those techniques. After students had a basic understanding of the principles behind hypothesis testing, they were introduced to the R Project for Statistical Computing [7], which they used to complete basic statistical analyses and graphing.

The biological component culminated in a research paper. Because these were all first-year students, we built in significant scaffolding by discussing and assigning the writing of a scientific paper one section at a time. The discussion of each section was followed by students bringing completed drafts of that section to class for peer-review and instructor feedback. We started with the Materials and Methods, followed by Results, Discussion, and finally the Introduction and Abstract. This allowed students to focus on the expectations and required elements of a relatively small segment. Sections that were simpler to write (i.e., Materials and Methods) were due before sections that required sophisticated technique, such as the citing of primary literature required in the Discussion and Introduction. The Writing Fellow provided instruction on portions of the writing, met with students to workshop ideas, and assisted in providing feedback on written drafts. Guidelines and rubrics for all assignments were made available well in advance of the due date for each assignment. Students were required to meet with instructors and Writing Fellow for help along the way.

### 9.3.4 Mathematical Modeling

The second major course component was agent-based modeling via NetLogo. NetLogo is a programming language and integrated development environment for modeling. It was designed to be ‘‘low threshold and no ceiling” and has an extensive modeling library available, making it ideal for first-year students. Our specific goal with this component was to have students transfer ideas learned from direct sampling of prairies to additional habitats. Students chose a habitat, created a model of community structure, and developed restoration strategies to mitigate the effects of invasive species. We chose to emphasize the role of invasive species after discovering the students’ field work demonstrated the extent to which invasive species had established in our prairie sites.

We discussed the importance of modeling in ecology followed by work with NetLogo. The class started familiarizing themselves with the program by working through tutorials [9]. The modeling process was scaffolded in a similar manner to the biological field work. Students began their investigations by working through several established NetLogo models. Each model became successively more complex; the first model involved an insect consuming grass (Bug Hunt Consumers), the next models added predators and invasive species that competed for similar resources (Bug Hunt Predators and Invasive Species), and the final model showed a relatively rich community structure of plant and predator species (Community Structure). At each step along the way, students completed short written assignments to describe modeling results, using prompts from the NetLogo model info tab. These write-ups emphasized the importance of sensitivity analysis and strengths and weaknesses of the modeling premises.

After students learned how to understand the established NetLogo models, we wanted them to code their own models. The Community Structure model became the template for this work. First, we parsed the code line-by-line and then we modified the Community Structure model to mimic our observations on prairies. Next, students self-selected into groups based on interest in investigating an additional habitat (e.g., boreal forest, Florida Everglades, emergent marsh). Groups were charged with researching each habitat via primary and secondary literature to isolate major plant and animal components, sources of habitat degradation, restoration approaches, and impactful invasive species. Students compiled this information into an annotated bibliography to ensure adequate background knowledge of their habitat. These details were used to create their own NetLogo models. In doing this, students saw the important role that modeling can play in predicting trajectories of habitat restoration and management.

The modeling project culminated in group research presentations. Each group presented a brief background of their habitat, as informed by their literature investigation, followed by an explanation of the creation of their model and a demonstration of their model in action. All models were required to include important producers and consumers, indications of which species were invasive, and important ecological interactions (e.g., predator-prey interactions). In this fashion, students not only acquired quantitative skills, students also gained and demonstrated oral communication skills. Students were evaluated on confidence in presenting information, good eye contact and vocal inflections, equal contributions by students, and quality of slides.

## 9.4 Reflections

We taught this course once as described. Students found the course to be a rigorous introduction to college, and it prepared them for upper-level STEM courses. Students honed writing, critical thinking, and quantitative skills in the course. Students also gained critical interdisciplinary experience. This occurred both because of the breadth of course components, and also because the course was co–taught by faculty from different STEM disciplines, each providing a unique perspective. Many of the students that performed well in this course were well poised to succeed throughout their academic career at Simpson. Within the first year following graduation, 44% of the students in this cohort are now either continuing their education in graduate school or medical school or have entered the STEM workforce.

While the course approach was effective, we felt that we were overly ambitious in trying to cover a large quantity of material. For example, student feedback indicated that the combination of the research paper and the modeling presentation was taxing. The workload was more appropriate for an upper level course. However, we do feel strongly that it is critical for students to gain authentic interdisciplinary experience early in their academic careers.

Thus, we revised the course for future offerings to be split into two separate sections of SC101, each emphasizing one of the two main disciplines, biology or mathematics. Although the courses were separate sections, there was significant overlap in content and course expectations, and substantial interaction throughout the semester. This was made clear to the students up front by having a shared syllabus. It was further achieved by having the two classes meet together regularly throughout early portions of the semester, before students chose their preferred disciplinary focus through the middle third of the semester. Then, toward the end of the semester, students worked on a final project in small groups composed of members from both classes. In this way, students could drive the interdisciplinary work by bringing their own nascent expertise into the discussion. We retained the scaffolding elements of class projects to incrementally build skill sets for technical writing and mathematical modeling. We feel that this approach allowed us to provide a similar interdisciplinary experience for students, but represented a more appropriate workload for early career students. Indeed, a comparison of student evaluations between the two iterations of the course showed an increase in almost all categories related to student perceptions of the course content and learning objectives (see Table 1).

An added benefit of splitting the course into two sections was that instead of being capped at 18 students, the two courses could have included as many as 36, doubling the number of students served. The course was also designed to be flexible in terms of the research question being addressed and the specific mathematical techniques and associated software being utilized. The second iteration involved two separate sections of SC101 with a biomathematical focus on water quality. The field work encompassed water chemistry and bioassessment sampling in local streams. The modeling component was similar but employed predictive modeling via R instead of agent based modeling via NetLogo. Students used regressions to model nitrate and phosphate levels in several watersheds around Iowa, which related to the field work they had completed. Students responded positively to having a choice in the specific discipline on which to focus. The main source of stress for students with the statistics and modeling stemmed from the learning curve

**Table 9.1.** Comparison of student evaluations between iteration 1 and iteration 2 of the first-year seminar course. These questions relate to student perceptions of course content and objectives and responses are on a 1 (low) to 5 (high) scale. Iteration 1 represents 15/16 (93.8%) of students in one first-year seminar section. Iteration 2 represents 32/36 (88.9%) of students and is an average of two first-year seminar sections.

| Student Evaluation Question: Course Perception | Iteration 1 | Iteration 2 |
|---|---|---|
| I have learned and understood the material for this course | 3.7 | 3.9 |
| The course stimulated my interest in the subject | 3.3 | 3.4 |
| The course helped me grasp connection between this and other areas of study | 3.6 | 3.6 |
| The required readings and other instructional materials were useful | 3.3 | 4.0 |
| The assignments were an essential part of the course's goals and my learning | 3.2 | 4.1 |
| The course clearly met its stated purposes and objectives | 3.9 | 4.2 |
| Overall, this course was a worthwhile learning experience | 3.5 | 3.9 |
| Means | 3.5 | 3.8 |

with R, but ample class time allowed them to be successful in conducting analyses. The interdisciplinary nature of the course was further enhanced organically by the enrollment of several students planning to explore non-STEM fields. This was a result of the course topic being of broad interest, and it added immensely to the breadth of perspectives in class discussions.

## 9.5 Bibliography

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## Appendix

### Guidelines and Grading Rubric for Final Research Paper

*This is the document that was provided to students as guidelines and grading rubric for the final research paper, which represents the culmination of the biological component of the interdisciplinary first-year course.*

**ABSTRACT:**

Write 1–2 introduction sentences explaining your topic, purpose, and research question(s). (1 pt)

Write 1–2 sentences describing your research methods (this may also include the type of data analysis you used). (1 pt)

Write 1–2 sentences describing the results and findings. (1 pt)

Write 1–2 sentences containing your conclusions and recommendations. (1 pt)

Writing was well organized, had good flow, and was free of grammatical errors and awkwardness. (1 pt)

**INTRODUCTION:**

Provided adequate background information for each component of the research (i.e., a paragraph at least for each dependent variable). Writing followed appropriate organization (10 pts)

Appropriate (peer-reviewed) literature was cited correctly to provide that background information (minimum of 10 sources, distributed between each of the relevant components). (5 pts)

All hypotheses were stated. (5 pts)

**MATERIALS AND METHODS (10 pts):**

Enough detail was provided for someone else to conduct the research.

**RESULTS (20 pts):**

Tables and Graphs: Did each one have a descriptive heading/caption in the right place?

Did each one correctly show important comparisons?

Were necessary elements included (units, axes, etc.)?

Was the order correct (following literature cited, Tables, then Graphs)?

Were tables and figures referenced (cited) properly in the text and in the correct order?

Were statements of statistical comparisons included, and were all relevant stats reported ($t$ stat, df, $P$)?

**DISCUSSION:**

Were the results interpreted (not simply restated) (10 pts)?

Were all hypotheses addressed and interpreted? Were alternate hypotheses supported?

For each hypothesis, you should include what it was, what the prediction was, and what the results were (in other words, did you support your alternative hypothesis or fail to support it?).

Did the section follow a logical, organized sequence of information?

Information put into context (10 pts)

Did you explain why the results were observed?

Did you explain how the results contribute to our knowledge? (in other words, do results match what is already known, or do they conflict with that information?)

If your results did not support the alternative hypotheses, why might that be? (List any potential weaknesses in experimental design or factors that might have affected results)

Was the minimum number (10 citations) met or exceeded using correct format?

Was the correct format used?

**LITERATURE CITED (10 pts):**

Was the correct and consistent formatting used? Was the correct number of sources cited?

**EDITING (15 pts):**

Did the student make every effort to incorporate suggestions, edits, etc. from previous drafts?

## Guidelines and Grading Rubric for Final Modeling Presentations

*This document was provided to students to serve as the guidelines and grading rubric for the final model presentation. The first portion of the document details a "model preview", or first draft, that was shown to instructors for feedback prior to the final model presentation. This was the final work associated with the mathematical component of this interdisciplinary first-year course.*

Students will be expected to show progress on model presentation during class on Friday, December 5th. Students should show evidence that a presentation has been created, as well as a plan for dividing presentation duties between group members. The remainder of the class period can be used for final editing/practicing of presentations.

Draft of presentation has been created and includes the following components:

1. Background (5 pts)
2. Model assumptions (including strengths and weaknesses) (5 pts)
3. Model outputs (5 pts)
4. Implications for management (5 pts)
5. A plan is in place for dividing presentation duties equitably between group members (5 pts)

Final Model Presentations DUE: Tuesday, Dec 9th from 3:15 – 5:15PM
Final model presentations will be given during the Final Exam period (listed above). All groups will present during that time. You will be given a total of 20 minutes, which includes a minimum presentation time of 12 minutes, maximum presentation time of 15 minutes, leaving 5 minutes for questions. Practice your talks to make sure they fall within this time frame or lose big time points.

Presentations need to include the following:

1. Background (15 pts)
2. Model assumptions (including strengths and weaknesses) (15 pts)
3. Model outputs (15 pts)
4. Implications for management (15 pts)

Presentations will also be judged on the following characteristics:

1. Effective oral communication was utilized (e.g., students were confident in presenting information, smooth transitions between sections of the talk/students, good eye contact, good vocal inflection, etc.) (10 pts)
2. Questions were addressed with logical, articulate answers (10 pts)
3. Slides were visually appealing, informative, and well prepared (free of typos, appropriate amount of text, etc.) (10 pts)
4. Students were equally involved in both delivering the presentation and answering questions (10 pts)

Reminder:

- Each student is expected to ask two questions during presentations of other groups.
- We are expecting equal contributions from each group member. You will be given a chance to provide anonymous feedback on group members and feedback on your own contribution. Grades may be adjusted accordingly.

# 10

# A First-Year Seminar Exploring the Mathematics of Sports Rankings

Jason Parsley

**Abstract** The modern era of Big Data is reshaping how decision-makers utilize information, leading to the developing field of analytics. This article describes a first-year seminar that explores these ideas through the lens of sports rankings. This seminar is structured around four main assignments. The final assignment, a research term project, requires students to thoughtfully develop their own sports analytics model; several term projects were presented at a local sports analytics conference. The article concludes with a reflection upon the successes and shortcomings of the seminar and a discussion of how one might modify the seminar for future iterations.

## 10.1 Background and Context

This article recounts a first-year seminar (FYS) on *The Mathematics of Sports Ranking*. This FYS has been taught so far just once, in the spring of 2016, at Wake Forest University. What follows details its structure and summarizes its successes and challenges; the appendix contains copies of the four main assignments for the course, including a research term project. This seminar provides a broad introduction to data and analytics through the viewpoint of sports, a popular topic which encouraged discussion and critique. One purpose in writing this article is to inspire others to run similar seminars: those interested are invited to contact the author.

Wake Forest University terms itself a *collegiate university*, one that blends aspects of a research university and a liberal arts college, with an undergraduate enrollment just over 5,000. Every student at Wake Forest must complete an FYS in order to graduate; enrollment is capped at 16 students per seminar. Each seminar meets for three contact/credit hours per week over a 14-week semester. These seminars are offered by permanent faculty across the College of Arts and Sciences, which oversees the First-Year Seminar program; most departments contribute a few first-year seminars per year. Instructors individually devise these courses, with approval from their department chair. Wake Forest lists four student learning outcomes [6] for these seminars; students will

1. read increasingly sophisticated texts critically
2. pose and respond to complex ideas
3. identify, analyze, interpret and evaluate different points of view and
4. construct cogent arguments in both written and oral form.

In addition, the author's proposal for this FYS enumerates the following seven goals.

1. Students will gain *mathematical awareness* and *fluency* via critically evaluating the implications of current data.
2. Students will understand topical applications of mathematics.

3. Students will undertake a *research experience* via their term project, which they will present at the Carolinas Sports Analytics Meeting (CSAM) in April.
4. Students will develop *presentation skills* by presenting their work both inclass and at CSAM.
5. Students will practice *technical writing*; the ability to write about technology and data is useful in many careers and this FYS provides a unique opportunity to hone these skills.
6. Students will interact orally; most class meetings focus upon class discussions and/or presentations.
7. This seminar topic should attract passionate students and be a lot of fun.

## 10.2 Mathematical Theme

In this seminar, students learn how mathematics and statistics can be used to rank and rate a list of objects, such as movies (e.g., Netflix), hotels (e.g., Tripadvisor), or search results (e.g., Google). Sports provides our primary source of examples, as data is plentiful; not only is ranking teams and individuals deeply ingrained in sports culture, but data analysis is currently reshaping how professional sports are managed, how players are valued, and how strategies are constructed. This course is built entirely around this theme; every reading and every assignment focuses upon rankings. To accomplish these goals, we must occasionally pause to introduce other mathematical or statistical ideas, e.g., standard deviation, linear regression, basic linear algebra concepts, . . . , needed to facilitate our analysis of rankings. Teaching this mathematical content accounts for roughly 10% of our overall classtime.

By design, this seminar is open to all first-year students and requires no prerequisites in mathematics. While students must learn some statistics and use *Matlab*, the seminar purposefully scaffolds these concepts to enable students of all backgrounds to accomplish our goals.

## 10.3 Course Structure

If we seek to rank the best pitchers in baseball this year, one natural place to start is with the Cy Young Award, determined by the rankings submitted by a group of sportswriters. If we instead seek to rank the best strikeout pitchers in baseball this year, we should instead look at the strikeout data for the current season. This dichotomy between *rankings generated by voters* and *rankings generated by data* naturally splits the course material for this FYS into two components.

Voter-generated rankings lead to studying different voting methods, along with their strengths and weaknesses. This seminar discussed six common voting methods: *plurality rule*, *Borda count*, *instant-runoff voting* (also known as *ranked choice voting*), *pairwise comparison*, *least-worst defeat*, and *ranked pairs* (also known as the *Tideman method*). Most sports ranking methods utilize a variation on Borda count, in which descending places on voter's ballot are worth decreasing amounts of points. One primary example, which students explore in the opening session of the seminar, is the Associated Press poll for college basketball (or college football) teams. Currently, the Associated Press releases each sportswriter's (voter's) ballot, which allows for detailed analysis of how the poll is constructed. This unit ends naturally with a discussion of Arrow's Impossibility Theorem, which asserts there is no voting method for three or more candidates that is perfectly fair.

Data-generated rankings, though not a new concept, drive the burgeoning field of *sports analytics*, which itself forms a small subfield of *data analytics*. Our seminar's journey through this material begins with linear algebra. Students need matrices and rudimentary knowledge of *Matlab* in order to compute rankings via the Massey, Colley, and Elo methods; see Langville and Meyer [3] for a readable explanation of these methods. From a scheduling perspective, March Madness (the college basketball playoffs) requires these methods to be firmly established before the middle of the semester; Section 10.3.3 describes our project on March Madness.

First-year seminars at Wake Forest feature significant amounts of writing; "writing to learn" is stressed above "learning to write". For our seminar, four "Major Assignments" accomplish this goal. The next four subsections describe these assignments, copies of which appear in the appendix. The final assignment is a term research project, worth half of the course grade, in which students devise their own sports ranking, compute it, and report on its findings. Seminar students may revise their written work after it is graded; each of the first three Major Assignments may be resubmitted up to twice. See the Appendix (p. 98) for the handout on revisions.

The second half of the course is focused on accomplishing this term project. Each group must present progress reports. Class meetings present statistical material that would be potentially useful in completing their research, such as expected value, distributions of data, standard deviations, and an introduction to linear regression.

In addition to these projects, the class features a number of topical readings, along with some more traditional homework assignments. See Section 10.3.5 for descriptions of some of these materials.

### 10.3.1 Major Assignment 1: *Moneyball*

American popular culture first encountered sports analytics through Michael Lewis's book *Moneyball* [4] and the subsequent film [9]. Exploring its themes, first in discussion and then in our first Major Assignment, serves as a constructive introduction to the course. How can statistical knowledge and modeling of sports lead to greater success on the field? What are its limitations?

For this project, students must choose a topic related to some aspect of the book, investigate it using additional sources, and write a short paper on their findings; the assignment description is provided in the Appendix (p. 93). The pedagogical motives for this assignment are varied and designed to lay groundwork for the rest of seminar. First, this paper forces students to, within the first two weeks of the semester, describe in their own words some facet of data analytics. In addition, students must write about a popular subject academically and produce a technical paper. There is significant value in students learning to effectively write technical reports; for many of them, their first job after college will require writing that is more technical and less like a humanities paper. Another benefit is that the instructor immediately gains an impression of the writing abilities of each seminar student.

Many other books could be substituted in lieu of *Moneyball*. One lesser known book, which the author has used in a different course and recommends, is *The Only Rule is it Has to Work*, by Lindbergh and Miller [5], two sports analytics journalists who ran a minor league baseball team one summer.

### 10.3.2 Major Assignment 2: Conduct Your Own Election

After learning about six different voting methods, students are tasked with their second project, to conduct their own election. See the description in the Appendix (p. 94). Each student must choose a topic with five or more candidates and then have at least 35 people vote on it. After tallying the votes, the student must determine the winner via each of the six voting methods studied. Analyzing the results is an important portion of this assignment. Did all the methods agree? If not, which one(s) gave the most believable results? Did any ties occur? Did any voting discrepancies occur?

This project in particular presents a formatting challenge. With a multitude of voters, there is a tremendous amount of data and calculations; we emphasize that the students' technical reports must be easily readable. The class makes extensive use of the file storage platform Google Drive, in order to choose topics, make ballots, and compute the results.

The seminar students chose a variety of topics for their elections. Some topics centered upon style (favorite dance move, favorite sports apparel brand), some upon entertainment (favorite music genre, favorite cartoons). Others were sports-focused (best basketball player, favorite sports league) or food-driven (favorite fruit, favorite Wake Forest dining option). This project received significant buy-in from the students.

### 10.3.3 Major Assignment 3: March Madness

The NCAA Division I men's basketball tournament, often referred to as March Madness, offers fans a chance to fill out a "bracket" predicting who will win each game. Deliberations on how to rank teams and build a better bracket have evolved as March Madness gained in popularity and as sports analytics matured. Currently, hundreds of different ranking methods have been suggested.

In this assignment, students must select a ranking method, other than the ones studied in class, to research. They then use that method to complete a bracket for March Madness. Students also fill out brackets based upon their personal views on who would win. Everyone in the seminar then posts their brackets to a common site, e.g., ESPN. The instructor, in addition, contributes additional brackets based upon the Massey and Colley ranking methods, as well as ones based upon the Associated Press poll, the coaches' poll, the seeded teams, and the ranking method that the NCAA uses in seeding basketball teams—in 2016, the RPI (*ratings percentage index*), currently the *NET ranking*.

The seminar students chose a variety of types of ranking methods and enjoyed considerable success. Before the Final Four, i.e., after 64 of 67 games in the tournament, our seminar group ranked 49th out of over 100,000 groups[1] on ESPN's bracket contest. Though no one individual bracket in our group did nearly so well, most of our brackets performed significantly above average.

Evaluating these results, a subsequent class discussion explored how statistical methods were powerful in the aggregate but could not always counter or explain outliers, for example, the urban legend of someone who selected teams based upon their mascots and wound up with a near-perfect bracket. Ultimately, our group did not retain its lofty ranking; ESPN heavily weights picking the champion and runner-up, which most of our brackets failed to do. A subsequent class discussion examined how this biases the final group rankings in favor of the champion's fan groups.

Students then wrote a paper on how their ranking method works and what the results were. The project description appears in the Appendix (p. 96). More than any other assignment, this project captured student excitement as they followed a popular event, one in which many of them were personally invested.

### 10.3.4 Major Assignment 4: Term Project

The second half of this FYS revolves around a term project. Pairs of students must develop a sports analytics thesis statement and then research it. They find data, analyze the results, and draw conclusions. Essentially, each pair is conducting an undergraduate research project that constructs their own sports ranking. The seminar significantly scaffolds this project; in addition to informal discussions in and out of class, students must present both their initial topic idea, for peer feedback, and a status report midway through this effort. See the Appendix (p. 97) for a description of the term project.

Students in this seminar were asked to present at the Carolinas Sports Analytics Meeting at Furman University in April 2016. Due to logistics, a majority of the students were unable to attend. Six students representing four partnerships presented posters at the meeting, which featured other students, professors, and professionals including an NBA analytics executive and a writer for ESPN. This undergraduate research presentation was a valuable learning experience which highlighted both academic and professional career paths.

By the end of the semester, each pair of students had to write a 10- to 20-page paper explaining their work. Those groups who did not present at the conference gave presentations to our class during our final exam period.

### 10.3.5 Other Readings and Assignments

In addition to the four Major Assignments, a number of additional readings and assignments introduce students to sports rankings. In the first portion of the course, students find the winner of sample elections via different voting methods and investigate the Olympic site selection method, which is a variation on instant-runoff voting and at times controversial. Students also read about the selection processes for both the NCAA football and basketball playoffs, and contrast these procedures with the voting methods studied. One assignment introduces *Matlab*, which they utilize when learning the Massey and Colley methods, to rank the schools within an NCAA basketball conference. Examining Steph Curry's shooting performances in the NBA, the class computes various statistical quantities: mean, median, expected value, histograms, etc. The students read many topical news articles on sports analytics, and later when preparing their term projects, on data visualization. The seminar also features several in-class writing assignments, often as lead-ins to class discussions.

As an example assignment, here is an actual situation where sports analytics would have helped an NFL coach.

**Question 1.** Suppose a football team, late in a game, is losing by 14 points; their only realistic hope is to score two touchdowns. Valiantly, they score one touchdown, worth six points, and then have a decision between kicking an extra point, worth one point, or attempting a two-point conversion. Which should they choose?

Kicking the extra point has a high probability $k$ of success; attempting the two-point conversion is more risky, with a probability $c$ near one-half. If the game is tied, the teams will play in overtime; call $p$ our team's probability of winning once overtime begins.

[1] This ranking used the average of the group's scores. In fairness, a large number of these groups did not have enough members to qualify for the ranking; one estimate is that approximately 42,000 groups were ranked.

This question has a straightforward answer. For simplicity in this article, let us assume $k = 1$, $c = 1/2$, $p = 1/2$, and that our team does score two touchdowns. If our team scores twice and kicks both times, then its chance of winning is $k^2 p = 1/2$. Instead, imagine the team goes for a two-point conversion. If successful, they should kick the second time, and then they win the game; if unsuccessful, then they would again go for two after the second touchdown in order to force overtime. The chance of the former scenario is $ck = 1/2$; the chance of the latter scenario is $(1 - c)cp = 1/8$. Adding these, the team's chance of victory, assuming two touchdowns are scored, is $5/8$ if they attempt a two-point conversion following the first touchdown.

By going for a two-point conversion then, under these simplifying assumptions, a team has a 12.5% greater probability of winning than if they kick the extra point. This exact question arose during a 2016 NFL playoff game, on the first weekend after our class began. Kansas City, down 14 points, scored a touchdown with just one minute left in their divisional playoff game at New England. Kansas City mistakenly did not go for two, and they did not score another touchdown; however, their strategical error did not go unnoticed. Benjamin Morris wrote a detailed analysis [7] for *FiveThirtyEight* incorporating more representative values for $k, c$, and $p$; attempting the two-point conversion is a robustly optimal decision, even as these values fluctuate. A few days after Kansas City's playoff game, the seminar first examined this question; after discussion, Morris's article was distributed; then as homework they computed the probability difference between the options, for varying $k, c$, and $p$ values.

## 10.4 Reflections

This first-year seminar has been taught once so far. One of its strengths was the availability of topical, relevant examples and materials, for instance the NFL playoff game described immediately above.

The students enrolling in this FYS differed from the overall student body in three noteworthy ways. First, they were all sports enthusiasts. Second, while the ethnic breakdown of the class mirrored that of the campus, only two female students enrolled. Perhaps better marketing of this seminar, possibly to women's sports teams, might avoid this gender imbalance. Third, exactly half of the students, 8 of 16, were varsity student-athletes, from the following sports: basketball, field hockey, football, soccer, and tennis. (I have included one team manager among this count.) This student-athlete population enabled knowledgeable, first-hand discussion of topics in sports and in particular in rankings. For instance, our discussion one day turned to the idea of ranking football and basketball prospects, which is a highly detailed process with a growing audience; it is accomplished first by grading high school players as 1-star to 5-star recruits, and then is refined to top-100 lists. We were able to hear from various athletes on how that ranking was done and what it meant to their circumstances and decision-making.

One success was the revision policy, see the Appendix (p. 98), which the author had not attempted before. Students are allowed to revise each of the first three Major Assignments, if they did not receive an A or A− grade. (The fourth Major Assignment, due at the end of the semester, leaves no time for any revisions.) As one primary advantage, students learn how to incorporate constructive feedback to improve their writing. Second, allowing revisions emphasizes that ownership of the course grade lies with the student; students unhappy with their grade have a direct means by which to better it. Third, allowing revisions frees the instructor to assign fair feedback and noninflated grades without losing student buy-in; the message to the student becomes "here are the strengths and weaknesses of this effort, and if the grade is less than desired, we can work together on how to improve it." The main disadvantage is increased grading time, but the author found grading the revisions to be a significantly faster process. Roughly half of the students submitted a revision to at least one assignment, though no one revised the same assignment twice.

Assigning a short introductory paper as the first Major Assignment was beneficial, as it forced students to grapple with our course themes and then write cogently about them, within the first few weeks of the semester. In letting students choose their own topics, our motivation was for students to develop a greater first-hand connection to the paper; this certainly occurred for some but not all of the students. One side effect was that the variety of papers made grading more enjoyable but slightly harder, especially when comparing efforts. Not only did the instructor gain an impression of each student's writing style, but the students received early feedback on how their projects would be evaluated and what the expectations of the course were.

The second project on voting is one the author has assigned in several different courses. This sort of experiential learning about voting methods is helpful for students' knowledge retention about these algorithms. And, the results provided valuable information, such as learning that the favorite dance move on campus that semester was *the Dab*.

One issue faced in the third Major Assignment was that several ranking methods that students selected lacked information about how they were constructed. Some seminar students were forced to start over using a different method. For future iterations of this seminar, the author intends to curate a list of 20-30 ranking methods whose algorithms are sufficiently public. Otherwise, this assignment was a success; it truly captured the students' interest.

The most significant setback to our seminar plans occurred when over half the class was unable to attend the CSAM conference, which was billed as a cornerstone of this course. Only one of the student-athletes was able to attend. Eventually, a deal was struck offering attendees sizeable extra credit. Arranging logistics for seven of us to travel to the conference was also an issue; eventually we settled on making it a long daytrip. Nonetheless, one main accomplishment of this seminar, in retrospect, was that six first-year students were able to present their work at a research conference. For future iterations of this seminar, instead of attending a conference, perhaps the class could schedule an on-campus event for presentations.

Another issue faced involved the textbook, *Who's #1?*, by Langville and Meyer [3]. This is an indispensible, wonderful book for anyone fluent in mathematics or data who wants to learn in detail about rankings. However, its level was too high for our first-year students, who possessed varying mathematical backgrounds. While there are other options for texts, the next iteration of this seminar will feature a better curated reading list and guided notes on the relevant voting, ranking, and statistical material.

Finally, how well did this FYS meet its goals and student learning outcomes, listed in Section 10.1? Two of the student learning outcomes, the second and the fourth, fit naturally with this seminar. The seminar students considered and responded to complicated ideas from the world of data analytics; they posed and developed their own topics for the term project. This seminar, through increasing students' quantitative literacy and critically examining statistical claims, satisfied the first outcome on critically reading. Finally, only in a broad sense (of voting as social choice) did students examine different points of view, the third outcome.

Meanwhile, our stated goals for this seminar were all clearly accomplished. The seminar students gained more fluency with data and mathematics in general via the course material, and our reading and assignments showed current applications of sports analytics. The term projects were a valuable research experience for the students; the seminar did stress technical writing, class discussion, and presentation skills. Finally, many students displayed a genuine passion for this course material.

Overall, this was an enjoyable seminar to teach, with enthusiastic students and a research experience that led to students presenting at a conference. The author believes it transfers well to other institutions and encourage others to utilize these ideas and materials.

## 10.5 Bibliography

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[8] Ken Massey. *Massey Ratings*. `masseyratings.com/cb/compare.htm`, accessed 09.29.2018.

[9] *Moneyball* (film), 2011, Bennett Miller (director), Columbia Pictures.

## Appendix

*Below are the project descriptions and the revision guidelines distributed to students in this seminar; they have been slightly edited for publication. Readers are invited to adapt these for your own usage. (If you do use any of these, please let the author know; feedback is welcome.) For reference, this FYS was taught in three 50-minute sessions per week.*

## Handout for Major Assignment 1: *Moneyball*

*This is an edited version of the handout distributed to students for the first project.*

After reading *Moneyball* [4], choose a (very narrow) topic pertaining to the book and sports analytics. Investigate this topic further, using at least two outside sources. Then write a 2-4 page coherent, succinct paper on your topic.

### Dates & Deadlines

- Class day 3: first class discussion, chapters 1–6.
- Class day 5: second class discussion, whole book.
- Class day 5: submit your topic statement and some details via Google Forms. Do this before you start the majority of your writing. Submit early for more feedback!
- Class day 7: upload your paper to our Google Drive folder, under Major Assignments.

### Objectives

1. Gain a first impression of your writing style, strengths and weaknesses.
2. Synthesize a topic from this book with some additional research and produce a well-written, coherent document.
3. Contrast the writing of technical papers with the writing of essays in the humanities, as we begin our journey towards efficient, successful communication.
4. Develop your writing about a subject [baseball, here] that is both (a) outside of usual academic coursework and (b) a passion (for some of you); ensure that your work is both objective and rigorous.

### Some possible topics

1. Explore one or two baseball statistics, and their correlation to wins and/or runs scored. Probably focus on a particular season, ideally the most recent one. On-base percentage is one key statistic in the book.
2. Follow the careers of 1-3 of the prospects named in the book. Compare their success with what was predicted.
3. Does Billy Beane still use analytical methods? If so, how and to what extent?
4. Have any teams rejected "Moneyball"-like tactics? Why?
5. Select a topic of your choosing.

Avoid the temptation to pick too broad of a topic. Maintain a sharp focus; aim for depth, not breadth.

## Handout for Major Assignment 2: Conduct Your Own Election

*This is an edited version of the handout distributed to students for the second project.*

In this assignment, you will make your own ranking by conducting an election and analyzing its results. Choose a topic for your ranking/election that has at least five candidates; five or six candidates seems ideal, but more are welcome. Have at least 35 voters participate in your election, with each one ranking all of the candidates in a preference order. You will analyze the results of your election using the ranking methods by voters that we have studied in this course.

### Topics

The choice of topic is rather important. Choose one that will give a variety of responses, and not one with an easily predictable result. For example, "preferred day of the week" is a bad topic, as most people would choose Friday or Saturday, and Monday will inevitably finish last. Be specific. Ideally, your topic will have close enough results to produce different winners via different voting methods.

*To do: Sign up for your topic on our Google spreadsheet.*

A good topic for this project has 5–6 clear, identifiable candidates, i.e., "favorite recording artist" is another bad topic, too many choices. If you have a great idea with more candidates, that is okay. If you are unsure about your topic, feel free to ask me or classmates for our opinions. You should not do the same topic as anyone else, or work together in any way on this project.

### Voters

Suggestion: have an odd number of voters, to avoid ties. All "voters" must come from the Wake Forest community. (I will not vote.) Ask your friends, too, and anyone else you know on campus. You are encouraged to have at least one non-student vote in your election. Feel free to draw on local campus "celebrities" as voters—extra credit may be given.

### Ballots

Use Google Forms to make your ballot. We will discuss how to do this in class. Your ballot should, most importantly, be easy to understand and complete. Feel free to make it aesthetically appealing and distinctive. You should list the candidates and ask the voters to rank the candidates in a preference order. Among the options on Google Forms, select these options:

- require university login
- automatically collect university username
- only allow one response per user (i.e., each person only gets one vote) and
- allow responders to edit their response.

When you are ready to tally the votes, it is best to have the form stop accepting responses.

*To do:* Share your form with me. (Do not just send the form to me; grant me access to your form and its responses.)

### Analyzing Your Results

As we have done in class, summarize the preference lists in one preference table. Construct the ranked pairs graph. Determine the overall ranking (of all candidates) via the following voter-based ranking methods: plurality rule, Borda count, instant runoff voting (IRV), pairwise comparison, least-worst defeat, and ranked pairs.

Is there a *Condorcet candidate*, i.e., one that defeats all other candidates head-to-head? If so, does that candidate win via each system above? Did you encounter any ties? (Ties can be difficult to handle with ranking methods; usually there is a separate tiebreaking procedure, which can be quite involved. Ask if you encounter ties for IRV or ranked pairs; each method has some fine print on how to break them; the other methods all simply declare a tie.)

### Your Report

In a typed technical report of at least three pages (not counting appendices), describe your election including all candidates. Then describe the overall ranking outcome (not just first place) via all methods above. Your document might begin with an *executive summary* of 1–2 paragraphs—aim for half a page—that quickly details your important findings.

Your report will include a lot of information: data, results, and computations. Strive to present this effectively and succinctly. Visual layout and presentation are rather important. You should utilize tables; feel free to include charts and color.

### Formatting Your Report

Technical reports are different than English papers and even scientific lab reports. Their purpose is to clearly present your findings in a well-written, easily understandable format; strive for both efficiency and completeness. Here are some suggested pieces of your report. None are mandatory; feel free to utilize your own formatting.

1. *Executive Summary*. Present quickly your topic and outline the main results. An overall results table is helpful. (This is similar to an abstract for an academic journal article, but less restrictive.)
2. *Introduction*. Describe your topic; it is ideal if you offer some context (e.g., why did you choose it? why is it of interest to you? to other people?)
3. *Data*. Make a preference table, or find another appealing way to present the voting data.
4. *Results*. Describe your process and how the results were obtained. You should complete all necessary computations to determine the winner under each system. Simple computations (e.g., for plurality rule, for Borda count, a pairwise comparison table, ...) should appear here. Put these in tables in general. If you need equations, it is frequently good to *offset* them. Anything too long should go in an appendix.
5. *Analysis*.
   - Compare and contrast the outcomes by the different methods. In your opinion, which method(s) best captures the will of the voters? Which methods are best suited to this type of election? Why?
   - If your election produced different winners by different methods, comment on the disparities. Which candidate do you believe should have won? Which method(s) correctly predicted this winner? Why so?
   - If applicable, address the following questions. Were there any irregularities when conducting the election? Did any voter fail to vote for all candidates or make unclear votes? How did you deal with these irregularities?
6. *Conclusion*. Depending upon how you write your analysis, you might omit this.
7. *Appendix*. Any lengthy computations should appear in an appendix.

### Advice

- Start early.
- Ask me questions as you go. If you ask questions on the night before the project is due, you may not get a response in time.
- Spreadsheets can compute the plurality and Borda rankings with little effort.
- Pairwise comparison can be done on a spreadsheet in multiple ways, or by hand.
- You should include a summary of the pairwise comparison results in your report. These might look good in a well-designed table.
- Instant runoff voting (IRV) will be the most complicated method. Remember that if a candidate has a majority of first-place votes at any point, they will win IRV—you can simply stop when this happens. You also may be able to justify eliminating several candidates at a time without fully making another preference table in between; if they cannot possibly get the most first-place, and you can argue why this is so, then proceed to eliminate them.

## Handout for Major Assignment 3: March Madness

*This is an edited version of the handout distributed to students for the third project, on March Madness.*

March Madness, the annual NCAA Division I men's basketball tournament, is upon us. We have learned the basics on how to create rankings, and it is now time to put our knowledge to the test of predicting the tournament outcomes.

In this assignment, you will create two *brackets* of your predicted outcomes.

1. Choose a ranking method, other than the ones covered in class (Massey, Colley, Elo, polls, . . . ). Use this ranking to construct a bracket, based solely upon the order of the teams.
2. Your own best guesses as to who will win.

You will post these brackets to our class bracket group on ESPN: You should name your brackets: YourName-RankingMethod and YourName. After the tournament concludes write a 3–8 page paper on the results. Include the following items:

- an executive summary,
- a description of how your ranking works, i.e., what is its process for inputting basketball results and outputting a ranking?—you will have to do some research here,
- an analysis and explanation: if the NCAA selection committee solely used your ranking method, would they have included any of the first four teams[2] left out of the tournament,
- a summary how your brackets performed (were they good at predicting upsets?), and
- copies of your brackets. (You can print your ESPN brackets as pdf documents.)

### Sources for Rankings

There are hundreds of different rankings posted for college basketball. Ideally, pick one with a detailed explanation of its methodology and that is based on sound mathematics.

- Ken Massey [8] maintains an online list of several college basketball rankings.
- FiveThirtyEight [2] predicts March Madness each year.
- BracketMatrix [1] predicts bracket seeding.

### Dates & Deadlines

*Note:* the NCAA Tournament typically begins just after Wake Forest's spring break—a logistical challenge. The relevant dates below reference that week.

- Tu.: sign up for your ranking method on Google Drive.
- W.: submit 1-2 paragraphs (typed printout or handwritten) describing the details behind the ranking method you have chosen. This will become a first draft of the description mentioned above. You do not have to fully explain your ranking; by Wednesday, submit your best attempt so far. Unless you grasp all of the details, you should conclude with two or more questions for me on how this ranking functions.
- Th., noon: tipoff and ESPN deadline for entering brackets. (This is a hard deadline; if you fail to enter brackets on ESPN, the highest possible grade on this assignment is a D.)

### Notes

1. Brackets will be posted, based on the (pure) Massey method, Colley method, polls, RPI, and seeds. Your goal is to beat them.
2. Grades are not based on which bracket does the best, but rather on how much effort you put into understanding another ranking method, and how well you describe it.
3. The best brackets, by the ESPN scoring system, earn extra credit.

[2] These four teams became the top seeds in the 2016 NIT: Monmouth, St. Bonaventure, South Carolina, Valparaiso.

## Guidelines for Major Assignment 4: Term Project

*This is an edited version of the handout distributed to students for the term project.*

In groups of two, extensively research a sports rankings topic of your choice. You will complete a term paper (10-20 pages), and present your findings in a poster at the 2016 Carolinas Sports Analytics Meeting at Furman University.

### Deadlines & Grading

- Class day 21: List three possible term project ideas, see below; present one to the class for feedback.
- Class day 22 [day before spring break]: Choose a term project topic. Complete the evaluation of your topic on Google Drive, see below.
- Class day 26: each group will present a five-minute status update.
- *Sa., April 9*. Present at the Carolinas Sports Analytics Meeting at Furman University.
- End of classes: term paper due.
- Final exam period: alternate presentation time for term projects.

The project grade is 65% term paper, 30% presentation, and 5% attendance/interim deadlines.

### Ideas for Term Project Topics

By the start of class day 21, each partnership should list at least three possible ideas to explore. Each group should post its topics on Google Drive; no fair stealing from other groups, at this point. You will present one of them, probably your favorite, to the class.

### Evaluation of Term Project ideas

Please list your topics below, and provide at least 1-2 sentences of detail. You should begin to research your topic, here are some things to consider.

1. What sport/level is the setting for your topic?
2. What sources of data are available?
3. How do you intend to use that data?
4. Is your topic already well-understood? E.g., ranking baseball pitchers by who throws the highest percentage of strikes can already be looked up. Try to find something new that you can contribute.
5. What ranking methods might you use? What other similar rankings to this exist?
6. Has there been any academic/sports analytics work done related to your topic?

*Goal*: Each group should decide on a topic before spring break. We will spend part of that day's class in working groups; bring your computers. Each group should post a plan online.

### Presentation Expectations

- Prepare a 20-minute talk.
- You may use Power Point, the board, props, ...
- Each partner should present some significant portion of the presentation; this does not have to be divided evenly.
- You should post your powerpoint to our Google Drive.

## Guidelines for Revisions

*This is an edited version of the revision guidelines distributed to students.*

All of our Major Assignments welcome revisions. You may submit up to two different revisions per assignment. You are welcome at any point to discuss your paper and its strategies with me. Revisions are not necessary, but they do provide a way to improve your grade on a Major Assignment.

### Submission Policies and Grading

1. Each revision must be significantly different than what was previously submitted; fixing a few grammatical errors is insufficient.
2. You must submit all versions previously submitted.
3. You are welcome to attach a short note (a half-page, at most), separate from the paper, explaining what was changed. This is not mandatory, or even expected.
4. The maximum possible grade for a revision is typically an A-, though I reserve the right to go above this for superior work. For instance, any paper that would normally earn an A+ (these are quite rare) will still earn an A+.
5. You receive the grade earned on your final submission; in the event that this is actually lower than the original grade, I will encourage you to do another revision. (This is the only way in which you might submit more than two revisions of a single assignment.)

# 11

# Introducing Graph Theory in a First-Year Seminar on Network Science

Brittney Miller

**Abstract** Network science has been a fast-emerging field with applications in many disciplines. With students wanting exposure to the latest hot topics, a first-year seminar on network science is an appropriate venue to gently introduce students to the underlying mathematical concepts in graph theory while helping them connect to the material through their interests in various areas of study. This connection to course content allows students to meaningfully engage with the ideas of network science and reveals the impact of mathematics in places they may not have imagined. There really is something for everybody since network science reaches into biology, business, chemistry, computer science, economics, history, physics, political science, psychology, and sociology. Students are exposed to graph theory through the book *Linked*, introductory activities to highlight and better understand the basics of graph theory, and writing assignments to help connect graph theory to students' interests.

## 11.1 Background and Context

Coe College is a private, residential, liberal arts college in Cedar Rapids, Iowa with approximately 1,400 students. Our First-Year Seminar (FYS) program is college-wide, and each FYS course is four credit hours, meeting four days per week during the 15-week fall term. The goals of our FYS are to introduce our students to the many dimensions of college life and to help them develop skills to be productive college students, including critical thinking and effective reading, writing, and speaking skills. Each student submits a portfolio as the final product in their FYS, which consists of two papers chosen by the student to showcase their work throughout the term. FYS is required for graduation and counts as one of five writing-emphasis courses students must fulfill for graduation. Each FYS section typically has 15 or 16 students. The topic for each section is determined by the instructor and explores issues from multiple perspectives within or across disciplines. Each department on campus is represented by at least one instructor in our FYS program, but the topic of each instructor's FYS is not necessarily related or specific to their discipline. Our FYS offerings do not have prerequisite courses, and students are placed into an FYS based on their indicated preferences on a summer survey.

## 11.2 Mathematical Theme

The title of my FYS course is "The Oracle of Bacon: How Everything is Linked" with a theme of network science. I first became interested in network science after reading Albert-László Barabási's book *Linked* [1], in which he introduces to a general audience how everything is connected to everything. He touches on some of the science used to investigate networks, and he discusses a variety of examples to illustrate these concepts. For example, Barabási explores the history of six degrees of separation and the small world phenomenon by demonstrating that the average distance between two nodes in a network is relatively small compared to the total number of nodes in the network. Indeed, six is the average number of degrees of separation between two people on our planet of billions of people [1, p. 30] and 19 is the average number of clicks between two documents on the World Wide Web in 1998 of approximately 800 million documents [1, p. 34]. While I am not an expert in network science, I am intrigued by this topic since almost every discipline has some network that can be studied and the science behind these networks is rooted in graph theory.

My students and I explore networks across different disciplines and investigate aspects of networks described in *Linked* (for the layman) and *Network Science* [2], a textbook also by Barabási. We spend about five weeks spread throughout the term (one third of the course time) discussing the basics of graph theory accompanied by activities related to nodes/vertices, links/edges, degrees, degree distributions, connectedness, and random networks/graphs. The understanding of these basic concepts allows for more in-depth discussions of specific networks and for closer reading of selected articles referenced in Barabási's books. Students also identify these elements of graph theory in three writing assignments, in which they reflect on networks that are important to them. The purpose of the course is not to provide a rigorous mathematical treatment of graph theory but rather to gently introduce the elementary concepts of graph theory to help students view their experiences through a new lens as they begin their liberal arts education. Network science is a perfect FYS topic because it reaches a range of students with different interests. It also introduces students to a different area of mathematics they might not have encountered before, and the possible assignments allow for flexibility based on students' interests while incorporating the practice of critical thinking, reading, writing, and speaking skills.

## 11.3 Course Structure

The design of the course uses *Linked* as the backbone, focusing on one chapter per week, and *Network Science* is used as an occasional supplement for more detailed and technical descriptions of examples. Other readings for each week include a selection of articles referenced in Barabási's books or that complement the material for that week. After completing a short reading quiz each day, we discuss the readings and complete activities to help students make a more personal connection to the material. Students also write three papers and give presentations.

### 11.3.1 Introductory Material

The course begins with an introduction to nodes and links, or vertices and edges, respectively, in graph theory. After reading brief descriptions of many networks in the first two chapters of *Linked* and *Network Science*, we discuss different networks that each of us is part of or that are important to us, and we describe the nodes and links. A few students have difficulty identifying the links in their personal networks, thinking of them as physical constructs instead of as interactions between nodes [2, p. 24]. Other students help the class realize that sometimes links can by physical, such as the cables in a power grid, but nodes may be connected by relationships or joint participation in an activity. We then discuss the difference between directed and undirected graphs. These discussions are the basis for our first writing assignment that will be discussed in Section 11.3.5.

### 11.3.2 Bridges of Königsberg

After establishing the nodes and links of various networks, we read about and discuss the Bridges of Königsberg and related problems [1, Chapter 2] [2, pp. 43–44]. With several different graphs where the nodes are pieces of land and the links are bridges, students try to figure out if one could walk across all of the bridges and never cross the same one more than once. Students quickly find solutions when they exist, but they struggle with explaining why the other graphs do not have solutions. We then discuss Euler's proof that there cannot be more than two nodes with an odd number of links, and we revisit the graphs and check that the ones that students couldn't find solutions for indeed have more than two nodes with an odd number of links [2, p. 44].

### 11.3.3 Random Graphs

The next activity we complete is the construction of random graphs to demonstrate how to reproduce properties of real networks [2, p. 74]. We construct some of these by hand together for students to gain a better understanding of the role of probability in graph theory and the structure of each type of random graph. We start with the Erdös-Rényi model, or the $G(N, L)$ model, in which $N$ labeled nodes are connected with $L$ randomly placed links [2, p. 75]. We start with five nodes and number all ten possible links. Then we use a random number generator to obtain three numbers between one and ten to place three random links. We also learn about the Gilbert model, or the $G(N, p)$ model, in which each pair of $N$ labeled nodes is connected with probability $p$ [2, p. 75]. Starting with five nodes again, we construct a random graph by connecting a pair of nodes if a randomly generated number between 0 and 1 is at least our chosen threshold value of $p = 0.7$.

This in-class activity of constructing random graphs reveals to students the different graphs they could obtain, and they notice that not all of their graphs were necessarily different. This discovery on their part is an excellent lead-in to the degree of a node, a graph's average degree, and a graph's degree distribution which helps us determine when to identify two graphs as equivalent. We calculate the average degree and identify the degree distribution of the graphs on the board with five nodes before turning to mathematical software to create several random (and more complex) graphs (see the Appendix, p. 103). This first take-home activity demonstrates to students that graphs with ten nodes are less easily identifiable as equivalent, but understanding some of the properties from the graphs can help us identify which ones are equivalent.

After completing take-home activity 1, my students and I discuss their findings. They are surprised at the difference in the graphs that were produced and the data they gathered on each graph (degree of each node and the average degree of each graph). We pick several of their graphs to determine their degree distributions before diving into the binomial and Poisson distributions. I emphasize the shapes of different curves, starting with linear and quadratic functions and eventually exponential functions, and then I explain the shapes of the binomial distribution and the Poisson distribution. Using our examples on the board, we also discuss the difference between discrete and continuous distributions, and why the use of the Poisson distribution with one parameter is preferable to the binomial distribution with two parameters, even though the binomial distribution is the exact distribution of random graphs [2, Section 3.4]. This conversation is challenging for many of my students because they are nervous about the quantity and level of difficulty of mathematics and probability we would be using throughout the term. I emphasize that we are interested in the big ideas and not necessarily all of the details of each distribution. I explain that the process of obtaining the degree distributions on randomly generated graphs could be done on a larger scale, (e.g., random networks with thousands of nodes), and the binomial distribution would be the result.

### 11.3.4 Degrees of Separation

Next we turn our attention to degrees of separation and the average distance between nodes in a network. For take-home activity 2, students begin by clicking on the "Random article" link on Wikipedia, and then they click a link in each new article in an attempt to find the shortest path to the Wikipedia page about Adolf Hitler (see the Appendix, p. 104). As a class, everyone shares the average number of clicks needed to link the random articles and Adolf Hitler, and they reflect on how few clicks (often fewer than six) are required to connect the two. They also notice how quickly they reach other links that they recognize will eventually lead to the last two or three clicks to Hitler such as United States → World War II → Hitler. This activity helps illustrate the concepts of degrees of separation, average distance between nodes, and clustering coefficient, or the relationship between a node's neighbors. Although we do not delve into the mathematics of computing the average distance between nodes in a graph, this activity helps students make sense of how to compute the average distances for different networks in *Network Science*.

Take-home activity 3 also illustrates the concept of degrees of separation in the "Oracle of Bacon" in which students play the Kevin Bacon game by finding different actors' Bacon numbers (see the Appendix, p. 105). Students navigate to the Oracle of Bacon website and type in the name of an actor. The database finds the shortest path between the chosen actor and Kevin Bacon by linking actors that have appeared in the same production. This exercise leads into a discussion about most real networks not being random, since the movie actor network exhibits growth and preferential attachment. Unlike random networks that have a fixed number of nodes that are randomly linked, the movie actor network continues to grow as new actors are cast in movies and these new actors prefer to link to actors with many connections. We revisit our discussion on degree distribution since real networks follow the scale-free model and have power-law degree distributions [1, Chapter 5] [2, Section 4.2].

### 11.3.5 Writing Assignments

Throughout the term, I ask my students consistently to identify certain characteristics such as nodes, links, and the average degree for every new network we encounter in *Linked* and *Network Science*. They also do this for each of their writing assignments. Because one of the objectives of FYS is helping students develop their writing skills, this course incorporates three types of writing assignments accompanied by presentations: 1) a personal essay about a network the student is personally part of, 2) a pair of argumentative papers and a debate on a topic that involved networks (e.g., multi-level management companies), and 3) a summative paper on different aspects discussed throughout the term of a particular network. Each writing assignment includes a presentation, and students receive feedback on their papers after submitting a first draft, occasional peer-review feedback, and feedback on presentations from their peers and me. Papers are graded with the most points assigned to the body of the paper based on how fully the student addresses the prompt. Consultants from our campus Writing Center give writing workshops on brainstorming, thesis statements, editing vs. revising, and plagiarism and citations. We also have a consultant from our campus Speaking Center give a workshop on effective presentations and slides. As a mathematics professor, I also incorporate discussions on building quantitative literacy skills while reading studies and interpreting data.

In the personal essay, students write about a network in which they are a node and create a graph representing their network (see the Appendix, p. 105). In addition to identifying and writing about the components of graph theory, they also include personal reflections on the importance of their network in their life. This assignment is completed soon after the beginning of the term and helps me get to know my students better as many of them write about their roles in their families or on sports teams. It also helps my students connect to the course content and gets them thinking about how everything is connected. For the second writing assignment, students write a pair of argumentative papers—one supporting and one refuting—about a controversial statement involving a network (see the Appendix, p. 107). Part of each argument for and against the statement must use network science, and they may even use the same graph theory concepts to support each of their cases. For example, one pair of students may write about whether or not Facebook is a safe platform for teens to post messages to their friend network. In addition to addressing the effectiveness of available security features of Facebook, students may argue that the degrees of separation between nodes in the Facebook network may or may not adversely affect users. Because social networks have small average degrees of separation, a Facebook user can efficiently disseminate information to their friend network. On the other hand, a Facebook user may find that the small degrees of separation between their friends and their friends' friends may result in their messages being shared with users who are not the original user's friends. Students learn how to use evidence and data to support their claims, instead of infusing their personal thoughts and emotions into a debate. The third writing assignment is a synthesis of information from the term about one particular network (see the Appendix, p. 108). This assignment provides a summary of the course and allows me to assess my students' understanding of the content.

## 11.4 Reflections

Having only taught this course once, I found it very helpful to build a foundation with the graph theory material. I thought the pace started off slow. However, when we reached the discussion regarding degree distribution after completing take-home activity 1, several students shared with me their high level of discomfort with mathematics, and I am glad we didn't rush through that. I was concerned and surprised by my students' feedback because take-home activity 1 only required that they count and find averages. I assured them that take-home activity 1 was the most complicated math they would complete in my FYS, but many of them still seemed nervous. I wish I spent more time helping my students be comfortable with mathematics, especially since the topic of my FYS should have made it clear to students that mathematics and network science are present in almost every field. In my regular classes, I discuss growth mindset to help students approach their math anxiety in a different way, and I think it would have been helpful to do the same in my FYS.

Overall, I think the class was a success for me, although I am not sure my students were as excited as I was about the material. Many of them did not read the topic description and expected the class to involve more bacon than it did. However, my students were still able to engage with the material in the course because they could focus on their own interests by investigating networks in their intended areas of study. As I hoped, my students appreciated the variety of readings from different disciplines since they could find a topic they were interested in and connect with the material in a way that could help them better understand the concepts we were discussing in class. We all discovered something new from a discipline we were not familiar with, and I suspect many students also identified disciplines they did not find exciting.

I deliberately filled the schedule with activities, assignments, and readings that would allow students to practice critically analyzing information and help develop their reading, writing, and speaking skills. We practiced identifying key information in the abstracts and introductions of articles from various disciplines, and I helped direct their attention to important information throughout the readings by providing reading questions (no more than five questions per article or chapter). The take-home activities helped my students become comfortable with the material, and they accurately wrote about the concepts we covered most in depth throughout the course. These exercises helped my students identify strengths in their own papers, and they used the readings as models in writing their papers.

On the whole, students were successful in the writing assignments in identifying the key concepts from graph theory that we discussed throughout the term to learn about network science. Holding peer-review sessions in class and workshops led by fellow peers who work as consultants at our Writing Center helped my students better engage with each other and produce better quality work. I think having a "near peer" to explain and discuss difficult concepts with can help develop a student's understanding in a different way than constantly listening and trying to learn from an instructor. Also, setting aside class time to work on writing assignments allowed them to organize their thoughts, get clarification on an assignment, ask questions about the material, and get started early. One of the most common struggles my students had was expanding on and explaining the context and significance of some concepts, for example, the average distance between two nodes on a graph. Often, they would provide the definition of the average distance and state the average distance for the particular network they were writing about, but they didn't always provide any context for the meaning or significance of the average distance in their chosen network.

I was challenged to find a way to balance class time spent on content I was excited about with activities to achieve the learning objectives, and ultimately I tried incorporating too much material. Trying to stick to the course calendar with the number of articles I assigned as readings did not allow for enough time to dissect all of the different networks I wanted to explore. If I were to teach this class again, I would provide my students with a list of options to choose from and have each of them become the expert on that network and present it to the class. I still think this is a great topic for FYS, and I look forward to teaching it again, helping my students be more comfortable with mathematics, and providing them a great first-term experience.

## 11.5 Bibliography

[1] Albert-László Barabási, *Linked: How Everything Is Connected to Everything Else and What It Means for Business, Science, and Everyday Life*, Basic Books, New York, 2014.

[2] Albert-László Barabási, *Network Science*, Cambridge University Press, Cambridge, 2016.

## Appendix

### Take-home Activity 1

*This activity is completed outside of class and is assigned during week 2 of the term. At this point, students already know the terms* node, link, degree, *and* distribution, *and they have been shown a demonstration of Sage during class.*

*Instructions:* Go to CoCalc.com, and either create an online account or connect via another account. Create a new project and a new Sage Worksheet. Complete the problems below. Print your code by clicking on the icon for "convert to PDF" and then "Generate PDF." Staple your code to the back of this page.

1. Create three random graphs, each with ten nodes and twelve links, using the Erdős-Rényi model described in Section 3.2 of *Network Science*. An example of what the Sage code looks like for this type of graph is:

```
A = graphs.RandomGNM(10, 12)
A.show()
```

   Click the "Run" button to display the output.

   Call each graph something different. For example,

```
B = graphs.RandomGNM(10, 12)
B.show()
C = graphs.RandomGNM(10, 12)
C.show()
```

For more information, see the Sage documentation for RandomGNM:
`doc.sagemath.org/html/en/reference/graphs/sage/graphs/graph_generators.html#sage.graphs.graph_generators.GraphGenerators.RandomGNM`

Complete the table below by finding the degree (or the number of links) of each node and the average degree of each graph.

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Graph 1 | | | | | | | | | | | |
| Graph 2 | | | | | | | | | | | |
| Graph 3 | | | | | | | | | | | |

2. Create three random graphs, each with ten nodes and probability $p = 0.4$, using the Gilbert model described in Section 3.2 of *Network Science*. An example of what the Sage code looks like for this type of graph is:

```
set_random_seed(n)
D = graphs.RandomGNP(10, 0.4)
D.show()
```

Replace $n$ with three different whole numbers of your choice to create your three random graphs. Call each graph something different.

For more information, see the Sage documentation for RandomGNP:
`doc.sagemath.org/html/en/reference/graphs/sage/graphs/graph_generators.html#sage.graphs.graph_generators.GraphGenerators.RandomGNP`

Complete the table below by finding the degree (or the number of links) of each node and the average degree of each graph.

| n | Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Graph 1 | | | | | | | | | | | |
| | Graph 2 | | | | | | | | | | | |
| | Graph 3 | | | | | | | | | | | |

## Take-home Activity 2

*This activity is completed outside of class and is assigned during week 3 of the term. At this point, students already know the terms* node, link, degree, distribution, *and* clustering coefficient, *and we have discussed examples of* small worlds *in class.*

*Instructions:* Go to Wikipedia and click on "Random article" on the left side of the page. Click on a link in the article that you think will get you closer to the Wikipedia article about Adolf Hitler. Click on a link in this second article that you think will get you closer to the Wikipedia article about Adolf Hitler. Continue clicking different links until you reach the Wikipedia article about Adolf Hitler. What is the shortest path from the random article to Adolf Hitler? Can you find one that takes six (or fewer) clicks?

Play this "game" three times to try to find the shortest path from a random article to Adolf Hitler. Record the links you click for three different paths in the tables below.

| **Random Article:** | | |
|---|---|---|
| **Path 1** | **Path 2** | **Path 3** |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

## Take-home Activity 3

*This activity is completed outside of class and is assigned during week 5 of the term. At this point, students already know the terms* node, link, degree, distribution, clustering coefficient, *and* small worlds, *and we have discussed examples of* collaboration networks, network growth, and preferential attachment *in class.*

*Instructions:* Go to the Oracle of Bacon website to play Six Degrees of Kevin Bacon. Pick a variety of ten actors not mentioned in the readings and find their Bacon numbers. Then find the average of these ten Bacon numbers.

Are the results what you expected? Why or why not? Write at least a full paragraph and use evidence from the readings to support your claims. Use in-text citations and provide a list of references in APA style. Type your response and follow the formatting guidelines in the syllabus.

| | **Actor** | **Bacon Number** |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| | **Average** | |

## Writing Assignment 1

*This assignment is completed outside of class and is assigned during week 1 of the term. At this point, students have read the summer book* Outcasts United*, they already know the terms* node *and* link, *and we have discussed examples of networks in class.*

### Prompt

Describe a network that you are personally part of and discuss its importance to you. Address characteristics of the network, such as its nodes, links, and size. How diverse (broadly interpreted) are the people in this network? How has this network grown and how will it continue to grow? What impact does this network have on you, society, etc.? How is this network similar/different to networks in *Outcasts United*? The primary goal of this assignment is for you to convey a personal narrative in the context of network science (our FYS topic).

### Grade Sheet

___ / **20 points** for first draft. The first draft should be a minimum of two pages. Be sure to address more than the questions posed in the prompt. Writing must use academic language and be checked for spelling and grammar. Include at least one figure on a separate page (which does not count toward the length of the paper). Include a list of reputable sources in APA format and use appropriate in-text citations throughout your paper. Your reference page should be a separate page which does not count toward the length of the paper.

___ / **30 points** for content. Describe a network that you are personally part of and discuss its importance to you. Address characteristics of the network, such as its nodes, links, and size. How diverse (broadly interpreted) are the people in this network? How has this network grown and how will it continue to grow? What impact does this network have on you, society, etc.? How is this network similar/different to networks in *Outcasts United*? The primary goal of this assignment is for you to convey a personal narrative in the context of network science (our FYS topic).

___ / **10 points** Introduction
___ / **15 points** Body
___ / **5 points** Conclusion

___ / **10 points** for style and grammar. Your final draft should follow the formatting guidelines, be well-written, and reflect work that has been edited multiple times. Writing must use academic language and be checked for spelling and grammar. Typos and grammar mistakes should be minimal and not detract from reading the paper.

___ / **10 points** for a figure. Include at least one figure on a separate page (which does not count toward the length of the paper).

___ / **10 points** for references and citations. Include a list of reputable sources in APA format and use appropriate in-text citations throughout your paper. Your reference page should be a separate page which does not count toward the length of the paper.

___ / **10 points** for the writing process. You must visit the Writing Center at least once during the writing process. Before you leave the Writing Center, request that a report of your conference be emailed to me. Then, reflect on your visit by thoughtfully answering the Writing Center questions. This reflection should follow the formatting guidelines and does not count toward the length of the paper.

___ / **10 points** for following the instructions. Pages of each draft stapled, figure (on a separate page), references (on a separate page), first draft materials (include all of the first draft items in the order above), Writing Center reflection (on a separate page), and final document uploaded as a single pdf file to Moodle.

___ / **100 points** total

### Presentation

___ / **10 points** for content, style, and grammar of the slides. Prepare slides for the presentation of debate points only, not for cross-examinations or rebuttals.

___ / **10 points** for content, style, and grammar of the presentation. The delivery should be well-enunciated, at an appropriate volume, and have minimal fillers (“um”, “like”, etc.).

___ / **10 points** for length. Each part of the presentation should be within 30 seconds of the allotted amount of time.

___ / **10 points** for following the instructions. Upload your slides as a single pdf file to Moodle.

___ / **10 points** for feedback.

___ / **50** total

## Writing Assignment 2

*This assignment is completed outside of class and is assigned during week 6 of the term. At this point, students already know the terms* node, link, degree, distribution, clustering coefficient, *and* small world, *and we have discussed examples of the prompt in class.*

### Prompt

With your partner(s), decide on a network and make a controversial statement about it. Each of you will individually write two papers—one supporting the statement and one refuting the statement. While you and your partner(s) may brainstorm together, you must write and develop your own ideas in each of your papers. Using the lens of network science, you should make your argument in each paper by providing and analyzing evidence obtained from reputable sources. The primary goal of this assignment is for you to write two argumentative papers using evidence to support the statement and to refute the same statement, regardless of your personal beliefs.

### Grade Sheet

**Paper**

___ / **10 points** for topic proposal. A topic proposal should be submitted and approved before turning in a first draft.

___ / **10 points** for conference at Writing Center, October 9–12.

___ / **40 points** for first drafts. Each first draft should be a minimum of two pages, which means each of your papers should go onto a third page. Be sure to address more than the topics posed in the prompt. Writing must use academic language and be checked for spelling and grammar. Include a list of reputable sources in APA format and use appropriate in-text citations throughout your paper. Your reference page should be a separate page which does not count toward the length of the paper.

___ / **10 points** for peer review.

___ / **10 points** for feedback conference.

___ / **60 points** for content. Decide on a network and make a controversial statement about it. Write two papers, one supporting the statement and one refuting the statement. Using the lens of network science, you should make your argument in each paper by providing and analyzing evidence obtained from reputable sources. The primary goal of this assignment is for you to write two argumentative papers using evidence to support the statement and to refute the same statement, regardless of your personal beliefs.

| Paper 1 | Paper 2 |
|---|---|
| ___ / **5 points** Introduction | ___ / **5 points** Introduction |
| ___ / **20 points** Body | ___ / **20 points** Body |
| ___ / **5 points** Conclusion | ___ / **5 points** Conclusion |

___ / **10 points** for style and grammar. Your final draft should follow the formatting guidelines, be well-written, and reflect work that has been edited multiple times. Writing must use academic language and be checked for spelling and grammar. Typos and grammar mistakes should be minimal and not detract from reading the paper.

___ / **20 points** for references and citations. Each paper must use at least two reputable sources—one from class and one that you find on your own. Include a list of reputable sources in APA format and use appropriate in-text citations throughout your paper. Your reference page should be a separate page which does not count toward the length of the paper.

___ / **20 points** for the writing process. You must visit the Writing Center at least once during the writing process. Before you leave the Writing Center, request that a report of your conference be emailed to me. Then, reflect on your visit by thoughtfully answering the Writing Center questions. This reflection should follow the formatting guidelines and does not count toward the length of the paper.

___ **/ 10 points** for following the instructions. Pages of each paper stapled, references (on separate pages), first draft materials (include all of the first draft items), Writing Center reflection (on a separate page), Speaking Center reflection (on a separate page), and final document uploaded as a single pdf file to Moodle.

___ **/ 200 points** total.

**Presentation**

___ **/ 15 points** for content, style, and grammar of the slides.

___ **/ 20 points** for content, style, and grammar of the presentation. The delivery should be well-enunciated, at an appropriate volume, and have minimal fillers (*um, like,* etc.).

___ **/ 20 points** for length. Each part of the presentation should be within 30 seconds of the allotted amount of time.

___ **/ 10 points** for following the instructions. Upload your slides as a single pdf file to Moodle.

___ **/ 10 points** for feedback.

___ **/ 75** total.

## Writing Assignment 3

*This assignment is completed outside of class and is assigned during week 12 of the term. At this point, students already know the terms* node, link, degree, distribution, clustering coefficient, small world, collaboration network, network growth, *and* preferential attachment, *and we have discussed examples of the prompt in class.*

### Prompt

Perform an analysis of a network of your choosing using concepts that we have read and discussed throughout the term. Address characteristics of the network, such as its nodes, links, size, type, etc. Describe how the topics in *Linked* and *Network Science* inform our understanding of your network, although not every aspect from *Linked* and *Network Science* will be appropriate to discuss with your network. You should find at least three academic sources that are not texts from class to incorporate into your paper, and compare and contrast the perspectives offered by each. As part of your 7-10 minute presentation, include a map of your network created manually or digitally by you. The primary goal of this assignment is to synthesize the knowledge you have acquired by applying it to a network.

### Grade Sheet

**Paper**

___ **/ 20 points** for topic proposal. A topic proposal should be submitted and approved before turning in a first draft.

___ **/ 45 points** for first draft. The first draft should be a minimum of five pages, which means your paper should go onto a sixth page. Be sure to address more than the topics posed in the prompt. Writing must use academic language and be checked for spelling and grammar. Include at least one figure on a separate page (which does not count toward the length of the paper). Include a list of reputable sources in APA format and use appropriate in-text citations throughout your paper. Your reference page should be a separate page which does not count toward the length of the paper.

___ **/ 110 points** for content. Perform an analysis of a network of your choosing using concepts that we have read and discussed throughout the term. Address characteristics of the network, such as its nodes, links, size, type, etc. Indicate how the topics in *Linked* and *Network Science* inform our understanding of your network, although not every aspect from *Linked* and *Network Science* will be appropriate to discuss with your network. You should find at least three academic sources that are not texts from class to incorporate into your paper, and compare and contrast the perspectives offered by each. As part of your 7-10 minute presentation, include a map of your network created manually or digitally by you. The primary goal of this assignment is to synthesize the knowledge you have acquired by applying it to a network.

- ___ **/ 20 points** Introduction
- ___ **/ 70 points** Body
- ___ **/ 20 points** Conclusion

___ **/ 15 points** for style and grammar. Your final draft should follow the formatting guidelines, be well-written, and reflect work that has been edited multiple times. Writing must use academic language and be checked for spelling and grammar. Typos and grammar mistakes should be minimal and not detract from reading the paper.

___ **/ 15 points** for a figure. Include at least one figure on a separate page (which does not count toward the length of the paper).

___ **/ 15 points** for references and citations. Include a list of reputable sources in APA format and use appropriate in-text citations throughout your paper. Your reference page should be a separate page which does not count toward the length of the paper.

___ **/ 15 points** for the writing process. You must visit the Writing Center at least once during the writing process. Before you leave the Writing Center, request that a report of your conference be emailed to me. Then, reflect on your visit by thoughtfully answering the Writing Center questions. This reflection should follow the formatting guidelines and does not count toward the length of the paper.

___ **/ 15 points** for following the instructions. Pages of each draft stapled, figure (on a separate page), references (on a separate page), first draft materials (include all of the first draft items in the order above), Writing Center reflection (on a separate page), and final document uploaded as a single pdf file to Moodle.

___ **/ 250 points** total

**Presentation**

___ **/ 40 points** for content, style, and grammar of the slides. You should summarize the main idea(s) in your paper to help your peers and me better understand the network you have analyzed. Your slides should be appealing, include visuals, and not be filled with too much text.

___ **/ 40 points** for content, style, and grammar of the presentation. The delivery should be well-enunciated, at an appropriate volume, and have minimal fillers (“um”, “like”, etc.). You should discuss the main ideas in your paper by clearly explaining your network and how topics from class readings and discussions apply to your network.

___ **/ 30 points** for length. The presentation should be 7-10 minutes with slides.

___ **/ 20 points** for the speaking process. You must visit the Speaking Center at least once during the speaking process. Before you leave the Speaking Center, request that a report of your conference be emailed to me. Then, reflect on your visit by thoughtfully answering the Speaking Center questions. This reflection should follow the formatting guidelines and does not count toward the length of the paper.

___ **/ 20 points** for following the instructions. Upload your slides as a single pdf file to Moodle.

___ **/ 150** total.

# Part IV

# Mathematics in Politics, Equity, and Social Justice

Mathematics of social choice, gerrymandering, apportionment, and fair division are rich topics for first-year seminars focused on utopia and dystopia, social criticism and commentary, and humanism. In addition, immigration, the civil rights movement, and mathematics literacy are all appropriate material for student work and discussion. Seminars in this section feature these topics and can be closely connected to objectives of experiential and community based learning.

# 12

# *Math and Politics*: An Interdisciplinary First-Year Seminar

Jacqueline Anderson

**Abstract** *Math and Politics* is a First-Year Seminar at Bridgewater State University that covers four major topics: social choice theory, gerrymandering, polling, and apportionment. In class, students frequently work in small groups on problem sets to learn the course's mathematical concepts, while outside of class students primarily read material to provide the accompanying political and historical content. The main challenge of the course is engaging students with widely varying levels of experience and interest in mathematics. In this article, I provide details about the course structure and content and discuss what went well in the course's first iteration, what did not, and what I have changed for the future.

## 12.1 Background and Context

Bridgewater State University (BSU) is a comprehensive public teaching university in Massachusetts serving approximately 11,000 students, 9,500 of whom are undergraduates. Our First-Year Seminar (FYS) program is a university-wide component of the core curriculum and is meant to provide topical courses that introduce students to academic thought and discourse. Every incoming undergraduate must take one FYS during their first year at BSU. Each semester, students have about 50 different courses to choose from, including (this semester) an anthropology course about *The Twilight Zone*, a geology course about the planets of the solar system, and a philosophy course about the ethics of Harry Potter. Courses meet for three hours each week for a 15-week semester and enrollment in each section is capped at 20 students. All FYS courses are designated as writing-intensive, which means that the course assignments must include at least 15 pages of writing with formative feedback and opportunities for revisions. To meet this requirement, some faculty assign three five-page papers, others assign eight two-page papers, and most assign a mix of longer and shorter writing assignments throughout the semester to meet the 15-page minimum. With the approval of their department chair and the FYS coordinator, any faculty member can propose to teach an FYS about a topic of their choice. While any department can design and offer FYS courses, most of the offerings tend to come from the humanities and social sciences. FYS courses offered by the Mathematics Department have been rare due to the writing-intensive requirement and the challenge of designing a mathematics course with wide appeal appropriate to students of varied mathematical backgrounds.

## 12.2 Mathematical Theme

Because of the dearth of mathematics offerings in our FYS catalog, I set out to create a course that would show the beauty, variety, and usefulness of mathematics to students who may not otherwise encounter it. My course is titled *Math and Politics*, and in it I weave together mathematics, history, and political science, with an emphasis on how we can use mathematics to better understand our political systems. In class, we spend about 75% of our time on mathematical content and 25% of our time discussing the relevant history and political science to motivate the mathematics and put it in context. The course is divided into four units: social choice theory, redistricting, polling, and apportionment. In the list below, I briefly summarize the time spent and the mathematical content present in each unit.

- Social Choice Theory (six weeks): Students learn logic and proof techniques as we look at different voting systems and determine whether or not they satisfy certain desirable criteria. We prove two impossibility theorems to end the unit.
- Redistricting (four weeks): Students use geometry, algebra, and data analysis to explore district maps and study how to detect various types of gerrymandering.
- Polling (two weeks): Students practice statistics and quantitative literacy as we analyze, interpret, and create political polls.
- Apportionment and fair division (two weeks): Students reinforce their logical thinking skills as we analyze the pros and cons of various apportionment methods.

## 12.3 Course Structure

I am about to teach this course for the second time. In this section, I describe the structure of the first iteration of the course, and in the next section I discuss the changes I am making for the second iteration.

### 12.3.1 Overall Structure

When designing the structure for the course, I had two constraints motivating my choices: the writing-intensive requirement and the wide array of students' mathematical backgrounds and interest levels.

#### Writing-Intensive Requirement

To meet the 15-page writing requirement, I assign two five-page papers and three two-page papers. In addition, students keep a journal in which they write weekly, one-page entries reflecting upon what they learned that week. Journals are collected at unannounced times, roughly once per month, and graded based on effort and thoughtfulness, not writing ability. The three two-page papers are inspired by these journal entries; at three points throughout the semester, I ask students to take one of their journal entries (or a portion of one of their journal entries) and expand it into a two-page reflection paper.

The first of the two longer papers is due after the first unit on social choice theory. Students choose a particular election—choices included local city council elections, presidential elections in various countries, the Best Picture Oscar, and the NBA All-Star teams. For their chosen election, students describe the method used to choose the winner(s). Then, they describe another social choice procedure that they think would be better to use for that particular election and make a persuasive argument as to why. In their comparative analysis, students have to refer to at least three of the voting criteria that we have discussed in class, and their arguments have to display an understanding of those criteria and how to prove whether or not a given social choice procedure satisfies them. See Appendix 12.4 for the assignment prompt.

The second of the two five-page papers is due at the end of the semester. While the first paper is a persuasive essay using mathematical reasoning, the second paper is more of a research paper. Students have to select a court case involving redistricting, describe the background for the case, the arguments made by both sides, the verdict, and the impact of the case. Students are told to pay special attention to how the lawyers arguing the case used quantitative evidence to try to prove that gerrymandering did or did not occur. To help students learn the writing process for longer papers, both of these five-page paper assignments include multiple intermediate deadlines and opportunities for feedback—students first submit their topic and meet with me one-on-one in office hours to discuss their plans, then they turn in an outline, then they bring a first draft to class for a peer-editing session, and finally they turn in their completed paper. In total, with all of these intermediate steps, the process for planning, outlining, and writing each paper is spread out over three to four weeks.

### Students' Mathematical Backgrounds

To address the fact that the student population varies widely in mathematical background and interest, I structure the course so that most of a student's time spent doing mathematics is in the classroom. Most days in class, we do small-group activities in which groups of two or three students used mathematics to analyze political structures. For homework, students do reading and writing assignments and finish whatever aspects of the in-class activity they did not complete during class time. I made this choice because I figured that students would be more able to do reading and writing on their own, while they may need help from me and from their fellow classmates to do the mathematics. In the classroom, I can manage each small group according to their needs. When I met with a pair of enthusiastic mathematics majors, I can ask them challenging questions that help them think beyond the contents of the assignment. When I meet with a pair of students who have weaker mathematics backgrounds, I can help them work through the algebra and explain the basic concepts more slowly. Students are graded on these in-class activities in terms of effort and progress as part of their class participation grade, so the group of math enthusiasts would not necessarily receive a higher grade than the group of mathematical beginners. By not tying grades to mathematical performance, I create an atmosphere where everyone is comfortable engaging with the material at a level appropriate to them.

### Grading

I grade students for their work on these activities in and outside of class in two ways: (1) an in-class participation grade in which students earn 0, 1, 2, or 3 points each day depending on their level of engagement with the material and (2) a grade for thoughtfully completing daily online surveys where students report their progress and ask questions. The advantage of an online survey is that it is a flexible medium, and I used it in different ways on different days to serve various purposes. If there is a reading assignment, I typically ask students how well they understood it on a scale from 1 to 5, give them a space to ask questions about the reading, and ask a couple of reading comprehension questions to ensure that they did the reading. If we are wrapping up an in-class activity, I ask them to report their results on the activity. I frequently check in with the students via the surveys to ask about the pace and workload for the class to make sure no one is too overwhelmed. The surveys are simultaneously assessment tools, feedback mechanisms, and spaces for students to ask questions privately. They also allow me to assess their progress in more detail without having to collect and grade every in-class activity. I prepare for each class by reading the survey results from the previous class, and begin each class by answering the questions they pose in the surveys. Oftentimes, the questions would serve as a perfect segue into the next topic.

The course grading breakdown is as follows:

- Participation – 20%
- Surveys – 10%
- Journal – 10%
- Reflection papers – 20%
- Voting systems paper (midterm) – 20%
- Redistricting case paper (final) – 20%

## 12.3.2 Examples of In-Class Activities and Content

In this section, I share examples of particular in-class activities and assignments for each unit of the course.

### Social Choice Theory

I begin the first day of the course with an activity meant to help students discover different social choice procedures. I provide a set of ranked-choice ballots with five candidates and ask students to devise a vote-counting system in which each candidate would win. While the initial brainstorming often leads to some silly ideas ("The winner is whichever candidate gets closest to 16% of all first-place votes," for example), when I modify the assignment to ask students for systems that they think would be reasonably good to use in real life, they are able to come up with many of the commonly-known systems: plurality, instant runoff, the two-round system, Borda count, Condorcet, and more. This leads to a discussion of the different social choice procedures and where they are used.

After learning about different procedures, students learn about a dozen different desirable criteria that a given social choice procedure may satisfy. As an example, one such criterion is the Pareto condition, which states that if every voter prefers candidate A to candidate B, candidate B should not win. We then discussed how to prove whether or not a social choice procedure satisfies a given criterion. In this unit, students learn the principle that they can prove negative statements with a counterexample, but proving positive statements is more difficult. For example, in one activity, I ask students to come up with a set of ballots to show that the Borda count (the method that assigns $n-1$ points to the candidate ranked first on a ballot with $n$ candidates, $n-2$ points to the candidate ranked second, and so on) does not satisfy the majority criterion: it is possible for a candidate to be the top choice for a majority of voters without winning. We then work together to show that the Borda count does satisfy the majority loser criterion: if there is a candidate who is the least favorite of a majority of voters, that candidate cannot win. In another example, we look at real election data to show that instant runoff voting does not satisfy monotonicity: it is possible for candidate X to win with a given set of ballots and then to modify some of the ballots by only moving candidate X up, resulting in a loss for candidate X. Students are baffled and intrigued by all the counterintuitive things that can happen under various systems.

Once we've learned about different criteria and which social choice procedures satisfy which criteria, students naturally ask if there's a perfect system. Every system we encounter seems to satisfy some of the criteria we want, but not all of them. At this point, we introduce Arrow's Impossibility Theorem [1] and the Gibbard-Satterthwaite Theorem [3, 5]. Arrow's Theorem essentially says that if there are more than two candidates, it is impossible for a non-dictatorship to both satisfy the Pareto condition and be independent of irrelevant alternatives. The Gibbard-Sattherthwaite Theorem essentially says that no non-dictatorship is invulnerable to strategy: it will always be possible that a voter may achieve a better outcome by voting differently from their true preferences. After proving these theorems together in class and seeing that there is no perfect social choice procedure, I have students decide which criteria they think are most important, and then they work to find a social choice procedure that satisfies all of their top-priority criteria. Through these activities, students are able to see that while they cannot devise a perfect social choice procedure, they can use mathematics to analyze different systems to make an informed choice about which system to use in a particular situation.

### Redistricting

We begin with an activity that was introduced to me at a Tufts University workshop [2] which was designed to give educators tools to teach about gerrymandering. In this activity, students take two copies of a $10 \times 10$ grid, with 60 squares colored yellow and 40 squares colored purple. They are tasked with dividing the grid into ten "districts" of ten squares each. On one sheet, they are told to draw the districts so the yellow party would win six seats and the purple party would win four. Students are able to do this without much trouble. On the other sheet, they are told to gerrymander: to draw the districts so that the purple party wins six of the ten available districts. They have to be more creative and use more convoluted shapes to make this work. We then discuss a number of ways in which we can measure how "compact" a district is and measure compactness scores for both maps. After completing this activity and learning about compactness measures, we look at real maps and measure compactness scores for actual Congressional districts using GIS software. Students are able to see the pros and cons of various compactness measures through these activities.

We also look at both racial and partisan gerrymandering. To explore partisan gerrymandering, we used 2016 election data in Michigan [4] to compute the efficiency gap (a quantity sometimes used to detect partisan gerrymandering) and to construct a seats-votes curve to visualize the partisan asymmetry of the map. In a seats-votes curve, the $x$-axis represents the proportion of votes received by a certain party statewide, and the $y$-axis represents the proportion of the state's seats in the House of Representatives won by that party. We chose to look at Michigan in 2016 because every House race was contested, and no race involved a significant third-party presence. After removing the small number of third-party votes from the data set, students plotted a point corresponding to what actually happened in 2016 on the seats-votes plane. Then, they plotted more points by uniformly raising and lowering the vote percentages in each district and observing the effect on the number of seats won by each party. This produces a curve from $(0, 0)$ to $(1, 1)$, seen in Figure 12.1. We discuss what a fair curve might look like: it should pass through $(0.5, 0.5)$ and be symmetric about the point $(0.5, 0.5)$ if both parties are treated equally by the district map. We debate what it should look like in

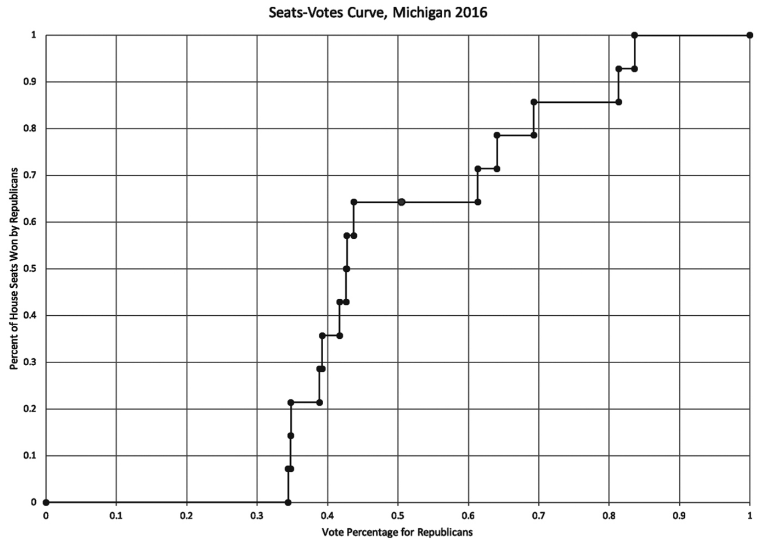


**Figure 12.1.** An Example of a Seats-Votes Curve

the middle: a straight line for perfect proportionality, or something steeper? Then, students use two fictional data sets to draw two seats-votes curves of their own to interpret. In both data sets, the party in question received 44.27% of the vote. In one set, that resulted in the party winning 7 our of 16 seats, while in the other set that party won 9 out of 16 seats. Students were able to see the evidence of gerrymandering by comparing the two curves. Through this activity, students learn how to create and interpret visual displays of data to help to illuminate an underlying problem. See the Appendix (p. 119) for the data used in this assignment.

### Polling

We discuss the importance of sample size, how to compute the margin of error for a poll and what that means, and how to interpret poll results. Students use quantitative reasoning to understand the difference between "Candidate X is polling at 60%" and "Candidate X has a 60% chance of winning" and other potentially nonintuitive aspects of probability and statistics. For a classroom activity, each student brings in a poll that they find online and analyze it. They compute the margin of error, write a news brief accurately summarizing the results of the poll, and analyze the strengths and weaknesses of the poll's methodology. We also do the classic sampling activity of estimating the distribution of colors in a bag of M&Ms. In this unit, students gain quantitative literacy and the confidence to read poll results (and news reports about poll results) and draw accurate conclusions.

### Apportionment

Students are skeptical when I first tell them we would be studying how states are allocated seats in the House of Representatives based on population. Isn't it just division with rounding? How interesting can that be? I began this unit by giving students a set of population numbers for a fictional country with four states and total population of 10,000. They have to decide how to allocate ten Congressional seats to those four states. Different students come up with different numbers, which leads to a debate about which is most fair. After this activity, students are on board with the idea that this is a more complicated question than it first seems. Next, we discuss the various apportionment models proposed by Hamilton, Jefferson, Adams, and Webster, finally ending with the Hill-Huntington method used today. We do another activity in which I give them county populations for Massachusetts and asked them to distribute seats for a 100-member Congress. I have students use a spreadsheet to compute the results using all of the different methods we learned. See the Appendix (p. 12.4) for this assignment. Students are able to see how some methods favor smaller states, while others favor larger states. We also look at 1990 census data to reapportion the House based on various apportionment methods, and see that different choices in apportionment method would have resulted in different results for the 2000 presidential election (Gore would have won had we still used Jefferson's method, while Hamilton's method would have resulted in an Electoral College tie). This unit shows students that something as seemingly benign as allocating seats in Congress based on population actually has a lot of sophisticated mathematics behind it, and our choice for which method to use has far-reaching consequences.

## 12.4 Reflections

At the end of the first iteration of this course, I solicited feedback from the students, asking how I could improve the course in future semesters. Students overwhelmingly enjoyed the class, but their biggest request was, to my surprise, for more math homework. Though I thought that I was closely monitoring their understanding of the mathematical course concepts by observing their in-class activities and reading their survey responses, students did not feel that their mathematics skills were being evaluated enough or given enough feedback. I thought that grading students based on their participation and survey responses, rather than by grading their mathematics work more traditionally, would best accommodate students at different levels, but in fact students wanted to be held more accountable for their work.

I was also dissatisfied with the results of the final paper I had assigned, in which students described and analyzed a court case about redistricting and gerrymandering. I was hoping students would be able to use what they learned about the many mathematical ways to analyze a district map for compactness and fairness in their analysis of the arguments of the case, but many cases simply didn't have much mathematical evidence in their arguments. Those that did were likely to be more recent, often ongoing, cases that were difficult for students to analyze because they were unresolved.

To fix both of these problems, while still adhering to the 15-page writing requirement, I dropped the final five-page paper and the two-page journal papers and replaced them with a portfolio assignment. Portfolios consist of weekly one-to-two page assignments that combine writing about course concepts, doing mathematics, and sharing their own views and reflections. I collect this assignment each week and provide formative feedback, which students then use to revise their assignment. Final, revised versions of the assignments make up their portfolio, which is collected at the end of the semester as their final assignment. I have retained the other five-page paper about social choice procedures because that writing assignment was effective in getting students to analyze the strengths and weaknesses of various procedures using mathematical reasoning.

I also plan to inject more basic civics and government information into the lessons. In teaching the class the first time, I came to learn that not all of my students are as civically engaged as I thought they would be. For example, I had to insert an unplanned lesson about the Electoral College and how it works, because so many of my students did not understand it and asked to learn more.

All in all, this course achieves the goal I set for it: to showcase the beauty, variety, and usefulness of mathematics to students who may not otherwise encounter it. Students experience a wide array of mathematics; in-class activities include algebra, geometry, statistics, probability, quantitative reasoning, visual displays of data, mathematical proof, and more. Whereas it might typically require four or five different traditional mathematics courses for students to see this many kinds of mathematics, in this class students are able to get a small taste of all of them in a way that is built around one cohesive theme. Perhaps a student isn't excited about mathematics, but is passionate about social justice. In this class, that student will learn that they can use mathematics as a tool to advance their social justice causes. Students leave this course seeing mathematics as an aid to help build and analyze the structures that govern our society.

## 12.5 Bibliography


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[4] Michigan Department of State, "2016 Michigan Election Results," `mielections.us/election/results/2016GEN_CENR.html` (accessed January 8, 2019).

[5] Mark Allen Satterthwaite, "Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions," *Journal of Economic Theory*, 10:2 (1975) 187–217.

# Appendix

## Midterm Paper Assignment: Comparing Social Choice Procedures

*This is the prompt and grading rubric given to students for their midterm paper, which is assigned toward the end of the unit on social choice procedures. This assignment of a five-page persuasive essay serves to help satisfy the writing-intensive requirement for all first-year seminar courses. It also helps to assess student understanding of the various social choice criteria that they studied and their ability to analyze a voting system using these criteria.*

For your first paper, you will answer the following prompt:

Choose a particular type of election (e.g., Massachusetts governor, MLB Hall of Fame, Best Picture Oscar, President of France, etc.) and research the social choice procedure currently used for that election. Select another social choice procedure of your choosing that you think would be better than the one currently used. Compare and contrast the two methods, and make a persuasive argument why your selected method is better than the actual method used. Your argument should incorporate at least three of the criteria discussed in class and should thoroughly analyze the pros and cons for each system.

**Requirements and Guidelines:**

1. Length: 5 pages, double-spaced, 12-point font, 1 inch margins. (An acceptable length is 4.5–5.5 pages. You will lose 5 points for every page above or below this range.)
2. Outline: Due February 12. Your outline should include your choice of election, the two social choice procedures youll be comparing, and the components of your argument in favor of your chosen system. Bring a hard copy of your outline to class on February 12. It may be typed or neatly handwritten (–10 points if outline not complete by Feb. 12).
3. First draft: Due February 21. Bring your draft to class for peer review (–25 points if draft not complete by Feb. 21).
4. Consultation with instructor: At some point in the paper-writing process, you should meet with the instructor in office hours to go over what you have and to ask questions (–5 points if not done).
5. Final draft: Due February 28. Submit an electronic copy on Blackboard and bring a hard copy (typed and stapled) to class (–15 points for every day it is late).
6. You must cite any sources you use. Feel free to use whatever citation format you prefer, but be consistent.
7. Your analysis of the two social choice procedures should incorporate at least three of the criteria we discussed in class, and these criteria should be used to support your argument. For example, instead of saying, "The plurality method could result in an unpopular winner," say, "The plurality method could result in an unpopular winner because it fails to satisfy the majority loser (or Condorcet loser) criterion."
8. Your final paper will be graded according to the following criteria:
   (a) Writing: Proper grammar, spelling, punctuation, and sentence structure (20%).
   (b) Organization: Thoughts and ideas are well-organized into paragraphs; the narrative is easy for the reader to follow. (20%).
   (c) Accuracy: All information in the paper is accurate and properly cited. (20%).
   (d) Persuasiveness: Your argument is logically sound, convincing, and well-supported by evidence. (40%).

## Partisan Gerrymandering Activity

*For this in-class activity, students learn how to use a spreadsheet to build a seats-votes curve to detect partisan gerrymandering. The first dataset represents a state that has been gerrymandered: the party in question receives about 44% of the vote on average, but wins seats in 9 out of 16 districts. In the second dataset, which represents a more balanced electoral map, the party in question also receives 44% of the vote but this time only wins 7 seats.*

| Set 1 District | Set 1 Votes | Set 2 District | Set 2 Votes |
|---|---|---|---|
| 1 | 0.521 | 1 | 0.600 |
| 2 | 0.412 | 2 | 0.511 |
| 3 | 0.242 | 3 | 0.546 |
| 4 | 0.553 | 4 | 0.649 |
| 5 | 0.532 | 5 | 0.337 |
| 6 | 0.397 | 6 | 0.567 |
| 7 | 0.573 | 7 | 0.346 |
| 8 | 0.404 | 8 | 0.368 |
| 9 | 0.578 | 9 | 0.546 |
| 10 | 0.127 | 10 | 0.331 |
| 11 | 0.502 | 11 | 0.454 |
| 12 | 0.528 | 12 | 0.162 |
| 13 | 0.273 | 13 | 0.392 |
| 14 | 0.337 | 14 | 0.502 |
| 15 | 0.543 | 15 | 0.431 |
| 16 | 0.561 | 16 | 0.341 |

1. Download the associated Excel file on Blackboard. [The spreadsheet includes the data in the above chart.]

2. Watch the demo for the first data set to see how to analyze it.

   (a) To fill the "amt to swing" column, take 0.5-(vote percentage). This is the shift needed for the seat to change parties. For example, if v = .61, then the amount to swing is −.11, which means that the party would need to lose 11% of the vote to lose this seat.

   (b) Select the four columns (district, votes, seats, amt to swing) and sort by amt to swing

   (c) Next, we're going to calculate the range of vote percentages that results in each number of seats (for example, if the party gets between 0 and n% of the vote, they'll get 0 seats, if they get between n% and m% of the vote, they'll get 1 seat, etc.). Here's how we do it:

      i. Start with 0 in the "min vote" column for 0 seats. In the "max vote" column take the actual average vote total (at the bottom of the "votes" column in $B$19) and add the smallest "amt to swing" number (cell D2). If you type "=$B$19+D2" and copy and paste it down the rest of the column (except for the last row), it will fill in properly. The "max vote" for 16 seats should be entered as 1.

      ii. Next, to fill in the rest of the "min vote" column, the minimum vote total for each seat should be equal to the maximum vote total for the previous number. In cell G3, type "=H2", then copy and paste the formula down the rest of the column to complete.

3. Now, we have a range of vote percentages for each seat, and we're ready to draw the seats/votes curve for this data set. Draw horizontal lines from "min vote" to "max vote" for each seat value below.

4. Repeat this process with the second data set. Compare the two. Do you see any evidence of gerrymandering?

## Apportionment Activity

*For this in-class activity, students use the different apportionment methods that they have learned to assign seats in a 100-member assembly to each Massachusetts county based on population.*

Here are 2012 population estimates for each Massachusetts county:

| **County** | **Population** |
|---|---|
| Middlesex | 1,537,149 |
| Worcester | 805,353 |
| Essex | 755,970 |
| Suffolk | 746,039 |
| Norfolk | 682,078 |
| Bristol | 550,856 |
| Plymouth | 498,393 |
| Hampden | 465,997 |
| Barnstable | 214,947 |
| Hampshire | 159,791 |
| Berkshire | 130,120 |
| Franklin | 71,535 |
| Dukes | 16,834 |
| Nantucket | 10,241 |

Total population of Massachusetts: 6,645,303.

Suppose Massachusetts decides to have a 100-member legislature, assigning some number of seats to each county. Each county must get at least one seat.

1. How many seats would each county get using Hamiltons method? Recall that for Hamiltons method, you compute the quota $Q = P/n$, where $P$ is the population and $n$ is the number of seats. Then, divide the population of each county by $Q$ and round down. Next, see how many leftover seats there are, and allocate them (one per state), first by giving one representative to any county with 0, and then in descending order based on the decimal part of $P_i/Q$.

| **County** | **Population** | $P_i/Q$ | **Seats** |
|---|---|---|---|
| Middlesex | 1,537,149 | | |
| Worcester | 805,353 | | |
| Essex | 755,970 | | |
| Suffolk | 746,039 | | |
| Norfolk | 682,078 | | |
| Bristol | 550,856 | | |
| Plymouth | 498,393 | | |
| Hampden | 465,997 | | |
| Barnstable | 214,947 | | |
| Hampshire | 159,791 | | |
| Berkshire | 130,120 | | |
| Franklin | 71,535 | | |
| Dukes | 16,834 | | |
| Nantucket | 10,241 | | |

2. Use the associated spreadsheet to determine the numbers for the methods of Jefferson, Adams, and Webster.

| **County** | **Population** | **Jefferson** | **Adams** | **Webster** |
|---|---|---|---|---|
| Middlesex | 1,537,149 | | | |
| Worcester | 805,353 | | | |
| Essex | 755,970 | | | |
| Suffolk | 746,039 | | | |
| Norfolk | 682,078 | | | |
| Bristol | 550,856 | | | |
| Plymouth | 498,393 | | | |
| Hampden | 465,997 | | | |
| Barnstable | 214,947 | | | |
| Hampshire | 159,791 | | | |
| Berkshire | 130,120 | | | |
| Franklin | 71,535 | | | |
| Dukes | 16,834 | | | |
| Nantucket | 10,241 | | | |

3. In your spreadsheet, compute POP/sqrt($n * (n - 1)$) for each county for $n = 2, 3, \ldots, 24$. Then, find the 86 largest values among those in that set. Use those to allocate seats using the Hill-Huntington method. What do you get?

| **County** | **Population** | **Seats** |
|---|---|---|
| Middlesex | 1,537,149 | |
| Worcester | 805,353 | |
| Essex | 755,970 | |
| Suffolk | 746,039 | |
| Norfolk | 682,078 | |
| Bristol | 550,856 | |
| Plymouth | 498,393 | |
| Hampden | 465,997 | |
| Barnstable | 214,947 | |
| Hampshire | 159,791 | |
| Berkshire | 130,120 | |
| Franklin | 71,535 | |
| Dukes | 16,834 | |
| Nantucket | 10,241 | |

4. Analyze your results. Do certain methods favor certain counties? Which method do you think produces the most fair result?

# 13

# *Mathematopia*: A Humanistic Math-Themed Freshman Seminar

Michael P. Saclolo

**Abstract** I describe a math-themed freshman seminar course conceived and developed through a humanistic approach. The seminar explores the theme of utopia/dystopia through the examination of three mathematically-oriented texts. The works use mathematical topics as a device to tell a story or to express social criticism and commentary. After a close reading, discussion, and analysis of each text, students complete a written assignment that serves, in part, to hone and assess intellectual skills desired of students at a college level. In this article, I explain how the three texts and accompanying written assignments can be effective in weaving together the course themes and achieving learning goals.

## 13.1 Background and Context

St. Edward's University is a private, Roman Catholic liberal arts university in Austin, Texas. The University enrolls approximately 5,000 students, of which about 4,000 are undergraduates. It has recently undergone a complete renewal of its general education program for undergraduate students. Full implementation of the program began in the fall semester of 2018. One of the products of this renewal is the Freshman Seminar program. Each freshman entering in the fall is required to enroll in a seminar as part of their graduation requirements. The program is university-wide, and although it is administered through the School of Arts and Humanities, faculty members from all academic units of the University are invited to develop and offer seminars that make for an intriguing entry point for college-level thinking. The seminar is a four credit-hour course with three contact hours per week. The course runs for an entire semester that is fifteen weeks long. While most courses at St. Edward's with three contact hours per week are normally attributed three credit hours, the fourth hour is allocated for required co-curricular activities that occur outside regular class meeting times. The seminar has a typical enrollment of about 20 students, conceived as forming a small community of learners. Two to four seminars are grouped in a cluster with a common theme to form a Learning Community or a Living and Learning Community, with students living in the same residence hall.[1] The seminars are designed to introduce students to college life and must have a significant out-of-class co-curricular component. They must also incorporate the freshman common theme and common reading, a book chosen for entering freshmen to read the summer prior to arrival at the University.

Potential instructors design a seminar related to their disciplinary field or area of expertise. They must agree to adopt the following learning outcomes, shared by all first-year seminars.

[1] These smaller seminars replaced a program called Freshman Studies where students enroll in a large lecture course (80-100 students) team-taught by two faculty members.

> *Students will join a community of learners and actively engage in academic and co-curricular exploration. As they do so, they will develop critical thinking skills necessary to become successful students and life-long learners by meaningfully confronting questions of social justice through the course materials and co-curricular experiences. To achieve this goal, students will:*
>
> - *Develop an emerging awareness of assumptions by engaging a variety of perspectives.*
> - *Interpret and then evaluate issues/evidence/sources central to course content.*
> - *Communicate effectively about multiple perspectives explored during the course.*
> - *Reflect on and apply knowledge developed in the classroom and co-curricular experiences.*

Thus the seminars are conceived so as to foster and build community among the students, to cultivate their critical thinking and communication skills, engage them in a variety of perspectives, and address questions of social justice. The clustering of several first-year seminars allows for students to interact with a wide variety of perspectives. Each seminar in a cluster takes the common thread and applies a unique perspective, either through a different disciplinary approach or flavor, consideration of different genres, or examination of narrower thematic focus.

## 13.2 Mathematical Theme

The seminar *Mathematopia* delivers a humanistic approach to implementing an entry-level university seminar. And though mathematically themed, the course is not conceived to teach formal mathematical content. It is part of a cluster of four seminars with the theme utopia/dystopia. Each of the other three seminars has a different focus, either in perspective or in the genre of works considered. One seminar is oriented towards American history and popular culture, and examines attempts at forming utopian societies. Another focuses on the depiction of utopias and dystopias across several types of media and literary genres. The third engages in the close examination of world-building in the formation of utopias and dystopias. While the four seminars are all stand-alone courses and meet separately for most of the semester, several common meetings and activities are also organized, convening all the students in the cluster, for the purpose of curricular and co-curricular interchange among the different thematic perspectives represented.

The concepts of utopia and dystopia evoke the dichotomy between orderliness and the ideal on the one hand and disarray and the deplorable on the other. The seminar *Mathematopia* explores this dichotomy through the mathematical notions of order, dimension, symmetry, classification, iteration, and chaos. The seminar is organized around three required texts that serve as the basis and incitement for class inquiry and discussion. The mathematical ideas mentioned previously are explored in conjunction with the reading and discussion of these texts.

All three texts chosen to anchor the *Mathematopia* seminar are especially compatible with the course themes and goals. All books express mathematical themes and address questions of social justice and inequality. Two of the readings are works of fiction and depict societies with characteristics of both a utopia and dystopia. Moreover, they both rely on extensive use of mathematical ideas, tropes, and imagery to propel their narrative. The third is a non-fiction work that exposes social justice questions arising from the use of mathematical algorithms in institutional decision-making.

## 13.3 Course Structure

A typical course meeting consists of discussions of a text involving both its content and related mathematical ideas. However, in-class activities exploring a mathematical concept are also interspersed throughout the run of the course. Course activities related to the the three texts, including the informal mathematical explorations, comprise about 70 % of the course. The other 30% is devoted to activities that link the seminars in a cluster or associated with the freshman common theme. These activities include the combined meeting of all seminars in the cluster, where all students are assembled for the purpose of discussion of the common (summer) reading, speaker, and the play associated with the common theme.

The seminar devotes three to four weeks to each text. Towards the end of that period, I assign a written assignment. The first assignment is a piece of creative writing. The other two assignments both require the students to analyze the readings and related materials critically, but they differ in design and are intended to accomplish different analytic goals.

### 13.3.1 An Encounter with Flatlanders

To many mathematicians *Flatland* by Edwin A. Abbott [1] does not need much of an introduction. This novella, narrated in the first-person by a character named A. Square, describes a society of polygonal beings living in a two-dimensional world. The first half of the novella chronicles the citizens' lives and customs within a class system imposed by one's polygonal size, with the highest order given to those whose "number of sides [...] so numerous, and the sides themselves so small, that the figure cannot be distinguished from a circle" [1, p. 28]. In the second half, the main character visits previously unknown other worlds, and the book culminates with the havoc created by the revelation of a higher-dimensional world and the attempts to suppress this knowledge. Abbott therefore manages to depict a society ruled by a totalitarian regime, utopian and orderly on the surface, but dystopian and chaotic in what lies beneath. Two animated film adaptations, both released in 2007, are available to supplement the reading. The feature-length version simply called *Flatland* is by director Ladd Ehlinger Jr. [3]. The animated short called *Flatland: The Movie* by directors Dano Johnson and Jeffrey Travis [5] is less faithful to the novella.

The concept of classification according to shape and the question of dimension in *Flatland* set up a natural opportunity to do informal classroom activities that further explore these ideas. These include examining dimension through the relationship between the scaling factor and the number of copies of a geometric object needed to form a larger version of the same figure. Another exercise was to consider the ways to classify or group the capital letters of the English alphabet (printed using the same font). For example, one may ask which of the letters can be drawn with one stroke. One can also categorize the letters according to the number of "holes" that they have (i.e., by topological genus). Yet another classifies the letters according to the types and number of symmetries (e.g., rotational or radial). A different investigation of symmetry involves those of the equilateral triangle. This exercise builds up to the construction of the Cayley table for the group of symmetries of the equilateral triangle, including the investigation of the associated binary operation among the rigid motions.

The written assignment associated with *Flatland* is to write a *lost chapter* of the novella in which the students must describe an aspect of Flatlanders' lives that is not depicted in the novella. Thus the students must take on the persona of the "author" and narrator A. Square. They are required to use the first person and address the readers directly in the second person. The students are given a list of suggested topics to consider: other jobs in Flatland, sports and leisure, food and dining, transport.[2] They must adhere to the physical laws of the Flatlanders' world, so they must have an good understanding of the limitations of the two-dimensional geometry of the land. In addition to written description, the students are required to provide an illustration of one aspect of their topic, just as A. Square did in the novella.

### 13.3.2 Pernicious Mathematics

Perhaps it is not immediately apparent to some how *Weapons of Math Destruction* by Cathy O'Neil [6] might fit with the theme of utopia and dystopia. Like many mathematicians, O'Neil's fascination with mathematics began early in childhood, developing it into a passion and the center of her professional trajectory. But as a former quant working on hedge funds, O'Neil became disillusioned by the ill-effects and negative social consequences of the use of algorithms in financial analysis as propelled by the financial crash of 2008. To O'Neil, "[t]he crash made it all too clear that mathematics, once [her] refuge, was not only deeply entangled in the world's problems but also fueling many of them" [6, p. 2]. Her book addresses the social inequalities created by the use of such mathematical models and the analysis of big data, not only in the domain of credit and finance, but also in the areas of education, insurance, commerce, the judicial system, and general civic life. Calling such mathematical models "Weapons of Math Destruction" or WMDs, O'Neil argues against the use of such models to decide the fate of ordinary citizens and does not hesitate to name what she deems as specific offenders, among them the IMPACT tool for assessing teacher performance, the Level of Service Inventory-Revised (LSI-R) to predict recidivism, and even the *U.S. News and World Report* college rankings. One can argue that the ideal (and impersonal) use of seemingly efficient algorithms contrasting with the regrettable disarray and injustice that it perpetuates reflects the dichotomy created by the interplay between utopia and dystopia.

For *Weapons of Math Destruction*, the written assignment is to compare, contrast, and provide a critique of two published reviews of the text. One purpose of this assignment is, in part, to have students engage in how they process

[2] The idea and much of the detail for this assignment was taken from an assignment posted by R. Jason Parsley of Wake Forest University, for an introductory mathematics course called *Explorations in Mathematics*. The teaching section of his web site includes a section on writing projects [7] for many types of mathematics courses.

the information that they are bombarded with and their sources. The first step of the assignment is to comb through published reviews of the book and choose two. The book is generally widely praised by critics. For this reason, one of the reviews to be chosen for comparison must contain some sort of negative criticism. Prior to the official announcement of the assignment, the students would listen to a presentation from a university librarian about how to critically choose and evaluate sources. The students are also instructed as to where and how to look for reviews of the book. In their report, the students are asked to provide the name and background of the reviewer, then summarize the main points of the review. Next, they are required to provide an analytic comparison of the two reviews. In the analysis the students must discuss the major points of the reviews and whether they agreed or disagreed with those views, taking into account the particular background or credentials of the reviewer. They may also say whether they feel that they need more information and identify what types of information they may need in order to agree or disagree with the reviewers.

### 13.3.3 We, the Mathematically Enlightened

Yevgeny Zamyatin's novel *We* [8] predates the more well-known totalitarian dystopia conceived in George Orwell's novel *1984* by 25 years. Orwell himself wrote a review of *We* in 1946, three years before his own novel appeared publicly. *We* depicts a futuristic totalitarian society united under the moniker OneState and ruled by a figure called the Benefactor. The society itself is described as mathematical and orderly, and all activities of its citizenry are presumably monitored. It is a society so regulated that even the sexual activities among its citizens are managed by the state: sexual partners must be officially assigned and sanctioned, and the registering of sexual acts is performed through the use of slips or coupons. The mathematics in *We* is primarily reflected in the use mathematical terminology and tropes. The main character's occupation is described as a mathematician and builder of a spaceship called the *Integral*. All citizens of this society are designated as "numbers" with the main character called D-503. One particular chapter (depicted as journal entries in the book) speaks of the narrator's childhood aversion to the square root of negative one [8, p. 39]. As a supplement to the novel, two film adaptations are available. One is a full-length feature that came out in 1982 in German, titled *Wir* by director Vojtech Jasný [4]. A more recent adaptation from 2016 is a short film in English called *The Glass Fortress* from director Alain Bourret [2].

The novel depicts the interconnection between the order and rules of a totalitarian state and the disorder and chaos posed by an impeding rebellion. Thus, after reading the novel, I introduce the students to the supplementary mathematical topic of fractals to show how iteration of simple mathematical rules may end up exhibiting or demonstrating chaotic phenomena. One of the classroom activities I facilitate in conjunction is to build a model of the Menger sponge using the material prescribed by the Mega Menger Distributed Fractal Build out of Queen Mary University of London.[3] Another activity is to have the students play with the numerous online fractal generators using their own computer devices.

The written assignment connected to this novel is to analyze several themes from the novel: the depiction of utopia and/or dystopia, the question of personal identity, and the figurative use of mathematical terms and concepts. The students must choose one particular chapter (referred to as "entries" or "records" in English translations of the novel) to analyze. For the chosen chapter, students are asked to discuss how the theme of utopia or dystopia manifests itself in the particular chapter. Next, they must pick a character in the chapter and scrutinize the notion of personal identity. Finally, they must write about the role of mathematical references in the chapter, citing specific examples. In particular, they can focus on the type of imagery or literary effect the mathematical references evoke.

## 13.4 Reflections

The three texts and their topics are well-suited to both the seminar and cluster themes. Their subject matter seems to be conducive to engaging students in the fruitful exploration of these themes, and these texts provide a solid foundation for accomplishing the course goals. Apart from their respective interesting and relevant content, there are also practical reasons why the texts work well in the design and organization of the course. All of them are written using shorter chapters, making it easy for me to divide the reading assignments and organize the discussion of content into easily

[3] See the web page `megamenger.com/`.

digestible segments. Another appealing aspect of this choice of texts is that they work well when presented in a specific order.

*Flatland* is an ideal choice to study first among the three texts. The piece is in the public domain and readily available to download from a number of digital archives. Thus it is readily accessible from the first day of class, and the free access mitigates the cost of purchasing reading materials for the course. Hard copy editions for purchase are also available, and for those instructors who wish for their students to have the benefit of reading assistance, an edition with notes and commentary by William F. Lindgren and Thomas F. Banchoff exists under the Spectrum series co-published by the Mathematical Association of America and Cambridge University Press [1]. Though perhaps the most mathematical in detail among the three books, its mathematical content is the most accessible. The creative writing piece assigned for *Flatland* also serves to gently draw the students into writing at the college level. Based on the pieces collected for this assignment, many students seem to embrace the idea of making a contribution to the story of this fictional two-dimensional world. And several have risen to the challenge of using their imagination and creativity in conjunction with the restrictions imposed by plane geometry.

As the only non-fiction work, placing *Weapons of Math Destruction* in the middle of the three serves to lessen the seemingly literary bent of the course, which I aim to do for this seminar.[4] The real-life and socially relevant topics discussed in *Weapons of Math Destruction* provide a sharp and immediate contrast to the fictional and almost fabulous story and imagery depicted in *Flatland*, while maintaining the mathematical theme and the inquiry into questions of social justice. For the written assignment on *Weapons of Math Destruction*, I find that compelling the students to consider other writers' opinions on and analyses of someone else's work is a good exercise in making the students critically consider their sources of information and opinion, even when that source of information is an authority figure like a professor or a best-selling book.

The third work, Zamyatin's *We*, is well-suited as the anchor to this collection of texts for several reasons. It is perhaps the hardest of the three to read and and analyze, and so it helps if the students have gathered earlier experiences and training in reading at the college level in the seminar and in their other courses. The novel's mathematical references, though not overly advanced, can be unfamiliar to entering freshmen who are not necessarily intending to specialize in mathematics or the physical sciences. For example, those who are unfamiliar with calculus may not automatically see the the significance of the symbolism behind the use of the term "Integral" to name a spaceship whose purpose is to conquer other worlds. Finally, among the three texts, *We* is the one with the clearest connection to the classical notion of dystopia in popular culture, and therefore it is a logical choice as a culminating text to tie in with the major themes of the course. As an earlier English translation and edition of the novel has recently been placed in the public domain, access has become easier for many readers. Concerning the written assignment connected to the novel, it produced the fewest successful outputs among the students who have taken the course. It is the most difficult text of the three to read analytically. As an exercise in literary analysis, the assignment greatly challenges the students to produce coherent and well-supported arguments, without the benefit of the type of instruction and training that they would get in an actual course in literature. Moreover, perhaps the amount of analysis required for such an assignment is too much for a shorter essay of about 500–750 words.

Perhaps the biggest challenge I faced in the implementation of the seminar is in organizing the course to seamlessly integrate those activities that are not related to the mathematical or cluster theme, but is nonetheless part of the freshman seminar program. For example, the freshman experience programming at St. Edward's University has a tradition of assigning freshmen the short and fun exercise of interviewing a professor, perhaps as a way to gauge their interest in a particular area of study. The freshman common theme and summer reading may or may not easily fit with the theme of one's seminar. As a point of comparison, I found the theme for one year, (individual) "identity", easier to tie in with the themes of the *Mathematopia* seminar, than the one from a previous year, which was "immigrant voices."

## 13.5 Bibliography

[1] Edwin A.Abbott, *Flatland* (W. Lindgren and T. Banchoff, eds.), Cambridge University Press and Mathematical Association of America, New York, NY, 2010.

[2] Alain Bourret, *The Glass Fortress*, Neva Productions, France, 2016.

[4] However, the instructor is quite open to designing a seminar around literary works of fiction with a mathematical theme.

[3] Ladd Ehlinger Jr., *Flatland*, Flatland Productions, USA, 2007.

[4] Vojtech Jasný, *Wir*, Zweites Deutsches Fernsehen, Germany, 1982.

[5] Dano Johnson and Jeffrey Travis, *Flatland: The Movie*, Flat World Productions, USA, 2007.

[6] Cathy O'Neil, *Weapons of Math Destruction*, Crown, New York, NY, 2016.

[7] R. Jason Parsley, Writing Projects in Mathematics, *Wake Forest University*, `users.wfu.edu/parslerj/projects.html`

[8] Yevgeny Zamyatin, *We* (M. Ginsburg, trans.), Harper Voyager, New York, NY, 1972.

## Appendix

### *Flatland* Assignment

*The first formal assignment is a piece of creative writing. It is assigned at the end of the third week of the semester and due two weeks later. After the initial evaluation and scoring, students are given the option to revise and resubmit the assignment to improve their grade.*

The purpose of this assignment is to produce a creative writing piece that illustrates an understanding of the world depicted in Flatland. To that end you will write a "lost" chapter to *Flatland*, in which you describe an activity or feature of the everyday life of Flatlanders and fill in any details that the author A. Square did not describe. Thus, your chapter must be consistent not only with the customs described in the novella but also with the limitations and conventions that plane geometry imposes.

You may consider a topic from the following list.[5]

1. What kinds of jobs, other than those explicitly mentioned in the novella, do Flatlanders perform?
2. What do Flatlanders do for fun? What are their parties like? Would they go on vacation? Where?
3. What types of sports might Flatlanders play? Describe such a sport.
4. A. Square mentioned that the visual arts are a bit boring in Flatland. What about the other forms of art? For instance, do they listen to music? How is music produced?
5. What might Flatlanders eat and drink? (Or, would there be a difference between eating and drinking?) How would their digestive system function?
6. What modes of transport could exist in Flatland? What would cars look like? How would they be powered? Could there be boats or trains (with tracks)? Could there be an analog to air travel?
7. Would it be possible to have natural calamities such as hurricanes, floods, and earthquakes? How would they occur? What damage would they cause?
8. Are there such things as plants and animals in Flatland? Could there be pets?
9. What might happen to A. Square if he gets out of prison?
10. If A. Square and his family escape Flatland and seek asylum in Spaceland, what sort of experience could they have?

More specific instructions and point distribution:

I You must write it as if the author A. Square is writing it. Notice that he uses the first person I' and often addresses the reader as You'. He also uses different capitalization conventions that we normally don't use. Try to reflect this style as much as possible. Finally, involve some aspect of geometry and the fact that there are some restrictions in the way Flatlanders would do things because of their two-dimensional world. The idea is to make the reader believe that it could have been written by A. Square. (50 points.)

[5] Most of these ideas were taken or adapted from the list provided by R. Jason Parsley of Wake Forest University, for an assignment for an introductory mathematics course called *Explorations in Mathematics* [7].

ii Give your chapter an effective title that is consistent with the style of the other chapter titles. Notice that a lot of the chapters begin with the words "of" or "how." (10 points)

iii Notice that in some parts of Flatland, A. Square provides an illustration. For your chapter, include one illustration depicting one aspect of what you have written. Do this by hand (like those in the novella), and it must include both a drawing and a handwritten note to describe the drawing. Again, this must be consistent with the way A. Square drew his illustrations. Do this on separate page (10 points).

iv Please type your chapter double-spaced with a word count of 750-1250 words (3-5 pages). Type your name at the upper left corner of the first page (10 points).

v Use correct grammar and punctuation. You do not need to use British English spelling (e.g., you can use the spelling color' rather than colour') (20 points).

Deadline: Save the written portion of the assignment as a pdf file. Scan or take a photo of your drawing and save it as a pdf, jpg, or png file. Upload both files into the corresponding Canvas assignment by September 27, 11:59 pm Canvas time. The late penalty is 10 points per day. No submissions will be accepted after October 2.

## *Weapons of Math Destruction* Assignment

*The second written assignment is a critical essay. It is introduced at the end of the seventh week of the semester and due in the ninth or tenth week. It is designed to demonstrate some dimensions of critical thinking and information literacy such as finding appropriate sources of information, evaluating evidence from outside sources, examining one's own assumptions as well as others', and formulating a position. As with the first assignment, students are given the opportunity to revise and resubmit after an initial evaluation.*

The purpose of this assignment is for you consider reviews and possible criticisms of the book *Weapons of Math Destruction.* This process is valuable to you as a consumer of information, as it can help you see more than one side of an issue, and eventually help you formulate your own opinion based on available information and viewpoints. To that end you will search for and write about two published reviews of the book, at least one of which should contain portions that express some sort of criticism of the book (though the overall tone of the review can still be positive).

**Step 1**: Find two reviews of the book. The length of the reviews should be at least about a page. Do not consider reviews that are just a few sentences or a single paragraph. At least one of the reviews should express some sort of criticism of the book, although its overall tone can still be favorable. The idea is to see what reviewers think of as good and bad aspects of the book. I strongly recommend that you have your two reviews chosen by October 19.

Where to search for reviews: below are a few ways to search for reviews. Some of them have direct links, but some merely provide the reference, and you will need to find the actual article/review yourself. Some link to the online version of a publication, but you may not be able to access the article unless you subscribe. In this case, check with someone at our library to see if there is another way to access the article. I can also help you look and figure out how to access the articles, but you will need to come visit me in my office to do this.

I Go to the SEU library main page: `library.stedwards.edu/`

Type the title of the book on the main search box and hit "return" or click on the magnifying glass icon to the right. Then once you see some results, look to the right and look for "Resource Type." Open the drop-down menu (if not already opened) and click on "Reviews." This will filter your search for reviews. Note that not all of the results will necessarily be immediately accessible, so make sure you are logged on to the site to maximize possible access.

II Below is a link to the American Mathematical Society web page for reviews of books, films, etc.:

`ams.org/news/math-in-the-media/reviews`

This page provides a list of many math-related media that have been reviewed. Scroll down until you see the title of our book. Below the title is a list of reviews with references. The links take you either directly to the article or to the main page of the website. Some of the articles are not accessible unless you subscribe. I do not recommend paying for a subscription, so if there is something you want to pursue, see if you can gain free access through our library.

III Below are lists of reviews from Cathy O'Neil's blog and the website for the book itself, so they may be especially curated to be overall favorable. Nevertheless, you may still find one to use as one of your reviews.

`mathbabe.org/2016/09/07/reviews-for-weapons-of-math-destruction/`

`weaponsofmathdestructionbook.com/about/`

IV Using a search engine to look for reviews is fine. You may consider (online) publications, commercial or organizational blogs, and even personal blogs. However, regardless of the source, you must be able to find out some information about the author of the review.

**Step 2**: Read the two reviews completely and thoroughly, making judicious notes of their main points.

**Step 3**: Write a report and analysis of the two reviews, and give your report an effective and interesting title. The report must be 500–750 words in length, typed double-spaced (2–3 pages). The following is a suggested outline. If you decide to organize your report in a different way, you must still ensure that it has all the required elements.

1. First paragraph/introduction: Introduce the notion that that you are writing about two reviews of the book *Weapons of Math Destruction* by Cathy O'Neil. Do not specifically mention the author and title of the reviews, as you will do that later, but give an idea of what you will be discussing in the subsequent paragraphs. That is, if the reviews praised some aspects of the book, mention this without giving too much away. The same thing goes for the criticism.
2. Next one or more paragraphs: In this part you will write about one of the reviews. If you have a review that is mostly favorable (rather than critical), I suggest that you start with that one. In your sentences and paragraphs make sure that you include:
   - Reviewer's name and a little bit of personal and/or professional background (e.g., education, current work, etc.).
   - Where the review appears and in what medium. Is it from an academic journal, magazine, (personal) blog, online news outlet, etc? Ask me if you are not sure.
   - The main points of their review. Note: some reviews include a summary of the book. You can mention this, but briefly. This is where you write about what they liked about the book (or not).

   You may use direct and/or block quotes from the reviews as necessary to illustrate what you want to highlight.
3. Next one or more paragraphs: Same thing as Part II, but for the second review. If this is the review that had more negative criticism of the book, make sure that you give this aspect ample room.
4. Next one or more paragraphs: This is where you compare and contrast the reviews and provide your own input. What were the commonalities (if any) of the two reviews? What aspects of both reviews did you agree or disagree with? Or, did you find it difficult to decide, and why? Is there one reviewer that you agree with more or not, and why? Given the backgrounds of the reviewers, can you say something about why they may have praised or criticized the book? You can also talk about what you need to do if you cannot quite decide whether you agree or disagree with the reviewers.
5. Concluding paragraph: Find a nice way to tie up what you have written. The concluding paragraph does not need to be long (perhaps 3-5 sentences), but it should give a sense of how you reacted overall to the reviews that you analyzed.

Include a list of references consisting of the two reviews. Use Chicago, APA, or MLA style for formatting.

Your report will be graded out of 100 points distributed as follows:

- Extent to which report contains all the requested content: 50 points
- Effective and interesting title: 10 points
- List of references with consistent formatting: 10 points
- Word count and double-spacing requirements followed: 10 points
- Grammar and punctuation: 20 points

Save your work as a pdf file and upload it into Canvas. The deadline is October 30, 11:59 pm Canvas Time. The late penalty is 10 points per day. No submissions will be accepted after November 2, 11:59 pm Canvas Time.

## *We* Assignment

*The third and final written assignment is a piece of literary analysis. It is given at the end of the twelfth week and due the fourteenth. Unlike the first two, no opportunity for revision is provided for this assignment.*

The purpose of this assignment is for you to analyze three major themes of the novel *We* by Yevgeny Zamyatin. To that end you will write a short essay where you will address the themes of utopia/dystopia, identity, and mathematics.

First, pick a particular entry/record from the novel. Choose from among the following entries/records: 2–4, 8, 10, 12, 15–19, 24, 25, 30, 31, 34, and 39. Do not choose an entry/record that is not on this list. Next, you will write a short essay of 500–750 words (double-spaced), with the following format and outline.

1. Essay Title: Choose an effective and interesting title for your essay that encapsulates the events or themes of the entry/record you chose.
2. Introductory paragraph: In this paragraph, identify your chosen entry or record number (in a sentence). Then give a brief synopsis of 3–5 sentences of what transpires in this record/entry. Finish your introductory paragraph by briefly describing what is to come for the rest of your essay.
3. Next paragraph (or two): In this section, discuss the notion of either utopia or dystopia within the context of this particular entry/record. What elements of the entry/section conform to the notion of either utopia or dystopia (or both). Be as specific as you can, citing particular elements or passages of the text.
4. Next paragraph (or two): In this section, discuss the notion of identity. Pick a character depicted in the entry/record. Most likely, it will be D-503, but you may also pick some other character as long as they are present or referenced within the entry/record. What do you learn about you chosen character's identity in reading this section? Again, be as specific as you can, and you are encouraged to cite particular passages.
5. Next paragraph (or two): In this section, discuss the role of mathematical references in this passage. Point out particular parts of the text where there is mathematical terminology and discuss how the author uses the terms. What imagery or literary effect do these mathematical references have in the unveiling of the narrative?
6. Concluding paragraph: End your essay with a concise paragraph that summarizes what you discussed in previous paragraphs.

The content of your essay must only include your own analysis. This is what I am interested in reading. Do not consult other references. (I may use a plagiarism detection service to check.) Also when you quote something from your text, you do not need to put a page number (as we may have different editions and translations anyway).

Your essay will be graded out of 100 points distributed as follows:

- Extent to which essay contains required content: 50 points
- Effective title: 15 points
- Word count of 500–750 words and double-spaced: 15 points
- Proper grammar and punctuation: 20 points

Upload your essay as a pdf file on Canvas. The deadline is November 29 at 11:59 PM Canvas time. The late penalty is 10 points per day. No submission will be accepted after December 5, 11:59 PM Canvas time.

# 14

# A First-Seminar Course on the Mathematics of Equity

Karen E. Clark

**Abstract** This article describes a First Seminar Program (FSP) course occasionally offered at The College of New Jersey with the theme of *The Mathematics of Equity*. The College of New Jersey (TCNJ) is a mid-size public liberal arts institution with all entering freshmen required to take an FSP course in the fall semester. All FSP courses at TCNJ are considered writing intensive, with an expectation that the theme of the class will lend itself to class discussions on issues that involve controversy. The *Mathematics of Equity* class includes the main topics of voting theory (including weighted voting), apportionment, and fair division. Students complete several writing assignments over the course of the semester along with more traditional mathematical activities involving calculation.

## 14.1 Background and Context

TCNJ is a selective public liberal arts college with approximately 7,000 undergraduate students. For a typical incoming class approximately 95% of freshmen live in dormitories on campus. The First Seminar Program (FSP) is within the purview of the Office of Academic Affairs, with each TCNJ department encouraged to offer at least one section of an FSP course each year. All first year students are required to take an FSP course in their first semester, and all FSP courses are designated as "writing intensive", having a course maximum of 18 students. Students are expected to do a significant amount of writing in the course, at least 20 pages, with opportunities for revision of assignments after receiving feedback. TCNJ has a 14-week semester (with final exams afterwards), and the FSP class meets twice a week, for 80 minutes each time. The class is one course unit. The College offers approximately 70 different FSP topics each year—incoming freshmen choose six possible FSP courses and are normally placed into one of their choices. What is unusual about TCNJ is that students have been housed according to their FSP choice, with most students in each section of an FSP living on the same dormitory floor. Each FSP section will typically include one or two commuter students. This policy of housing students according to their FSP course is in the process of review, so it is not clear if the housing component of the FSP will continue in the future.

Most students at TCNJ will have declared a major during the admissions process, and arrive as freshmen in the fall having been placed in a major program. The FSP is intended to be a course outside of the student's major, and all FSP courses are expected to have the learning goals of fostering intellectual curiosity, introducing students to college level assignments and expectations in terms of writing, reading, research, and oral presentations, and to improve students' ability to think critically about their world. In addition, in the *Mathematics of Equity*, FSP additional goals are to introduce students to resources on campus such as computer labs and tutoring services, and to expose students to mathematical topics that have applications in the real world.

## 14.2 Mathematical Theme

The mathematical theme of this course is ways of mathematically measuring "fairness" in different real-world contexts. Approximately the first third of the semester is spent studying voting systems, fairness criteria, and weighted voting, with a more in-depth treatment than what might comprise a single topic in a traditional liberal arts math course. Throughout the semester historical examples are intertwined with mathematical calculations to illustrate outcomes that students can analyze and discuss in the context of measuring equity. At the end of this topic there is a brief discussion of Arrow's Theorem [9].

In the second part of the course, we cover apportionment and resulting paradoxes, again with concrete examples whenever possible to demonstrate the underlying concepts. In this middle part of the semester, students are introduced to spreadsheets to perform simple calculations and are expected to become comfortable with working with larger data sets than in the earlier weeks of the semester.

The course ends with a short unit on fair division—we begin this topic with a discussion of basic cake-cutting techniques, and then move to analysis of methods to divide discrete assets, such as in divorce. The last class meetings are devoted to student presentations of their semester-long projects. In the three different topics that make up this FSP course, the class repeatedly considers the concept of what "fair" means, and how one can use mathematics to measure fairness in order to assess different possibilities and outcomes.

The mathematical content for the course is introduced to the students at the level of a typical liberal arts math class. Approximately one-fourth of the class is devoted to the mathematical calculation, with the remainder for discussion, writing, presentation, and analysis.

## 14.3 Course Structure

The relatively long 80-minute class meeting time for the FSP course allows for a variety of in-class activities, with a typical class consisting of a short lecture on the topic or mathematical calculation being introduced, followed by group work on problems or activities that allows students to perform calculations themselves. Most classes end with a period of class discussion on the group work and how it connects to the course theme of measuring equity.

In addition to the weekly homework assignments, short writing assignments, and paper and presentation, students are required to take two traditional in-class exams during the semester, and have to take a final exam during the final exam period. The exams are structured so that approximately half of the questions involve calculation, and the other half involve explanation of concepts discussed in class in short answer/ short essay type questions.

Finally, while the vast majority of class time and effort is spent on investigating mathematical topics, some class periods include time spent on discussing typical freshman issues and college adjustment. This includes discussions about registration, tutoring, changing majors, and a half-class spent on going through the College Academic Integrity Policy. In the subsections below I discuss a few of the out-of-class and in-class assignments that were incorporated into the FSP. The specific assignments are included in the Appendix.

### 14.3.1 Reading and Writing Assignments

Students spend the first month of the semester reading the book *Gaming The Vote: Why Elections Aren't Fair* (*and What We Can Do About It*) by William Poundstone as part of their homework outside of class. This 2009 book describes, through specific historical examples, how spoiler candidates have affected election outcomes. Further, it describes different voting methods, and gives an overview of the main topics that are studied at the beginning of the course. The students respond to the information in this book in three separate writing assignments over the first few weeks of the semester, while spending class time investigating the specific voting methods discussed applied to simple examples. As the book is organized into three separate parts, students complete a writing assignment as homework after they read each part, to partially satisfy the writing intensive requirement in place for all FSP courses. After reading Part I, students are asked to write a one-page summary of the first part of the book and say what the main idea is. Students often struggle with this initially and have difficulty identifying the "big picture" idea in a concise way. I provide feedback on the initial draft, so that students can revise and resubmit an improved final draft.

The class repeats this assignment after reading Part II of the book, and students are then required to bring their drafts to a class meeting. During this class period, I have been fortunate to be able to arrange for a team of peer tutors from TCNJ's writing center to attend the class and in small groups the tutors train the FSP students to peer review each other's essays. The aim of this activity is for students to learn to peer review writing, and to become familiar with the tutoring resources that are available for them on campus. It is essential that I schedule this activity at the beginning of the semester, when the tutoring center is less busy and can spare the tutors for a class session. After the FSP students receive feedback from their classmates in this peer review session, they then revise their drafts as homework and submit them as their second writing assignment. For the third and final final writing assignment on the *Gaming the Vote* book, students are asked to write a two-page response paper to analyze the claims of the book and in this paper each student must say if they agree with the author's conclusion, and of course back up their opinion with an argument. This is a very different type of task than the first two short essays were, and after the third writing assignments are due one entire class period is spent discussing their responses to the prompt given for this final paper on the assigned book. In the three writing exercises what most students seemed to struggle with is the short page limit on the assignments. The students can summarize the reading, but have trouble doing so in a concise way that only highlights the most important ideas and does not include unnecessary detail.

### 14.3.2 Voting Theory In-class Activity

During the first part of the semester, some class time is spent introducing the various voting methods discussed in the *Gaming the Vote* book in a more concrete way, with students using calculators to do calculations that illustrate the various methods. Many of the examples are modeled after those included in standard liberal arts math textbooks, most of which have brief chapters on voting methods and fairness criteria. The mathematical background and comfort level with basic mathematical concepts is extremely variable in this course—I typically have FSP students who are enrolled in Calculus 2 at the same time, and other students who have difficulty with basic fraction calculations. Keeping this in mind, much of the in-class activities are formulated to include a group component. The benefit of having students work through problems in groups during class time is that they are able to identify mistakes and get feedback immediately, and given that most students are living in the same dorm they seem comfortable working in groups both on homework and during class.

After spending some time performing calculations independently and then reflecting with their group, the class then comes together to discuss different fairness criteria in light of the numerical results each group has obtained. The emphasis is on this final group discussion, when students can express which method they feel is more "fair" in each situation. For instance, in the sample in-class assignment in the Appendix (p. 139), students work through problems that increase in level of difficulty, and motivate questions that can be discussed first in a small group and later in the large class setting. In the first problem students are given a preference schedule for an election and have to do a simple calculation. This insures that everyone has mastered the mechanics of the voting method (in this instance, plurality), and the first fairness condition (the Condorcet condition). This is followed by the much more difficult task of the student having to create a preference schedule in which a desired outcome occurs (a certain candidate wins the election, and the Condorcet condition holds). Students are asked to compare answers within their group and discuss what it means for a fairness condition to hold or not for a particular voting method. This is a sophisticated and rather abstract idea for many of our students, and it introduces the concept of counterexample, which will be something they will have to become very comfortable with in the course. The questions that follow are similar in structure, but for a different voting method and fairness criterion. As part of this assignment the groups are asked to come up with explanations of why a particular voting method does satisfy a fairness criterion, so that when the class comes together at the end to discuss the group work, the whole class can compare and critique the responses from each group. This part of the exercise reinforces the writing component goals for the course, but in the context of responding to mathematical calculations that each person or group has performed.

### 14.3.3 Apportionment Activity in the Computer Lab

For the apportionment unit in the middle part of the semester, the first in-class activities to introduce the topic include some simple calculations that can be done with a calculator. However due to the nature of the subject matter the examples are more interesting with larger data sets. After the students have been introduced to the basics of apportionment through the earlier in-class activities, the class spends one class period in an instructional computer lab on campus to work on concrete examples using census data. The specific assignment given is included in the Appendix (p. 139). During this class period, approximately the first 20 minutes are spent with students learning the basics of how to use a spreadsheet, with an introduction to the commands they will need. Students then work in groups on the assignment given, which requires them to study a larger quantity of data than they would be able to tackle by hand with a calculator. In this assignment, the class is provided a spreadsheet with actual census data for the populations of all 50 states in the last available census, and students must then use this information to determine how many congressional representatives each state would be allotted using the different apportionment methods the class has been studying. Most apportionment methods work by rounding decimals in different ways, so students need to code this in the spreadsheet. This particular class meeting is beneficial for students in that it gets them comfortable with using the campus computer labs, and exposes them to some basic technical skills they will need later on in their college career such as downloading documents from a shared drive, submitting an assignment electronically, and becoming comfortable with spreadsheets. They are able to experiment with different calculations and scenarios easily, and are able to be asked more challenging and thought-provoking questions after they have the opportunity to "play with" the data.

This class ends with a discussion of the Hill-Huntington Method [10], which is the apportionment method that is currently used in the US, and is complicated enough that it would not be able to be discussed without using the spreadsheet. The class meeting following this lab "field trip" is spent in the usual classroom discussing student observations from their data experimentation. This class experience in the computer lab is highly engaging for some students who are resentful of having to take a math-themed FSP—they indicate their impression that becoming comfortable with using spreadsheets would be useful for them later on, no matter what their major, and the level of interaction and enthusiasm is highest during this particular class period, both times I have taught the FSP.

### 14.3.4 Fair Division Classwork

In the last part of the semester, students study different methods of fair division. It seems like this is not as connected to the earlier topics as it should be. However, I have found that students tend to like the topic a lot, and it has been a good way to engage the class in lively discussions at the end of the semester. The presentation of this unit begins with cake-cutting strategies, and ends with ways to divide assets (land, possessions) among competing parties. Some of the methods presented here require students to be comfortable with very simple algebra (solving one equation in one variable), so again it is beneficial for students to be able to work problems in groups to become comfortable with the calculation aspect of the subject matter. I have included a sample class activity for this unit in the Appendix (p. 140), where the task is set up so that students begin by individually doing a calculation just to make sure that everyone in the class understands the method being used. In the entire unit on fair division we make the basic assumption that the players involved in the examples and exercises the students are given behave in an ethical way, and in this assignment we initially assume that all bids on items are honest bids. Once the students have determined how the assets are divided in a fair way, they explore first individually, and then in groups, how the division would change if the players were to strategize instead of submitting honest bids. This is a concrete way for the class to arrive at a better understanding of the method (in this case, sealed bids) and how this unit connects to the course theme of analyzing fairness.

### 14.3.5 Course Project

At the end of the third week of the semester, students are assigned a semester-long project which consists of a short paper (approximately five pages) and an in-class presentation of their paper topic. The paper is due in the second to last week of the semester, with the presentations occurring during the last three or four class meetings. The paper topics are chosen by the students, with the only requirement that proposed topics have to in some way relate to fairness in voting. Paper topics have included voting methods not discussed in class (e.g., approval voting), sore loser laws, and insincere voting. Students sometimes try to tie their paper topic to whatever their major is. Students are required to submit a proposal for a paper topic early in the semester, and after approval submit a first draft mid-semester for feedback before the final paper is due. Combined with the short answers that are required as part of the more traditional weekly homework assignments, these satisfy the College's requirements for the course to be designated writing intensive.

## 14.4 Reflections

I believe that my FSP sections successfully achieve the learning goals that are expected of seminars in the program. The nature of studying different voting methods and the required reading at the beginning of the semester give students plenty of opportunities to reflect on abstract issues about what it means for a system or process to be fair. Certainly, this is reinforced in the last material on fair division. The writing assignments and short homework essays that require students to interpret calculations provide a level of challenge that is different from most students' high school experience. The amount of work that has to be done independently, over longer periods of time than what they are used to, sets the tone for the difference between college and high school work.

I have taught this course twice and am hoping to teach it again in the near future. Some factors have changed on a college level at TCNJ with regard to the FSP courses since I last taught this seminar, so I suspect that the next time will be a different experience. In particular, this course will no longer satisfy the quantitative reasoning requirement at TCNJ, which was a large motivating factor in the past for freshmen to choose this FSP section. In addition, it is not clear that TCNJ will continue to house students according to their FSP selection, which would surely create a different class dynamic. Usually by mid-semester the students in the course section tend to be very comfortable with each other in a way that is not typical in the other more standard math courses I regularly teach, because these first-year students are seeing their classmates regularly outside of class time by virtue of their living on the same dormitory floor. I believe this is beneficial for students who might be struggling with homework assignments, as I know several of them rely on fellow classmates for help.

One benefit of having several mostly disconnected topics put together to form this course is that students feel they are starting each new unit with a "clean slate" and can recover if they had trouble with an earlier topic. Having the course consist of very different types of assessments allows students who might do better on different types of assignments or exams the opportunity to do well in the course.

There are many things I would do differently the next time I teach this class. At the beginning of the semester, students have a challenging workload as they are simultaneously reading a book and doing writing assignments, and in addition completing traditional homework that has a more calculation focus. Once the book reading part of the course is done, I intend to structure much of the rest of the semester in a more flipped-classroom style, with readings or videos assigned as homework, and more class time devoted to group work on the assigned readings or videos that I have posted.

The examples presented at the beginning of the semester tend to come almost exclusively from political elections and that is interesting and appealing to many of our students. However, often students are motivated by examples that come from sports or entertainment (e.g., Academy Awards) and I intend to try to incorporate a more diverse set of examples in the course next time.

In general I observe that students find the fair division topic more interesting than the earlier voting theory material as they indicate that they believe dividing assets is more relevant to their own lives, while the political implications of voting methods and apportioning representatives is quite a bit more esoteric.

I will probably eliminate the student presentations given at the end of the semester on the paper topics. Unfortunately it is common for the presentations to focus on style and "flash" rather than substance, and it is difficult to engage students in the audience in the presentations of their classmates at that point in the semester when they are all feeling overwhelmed. The FSP course is expected to include a presentation component; however I think it will be possible to incorporate this earlier in the semester, in a way that is more connected to the course material that all students are learning. It is important in this class to incorporate as many recent examples as possible, in addition to the historical examples (for our freshmen the 2000 election is a historical example!). A weekly or bi-weekly current events component to the course would enrich the course experience, and could serve to satisfy the student presentation component at an earlier point in the semester.

As the students are finishing the last part of the book, several class periods are spent studying the topic of weighted voting systems and measuring power. This is mathematically the most sophisticated and abstract topic in the course, and at the end of the course most students indicate that they find this to be an incredibly challenging unit. We are able to spend a class period discussing the UN Security Council, which is a way to incorporate a real-world, current subject into the course. While there is value in students being exposed to this level of mathematics in the course, and the ability to include a timely issue into the class is attractive, I believe this unit is too difficult for many of the students in the way that I present it. In the future I would add at least one or two class meetings on simple combinatorics to lead into this topic, so that students are more comfortable with the concept of forming committees, and counting arguments in general.

A major difficulty for students in this course is the lack of a text. Many of the available books on voting theory are on too high a level for this group of students. Using the course learning management system to post sources of information that are publicly available on the course topics is beneficial for students who struggle without a formal text.

In the short period of time since I have taught this course, I believe that student access to computer resources has changed quite a bit at my school—I observe that today most of the freshmen arrive on campus with laptops, and my department has loaner laptops that can be borrowed during class time for the few students who are without. Given this trend for students to have better access to computer resources, I plan to revise the in-class assignments I have been giving to have students utilize spreadsheets rather than pencil, paper, and calculator. The use of technology would also free up some class time which can be spent on adding another topic to the course.

Finally, I would like to include a unit on gerrymandering the next time I teach this class, as it is a topic that is in the news regularly, and there are now resources available so that the subject can be presented on a level that would be appropriate for my freshmen seminar students. The class time required to add this topic and the combinatorics background for weighted voting would come from a reduction in the time spent on student presentations at the end of the course and on efficiencies coming from greater reliance on computer technology.

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## Appendix

### Writing Assignments

*These are the three writing prompts that students had to complete as homework assignments during the first part of the semester, in response to the reading. Writing assignment I was due in week 3 of the semester and students received feedback from the instructor. Writing assignment II was due in week 4 of the semester and students provided peer feedback during class time. Writing assignment III was due in week 5 of the semester as a final overview of the class reading.*

1. **Writing Assignment I** Please write a one-page summary of Part I of *Gaming the Vote*. Try to be as precise and comprehensive as one page allows. What were the most important themes of this part?

2. **Writing Assignment II** Please write a one-page summary of Part II of *Gaming the Vote*. Try to be as precise and comprehensive as one page allows. What were the most important themes of this part?
3. **Writing Assignment III** Please write a two-page response paper for the entire reading. A response paper should discuss and analyze the themes and claims of the book. Is the book persuasive to you? You will need to support your opinions and analysis with direct references to the book.

## Sample Voting Theory in-class Assignment

*These are problems that students worked on individually and in groups of 2-3, during one class period in the first few weeks of the semester.*

You are meant do do problems 1, 2, 5, 6 individually, and then discuss the other problems with your group.

1. Consider the preference schedule below:

| | | | |
|---|---|---|---|
| A | A | B | C |
| B | C | C | B |
| C | B | A | A |
| 3 | 1 | 3 | 2 |

   Determine the election winner using plurality, and explain why this violates the Condorcet condition.
2. Come up with an example of a preference schedule where candidate A wins the election using plurality, and the Condorcet condition holds for your election.
3. In your group, compare and discuss your answers to questions 1 and 2. What do we mean when we say that a voting method does not satisfy a fairness condition? Does it mean that the violation happens all the time?
4. In your group, discuss why plurality always satisfies the Majority Criterion. Together come up with a two-sentence explanation that you can share with the whole class. Compare with your discussion for question three.
5. Borda Count does not satisfy the Majority Criterion. Come up with a preference schedule for 3 candidates in which the winner using Borda Count does not receive the majority of votes.
6. Now come up with a preference schedule for three candidates in which the winner using Borda Count does receive the majority of votes.
7. In your group, compare and discuss your answers to questions 5 and 6.
8. In your group, discuss why Borda Count always satisfies the Monotonicity Criterion. Together come up with a two-sentence explanation that you can share with the whole class.

## Sample Apportionment Lab Assignment

*This is an in-class activity that students worked on in an instructional computer lab. Students were given a short introduction to Excel before working on these problems.*

**Part I** Download the spreadsheet posted with population data from the 2010 census. Clear any information you won't need to determine the apportionment, and from the column that gives the number of electoral votes for each state determine each state's number of congresspersons in the House of Representatives. Remove Washington DC. There are 435 seats in the House—make sure that you get this total number in your spreadsheet.

**Part II** You are meant to do problems 1-4 individually, and then discuss problems 5-7 with your group. Arrange your spreadsheet so that each answer is in a different column, and make sure your columns are clearly labelled.

1. Determine the apportionment using Hamilton's method. Make sure to sort at the end so the data is given in the same order as you were given (according to state population, largest to smallest).
2. Determine the apportionment using Jefferson's method.
3. Determine the apportionment using Adams's method.

4. Determine the apportionment using Webster's method.
5. Look at your results. Do any of the methods match the actual apportionment? If not, which comes closest? Discuss with your group.
6. Which method seems to favor big states? Discuss with your group.
7. Which method seems to favor small states? Discuss with your group.

**Part III** The *arithmetic mean* of two consecutive numbers is what we think of as the average—we add the two numbers together and divide by 2. So the arithmetic mean of 4 and 5 is $\frac{4+5}{2} = 4.5$. When we determine an apportionment using Webster's method, we calculate the quotient and round up or down as usual, by rounding at the arithmetic mean. So if a state's quotient is 4.48, the state would get four representatives because 4.48 is less than the arithmetic mean of 4.5. If a state's quotient is 4.56, the state would get five representatives because 4.56 is greater than the arithmetic mean of 4.5.

In the **Hill-Huntington Method**, which is the method that is actually used to determine the apportionment of the House of Representatives today, a state's quotient is rounded up or down according to the *geometric mean* rather than the arithmetic mean.

The *geometric mean* of two numbers is the square root of the product of the numbers. For example the geometric mean of 4 and 5 is $\sqrt{4 \cdot 5} = \sqrt{20} \approx 4.47$. So when we use the Hill-Huntington method, if a state's quotient is 4.48, the state would get five representatives, because 4.48 is greater than the geometric mean of 4.47.

1. Find the geometric mean of 8 and 9. Compare with your group.
2. Find a quotient between 8 and 9 that would be rounded up using the geometric mean and rounded down using the arithmetic mean. Compare with your group.
3. Determine the apportionment of the fifty states in your spreadsheet using the Hill-Huntington method. Does it match what you were given as the actual apportionment?

## Sample Fair Division in-class Assignment

*This is an in-class activity that students worked on in groups of 2-3 during a class period at the end of the semester, as part of the unit on Fair Division.*

Three roommates will be using the method of sealed bids to divide assets that they purchased jointly when their lease is up. Each roommate values the items as shown in the table below.

| | Sam | Alex | Morgan |
|---|---|---|---|
| Microwave | 20 | 35 | 30 |
| Armchairs | 60 | 65 | 70 |
| Rug | 70 | 85 | 120 |
| Toaster Oven | 30 | 25 | 20 |

1. Determine the final settlement. Once everyone in your group is done, compare answers to make sure you all understand the calculation.
2. Suppose the chairs are worth \$65. Furthermore, suppose Alex absolutely does not want the chairs, and puts in a bid of \$1. What is the outcome of the settlement in this case? What is the effect on Alex of her putting in this insincere bid?
3. Suppose Alex really does want the chairs, and puts in a bid of \$120. What is the outcome of the settlement in this case? What is the effect on Alex of her putting in this insincere bid?
4. Discuss your answers to questions 2 and 3. with your group.
5. Suppose Sam does not want the rug, and she has knowledge of both of her roommates' bids for the rug. What should she bid for the rug in this instance?
6. Suppose Sam does want the rug, and she has knowledge of both of her roommates' bids for the rug. What should she bid for the rug in this instance?
7. Discuss your answers to 5 and 6. with your group.

# 15

# Exploring Immigration Through Statistics and Experiential Learning

Kathryn Cerrone

**Abstract** A first-year seminar conducted as part of a statistics learning community at The University of Akron was run for several years, the last two of which incorporated an experiential learning project. The goal of incorporating experiential learning in the courses is to improve student engagement. This paper will discuss the initial goals of the learning community, the implementation of an experiential learning project and reflections on the project. Based on comments in the students' final reflection papers, student engagement increased.

## 15.1 Background and Context

The University of Akron is a large, public Midwestern research university in northeast Ohio. Enrollment has ranged from 22,000 to 27,000 over the past decade. About 75%—80% of the students are undergraduates. Over 200 undergraduate majors are offered at the main campus ranging from engineering and STEM to English, music, and fine arts. Graduate degrees in many fields are also offered at the Masters and PhD level. The university is on a 15-week semester schedule with 50-minute class periods per credit hour.

The first-year seminar at The University of Akron is intended to introduce incoming freshman to college life, emphasize academic expectations, develop study skills, provide support for underprepared students, allow students to explore different majors and careers, address information literacy, and encourage all students to get involved in the campus community. Underprepared and exploratory students, those who have not selected a major at the time of enrollment, are required to take a first-year seminar and all others are strongly encouraged to. First-year seminars are run through the Division of Student Success in the Office of Student Affairs. Each fall, an average of 50 to 60 sections of 20 to 30 students each are conducted and about 20 sections of the same size are offered in the spring semester. The course is run as a two-credit hour class typically meeting twice per week for 50 minutes each class.

About one-third of the FYS sections are run as learning communities (LC) in which students take at least two courses together that have an integrated theme. Faculty teaching LC courses are expected to meet regularly throughout the semester to develop co-curricular activities and track student progress across the courses. The statistics LC was originally comprised of students taking the introductory level *Statistics for Everyday Life* (*SEL*) course and the first-year seminar, *The Akron Experience: University 101*. *English Composition I* was added to the learning community to improve the students' writing when the experiential learning project was introduced. The LC is only run in the fall semester. Students enrolled in the courses are from diverse majors from across campus including education, nursing and political science. There are also several exploratory students in the courses. The students are placed in the LC by their academic advisors.

## 15.2 Mathematical Theme

The theme of the LC is *Statistics in Everyday Life*. The goal is to introduce students to entry level statistics and minimize math anxiety so they can become informed citizens who can think critically about the information they encounter on a daily basis. Initially, the SEL LC first-year seminar was developed to be a support course for *Statistics for Everyday Life* where, in addition to the standard first-year seminar curriculum, there is a focus on how the topics covered in class apply specifically to math and statistics. Approximately 10% of the FYS course was dedicated to reviewing the introductory statistics content for the SEL course.

One goal of the LC is to improve student completion in the *Statistics for Everyday Life* course. Students attend math anxiety workshops and review days are held before each statistics exam. However, the professors involved are also looking for a way to engage the students on a deeper level. After several years of successfully teaching the LC, the teaching team decided to incorporate experiential learning and a larger cohesive project into the courses in the aim of improving student engagement. A description of the updated course structure follows.

The SEL LC was reorganized in order to incorporate the experiential learning project. First, the statistics course created a new project to incorporate the experiential learning activity data. Second, the first-year seminar curriculum was updated to adjust for the time needed to prepare for and complete the experiential learning project. Finally, a section of English Composition I was added to the LC.

The LC faculty met with the university's service and experiential learning coordinator to brainstorm potential project ideas. Originally, it was planned to have the students work with the Summit County Re-entry Network,[1] a group that assists those recently released from prison in finding housing and employment. The students were going to gather and analyze data for the network that could be used on their website and in publications distributed to the public. This would have fit with the freshman reading assignment of *Orange is the New Black* by Piper Kerman [1]. Unfortunately, the partnership fell through right before the start of the semester and the team regrouped. Ultimately, the topic of immigration was chosen for the project not only because it is a trending topic of discussion with the ensuing refugee crisis and changing immigration policies, but because it is one with which many people have little first-hand experience.

## 15.3 Course Structure

The experiential learning project arced across all three courses, *Statistics for Everyday Life*, *The Akron Experience: University 101*, and *English Composition I*. Each course had separate assignments that tied together with the common theme. In the statistics course, students learned about data literacy and computation. In the *English Composition I*, course the students focused on reading, analyzing and creating texts. The FYS, taught by the author, concentrated on student engagement. Although the following curriculum spanned across three separate courses, it could be used in a single course based on time constraints and student preparation. Alternatively, parts of the assignment could be implemented individually as time and resources permit.

### 15.3.1 Statistics for Everyday Life

*Statistics for Everyday Life* is an introductory level, general education statistics course covering a conceptual approach to the basic ideas and reasoning of statistics. Topics include descriptive statistics, probability and uncertainty, and statistical inference (estimation and hypothesis testing). The course is four credit hours including a 50-minute weekly lab where students analyze applications using the computer software *StatCrunch*.

In the SEL course, the students are taught descriptive and inferential statistics as well as how to evaluate them using *StatCrunch*. The statistics faculty member accessed data on the local population from the 2012 American Community Survey. The data was narrowed down to the county in which the university resides and only the following social and economic characteristics were included: citizenship, place of birth, length of time to become a citizen, employment status and sector, level of education, use of public assistance, and ability to speak English. The demographic information gender and age were also included. During the statistics course, students reviewed 13 potential research questions (see Appendix 15.4). Student groups of three or four are each assigned a research question. The students are then provided the data during their lab time and asked to use descriptive statistics to answer their research question. Specific

[1] summitcountyreentrynetwork.org/

instructions are given based on whether the data is quantitative or categorical.

1. If the two variables are both categorical, the students are asked to create a contingency table with the two variables, create two separate bar charts for the categories of the explanatory variable and analyze the table, the proportions, and the charts making comments on any small or large differences. Finally, students are asked to conclude whether the variables are associated (dependent) or not associated (independent).

2. If the two variables are both quantitative, the students are asked to create a scatter plot with the sample regression line, calculate the correlation coefficient, make an interpretation of the slope and the correlation coefficient, and comment on whether there is a strong or a weak correlation between the two variables and whether the relationship is positive or negative.

3. If the response variable is quantitative and comparing the groups of a categorical variable, students are asked to get the summary statistics for the quantitative variable for the two categorical groups, create side-by-side box plots of the quantitative variable for the two groups, and comment on anything interesting found from the summary statistics and the box plots (skewness, outliers, quartile differences). They are asked to analyze and comment on whether there is a large or relatively small difference between the means of the two groups.

The statistics professor then shared the results for all of the questions with class after the reports were submitted.

This project could be modified or abbreviated if there is not enough time to cover the statistics in full. For instance, the analysis portion could be provided by the instructor rather than compiled by the students. It would be expected that the students have some background in statistics and there would need to be thorough discussion about what the statistics were describing.

### 15.3.2 FYS

The FYS is structured to incorporate assignments on study skills, time management, and engagement with the campus community and community at-large. To align with the SEL course as part of the LC, the FYS has tailored assignments to support the students in their study of statistics. During the first few weeks of the semester the students are required to attend a math anxiety workshop. Math study skills are discussed during class and in discussion board posts throughout the FYS and study sessions for the statistics exams are held before every exam. Writing assignments related to the statistics mini-projects are assigned in the FYS. For instance, the statistics class had a mini-project on textbook costs. The students conduct a short survey among their friends and run descriptive statistics on the data compiled from the entire class. In the FYS, the students discuss some of the reasons for high textbook costs and how it impacts their budget for school. The students also complete activities unrelated to the statistics theme such as attending the freshman reading lecture, planning a budget, applying for financial aid, and meeting with their peer mentors. Assignments include:

- a photo scavenger hunt to help students explore campus and identify the locations of important buildings and departments such as advising and financial aid
- meetings with their advisor as well as reflections on their major and course plan
- time-management exercises where students record their daily habits then make a schedule for each coming week along with reflections on how they use their time.

As students are learning about analysis in the SEL course, they are discussing immigration and refugee status and how it impacts the local community in the FYS. They are also preparing for their experiential learning activity and visit to a local immigration support organization, the International Institute of Akron (IIA)[2]. The FYS faculty member developed a partnership with the IIA, meeting with the outreach director to discuss the best way to have the students visit and get involved. Students are assigned to visit either an English for Speakers for Other Languages (ESOL) or citizenship test preparation class for recent immigrants and refugees from Nepal and Myanmar in order to give the students a fuller picture of the process of resettlement in the United States. No more than three students attend any given class as to not overwhelm the participants. Five different days and times are selected over the span of two weeks in order to fit within the students' schedules. Two of the FYS and two of the *English Composition I* scheduled

[2] iiakron.org/

class periods as well as an additional non-class time are offered as options for attending the sessions at IIA. Prior to attending, a discussion is held during class to address proper behavior and what is expected of the students during the visit. Students began by observing the class and then help with the learning activities either by playing a game or sharing about themselves. Once everyone had visited the IIA, a discussion is held in the FYS and *English Composition I* courses about what the students noticed, what they learned, how they felt about the experience and how it related to the statistics they had run in SEL. In-class and online discussions are held about how math and statistics can provide a model of a situation and how narrative and personal interactions fill in the full picture with color and context.

### 15.3.3 English Composition I

To pull the whole project together, students discuss the concept of the "American Dream" in their *English Composition I* course. They read Amy Tan's *Mother Tongue* [2] and have in-class discussions about how the data they are analyzing and their visit to the ESOL and citizenship classes fit in the "America Dream." The students then have to write a five-page essay tying the statistics, their visit to the IIA, discussions held during the English and FYS courses, and essays read in the English course together (see the Appendix 15.4). Students are asked to explore what the "America Dream" means to them and how they see it changing in the future, especially how it has an impact on those they encounter at the IIA.

In the final reflection the students make astute observations. When discussing the statistics comparing one's ability to speak English with one's income as shown in Figure 15.1 a student wrote, "As a person's ability to speak English increases his or her income also increases. This Statistic (sic) shows that it is difficult for someone to actually have the American Dream if they don't have the right tools (English and income) to help them achieve their goals." She went on to discuss her visit to the IIA and how it impacted her view on immigration, "That visit showed me that their (sic) are people that want to be apart (sic) of America because they want a better life and love this place... It was amazing to see how different life is for them than for me. It was a great experience to interact with them, teach them new words and how to write, it was like teaching a child, it was incredible."

Another student commenting on the same statistic noted,

> Judging by [Figure 15.1] there is a direct correlation between an immigrant's income and how well they speak English. This means that, if an immigrant speaks English poorly, then most likely his or her income

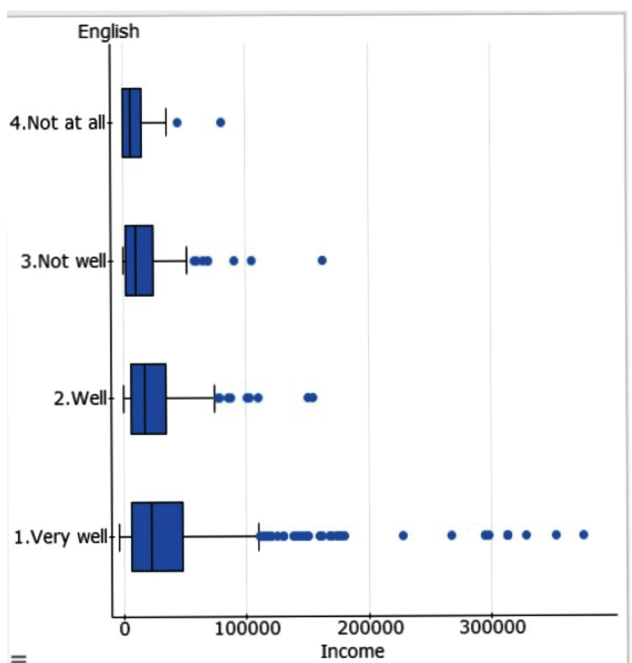


**Figure 15.1.** Visual representation of the association between English proficiency and income produced by students using *StatCrunch*

> will be very low. However, if an immigrant speaks English very well, then his or her income will be higher. This just goes to show why the immigrants in the International Institute were starting with the basics, because it is very beneficial for them to learn our language for job opportunities. I find this the most challenging aspect of being an immigrant in search of the American dream, because if one's interpretation of it is to be wealthy, then they must speak English very well.

Other students explored the impact of English proficiency on educational attainment noting that,

> [E]ducation limits the American dream tremendously. Depending on the extent of education you have is the deciding factor on if you are able to live the American dream or not. According to (sic) statistics(,) how well you speak English can affect the education level you will attain. If you can speak English very well the education level you can attain will also increase. There is (sic) only about 3% of people that speaks (sic) English well but has (sic) no schooling. On the other hand, there is (sic) about 24% of people that are college graduates.

This statistic was particularly notable considering the students were embarking on their own journey in higher education.

Although this is an experiential, rather than service, learning experience, the students are encouraged to volunteer at the IIA after completing the project. One student noted that he planned to do so in his final paper but the instructors were unable to verify if he did.

## 15.4 Reflections

The immigration experiential learning activity took quite a bit of time, discussion, and effort to bring to fruition. Having a good service or experiential learning coordinator was key to making the project a success primarily because it is important to have good buy-in from the partnering organization. The first attempt at developing a service learning project fell through and it was due to the resources provided by the service learning coordinator that a new project was able to be developed in such a short time-frame.

It is also imperative to have buy-in from the students and to ensure that they take the project seriously. This can be done by setting up clear expectations for the students, holding discussions on how to act appropriately, explaining what they should pay attention to during the excursion, and discussing how the statistics and experience are intended to fit together in order to have a successful learning experience. The experiential learning project and format for the class were used successfully for two years, with engagement in the project improving from the first to second year because of an improved relationship with the partnering organization and clearer instruction on what was expected of the students.

There were also some changes to the LC over several consecutive years that impacted the success of the project. When the LC was started, before the experiential learning project was implemented, there was one section of the SEL course with approximately 70 to 80 students. In order to accommodate all of the students, three sections of the FYS with 20 to 25 students each were offered. One section was taught by the statistics professor, another was taught by a math professor, and a third by an academic advisor with a background in statistics. This allowed for all of the students in the SEL course to be simultaneously enrolled in a section of the first-year seminar.

Having all of the students enrolled in all of the learning community courses helped to make it successful. Unfortunately, after two years of teaching in this manner, it became difficult to find faculty or staff willing to teach in a statistics-focused learning community. By the time the experiential learning project was introduced there was only one section of the FYS with 25 to 30 students, taught by the original mathematics instructor. There was also only a single section of *English Composition I* with 20 students in the learning community. However, the SEL course had grown to over 80 students, meaning only some of the students in the statistics course were enrolled in the other two LC courses. Ultimately, only 11 students were enrolled in all three of the courses in the LC. There were several others who were enrolled in two of the three courses (the FYS and either SEL or *English Compositon I*). This made it more difficult to provide incentive for the students not in the LC to attend the visits to the IIA and to make sure that all students in the FYS who were not enrolled in the English course did the readings assigned for that class.

In an attempt to remedy some of these drawbacks, the FYS and *English Composition I* instructors worked closely to overlap readings and assignments between courses so that pertinent information was available to all students. Only students in the FYS and English course were required to complete the experiential learning visit to the IIA. Not all of the statistics students attended. Having all of the students attend would have also proven difficult due to the small capacity at the IIA.

Having students in the FYS enrolled in one, two, or all three LC courses also added challenges to the final writing assignment (see Appendix 15.4) aimed at drawing from all three courses. To make sure that each group had all of the information necessary to complete the final writing assignment, groups were assigned in a jigsaw manner so that at least one person was also in the statistics course and one was from English course so they could explain their portion of the assignment from the other class.

Another issue that arose due to the lack of co-enrollment between classes was that of scheduling when the experiential learning activity took place so that it did not conflict with the students' other classes. The instructors for the FYS and English courses scheduled the visits during two of their classes each. An additional time was selected based on the availability of several students in an effort to keep the number of students per visit to no more than three, per the request of the institute. The students were only required to spend the 50-minute university class period at the IIA rather than the full two-hour ESOL or citizenship class. Relatedly, the IIA was about a 10-minute drive from campus. Therefore, driving time and transportation to the institute had to be addressed. The University of Akron is predominately a commuter school so most students were able to drive themselves. However, some residential students carpooled with classmates or the instructor to the institute.

Despite some of the drawbacks, the project successfully improved student engagement, based on responses in the students' final reflection papers. The faculty teaching the course would have liked to see it continue. Unfortunately, the FYS professor was unable to continue teaching the course due to teaching load policy changes at the university, and a replacement was not found. The instructors felt like the project did a thorough job of having the students learn about and analyze the data and also helped them put the statistics into context. It also helped them gain perspective on the struggles that immigrants and refugees face when moving to a new country.

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## Appendix

### Questions Used in the Statistics Mini-project

*These are the questions used, along with data collected from the 2012 American Community Survey, for the SEL mini-project given halfway through the semester. The data included responses to the questions of citizenship, place of birth, length of time to become a citizen, employment status and sector, level of education, use of public assistance, and ability to speak English as well as demographic information of gender and age.*

**Categorical vs. Categorical**

- Is how well someone speaks English associated with his or her highest level of education attained?
- Is place of birth associated with grandparents living with their grandchildren?
- Is a person's citizenship associated with his or her employment status?
- Is a person's citizenship related to whether or not he or she has health insurance?
- Is a person's citizenship related to whether or not he or she received public assistance?
- Is a person's citizenship associated with his or her employment sector?
- Is a person's gender associated with how well he or she speaks English?

**Quantitative vs. Quantitative**

- Is an immigrant's age associated with the length of time he or she took to become a US citizen?
- Is the length of time an immigrant has been in the US associated with the length of time he or she took to become a US citizen?
- Is a person's age associated with his or her income?

**Quantitative vs. Categorical**

- Is how well someone speaks English associated with his or her income?
- Is a person's gender associated with the length of time he or she took to become a US citizen?
- Is the part of the world in which a person was born associated with the length of time he or she took to become a US citizen?

## Essay Prompt Used in the *English Composition I* and FYS courses

*This is the document that was provided to students with the essay prompt and writing guidelines used as a joint final project for the English Composition I and FYS courses of the LC.*

Essay: The American Dream

Investigate the American dream. What is it? Is it the same for all Americans? Is it a myth? Is it simply a quest for a better life? How has the American dream changed over time? What is left of the American dream for you and for those of your generation? Consider these questions, choose a "way into this discussion" that narrows your argument (i.e., the American dream and immigration, the American dream and gender, the American dream and race, etc.) and analyze the current state of the American dream. Drawing on at least two of the essays from our text, the statistics you gathered in the SEL class, and any information you can draw upon from your visit to The International Institute of Akron, write an analysis of the American dream and its future.

Your challenges and guidelines for this assignment are as follows:

- Remembering that this is an argumentative essay and you must have a solid thesis that answers the "so what?" of your interpretation.
- Remembering to integrate all quotes with your own words.
- Remaining objective in your argument, avoiding the first person, while keeping in mind our Rhetorical Concepts.
- Observing all the conventions of formal essay writing.
- 5–6 pages.
- APA citation.

# 16

# A Civil Right: Math Anxiety to Math Literacy

Gretchen Wilke Whipple

**Abstract** The first-year seminar *A Civil Right: Math Anxiety to Math Literacy* challenges students, both those who describe themselves as math anxious and those who aspire to be math educators, to develop an understanding of the value of math and of math anxiety. The readings, and the questions inspired by them, offer opportunities to explore math literacy and the civil rights movement. The content allows assignments to address the college-level learning goals for this course without requiring the professor to be an expert on areas outside of their discipline. This article will describe the major readings, the informal writing, and the formal writing assignments of this course.

## 16.1 Background and Context

Located near Asheville, North Carolina, Warren Wilson College is a private institution offering primarily undergraduate academic programs with a liberal arts emphasis to fewer than 800 students. Its unique general education program requires that all students work on campus a minimum of ten hours per week and participate in substantial off-campus community engagement experiences. A four-credit first-year seminar (FYS) is required of all incoming first-year students. Each FYS has a maximum enrollment of 16 students.

Instructors across campus design and offer courses that strive to satisfy the college-level learning goals for FYS, which include

- introduction of a discipline or an interdisciplinary topic
- development of a sense of civic identity
- improvement of written communication and argumentation skills
- transitioning to college.

The college has chosen to emphasize the growth and development of each student's civic identity. Thus each FYS must be a service-learning course. Additionally, each FYS is a writing-across-the-curriculum course and must involve extensive informal and formal writing. Finally, the seminar must appeal to incoming students.

## 16.2 Mathematical Theme

This article describes the FYS entitled *A Civil Right: Math Anxiety to Math Literacy*. Many people avoid math or believe that they cannot do math. Math anxiety, the symptoms of which range from avoidance of all things computational to physical illness when faced with numbers, limits the opportunities of this large population and this in turn limits society, especially in science, technology, engineering, and mathematical disciplines. This FYS explores math anxiety and its consequences. To motivate our study of math anxiety, students build an understanding of math literacy and its value. Thus the course challenges students to develop

- a definition of math literacy
- an understanding of why math literacy is necessary to all in our society
- an understanding of the symptoms of math anxiety
- empathy for those that math anxiety prevents from achieving their full potential
- a familiarity with the theories of the causes of math anxiety
- an idea of potential techniques for addressing math anxiety.

To pursue these goals students engage in a variety of reading, writing, and service assignments. Professor and students explore specific mathematical concepts and activities for teaching them.

While mathematical content is not explicitly one of the goals of this course, the professor integrates a wide variety of mathematics. The topics explored will vary based on the students' background and the questions that arise organically from the readings. Each of the three times that this has been taught, students have explored, at a minimum, the pedagogy and a conceptual understanding of operations with signed numbers, techniques for approximation, and the idea of limit in the context of measuring change in calculus.

## 16.3 Course Structure

The class meets two days a week for 80 minutes throughout a 14-week semester. Students focus on math literacy and the diversity and importance of mathematics for the first third of the semester. Readings concerning the characteristics, causes, and mitigating techniques of math anxiety comprise the content for the second third of the semester. During the last third of the semester, students explore supplemental readings and complete their final project.

A set of questions, initially suggested by the professor, then modified by student input, guides our explorations. The following is our initial set of questions:

- What is math literacy?
- What is math anxiety?
- What are some potential causes of math anxiety?
- What are some of the potential techniques to mitigate math anxiety?

Most class sessions evolve as a response to a reading assignment. Students describe the main points of the reading as it addresses one of the seminar's main questions. Students discuss their responses to the reading and evaluate the author's arguments. Students write during class each meeting. In-class writing is often collected and read for continued engagement with the material. However, grammar and spelling are not graded on these assignments.

Throughout the first weeks of the semester, students read about math literacy. These readings inspire discussions about what math is and why it is important for individuals and society. The question "Should math literacy be a civil right?" is added to our short list of guiding questions. The professor does not need to answer these questions. By mid-semester each student writes his or her own description of mathematics, math literacy, and the need for math literacy. Then a variety of readings about math anxiety introduce students to the nature of academic dialog. Students read, write, and discuss various descriptions of math anxiety, potential causes, and mitigation techniques. Students build their own descriptions of math anxiety, their individual theories as to its causes, and suggest a list of techniques to avoid or address it.

To satisfy the college-level writing learning goals, students write three formal papers. The final is a group project: the class designs and presents a workshop on math anxiety.

### 16.3.1 Reading Assignments

Each of the reading assignments address several course questions. In order to emphasize that students are joining an academic discourse, a wide variety of authors are included: a civil rights activist and educator, a British math educator, a college administrator, a neurologist and educator, and psychologists with varying specialties.

### Moore/Hall/Vincent and Moses/Cobb

A short reading from the memoir, *same kind of different as me*, by Denver Moore, Ron Hall, and Lynn Vincent [5], begins to build an understanding of the need for math literacy. Denver Moore was raised on a sharecropping plantation in Louisiana. He was not taught to read or write. He was an adult, in the 1960s, before he learned that he could have gone to school. Moore, in two paragraphs, captures how helpless one can be without essential math skills [5, p. 12]. Students recognize that this experience could apply to persons who do not have confidence in their math skills: they can be easily taken advantage of and their choices may be severely limited.

To continue their exploration of the description and value of mathematics, students read *radical equations: Math Literacy and Civil Rights* by Robert P. Moses and Charles E. Cobb, Jr. [7]. In this text Moses's work organizing voter registration in Mississippi in the 1960s is paralleled to his work founding The Algebra Project decades later. This memoir presents an argument that overcoming difficulties in math is valuable on an individual level and is essential on a societal basis. The authors assert that math literacy is this generation's civil rights issue [7, p. 5]. This discussion of civil rights activism diversifies content of the course and appeals to the non-math inclined.

Reading the Moses and Cobb book, which discusses the civil rights movement, math avoidance, and activities for learning mathematics, naturally leads to activities that satisfy many of the college-level goals of the course. To understand the references to Moses's voters registration work in Mississippi, students must explore voter repression and the civil rights movement. When students are unfamiliar with a historical reference, assigning a short, low-stakes, research, writing, and presentation activity provides an excellent opportunity for them to contemplate factors that contribute to the development of a civic identity and to improve research and presentation skills. Questions are listed on the board as they arise and each question is assigned to two students at the end of class. Students submit their properly formatted sources and teach their classmates about the topic during the following class. Thus students teach the professor and each other in areas where the professor is not an expert.

Discussing *radical equations* allows students to explore their place and time in history. Students consider their experiences beside those of Bob Moses, an educated Chicago black man who was in Mississippi in the 1960s organizing voter registration. They recognize that Moses embodies the mature civic identity that Warren Wilson College has embraced as an admirable quality. His dedication, both to voter registration in the 1960s and to enabling young people to develop the math background to succeed in the twenty-first century, model constructive citizenship.

### Tobias and Buxton

Sheila Tobias and Laurie Buxton were among the first academics to formally explore the concept of math anxiety. Reading the work of these two authors allows students to appreciate the development of knowledge and understanding over time. It also places our participation in this academic discourse in a historical context.

Reading "Mathematics: Its Nature and Purpose" in *Mathematics for Everyone* by Laurie Buxton [2, pp. 208–227], and "The Primacy of Mathematics, or If I Could Do Math, I Would ..." in *Overcoming Math Anxiety* by Sheila Tobais [9, pp. 31–49], students continue their investigation of what mathematics is and arguments why mathematical skills are valuable.

Additional readings in *Overcoming Math Anxiety* [9] and in Buxton's *Math Panic* [3] allow students to detail the symptoms, potential causes, and potential techniques for overcoming math anxiety as these early researchers understood them.

### Willis

Modern psychological studies confirm and build on these understandings. By reading the works of a neurologist and educator and a variety of psychologists, students explore research techniques of additional disciplines and investigate a diversity of descriptions of the issue. This offers opportunities for guest presentations from colleagues across campus.

Judy Willis is an authority in the field of learning-centered brain research and classroom strategies. Her book *Learning to Love Math* [10] contains the appendix, "The Brain Owner's Manual" [10, pp. 165–174 ]. Reading this appendix enables students to understand, on a fundamental level, how the brain processes new input and, most importantly, that initial reactions can be modified. Students learn through the Willis text how boredom and frustration can lead to the same fight, flight, or freeze reaction as danger and how this can close our brains to learning. Willis describes the need for activities that challenge, and thus don't lead to boredom, and that are achievable, and thus don't frustrate [10, p.

20]. Students recognize and understand this necessary balance. Discussions of this reading can help students recognize what they must do to promote their own learning.

A potential schedule of reading assignments for the majority of a semester is displayed in the Appendix on p. 155.

### 16.3.2 Assessments

Student learning is measured through a participation and engagement grade, journal writing, three formal papers, and a final project. Participation in service opportunities and transitioning to college events are also assessed, but these will not be discussed here.

#### Participation and Engagement

A broad evaluation of participation and engagement is assigned weekly. A three-point scale is used to evaluate in-class discussion participation and engagement and in-class quick writes, described below:

| | |
|---|---|
| actively and constructively engaged | 3 pts |
| constructively engaged | 2 pts |
| actively engaged | 1 pt |
| not engaged | 0 pts |

In-class quick writes are writing-to-learn activities. Students write a paragraph or two in five minutes, with the best spelling and grammar they can muster, addressing or summarizing a reading, a discussion, or what they know about a topic. These writing assignments are essential to discussions because students must refresh their memories and organize their thoughts to write. Also, requiring students to write provides structure for very shy students to contribute, they may simply read their paragraph, and encourages the students inclined to ramble to make their point succinctly. Some examples of quick write prompts are listed below.

- Address (an open-ended or exploratory question that reviews previously discussed material or stimulates interest in what is coming)
- Summarize the most important point in the assigned reading
- Summarize the main points of the discussion in your own words (mid-class this can be used to refocus or cool off)
- Express confusion or ask a question
- Address the question: What is the most significant thing you learned today?
- Address the question: What question is uppermost in your mind about today's topic?

Quick writes are collected and read. This daily writing satisfies the college-level writing learning goals and a professor's need-to-know that students are completing the reading and engaging with the material.

#### Journal

Students respond to all readings and discussion with summaries, questions, and arguments in a journal. Students complete this assignment either electronically or in a hand-written paper journal. Students are encouraged to refer to specific points, definitions, or descriptions from the reading or discussion as the content addresses one of the seminar's main questions. The journals are collected and assessed bi-weekly. They are assessed using a three pronged three-point scale. Clarity of writing, continuous effort, and evidence of engaged thinking are each graded as excellent (2 pts), good (1 pt), or no evidence of (0 pts).

Regular feedback enables students to evaluate their improvement, or lack thereof, their increased understanding of our topic, and their participation, even informally, in an academic discussion.

### Formal Papers

Three formal writing assignments, with at least one five-page minimum, are included in the college-level writing expectations for an FYS. The scaffolding of the college-wide writing expectations suggest that formal research papers are not necessary at this level. Thus each of these papers relies solely on the reading assignments.

The first formal writing assignment is a two-page paper addressing the argument for the importance of math literacy made in the Moses and Cobb text [7]. The second formal writing assignment allows the students to choose to write an opposing viewpoint in response to one reading or to write an interview with one of the authors. The last paper is a compare and contrast paper. Students choose two authors and write a five-page paper addressing how each author's work has added to the academic dialog on the subject of math anxiety.

Details of these assignments and their corresponding rubrics are found in the Appendix.

### Final Project

The final assessment is a group project. The students design and present a workshop on math anxiety. This project is very student-driven. It requires students in the class to work as one team to determine, at a minimum:

- who the audience will be
- what the content of the workshop will be
- how the content of the workshop will be presented (including a balance of presentation responsibilities across the students in the class)
- any logistics.

During the last weeks of the course, work on this project is the in-class activity. Specific out-of-class assignments may be designated by the group. The workshop will be presented either to the audience determined by the class, if that audience is available, or to the college math tutors.

## 16.4 Reflections

I have taught six versions of FYS before teaching this appealing topic. The civil rights aspect of this course attracts a wide variety of students beyond potential math majors. The math anxiety aspect attracts potential science majors who recognize their need to address their reluctance to do math. Also, math majors desiring to be more empathetic to their peers enroll. Thus the topic helps a math theme to appeal to incoming students.

The individuals in a class influence the flow of the material. A talkative group may engage with the material in class, but may be inclined to forget details. A less talkative group may engage deeply with the material in writing, but require an artificial conversation starter, such as going around the room to each person in turn for comments on the reading. Regular writing, and prompt feedback on that writing, appears essential for both types.

Reading student journals regularly alerts the professor to the need for clarification. It also deters students from neglecting to interact with the readings regularly. Allowing students to choose to submit paper or electronic journals improves student engagement. Two logistical issues require navigation when paper journals are allowed. The first is illegible handwriting. The second is that during the grading times students do not have access to their (paper) journal to continue responding to the ongoing course work. Requiring electronic submissions addresses both of these issues.

In the first offering of this course students knew very little about the history of slavery, Reconstruction, Jim Crow Laws, and the civil rights movement. Thus the questions inspired by Moses's civil rights work caused us to spend more time on history than on math literacy. For the second iteration of this course, I included a historical reading to address this. However, this provided answers before the students had formed questions. In the next iteration, I plan to assign the history reading only in response to their questions.

Fewer readings, which students found repetitive, and an increased emphasis on exploring specific math concepts and activities for teaching or learning them, as found in the Moses and Cobb text [7] and in the Willis text [10], will diversify the content and also lead to a variety of class activities.

The quality of the interview and the compare and contrast papers varied widely based on the strengths of the individual students. In contrast, the opposition papers were generally very weak: students often reacted to a misunderstanding of the author's work. This suggests a modification of this assignment is necessary.

While the loose structure of the assignment feels risky, the math anxiety workshop project was a very successful assignment. During planning and preparing conversations, students demonstrate what they have learned and how well they have learned it. In-class discussions and preparation allows the professor to observe how much of the content, and empathy, individuals have learned. Additionally, the professor can guide the distribution of the work. Finally, the presentation is excellent experience for the students and can benefit the audience.

In conclusion, without requiring the professor to pretend to be an expert on content outside of mathematics, this successful seminar topic is interdisciplinary. We who love math should regularly be reminded, as one of my students bluntly stated, “math anxiety is real, and it can be overcome. Fear can be overcome, stupidity cannot.” Teaching this course helps us to recognize the need to help students overcome this anxiety. The material helps students understand themselves, their learning, and their classmates.

The author would be more than happy to share more details or discuss this topic.

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# Appendix

## Reading Schedule

*This tentative schedule of reading assignments was handed out on the first day of the course. This allows students to plan their reading around their other coursework.*

| | | |
|---|---|---|
| Week 2 | Tobias (1978) pp. 11–16, 23–43 | Buxton (1984) pp. 208–218 |
| Week 3 | Moses/Cobb (2001) pp. vii–xv, 3-22 | Moses/Cobb (2001) pp. 23–87 |
| Week 4 | | Willis (2010) pp. 165–174 |
| Week 5 | Moses/Cobb (2001) pp. 91–113 | Moses/Cobb (2001) pp. 114–133 |
| Week 6 | Tobias (1978) pp. 44–69 | Tobais/Piercy (2012) pp. 1–4<br>Buxton (1981) pp. 3–36 |
| Week 7 | Buxton (1981) pp. 113–130 or<br>Buxton (1981) pp.175–191 | Beilock/Willingham or<br>Furner/Berman |
| Week 8 | Willis (2010) pp. 1-16 | Willis (2010) pp. 16–32 |
| Week 9 | Willis (2010) pp. 33–46 | Willis (2010) pp. 47–68 |
| Week 10 | Willis (2010) pp. 69–93 | Buxton (1981) pp. 113–130 |
| Week 11 | Buxton (1981) pp. 147–165 | |

## Importance of Math Literacy Paper Assignment

*This is one of three formal writing assignments each of which satisfy college-level writing requirements. It is assigned as we finish reading and discussing the Moses/Cobb text. Students are required to recognize the implicit argument concerning the need for math literacy.*

In *radical equations: Math Literacy and Civil Rights*, Moses and Cobb make an argument for the importance of math literacy. They do this both in words and by the narrative of Moses's life. Write a two-page paper summarizing and evaluating this argument. Use direct quotes from the text and be sure to properly cite them. Be sure to address:

- what is the most convincing point, said or described?
- how do the authors use Moses's work to make this argument?
- is the argument convincing? why or why not?

## Importance of Math Literacy Paper Rubric

*This is the document that was used to evaluate the final version of the math literacy paper. Students received this with the feedback on their draft version.*

Three major themes explicit in text:

- math literacy needs to be a civil right; it is necessary to be a full citizen; it is a gate keeper to citizenship, higher education, certain careers and opportunities
- math literacy is a tool of liberation; it can provide hope and opportunity to the younger generation
- math literacy will be required in the workplace just as reading literacy is now

| What | Outstanding (100%) | Excellent (95%) | Room for Improvement (75%) | Unsatisfactory (65%) |
|---|---|---|---|---|
| **Summary 65 pts** | Includes all three major themes and mentions others or highlights one major theme, exploring it thoroughly. | Mentions two major themes and explores one major theme well. | Discusses other strong arguments without mentioning any of the major themes. | Focuses on a minor issue; misses major themes and even strong arguments. |
| **Critique 25 pts** | Displays understanding of the argument; addresses the strengths and weaknesses of the argument; includes responses and reactions to the argument. | Displays understanding of the argument; addresses some of the strengths or the weaknesses of the argument; includes responses and reaction to the argument. | Addresses some strengths and/or weaknesses of the argument; includes responses and reaction to the argument. | Barely addresses strengths and/or weaknesses of the argument; includes responses or reaction to the argument. |
| **Organization 5 pts** | Logical sequence of topics; clear summary separated from or incorporated with critique. | Logical sequence of topics; reading flows smoothly. | Critique and summary are blurred together; unclear writing. | Unclear writing. |
| **Grammar, Mechanics, Neatness 5 pts** | Complete sentences; a variety of sentence structures; 0–2 spelling errors; 0–2 grammar/formatting errors. | Complete sentences; a variety of sentence structures; 3–5 spelling errors; 3–5 grammar/formatting errors. | 1–2 incomplete sentences; run-on sentences; 6–8 spelling errors; 6–8 grammar/formatting errors. | More than 2 incomplete sentences; more than 2 run-on sentences; over 9 spelling errors; over 9 grammar/ formatting errors. |

## Short Formal Writing Assignment

*This is one of three formal writing assignments each of which satisfy college-level writing requirements. It is assigned at the end of week 7. Students receive the short description and the documents used to evaluate the paper. They have two weeks to complete the paper.*

Complete one of the following by writing a formal two-page paper.

**Interview**

- Choose one reading and then write an imagined interview with the author.
- Write a sentence or two describing the main point(s) of the reading.
- Based on the reading and the class discussion, pose three to five questions to the author.
- Questions should be specific, address the topics we are interested in, and either be answered in the reading or be reasonably extrapolated from the reading.
- Using the reading as a beginning point, write a response to your questions as you think the author would respond.

**Rubric for Interview**

| Percent | Excellent (100%) | Good (85%) | Fair (70%) | Poor (65%) |
|---|---|---|---|---|
| **Reading Choice 5 pts** | Appropriate and interesting. | Appropriate or interesting. | Insufficient to inspire questions. | Insufficient and uninteresting. |
| **Content 75 pts** | Very detailed questions about the reading, 4 or 5. Questions posed address interesting, appropriate issues. Answers demonstrate an excellent understanding of the author's point of view and the content. | Clear questions about the reading, 3 or 4 or 5. Questions posed address interesting or appropriate issues. Answers demonstrate a good understanding of the author's point of view or the content. | Questions are vague or uninteresting, 3 or 4. Few specifics are provided in the answers. Answers indicate only a vague understanding of the content. | Does not address any of the elements of the assignment. No direct questions about the content of the reading. Answers indicate a lack of understand the content. |
| **Accuracy 10 pts** | Answers provide a scholarly voice. Could have been written by the author. | Answers could have been written by the author. | Unlikely that the author would have answered in this tone or with these words. | Unlikely that the author would have answered in this tone and with these words. |
| **Grammar, Mechanics, and Neatness 10 pts** | The interview questions and responses contain very few errors in grammar, punctuation, or spelling, or no errors at all that distract the reader from the content. | The interview questions and responses contain a few minor errors in grammar, punctuation, or spelling that distract the reader from the content. | The interview questions and responses contain several errors in grammar, punctuation, or spelling that distract the reader from the content. | The interview questions contain many errors in grammar, punctuation, or spelling that greatly distract the reader from the content. |

**Opposing viewpoint**

- Choose one reading, write a paper expressing an opposition viewpoint
- Write a minimum of one page summarizing and supporting the main points the author makes in the reading.
- Then write a minimum of one page expressing doubt, questioning, and/or posing an alternative point of view.

**Rubric for Opposing Viewpoints Paper**

| **Percent** | **Excellent (100%)** | **Good (85%)** | **Fair (70%)** | **Poor (65%)** |
|---|---|---|---|---|
| **Reading Choice 5 pts** | Appropriate and interesting. | Appropriate or interesting. | Insufficient to inspire questions. | Insufficient and uninteresting. |
| **Content 75 pts** | Very detailed, clear summary of the main points of the reading. Demonstrates an excellent understanding of the author's main points. Doubts and alternative view point address interesting, appropriate issues. | Very detailed summary of the main points of the reading. Demonstrates a good understanding of the author's main points. Doubts and alternative view point address interesting or appropriate issues. | Summary is vague and misses key points of reading. An understanding of some of the author's key points is evident. Doubts and alternative view point weak or uninteresting. | Summary is vague and misleading; incorrect. A lack of understanding is demonstrated. Doubts and alternative view point is missing or unclear. |
| **Accuracy 10 pts** | Submission provides a scholarly voice. Summary is strongly, convincingly supported. Alternative view is strongly, convincingly supported. | Submission provides a clear voice. Summary is convincingly supported. Alternative view is convincingly supported. | Submission provides a clear voice. Summary is not supported. Alternative view is supported by opinion only. | Submission is sloppy or personal. No support for the summary. Informal opinion is all that is offered. |
| **Grammar, Mechanics, Neatness 10 pts** | Submission contains very few errors in grammar, punctuation, or spelling, or no errors at all that distract the reader from the content. | Submission contains a few minor errors in grammar, punctuation, or spelling that distract the reader from the content. | Submission contains several errors in grammar, punctuation, or spelling that distract the reader from the content. | Submission contains several errors in grammar, punctuation, or spelling that greatly distract the reader from the content. |

## Long Formal Writing Assignment: Compare and Contrast Paper

*This is the last of three formal writing assignments each of which satisfy college-level writing requirements. It is assigned at the end of week 10. Students receive the description and the document used to evaluate the papers.*

**Background**

We have read and discussed substantial portions of Laurie Buxton's *Math Panic*, a scholarly report of an educational study, Sheila Tobias's *Overcoming Math Anxiety*, a book written for the general public, and Judy Willis's *Learning to Love Math*, a book written for middle school teachers. Each author contributes to the academic discourse on the difficulties many students face when studying math. Each author addresses some of our questions:

- What mathematics is important?
- Why is mathematics important?
- What are the symptoms of math anxiety?
- What are potential causes of math anxiety?
- What are some potential techniques for mitigating math anxiety?

Choose (a minimum of) two of these authors and write a five-page compare-contrast essay addressing a theme relevant to our discussions. Your audience will be your classmates and your instructor.

You have engaged in this academic discourse in class. Often in a discussion you need to ask your colleagues to re-explain their point of view. You will need to do this by rereading parts of our readings and perhaps reading beyond what we have read. (Books are on reserve in the library.)

By writing this paper you will be continuing the scholarly academic discourse about math anxiety. Thus this paper should be formally written, with accurate grammar and spelling. Avoid first-person references and opinions. You should include, correctly documented, specific examples and quotations.

**Purpose for this formal writing**

Our syllabus states that student learning outcomes will include:

- Enhance critical thinking skills.
- Demonstrate knowledge of causes and symptoms of math anxiety.
- Demonstrate knowledge of techniques to avoid, reduce, or manage math anxiety.
- Recognize studying as participation in intellectual discourse.

This paper should address each of these outcomes by requiring students to summarize the conclusions of two of our authors, synthesize other readings and our discussions in order to compare and contrast the authors' conclusions. A compare and contrast essay requires understanding of both texts, the point of view of the authors and the varied audiences.

**Format**

- Paper length: minimum of five pages
- Margins should be no wider than 1 in. top and bottom; 0.75 in. right and left; moderate in Word
- Name in the header, justified right; the page number in the footer, justified right

## Long Formal Writing Rubric

*This is the document used to evaluate the final version of the compare-contrast paper. It is available to students when the assignment is given.*

| | **Very Good (5 pts)** | **Good (4 pts)** | **Proficient (3 pts)** | **Needs Work (2–0 pts)** |
|---|---|---|---|---|
| **Draft I** | Strong thesis statement clear. Paper follows one of the two compare-contrast structures. Well written, logical order. A complete draft; five pages, format guidelines followed. | Thesis statement clear. Paper follows one of the two compare-contrast structures. Well written, logical order. A near complete draft, format guidelines followed. | Thesis statement unclear. Paper mixes the two compare-contrast structures. Writing needs work; outline and order needed. A near complete draft. | No clear thesis statement. Paper mixes the two compare-contrast structures. Incomplete draft; barely begun. |
| **Participation in Peer Review** | Thesis identified correctly. Several well-chosen important points starred. Correct structure identified. Thoughtful feedback outlined. | Thesis identified. Several important points starred. structure identified. Thoughtful feedback. | Thesis identified incorrectly. Some important points starred. Structure incorrectly identified. Thoughtful feedback. | Thesis identified incorrectly. Few important points starred structure not identified. Not much feedback. |

**Long Formal Writing Rubric (cont.)**

| | **Very Good (5 pts)** | **Good (4 pts)** | **Proficient (3 pts)** | **Needs Work (2–0 pts)** |
|---|---|---|---|---|
| **Organization & Structure** | Paper breaks the information into whole-to-whole similarities to differences or point-by-point structure. It follows a consistent order when discussing the comparison. | Paper breaks the information into whole-to-whole similarities to differences or point-by-point structure but does not follow a consistent order when discussing the comparison. | Paper breaks the information into whole-to-whole, similarities-to-differences, or point-to-point structure, but some information is in the wrong section. Some details are not in logical or expected order, and this distracts the reader. | Structure is neither whole-to-whole similarities-to-differences, or point-to-point. Many details are not in logical or expected order. There is little sense that the writing is organized. |
| **Content** | Writing shows understanding throughout, noting three similarities and two differences. Strong interesting claim. | Writing shows concrete understanding of similarities and differences. Interesting claim. | Writing shows some understanding of similarities and differences. Clear claim, but neither strong or interesting. | Writing does not show understanding of similarities and differences. Content shows several inaccuracies and confusion. Thesis is unclear. |
| **Purpose and Support Details** | Paper compares and contrasts items clearly. Paper points to specific statements to illustrate the comparison. Paper includes only the information relevant to comparison or contrasts. | Paper compares and contrasts items clearly, but the supporting information is general, not specific. Paper includes only the information relevant to the comparisons or contrasts. | Paper compares and contrasts items clearly but the supporting information is incomplete. Paper may include information that is not relevant to the comparison or contrasts. | There is no supporting information or the supporting information is incorrect or incomplete. |
| **Transition** | Paper uses compare-contrast transition words to show relationships between ideas or authors. Paper uses a variety of sentence structures and transitions. | Paper moves from one idea or author to the next, but there is little variety. Paper uses comparison and contrast transition words to show the relationship between ideas or author. | Some transitions work well; but connections between other ideas or authors are fuzzy. | The transitions between ideas or authors are unclear or nonexistent. |

**Long Formal Writing Rubric (cont.)**

| | **Very Good (5 pts)** | **Good (4 pts)** | **Proficient (3 pts)** | **Needs Work (2–0 pts)** |
|---|---|---|---|---|
| **Format** | Moderate margins; name, justified right, in header; page number in footer, justified right. | 5 pages and 2 of moderate margins; name, justified right, in header; page number in footer, justified right. | 5 pages and 2 of moderate margins; name in header; page number in footer. | 5 pages and 1 of moderate margins; name in header; page number in footer |
| **Flow of Writing** | Paper shows thoughtful, logical, and clear layout of ideas. | Writing is logically and clearly organized. | Writing is somewhat organized. | Writing is disorganized and hard to follow. |
| **Correct Grammar and Spelling** | Paper has been proofread and has no grammatical errors or spelling mistakes. | Paper has minor grammatical errors and few spelling mistakes that do not take away from reading comprehension. | Paper has several grammar and spelling mistakes that may somewhat interfere with comprehension. | Paper has numerous mistakes in grammar and spelling, making it difficult to read. |
| **Bibliography** | All referenced works are listed and are accurately formatted. | Some of the works referenced are accurately formatted. | Few of the works referenced are listed or they are inaccurately formatted. | Missing bibliography. |

# Part V

# Mathematics in Pop Culture and History

What role does mathematics play in our personal lives? in our society? in our history? How have scientists and mathematicians been portrayed in recent books and films? How have contributions from diverse groups shaped the development of mathematics and related fields? These interesting questions are at the heart of the first-year seminars described in this section.

# 17

# Mathematical Identities: Diverging from the Stereotypes

Jennifer B. Schaefer

**Abstract** I discuss my experience at Dickinson College teaching a first-year seminar investigating the portrayals of mathematicians in popular culture. I focus on how I combined the content of this topic with writing instruction and writing assignments to address the college-level learning goals of student development of critical analysis, writing, and information literacy skills.

## 17.1 Background and Context

Dickinson College is a highly selective liberal arts college in central Pennsylvania with approximately 2,400 students. First-year seminar (FYS), the first tier of the Writing Program at Dickinson, is designed to help students make the transition to college-level academic culture. All students take an FYS their first semester at Dickinson. Each one-credit course is capped at 16 students and is usually scheduled in blocks of 50 minutes, three times a week for the entirety of the 15-week semester. Students select six seminars they are interested in from a list of approximately 40. Dickinson then assigns each student to a seminar on their list, while attempting to maintain course capacities and to balance gender and the number of international students. There are no prerequisites which means students with a wide variety of backgrounds and interests can register for this course. Of the 16 students who registered for the first iteration of my class, eight were interested in STEM related fields, five in the social sciences, and three in the humanities. Faculty from across the college are required to teach an FYS each sabbatical cycle, approximately every five to six years. Faculty are free to choose their own topic but the student development of critical analysis, writing, and information literacy skills are the college-level learning goals of every seminar.

## 17.2 Mathematical Theme

The theme of my FYS, entitled *Mathematical Identities: Diverging form the Stereotypes*, is portrayals of mathematicians in popular culture. In particular, my students are tasked with analyzing how portrayals of mathematicians involving gender, mental health, race, and sexual orientation affect who is attracted to, and often welcomed into, the field of mathematics. We often think our discipline is unaffected by social issues. In fact, David Hilbert once said, “Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country” [6, p. 136]. However when thinking of mathematics in terms of the culture of the mathematics community or its place in society, nothing could be further from the truth. The stereotypes surrounding mathematics and mathematicians portrayed in popular culture, and consequently deeply ingrained in society, have very real implications. For example, society and the media have linked mathematical ability with innate “brilliance”—either one is born a “math person” or they are not. When a student internalizes this culture belief and encounters a mathematics problem they cannot easily solve, the student may immediately assume they are not a "math person," causing them to avoid mathematics in the future. In addition, mathematicians are often portrayed as older, white men with poor social skills. Again, these stereotypes may inadvertently affect the number of individuals from underrepresented groups who continue in mathematics because they may feel they do not fit in with the community. Other implications include explicit or implicit biases towards individuals who do not fit the stereotypes, stereotype threat in mathematics, impacts of the model minority stereotype, and more broadly, linking brilliance and mental health and connecting mental health with violence. By the end of the course, I want my students to challenge these stereotypes, to recognize their implications, and to begin to see how stereotypes affect many facets of our lives. It is important to note that teaching mathematical content is not a focus of this course.

## 17.3 Course Structure

The course theme is split roughly into four topics associated with portrayals of mathematicians in popular culture: mental health, gender/sexual orientation, race, and implications of stereotypes of mathematicians related to these areas. These topics are incorporated into almost all course readings and films, class discussions, and writing assignments. The main texts and films used to introduce the topics are *The Housekeeper and the Professor* [13], *A Beautiful Mind* [8], *Arcadia* [17], *Proof* [2], *The Imitation Game* [18], *Top Secret Rosies: The Female "Computers" of WWII* [5], *Mathematics in Popular Culture* [15], *Stand and Deliver* [12], and *Beyond Banneker: Black Mathematicians and the Paths to Excellence* [19]. The class readings and course discussions are supplemented with research articles (see [3], [4], [7], [9], [10], [16]) and presentations by faculty experts.

While investigating stereotypes of mathematicians in popular culture is a worthy goal in and of itself, the principal requirement of the course is to address the college-level learning goals of student development of critical analysis, writing, and information literacy skills. In the subsections below, I describe how I utilize a variety of resources and assignments to teach these skills to students while helping them develop a formal writing process.

### 17.3.1 Teaching Critical Analysis, Writing, and Information Literacy Skills

Dickinson's FYS Program offers a variety of resources to help faculty teach and the students learn critical analysis, writing, and information literacy skills. These resources include the text *Writing Analytically* [14], library liaisons, and student writing associates.

#### *Writing Analytically*

*Writing Analytically*, a text adopted by the FYS Program, is "designed to be used in first-year writing courses or seminars, as well as more advanced writing-intensive courses in a variety of subject areas" [14, p. xxiv]. The text provides readings and assignments with the goal of moving students away from passive summary, mere personal reflection, and the formulaic five-paragraph essay and towards "the tools they need in order to engage in the analytical habits of mind that will be expected of them in their courses and in the world they encounter after graduation" [14, p. xxiii]. Using the *Writing Analytically* writing syllabus developed by Dickinson's Writing Program Director as a guide, I assign daily reading assignments that address content and skill. Some of the readings I assign cover topics such as analyzing evidence, "asking 'so what'," personal response, compare/contrast, reasoning from evidence and claims, making a thesis evolve, and establishing focused research questions. Simultaneously, I assign short writing assignments to be completed in journals so that students can put the skills they read about into practice (see the Appendix, p. 171).

#### Writing Associates

Writing associates are trained, undergraduate peer writing tutors. Faculty can request a writing associate to assist them with teaching writing courses. My writing associate leads short in-class lessons on elements of the writing process, including developing a thesis statement and effective strategies for peer review. He facilitates peer-review exercises for each writing assignment and holds office hours to assist my students with writing and to clarify my expectations and feedback. Because he is a peer tutor at the Writing Center, my students often set up one-on-one meetings with him as they work on drafts of their papers.

### Library Liaisons

Each FYS is assigned a library liaison. Their role is to work with faculty to incorporate information literacy into their course activities and assignments and to help locate library materials related to their course topic. Several times during the semester students participate in library learning modules led by our library liaison. These modules introduce various aspects of the Dickinson College library system and the research process, including finding and accessing library resources, evaluating resources and research as a process, and establishing focused research questions and making a thesis evolve. She is also available for small-group or one-on-one meetings with students outside of class. The information literacy assignments from the learning modules, together with class participation, count as 10% of the final grade.

## 17.3.2 Applying Critical Analysis, Writing, and Information Literacy Skills

To help students put their newly acquired analysis, writing, and information literacy skills into practice, I assign four writing assignments that constitute an additional 80% of the final grade: an analysis paper (15%), a comparative analysis paper (20%), an exploratory essay (20%), and a final research synthesis paper (25%). The remaining 10% of the final grade is determined by journal assignments given on a regular basis, either in response to reading assignments from *Writing Analytically* or short reflective writing prompts assigned in class.

When teaching introductory writing, it is important to ensure that each assignment is at the appropriate level and focuses on addressing a limited number of skills, with each subsequent assignment building on the skills previously assessed. This idea, outlined in *How Learning Works: Seven Research-Based Principles for Smart Teaching*, is known as scaffolding and refers to

> the process by which instructors give students instructional supports early in their learning and then gradually removing the supports as students develop greater mastery and sophistication. One way to apply scaffolding to a more complex assignment is to ask students to first practice working on discrete phases of the task and, later, ask students to practice integrating them [1, pp. 146–147].

In the following paragraphs, I describe my writing assignments and highlight the skills addressed. Each writing assignment is assessed using the FYS Writing Rubric, developed by the Dickinson College Writing Program, which I modify to align with the expectations of each assignment.

### Writing Assignment #1, Analysis Paper

The first assignment asks each student to write a short (three to four pages) paper that takes a critical and well-informed position and argues a particular interpretation of *The Housekeeper and the Professor* reached through analysis. More specifically, the students are asked to analyze the following question: “How is the Professor’s view of mathematics reflected in his relationship with the Housekeeper and/or Root?” This assignment requires students to think more deeply about a specific theme in *The Housekeeper and the Professor* and for them to demonstrate their ability to develop a clear interpretation of a text based on evidence from the text itself. The two overarching skills addressed in this assignment are close reading and using evidence to guide analysis. As the prompt asks the students to respond to a question regarding one of the course texts, no additional sources or research are required.

### Writing Assignment #2, Comparative Analysis Paper

Prior to the second assignment, the class reads two comparative analysis pieces in *Mathematics in Popular Culture*: “The Mathematical Misanthrope and Popular Culture” [15, pp. 198–218] that analyzes the varying ways the media portrayed mathematicians Ted Kaczynski and John Nash and “Smart Girls: The Uncanny Daughters of *Arcadia* and *Proof*” [15, pp. 172–186] that focuses on the similarities and differences of *Arcadia*’s Thomasina and *Proof*’s Catherine. The second assignment calls on students to select two of the readings or films from class and, in terms of portrayals of mathematicians in these works, to identify two to three of the most revealing similarities and two to three of the most revealing differences. Using these similarities and differences, each student is asked to write a paper (four to five pages) that analyzes the following questions: “Where do the portrayals of smart mathematical women overlap with the stereotypes of the mathematical misanthrope? Where do they differ? So what?” The two additional overarching skills addressed in this assignment are identifying difference within similarity and making compare and contrast arguments more analytical. As with the first assignment, no outside sources or research are required.

Throughout the semester, our class analyzes portrayals of mathematicians in popular culture and considers the following questions: “Are these portrayals accurate? How do these portrayals affect not only the mathematical community but society as a whole?” The final two writing assignments ask the students to identify an implication of the stereotypes of mathematicians that is of interest to them and to pose a significant and interesting problem or question related to this implication that requires both library research and their own analytical skills to answer.

#### Writing Assignment #3, Exploratory Essay

The first part of the final research project is a formal exploratory essay[1] (five to six pages), a first-person, thesis-seeking, chronologically-organized account of the student’s research process as they explore possible solutions to their proposed problem or question. In their essay, the students need to describe their problem or question, how and why they became interested in it, and why it is problematic for them. They are asked to contemplate their question and describe the extensive research process they undertook to further understand their problem. As they wrestle with their problem’s complexity, they are to narrate the evolving process of their thinking, including both external details (people they talked to, what sources they read, how they found these sources, etc.) and internal details (what were they thinking about, how did their ideas evolve, etc.) This assignment stresses the idea that research is a process and shows how new information can lead one to reformulate their thesis through expansion, narrowing, and/or shifting of focus. This is the first writing assignment that requires students to locate outside resources using the information literacy skills developed with help from our library liaison.

#### Writing Assignment #4, Research Synthesis Paper

For their final essay (six to eight pages), the students are to write a thesis-directed response to the question or problem they identified, by bringing together multiple sources and synthesizing multiple perspectives on their topic. Their essay is to be both informational and analytical, supported by research. The students are to present the results of their library research and their own analysis from their exploratory essay, and then go further by analyzing and interpreting the information to explain its implications and significance while make interesting connections between ideas. This final assignment requires students to combine all of the critical analysis, writing, and information literacy skills they have learned in this course.

### 17.3.3 Writing as a Process

In addition to teaching the above writing skills, I want my students to develop a writing process that will serve them well, both at Dickinson and as a future professional. Thus for each paper I implement a structured writing process that includes prewriting activities, drafting, giving and receiving feedback, reflection on feedback, and revision. To analyze their source material and begin developing a thesis, students complete prewriting activities modeled after the *Writing Analytically* readings and assignments. Once their first draft is complete, students participate in a peer-review exercise led by the writing associate and/or me. Each student reads two of their peers’ papers and completes a peer-response worksheet[2] for each paper they review. This worksheet asks the reviewer to focus on specific elements of their classmate’s writing and serves to model the types of feedback writers find helpful (see the Appendix, p. 171).

After receiving feedback from their peers, each student is faced with the difficult question that all writers face: what do I with this feedback? To ensure that students thoughtfully consider the feedback they receive and are intentional about how they use it, I require them to complete a revision memo. In this memo students summarize the feedback they received on their first draft and describe what feedback was helpful (see the Appendix, p. 172). They explain the changes they made moving from their first draft to the final draft and which changes were prompted by the feedback from their writing group and which were made using their own judgment. They are also asked what elements they would work on if they had the time to revise their paper again. This last question is included to emphasize the fact

[1] See the Purdue Online Writing Lab (`owl.english.purdue.edu/`), a wonderful online source for writing resources and instructional materials for faculty and students, for a more detailed description and organization of exploratory papers.

[2] I got the great idea to incorporate peer-response worksheets and a revision memo from Catherine Craft-Fairchild, Professor of English, of the University of St. Thomas.

that two drafts is almost never sufficient. They are required to submit their prewriting activities (with evidence that they used one or more of the analysis techniques from *Writing Analytically*), their first draft with both peer-response worksheets, and their revision memo with their final draft.

## 17.4 Reflections

I have taught two FYSs, but this is the first time I used the topic of portrayals of mathematicians in popular culture. When I was first charged with teaching an FYS, I was advised to be cautious about selecting a math-related theme. Several colleagues had done so in the past with less than ideal outcomes. Given this advice and my family's agricultural background, I decided to teach my first FYS on sustainable agriculture and the small family farm. This seminar was successful in terms of student interest and adequately achieved college-level learning goals. However, teaching academic writing through content outside of my area of expertise was extremely difficult. As a woman in mathematics, I have encountered many challenges, from having my mathematical abilities questioned to being told repeatedly, by a variety of people, that I "don't look like a mathematician." Through these experiences, I have developed a passion for improving diversity and equity in mathematics. The topic of portrayals of mathematicians in popular culture provided me with a course theme aligned more closely with my academic and personal expertise. In addition, it lent itself perfectly to the discussion-based, writing-intensive nature required of an FYS and provided content to address the college-level learning goals.

Conversations about gender, mental health, race, or sexual orientation are not easy to engage in. In fact, they are difficult and uncomfortable, and often very personal. As the facilitator of such discussions, it is important to create an open and comfortable classroom environment conducive to discussion and self reflection. I found reading *How Learning Works: Seven Research-Based Principles for Smart Teaching* [1] and attending several humanities or social science colleagues' classes beneficial resources in this regard. Being patient and nonjudgmental as I persistently ask students to grapple with new ideas was essential because I was more than likely challenging some of my students' deeply held beliefs. Furthermore, displaying my own vulnerability was especially valuable as it conveyed to the students that I was still learning and struggling with these complex issues as well.

My students were more open than I had expected to questioning previously held perspectives and considering new ideas related to gender, mental health, race, and sexual orientation. I think this may be in part due to the fact that we discussed these course topics in a mathematical context. Through class discussions and writing assignments, I saw my students begin to challenge the stereotypes of mathematicians portrayed in popular culture and to recognize their implications. In addition, the following quote from one of my students showed that their learning had extended beyond the mathematics community:

> Mathematical Identities was formative in helping me develop my personal identity as a mathematician and motivated me to examine how intersections of gender, race and personality impact people's experiences in all STEM fields. Starting my experience at Dickinson with such an interdisciplinary course encouraged me to find connections in my math and computer science courses and the material in my social science courses relating to bias and discrimination. Whether or not I continue directly in mathematics after college, I know that I will carry the lessons from the FYS regarding the importance of diverse representation in breaking barriers and unlocking individuals' full potential.

Given my identity as a white, heterosexual woman who has had periods of anxiety, I was most comfortable leading conversations surrounding gender and mental health but felt less equipped leading conversations involving race and sexual orientation. This, together with the limited portrayals of racially and sexually diverse mathematical persons in mainstream popular culture, meant that the race and sexual orientation units were not as strong. I have worked to improve my understanding of these issues for future iterations through education and conversation. I have also continued to search for new resources to add to the course syllabus. One popular film I plan to add is *Hidden Figures* [11]. This recently released film presents many issues intersecting race, gender, and mathematical ability that align perfectly with the course's theme.

In my first FYS, the mini-modules led by my writing associate and by my library liaison were the primary modes of writing instruction. It was no surprise then that my students struggled throughout the semester with basic analysis and writing skills not covered in these modules. Incorporating readings, exercises, and assignments from *Writing*

*Analytically* greatly enhanced my writing instruction, which in turn improved my students' analysis skills and writing. The readings and exercises from *Writing Analytically* also helped me facilitate more effective and engaging class discussions.

Clearer prompts and better scaffolding of writing assignments produced stronger student papers than in my previous FYS. Assigning an exploratory essay versus a more common annotated bibliography paper and requiring my students to complete pre-writing activities, peer-response worksheets, and revisions memos for every assignment emphasized that writing is a process. Unfortunately, the prompt for the final writing assignment produced mixed results. I wanted to give students the freedom to pick an implication that was of interest to them, pose a significant question, and use their analytical skills to answer that question, but this flexibility meant I received unfocused, sometimes off topic, research papers. In the future, I will provide a more well-defined prompt to clarify the expectations of this final assignment and restrict the number of skills assessed.

I was able to provide more instructive feedback on my students' writing by adapting the FYS Writing Rubric for each writing assignment. However given my limited experience in assessing writing assignments, I felt unsure if I was transferring my assessments to assignment grades appropriately. I subsequently volunteered for a Writing Program assessment project that allowed me to gain more experience using the FYS Writing Rubric and to compare my evaluations of FYS papers with the assessments of other more experienced instructors. I highly recommend such an experience to mathematics faculty looking to improve their grading of writing assignments.

## 17.5 Bibliography


[1] Susan A. Ambrose, et al. *How Learning Works: Seven Research-Based Principles for Smart Teaching*, Jossey-Bass, San Francisco, 2010.

[2] David Auburn, *Proof*, Faber and Faber, New York, 2001.

[3] Andrei Cimpian and Sarah-Jane Leslie, "Response to Comment on 'Expectations of Brilliance Underlie Gender Distributions across Academic Disciplines'," *Science*, 349:6246 (2015) 391-c.

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# Appendix

## Sample Journal Prompts

*The following is a collection of journal prompts that students are asked to respond to in their journals. These short writing assignments allow students to put the skills they read about in Writing Analytically into practice.*

- After reading *Writing Analytically*, Chapter 1, "Fourteen Short Takes on Writing and the Writing Process" and "Some Crucial Differences between High School and College Writing" by Joseph M. Williams and Lawrence McEnerney, what do you think you will need to do differently in college than you did in high school when it comes to reading, researching, and writing?
- After reading *Writing Analytically*,"Notice and Focus" (pp. 24–26) choose six key sentences from the reading—two strange, two interesting, and two revealing—and write them in your journal. Be sure to label them.
- After reading *Writing Analytically*, "Thesis Statements" (pp. 326–327) and "Analyzing Evidence" (pp. 207–213) choose three interesting ideas or themes you would like to analyze from *The Housekeeper and the Professor* and briefly describe why you are interested in these ideas or themes.
- Perform the analysis technique 10 on 1 on an example from *The Housekeeper and the Professor* that is applicable to your topic for Analysis Paper #1. I suggest rereading pages 213–215 in *Writing Analytically*, as it has a good example of 10 on 1. You should not do this writing assignment in your journal because you can turn it in as one of your pre-writing activities for Analysis Paper #1.
- Read *Mathematics in Popular Culture:* "Smart Girls: The Uncanny Daughters of Arcadia and Proof." Write a one paragraph personal response to today's reading. (Refer to *Writing Analytically* "Personal Response" (pp. 153–156) before you begin.)
- Choose a brief (1–3 sentences) passage from the reading and practice Paraphrase x 3. Remember to complete steps 1–4 on page 36 of *Writing Analytically*.

## Peer-Response Worksheet

*After a student reads a peer's paper, they are required to complete the following peer-response worksheet. This worksheet asks the reviewer to focus on specific elements of their classmate's writing and serves to model the types of feedback writers find helpful.*

**Peer-Response Worksheet**
**The author is responsible for handing in this sheet with their final draft!**

Author's name: ______________________________________________________________

Writing group member's name: ________________________________________________

What is the writer's **thesis** or main/controlling idea? Try to restate the thesis *in your own words*. If the thesis is not immediately apparent, what appears to stand out as the central focus, or best idea, after you have read the entire essay?

*Does the thesis fulfill the requirements of the assignment?* Y N

*Does the thesis offer a sufficiently unified, narrow, and specific focus for the whole essay?* Y N

If you answered "No" to either of the above, can you help the writer develop a better thesis from material in other parts of their paper?

**Originality:** Did the writer tell you something you didn't already know just by reading the class texts?
Y N

(An effective essay "develops an authentic, fresh insight that challenges the reader's thinking. The paper shows a complex, curious mind at work. Caution ... 'fresh' should not be confused with 'startling'," Dr. Michael Mikolajczak, Professor Emeritus of English, University of St. Thomas.)

**Coherence:** Do all the paragraphs of the essay relate to the thesis and to each other? Y N
If not, is there any way to connect the material that seems disconnected?

**Organization:** Does the structure of the essay make sense—was it smooth and easy to follow? Were points made in a logical order, and made only once rather than repeated? Y N

**Textual evidence:** Did the writer cite the class texts fully enough so that you could readily see what they based their observations upon? Y N

Does the writer make clear the relevance of the passage(s) chosen to the point being made? Y N

Please provide the writer with two additional pieces of textual evidence that would support their point that they didn't think of.

1.
2.

What has the writer done a particularly good job on?

What should be the writer's **top priorities** as they revise? Please be as specific as possible.

Note: Comments on the worksheet should not focus on grammar. As you read the drafts, if you notice errors in punctuation, grammar, or spelling, you may wish to mark them on the draft so that the writer can consider them later, but do not spend time here addressing editorial issues—doing this could leave a writer feeling that you are not interested in the content of what they have to say!

## Revision Memo

*To ensure that students thoughtfully consider the feedback they receive from their peers and are intentional about how they use it, I require them to complete the following revision memo. This memo asks students to summarize the feedback they received on their draft and describe what feedback was helpful.*

**Revision Memo** (Please attach a revision memo to your final draft.)

Name: ____________________________________________________________________________

Essay title: ________________________________________________________________________

1. Summarize the comments and suggestions your peers made about your first draft. Was the feedback helpful? What suggestions do you have for your team that might improve their feedback for your paper? (e.g., it would have been helpful if they had done the following ... )
2. What did you change in moving from the first draft to the final draft? Which changes were prompted by the feedback from your writing group? Which were ones you made using your own judgment?
3. If you had the time to revise your paper again, what elements would you work on?
4. What do you think the strongest aspect of your writing is in this paper? What insights about your writing or process of writing resulted from working on this assignment?

# 18

# Mathematics and the Pursuit of Happiness

Meghan Sleezer

**Abstract** This first-year seminar course combines mathematics and the pursuit of happiness by exploring various mathematical concepts and virtues for living "the good life" by way of Sunil Singh's *Pi of Life: The Hidden Happiness of Mathematics*. Class activities include reading discussions, problem solving, interactive lectures, and videos. The final project brings students together to explore mathematical themes and virtues for living "the good life" in popular American films.

## 18.1 Background and Context

I teach at Concordia University Chicago, a Christian liberal arts university with about 1,000 undergraduate students. Our first-year experience program is housed in the College of Arts and Sciences and is required for all undergraduate students at the university. The program is titled "The Noetic Experience," meaning the transformative process of engaging in the Life of the Mind and pursuit of the Good, the True, and the Beautiful. This theme is explored through two courses over consecutive semesters: first, through a Humanities (HUM) lens in the fall, and then through a Social and Behavioral Science (SBS) lens in the spring.

In the fall semester, the HUM courses are taught in lockstep and focus on the theme "The Good Life," an interdisciplinary exploration of the fine arts, philosophy, and culture in their historical and ethical context. The course partners with a retention course called College Success and Vocation (COL). In the spring, each SBS instructor chooses a special topic from their specific discipline and the corresponding course content. Currently there are 15 instructors each offering a different topic from a variety of fields, and I am the only professor to offer a section focused on mathematics. Students are allowed to enroll in the topic of their choice as space allows, but each section has an enrollment cap of 24 students. There are no mathematics prerequisites for the course and so students come with varied mathematics backgrounds and from all sorts of academic disciplines.

The mathematics section meets the college-level learning goals of understanding the ways in which logic, ethics, and evidence are essential to quantiative and qualitative research, and creating and presenting effective written and oral arguments based on sound logic and quantitative/qualitative, empirical evidence. There is a required four-page final paper in each SBS section loosely based on "The American Dream," mirroring the same final paper requirements of HUM. The 16-week courses are three credit-hours each, and my specific course is offered Mondays, Wednesdays, and Fridays for 50 minutes each class meeting. The college grants Noetics professors an additional (overload) credit to teach the course as a way of showing appreciation for the extra preparatory work and grading involved, and to entice and retain capable professors to teach the course outside of their typical load.

## 18.2 Mathematical Theme

The course explores mathematics in media, politics, and nature, along with virtues of living "The Good Life"—humility, simplicity, courage, curiosity, gratitude, health, power, resiliance, laughter, and hope—specifically focusing on the pursuit of happiness, a right afforded to Americans in the Declaration of Independence. The students are introduced to topics in mathematics that every productive citizen needs to know to process, evaluate, and understand the numerical and graphical information in our society. They do this by investigating applications of mathematics in marketing, news sources, finance, probability, statistics, geometry, population growth, and voting schemes through the reading of texts, the use of appropriate technology, and individual and cooperative problem solving. The goal is for students to see mathematics in a new and exciting way, leading to an appreciation for and an understanding of the world around them that is not otherwise achieved through the curriculum of traditional mathematics courses. Thus, approximately 50% of each class meeting is dedicated to doing mathematical processes, and the other 50% is composed of lecture and discussion about the applications of the mathematics.

## 18.3 Course Structure

The course has three main components: course readings, class discussion/activities, and final project. Each week students read a chapter from the textbook and submit a 250-350 word reflection (guidelines and rubric for the reflection are in the Appendix, p. 181). Students come to class each Monday ready to discuss the reading, and I choose a few ideas from the chapter on which to base the lecture, videos, and activities for the week. There is also a short, simple quiz at the beginning of class each Friday based both on the reading and class discussion/activities. The final project requires students to watch a movie, write a four-page paper discussing the mathematics and virtues in the movie, and then present their findings with a group of classmates who also watched the same movie.

### 18.3.1 Course Readings

The course readings come primarily from Sunil Singh's book *Pi of Life: The Hidden Happiness of Mathematics* [20]. It explores various mathematical concepts (e.g., Golden Ratio, fractals, probability) and virtues for living happily (e.g., simplicity, curiosity, resilience) and their connection to us on a personal, national, global, and universal level. One should be warned, however, that the intended audience of the book—especially for the author's anecdotes—does often seem to be mathematics educators and parents, but the content is invaluable despite that issue. In teaching the course, I cover roughly one chapter from the textbook per week, and then supplement the textbook with about two weeks of exploring the mathematics of voting and social choice. Through all of the readings and activities, my hope is that students will come to see the freedoms afforded by a meaningful study of mathematics, the disadvantages of mathematical illiteracy, and the happiness that is achieved by studying mathematics. Singh connects mathematical concepts and virtues for living "the good life" in the book's Introduction: "It's time to flip [the] lens and examine what mathematics can teach society—beyond facts, figures, and applications. It can reflect values and traits that we, as a society, cherish and strive to hold" [20, p. xxi]. Singh also references Francis Su, who speaks widely on the beauty of mathematics. In one interview, Su sets the stage for studying mathematics: "Math is the science of patterns, and the art of engaging meaningfully with those patterns. ... If you learn to do mathematics, you are much better equipped to understand the world around you in many different ways" [5].

Chapter 1 of the book, "Zero (Humility)", discusses the various reasons why you cannot divide by zero, including repeated subtraction and missing factor. Students seem to have such an instrumental understanding of this fact, and so they come to discover different ways of seeing why dividing by zero is "undefined." The concept of infinity comes up in this chapter, so it makes for a good opportunity to explore why dividing by zero is not "infinity," nor "indeterminate," for that matter. Singh includes in this chapter an anecdote about how little mathematics he knows [20, p. 11-12], which feeds into the the virtue of "humility." Students need to know that it is okay to admit to confusion, to feel free to ask for help, and that mistakes are opportunities to learn. The book references Francis Su, who speaks widely about this, connecting it to the idea of human flourishing [7], and it also includes a quote by Eric Greitens:

> I begin with humility, I act with humility, I end with humility. Humility leads to clarity. Humility leads to an open mind and a forgiving heart. With an open mind and a forgiving heart, I see every person as superior

> to me in some way; with every person as my teacher, I grow in wisdom. As I grow in wisdom, humility becomes ever more my guide. I begin with humility, I act with humility, I end with humility [20, p. 15].

Chapter 2, "One Plus Two (Simplicity)," really drives home the concept of "endless play and joyful struggle" [20, p. 18] with the inclusion of several engaging mathematical concepts such as the Sum of Odds problem, the Handshake Problem, and the Gaussian Sequence and Triangular Numbers. Students really enjoy engaging in these problems, especially when shown visual illustrations and allowed to really grapple with generalizing the patterns algebraically. This chapter has a similar feel to Chapter 6, "Seventeen (Health)," in that it also celebrates the virtues gained by the problem solving process. In this chapter, Singh also outlines the criteria for "flow" that Mihaly Csikszentmihalyi's research discovered. I find this to be an eye-opening read for the students as they are able to objectively identify what problem solving provides for the human spirit. Chapter 8, "ABC'd (Resilience)," explores topics in a similar vein in the sense that it allows students to explore the beauty of patterns in mathematical concepts such as the Collatz Conjecture and the abc Conjecture.

The concept of proof is explored in Chapter 3, "Q.E.D. (Courage)," through mathematical proofs, proofs in movies, and even logic in nursery rhymes. Specifically, the author discusses proofs of the Pythagorean Theorem and Pythagorean Triples, prime numbers, and Fermat's Last Theorem, along with the burden of proof needed in court proceedings à la *12 Angry Men* and unfounded assumptions in the tale of Hansel and Gretel. Students feel like a "real" mathematician by working through these examples, helping them get a taste of the living nature of mathematics and abating any preconceived notions that mathematics is a dull, sterile subject. Singh quotes Paul Lockhart, who has a compelling argument for studying proofs:

> Mathematics is the art of explanation. If you deny students to engage in this opportunity—to pose their own problems, to make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, to cobble together their own explanations and proofs—you deny them mathematics itself [20, p. 31].

Chapter 4, "Infinity (Curiosity)," takes the students on an exploration of very large numbers, challenging them to try to imagine the size of the universe, Graham's Number, and other seemingly infinite values. For example, the author poses the question: will we ever run out of new music? The goal of this chapter is to push students to think about these topics, possibly for the first time ever, and to breed a new sense of curiosity about the world—and universe—around them. The purpose of this chapter is also to help students see the difference between facts and figures presented to them in the news and social media. For example, values of a million, billion, and trillion, values are thrown around so much in reference to things such as population and national debt, but are not often truly understood. Students need to be responsible consumers of information and active members of society. Knowledge is key to this, but it is also necessary to have a curiosity and drive to dig deeper into facts and figures presented in the media.

In the same vein as the mind-bending study on "infinity," Chapter 5, "Negative Square Root (Gratitude)," presents the ideas of fractals and imaginary numbers. The author points out in this chapter that Mandelbrot's ideas about fractals in the 1970s quickly grew into other fields including medicine, economics, urban planning, film, and even public policy. There is a example of these applications described in the book by way of Dr. Ron Eglash, who discovered fractal designs in African societies [6]. In this chapter, students are encouraged to view the world from a distance, but also to give close attention to something as small and simple as a blade of grass, which "becomes a mysterious, awesome, indescribably magnificent world in itself" [20, p. 55].

One of the most engaging and applicable chapters is 7, "ME (Power)," a chapter focusing on probability and mathematical expectation. Yet again, the students are faced with situations that appear in their everyday lives through leisure activities, consumer goods, and media; for example, the concept of mathematical expectation in the lottery and extended warranties. This chapter makes a clear case for the freedoms afforded by a meaningful study of mathematics and the disadvantages of mathematical illiteracy (innumeracy); the students see how mathematical knowledge really is power.

As was stated earlier, the intended audience of *Pi of Life* seems to be mathematics educators. This is most evident in Chapter 9, "Thirty-Seven (Laughter)" and in Chapter 10, "Phi (Connection)," so much so that I do not include the last half of chapter 9 and the entirety of chapter 10 in required readings for the course. Singh's lament about the current state of mathematics education in chapter 9 is interesting and important, but not directly applicable to the audience of students. However, the first half of the chapter celebrates the happiness of mathematics by showing how it can be used for humor, so I assign that reading to students in the hopes that they, too, are able to find mathematical humor in their everyday lives. Also, the ideas of phi and the Golden Ratio presented in chapter 10 are, of course, pertinent to the goals of the course, so I do take the opportunity to spend plenty of class time exploring those concepts the students were so excited about during the first week of the course.

The final and 11th chapter of the book, "Pi (Hope)," provides a brief history and derivation of the constant $\pi$. The chapter includes more editorial from the author about the state of mathematics education, but also a discussion about how the iconic symbol $\pi$ has brought joy and connection to the globe through the beauty of mathematics.

### 18.3.2 Class Activities

Regardless of past experiences with mathematics or current attitudes about mathematics, I find that I "win over" students almost immediately in the first week of class by introducing the idea of the beauty of mathematics by way of the Fibonacci sequence and Golden Ratio. I am amazed at the enthusiasm the students display after seeing the pattern and applications of this mathematical concept. Establishing a productive classroom culture is also imperative in the first week, and I do that by allowing students to engage in problem solving activities through productive struggle and respectful discourse. One great entrypoint to problem solving are KenKen puzzles[1] that come in a full array of level and difficulty. Through these, the students feel the satisfaction that comes with solving a difficult problem and gain a sense of confidence in their abilities to conquer other problems in their life they might face. I present the students with various problems throughout the semester, including the "Age Problem" and "The Die Hard" problem, detailed in the Appendix (p. 182). The section on probability is ripe for problem solving opportunities, and the textbook includes several such problems. I also present the following problem: "A mother tells her friend she has two children. She says she has at least one girl. What is the probability she has two girls?" Once we grapple with and solve it, I lay a more challenging problem on them, "The Family Feud Problem" from kenkenpuzzle.com, also included in the Appendix (p. 182).

I employ videos with great frequency throughout the course, especially those by Numberphile, a content creator that Singh refers to quite often as well. While much of the content of the Numberphile videos is beyond the scope of my intended level of mathematics for this course, they do present mathematical concepts in a clear manner all while illustrating the beauty of mathematics. For Chapter 1, "Zero," I use their video, "Problems with Zero" [17], where they provide a few different explanations about dividing by zero. I do not show this video in its entirety since it eventually talks about limits; I aim to keep the scope of the course accessible to students with all levels of mathematics backgrounds. However, I am sure to deliniate the terms "undefined," "indeterminate," and "infinity," as they are so often confused. Another great resource of videos are by the late Dr. Jeffrey Bergen from DePaul University, who was a sort of March Madness Bracket Probability celebrity, at least in the Chicago area, appearing on all the local news stations around that time each year to discuss the probability of filling out a perfect bracket. One such video is searchable on the NCAA website [9], and he also has a narrated PowerPoint posted on YouTube [14]. Dr. Bergen also did a radio spot in 2017 on WGN Chicago radio's Bill and Wendy show, that illustrates the lack of number sense in even the most public of figures [4]. Finally, there are several beautiful videos and visualizations that help to illustrate $\pi$ in the final chapter. I show the Numberphile video, "Pi" [15] to illustrate Archimedes' method of estimating $\pi$ by finding the perimeter of polygons with increasing number of sides. And then, now that the students have seen the infinite nature of the decimal representation of $\pi$, I show them the images in the 2015 *Washington Post* article, "10 Stunning Images Show the Beauty Hidden in Pi," to illustrate the beautiful randomness of the digits of $\pi$ [22]. Along with those images I show the video from StarTalk with Neil DeGrasse Tyson, "The Beauty of Pi," in which mathematician Ken Ono explains how Ramanujan "tamed" $\pi$ [1]. And then, to stir up a little more curiosity, I show Vi Hart's video, "Pi is (Still) Wrong," in order to introduce the concept of *tau* [16].

Hands-on activties are key to student engagement, and so I incorporate them as often as possible. In Chapter 5, "Negative Square Root," I have students create their own Sierpinski Triangles, Sierpinski Carpets, and Koch Snowflakes. (For extra credit I allow a group of students to construct a large three-dimensional display of a Sierpinski Gasket made out of paper.) During the section on probability, I am sure to illustrate the difference between theoretical and experimental probability and also the law of large numbers through trials with dice, coins, and playing cards. For the

[1] www.kenkenpuzzle.com; a Sudoku-like puzzle that includes mathematical operations, developed by Japanese mathematics instructor Tetsuya Miyamoto who coined "The Art of Teaching Without Teaching"

section on *phi*, there are, of course, numerous opportunities for hands-on activities. Having plenty of rulers on hand, I guide students through the NCTM Illuminations activity, "A Ratio that Glitters: Exploring the Golden Rectangle" [19]. We begin with Golden Rectangle Activity Sheet 1, "The Golden Ratio in Architecture and Art," where students draw a rectangle around pictures of the Parthenon and the Mona Lisa and see whether the rectangle they constructed is close to a Golden Rectangle. Then I hand out centimeter grid paper for students to construct a Golden Rectangle (and subsequently a Golden Spiral) with the help of Golden Rectangle Activity Sheet 2, "Adding Squares to Make Rectangles." Then I have students measure the various dimensions of their body, hand, and arm using tape measures, measuring tape, and rulers following "The Golden Ratio" activity. (I included the list of measurements and ratios in the Appendix, p. 182.) To investigate the Golden Ratio in a face, I print out a picture of Brad Pitt for them to measure and include in their activity sheet. Finally, in order to explore the title concept of *pi*, I bring a large variety of "circular" (mostly cylindrical) objects of assorted sizes and dimensions for students to measure. I offer the students tape measures, rulers, string, and tape to aid in getting accurate measures. The goal is to discover that the ratio of the circumference and diameter is always very close to $\pi$.

I take a break from the textbook for a few weeks during the semester to focus on the mathematics of voting and social choice. I obtain most of my teaching materials from old Princeton lecture notes I found on the web [3] that are based on the textbook *For All Practical Purposes* [8]. I cover only the material that is practical and hands-on, avoiding any theoretical-based mathematics or complicated notation. I first focus on the different systems in existence that can select a "winner": plurality, plurality with run-off, sequential run-off, and Borda count, using a couple different YouTube videos to further illustrate the systems. In order to practice these concepts, the students complete a lab based on voting for various beverages. (The referenced Princeton website contains this lab and other very helpful examples and practice problems.) I then shift the focus to the Banzhaf power index where voters do not have equal standing, as in the case of shareholders or the Electoral College. We also briefly discuss fair division and watch a video that illustrates the cutting and sharing of chocolate cake. Again, I keep things simple and tangible, avoiding complicated mathematical equations or notations. The goal of this unit is simply to expose students to these concepts in the hopes that they become more productive citizens and better consumers of information.

### 18.3.3 Final Project

The concluding two to three weeks of the course are spent focusing on the final project, "Mathematics and Virtues in American Cinema." Students are given a list of movies from which to choose: *Hidden Figures* [10], *The Pursuit of Happyness* [18], *Moneyball* [13], *The Big Short* [2], *The Imitation Game* [11], *Stand and Deliver* [21], and *The Man Who Knew Infinity* [12]. I allow up to three people to watch each movie. Students are responsible for acquiring the movie and watching it on their own time. They then work with their "movie group" to prepare a presentation that discusses both the mathematics present in the movie and also the virtues for living a happy life that are exhibited in the movie. Each student is responsible for a four-page paper detailing this information that they write and submit individually. The instructions for the paper and presentation are included in the Appendix (p. 183).

## 18.4 Reflections

I have now taught the course two times, and I have really enjoyed teaching it; teaching past iterations of the FYS have always felt out of my comfort zone. It has been exhilarating to not only develop my own course, but also to be allowed to spend as much or as little time on mathematical topics as I want. We are able to delve into concepts in a way that never gets to happen in traditional college mathematics courses, and I know the students really enjoy it as well. In fact, I had one student who was waffling between majors who decided—for certain—that they now want to teach mathematics because of the experiences in this course. Many students expressed throughout the previous semester that they never realized these mathematical connections, or expressed how they wished they had opportunities for learning mathematics like this in their past schooling. Overall the students have expressed great satisfaction in getting the chance to grapple with the mathematical concepts covered in the course.

Since the first time I taught the course I made a few significant changes. Despite all the positive comments and obvious enthusiasm by many students in the previous semester, there was still a notable amount of student apathy during class discussions and activities. I attribute this to the fact that I approached the course with a "love of learning" angle. Even though I explained my philosophy at the beginning of the semester—how there are no tests or quizzes and that everything we do in class is for the love of learning—and explained there would be a participation grade, the philosophy did not make for good discussions, and I found many students needed external motivation to truly put effort into problem solving. And thus I realized the need to give a short quiz at the end of each week just to motivate the students to pay more attention to the reading and class activities. So far it seems to be achieving its intended goal; students take notes on their own volition during lecture, and videos and the general level of classroom discourse is dramatically improved. I must note that I aim to write the quizzes so that they are simple for students who do the reading and pay attention in class, but challenging for those who do not (a tough balance to strike, but a goal all the same). The last thing I want as a result of this course is for students to dislike mathematics more than they already might; I want it to be an inspiring and positive experience all around.

The weekly reflection papers seem to achieve their intended goal of compelling the students to read the textbook. However, I have the sense that some students read only just enough of the chapter to help prompt a couple paragraphs of reflection. I do believe that the short weekly quizzes help remedy this issue, though, because I make clear that the quizzes will cover both the readings and class activities.

I mentioned previously that all SBS sections at my university are to base their final paper on the American Dream, but those guidelines were relaxed before the second iteration of my course. Instead I have chosen to focus the theme on the "pursuit of happiness" aspect of the American Dream, that is addressed nicely by the virtues outlined in the textbook. As such, I have added in the suggestion to students to relate the reading each week to one's pursuit of happiness.

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## Appendix

### Reading Reflection Guidelines

*These guidlines were provided to students on a half-sheet of paper for their reference while composing their reflections each week.*

While reading:

- Underline words, concepts, and references about which you have questions or want to look up on your own.
- Make notes in the margins or on a separate sheet of paper.

After reading:

- Write a reflection addressing one or more of the following questions:
  - How has your thinking changed after reading this chapter?
  - How does something you read relate to other fields or disciplines?
  - How does something you read relate to one's pursuit of happiness?
- This reflection is not a summary of the information read. It is also not an editorial about your feelings about the content.

Formatting:

- 250-350 words in length
- APA style (Times New Roman, 12 pt. font, double-spaced, 1 in. margins)
- In-text citations (see Purdue OWL APA website)

## Problem Solving Problems

*These are some of the problems that the class engages with throughout the semester.*

### The Age Problem

John spent a quarter of his life as a boy growing up, one-sixth of his life in college, and one-half of his life as a teacher. He spent his last six years in retirement. How old was he when he died?

### The *Die Hard* Problem

In this activity you will be asked to develop a strategy to bring up from river a certain amount of water when you have only two containers, a 4-quart bucket and a 9-quart bucket, with which to measure.

1. How can you bring up from the river exactly 6 quarts of water?

2. How can you bring up each of the indicated quarts of water?

   (a) 1 quart
   (b) 2 quarts
   (c) 3 quarts
   (d) 5 quarts
   (e) 7 quarts
   (f) 8 quarts

3. Develop a strategy to bring any amount, $n$, of quarts of water when you have only two containers, a 4-quart bucket and a 9-quart bucket, with which to measure.

### The Family Feud Problem

A newlywed couple is planning their family. They'd like to have four children, a mix of girls and boys. Which is more likely: (1) two girls and two boys or (2) three children of one gender and one of the other? (Assume that each birth has an equal chance of being a boy or a girl.)

## The Golden Ratio Activity

*This is the list of measurements and ratios referred to in the Golden Ratio Activity.*

**Full body measurements:** (a) distance from the ground to your belly button (b) distance from the ground to your knees (c) distance from your belly button to the top of your head (d) distance from shoulder line to the top of your head.

**Hand/arm measurements:** (e) distance from fingertip to elbow (f) distance on pointer finger from fingertip to first knuckle (g) distance between first and second knuckle (h) distance between second and third knuckle (i) length of your hand (wrist to fingertips) (j) distance from your wrist to your elbow.

**Head measurements:** (k) length of face (bottom of chin to top of head) (l) width of face (m) distance between where your lips meet and where your eyebrows meet (n) length of nose (nose tip to between the eyes) (o) distance between tip of jaw and where the eyebrows meet (p) width of mouth (q) width of nose at widest point (r) distance between your nostrils (s) distance between your pupils (t) distance between eyebrows.

**Ratios to find:** a/b, a/c, c/d, e/j, g/f, h/g, j/i, k/l, m/n, k/o, p/q, q/r, s/t.

## Final Project Description

*This document is provided to students as a guideline for the final project, which represents the intersection of mathematics and virtues for living a happy life.*

- **Part 1—May be completed individually or as a group**

  Watch one of the following movies as identified by your group:

  - *Hidden Figures* (2016)
  - *The Pursuit of Happyness* (2006)
  - *Moneyball* (2011)
  - *The Big Short* (2015)
  - *The Imitation Game* (2014)
  - *Stand and Deliver* (1988)
  - *The Man Who Knew Infinity* (2015)

- **Part 2—To be completed as a group**

  Identify topics related to the movie that each group member will investigate:

  - What mathematics is in the movie? Research as well as you can.
  - Which virtues from our textbook are present in the movie, and how are they exhibited?

- **Part 3—To be completed individually**

  - Research the topics using a minimum of two to three outside sources.
  - Write a paper (approximately 1000 words, or three to four pages) summarizing your findings.
  - Your paper must explain/discuss the mathematics in the movie and also explain which virtues are present in the movie and how the virtues are exhibited.
  - You may include a summary of the movie in your paper. You should spend no more than 250 words doing so.
  - Papers are to be submitted electronically, including APA citations and a reference page.
  - Papers must be done individually; plagiarism will not be tolerated.

- **Part 4—To be completed as a group**

  - Prepare a 12-minute presentation that will be shared with the class during the last week of the semester.
  - The group presentation must explain the mathematics in the movie and also explain which virtues are present in the movie and how they are exhibited.

# 19

# Math in Pop Culture: A First-Year Writing Seminar on Mathematics

Mark Kozek

**Abstract** In this article I describe a first-year writing seminar in which students learn about mathematics by reading or watching mathematical stories that appear in mainstream media and popular culture. I discuss several of the mathematical pieces the students read or watch, assignments in which students write about a mathematical film and a mathematical book, and how, as a mathematician, I approach teaching *writing* in a first-year seminar.

## 19.1 Background and Context

Whittier College is a small liberal arts college located in the greater Los Angeles area. Almost 70% of our approximately 1,650 students belong to minority or underrepresented groups. Many are first-generation college students, first-generation Americans, or belong to several other demographics that may not often be present in large numbers at other private colleges or universities.

All first-year students must enroll in *INTD 100: College Writing Seminar* (CWS) in the fall term.[1] It is a three-credit course that meets for three contact-hours per week during a 14-week semester. Its course description states, "Students read complex texts chosen to sharpen critical reading and thinking skills. Writing assignments are designed to teach and practice persuasion, description, narration, exposition, and research-based writing. Extensive revision is emphasized."[2]

All permanent faculty are expected to teach CWS regularly (at least once before applying for tenure and again prior to promotion) and determine their section's theme. CWS also satisfies our *Community* Liberal Education requirement, so the students must co-enroll in a second (partner) course that need not be related thematically to their CWS. The CWS instructor serves as the students' academic mentor and supports their transition to college life. In addition, each section, capped at 16 students, has an embedded student assistant—a junior or senior who has undergone formal training, both as a writing peer mentor and as a student life peer mentor.

Students choose their CWS section during summer registration based on the course title, a brief description, their intended major, or its partner course. CWS is coordinated by the Director of College Writing Programs in collaboration with the Dean of First-Year Experiences, who monitor enrollment in each section to maintain balances in gender, demographics, non/residential students, and prospective majors. The Director of College Writing mentors first-time CWS instructors during the previous spring and during a day-long instructional session the week before classes begin.

[1] INTD = Interdisciplinary Studies Department.

[2] From the online Whittier College Catalog, available at `catalog.whittier.edu`.

## 19.2 Mathematical Theme

The theme of my CWS is *Math in Pop Culture*. My goal for this course is to use mathematical narrative (both fiction and non-fiction) associated with stories that appear in mainstream media and popular culture to hook students into caring more about mathematics and to realize that their personal connection to mathematics is closer than they realize.

While learning mathematics might be challenging to some first-year students, learning about mathematics history should be accessible to most, especially if the medium is through popular culture. Also, since most first-year students have not been exposed to mathematics history previously, they are able to start at the same place and undertake this adventure together, reinforcing the *Community* Liberal Education aspect of CWS. Approximately one-third of the course is devoted to teaching the students about mathematics history or mathematical ideas that relate to the reading or viewing (such as logic, set theory, infinity, Goldbach's conjecture, the Clay Millennium problems, cryptography, non-orientable geometry, the fourth dimension, etc.). The rest is devoted to writing exercises, to discussing the reading or viewing, or to elements of student life and the first-year experience.

We start with a historical fiction graphic novel and with a thriller film, and we learn about mathematics and mathematics history to understand the novel and the film better. After these units, and their discussions, the students start writing about mathematics, first by writing their mathematical autobiography, then by writing about a mathematical film, and finally about a mathematical book of their choice. So, without realizing it, the students are reinforcing their personal connection to mathematics. And in a perfect world, something new the students read or watch in my course will inspire them to want to learn more about mathematics on their own.

## 19.3 Course Structure

*Math in Pop Culture* is a three credit-hour course which meets for three 50-minute periods each week (or two, 80-minute periods). It is divided thematically into five units whose readings/films introduce students to mathematical biographies, famous mathematical problems, mathematics in World War II, women in mathematics[3], and mathematics education. Its partner course is *MATH 141: Calculus I* taught by a different math professor; other than sharing students, these courses are separate.

I start each unit by assigning one of the following: the graphic novel *Logicomix: An Epic Search for Truth* [1], the Spanish film *La Habitación de Fermat* [22], the documentary *Top Secret Rosies: The Female Computers of WWII* [7], the documentary *Calculating Ada: The Countess of Computing* [25], and the film *Stand and Deliver* [15]. By featuring documentaries, I ensure that I do not accidentally assign a film that one of my students wishes to write about later in the course. Further readings come from *Mathematics in Popular Culture: Essays on Appearances in Film, Fiction, Games, Television and other Media* [26]. (See the Appendix, p. 191, for additional readings and films.)

Students have three types of writing assignments that amount to approximately 30 pages and make up 70% of the course grade. The remaining 30% of the course grade is comprised of reading/film comprehension quizzes; other homework, group work, or problem sets; attendance; and a completion grade that is awarded based on turning in preliminary drafts, meeting with the student assistant to discuss the drafts, and promptly returning feedback from peer-review exercises to the original author.

### 19.3.1 Readings and Films

At first, the students are mildly amused by the notion of reading a graphic novel about mathematics in a college-level writing course. But without fail, there is always something about *Logicomix* that draws them into its story. I assign *Logicomix* because its historical framework provides the students with a solid foundation upon which to build their mathematical body of knowledge. Furthermore, *Logicomix* includes a glossary of all its main mathematical characters, theorems, and ideas that they frequently refer back to throughout the course.

Next, although it is a film in Spanish, the students enjoy watching *La Habitación de Fermat* (Fermat's Room). Given the demographics in Southern California, many students appreciate that the film is in Spanish. This viewing experience also offers non-Spanish speakers a glimpse of the struggles non-native English speakers face as they navigate American college life. Without going into the plot, the main characters, one of whom proved Goldbach's conjecture, find

[3] Gender in mathematics, and to a certain degree, race and languages, are embedded throughout the *entire* CWS, not only in the unit on women in mathematics.

themselves having to solve math puzzles throughout the film. Prior to watching *La Habitación de Fermat*, which is the only film we screen during class time, I have the students work in groups to solve a problem set comprised of the same puzzles that appear in the film, but the students do not know this in advance. Sooner or later, the students recognize their puzzles in the film, which engages them by either watching how the characters struggle to solve a puzzle they already solved, or watching intently to learn the solution to the puzzle the students have not yet solved for homework, which is due the following class period.

For the next two units, mathematics in World War II and women in mathematics, we watch the documentaries *Top Secret Rosies: The Female Computers of WWII* and *Calculating Ada: The Countess of Computing* instead of mainstream movies or television series about mathematics because I want to give the students the greatest chance of choosing a mathematical movie they connect with to write about in their second short essay. I have been using *Top Secret Rosies* in various interdisciplinary courses since it was released in 2010; students not only enjoy it, but it always impacts the students deeply. And what is not to love about the true story of an elite, top secret team of World War II heroines who were recruited from their high schools or colleges because they were the best at math!

We watch *Calculating Ada* because it tells the story of Ada Lovelace (Lord Byron's daughter), who might have written the first computer program, but also because it brings mathematics online, another popular culture medium. *Calculating Ada* is presented by Dr. Hannah Fry,[4] a mathematician who maintains a robust social media presence, whose videos have millions of YouTube views, and whose *TEDx* Talk "The Mathematics of Love" [9] went viral. Watching *Calculating Ada* online opens a back-door to other online media such as TED talks, Numberphile videos, etc.

The last required film is *Stand and Deliver*, which is a natural complement to the unit on mathematics education. Since *Stand and Deliver* is set in East Los Angeles, not far from Whittier College, I have always had a student in my class who graduated from a high school in East Los Angeles (but not yet Garfield High School, where these events took place). Invariably during our class discussion, this student shares all the ways the film is accurate and all the ways it is not. *Stand and Deliver* is a film that has inspired a considerable body of writing. I have the students research this scholarship and bring an article to class for discussion and (although they do not realize it yet) to serve as a model of scholarly writing on mathematics in popular culture.

I try to complete the five units and all assignments, except the final paper, with about two weeks to go in the semester so that the students can focus exclusively on their final paper at the end of the term. During these last two weeks, if we are not doing an in-class peer-review exercise for their final paper, I am able to include a few bonus screenings. Because we can watch and discuss an entire episode in one class period, I show mathematical episodes from recent television series. Some examples include: the *Numb3rs* episode "Prime Suspect" (CBS 2006-2011, Season 1, Episode 5) that features the Riemann Zeta hypothesis, the *Elementary* episode "Solve for X" (CBS, 2012–2019, Season 2, Episode 2) that features P vs NP and a female mathematician, or the *Futurama* (FOX 1999-2003, Comedy Central 2007-2013) episodes "Prisoner of Benda" (Season 6, Episode 10) that gave us the Futurama theorem and "2-D Blacktop" (Season 7, Episode 15) that features various geometric concepts, including a Möbius strip, the fourth dimension, and a visit to a flat, two-dimensional world.

### 19.3.2 Writing Assignments

Students have three types of writing assignments: reflections, two short essays, and a final paper. The reflections serve as a scaffold to the short essays, which in turn serve as a scaffold to the final paper.

A reflection is a written response to a question or topic I pose. It is meant to be more formal than a journal entry, but not as formal as an actual essay. A reflection is one single-spaced page long. Reading the reflections gives me a sense of how deeply the students are actually thinking about the topics we discuss in the course. They allow my student assistant and I to provide regular feedback on the students' writing, and we often use the reflections for peer-review exercises. I end up assigning about ten reflections through the course, and each short essay starts out as a reflection. The reflections are graded based on completion, not accuracy, and count towards 15% of the final grade.

The two short essays are four to five pages long (or 1200-1500 words), and each counts 15% towards the final grade. The first short essay tasks the students with writing their mathematical autobiography (so, a memoir) and follows the unit on mathematical biographies. Often, upon arriving to campus, first-year students tend to write or talk about who they are or where they come from. In a sense, this essay encourages the students to focus on themselves, so that in

[4]For more on Hannah Fry please see `hannahfry.co.uk/`.

all subsequent essays they can write about (and only about) the assigned topic. In addition to *Logicomix*, which is about Bertrand Russell, the students read a few other mathematical memoirs or biographies to use as models (such as Sarah Flannery [10], G.H. Hardy [11], or Julia Robinson [20]) and the essay "Rise of Narrative Non-Fiction" [6, pp. 183–186]. What begins as a reflection on "mathematics in my life" becomes a more directed autobiographical essay (see the Appendix, p. 192, for the essay prompt). After I distribute and discuss the prompt, the students are required to meet with the student assistant outside of class to discuss their initial draft. Based on this meeting, each student makes changes to their paper and then submits their first draft. I review their essays and provide individual student feedback. The final draft is due about two to three weeks after the initial reflection is assigned.

For their second essay, the students each write a scholarly review of a mathematical movie (or two mathematical episodes of a television series) of their choice and approved by me. No two students may write on the same movie. I give the students some scholarly book and film reviews to serve as models for their paper, but the prompt also asks them to find a scholarly article about their movie to cite in their essay. So, while the students may think this assignment is about mathematical movies, it is really an exercise in research methods. The second essay has a similar time-line as the first essay. (See the Appendix, p. 192, for the essay prompt.)

For their final paper, which is eight to ten pages long (or 2500–3000 words), the students choose a mathematical book, either fiction or non-fiction, and approved by me, and use evidence from the text to determine whether their book exhibits the four characteristics of paramathematics described in "Embedding Mathematics in the Soul: Narrative as a Force in Mathematics Education" [4]. This assignment is a textual analysis, and it counts as 25% of their final grade. Again, no two students may write about the same book. Often students choose the book their film was based on; if so, there is also a compare and contrast element to their paper. Their book lottery occurs around the midpoint of the semester, and the overall time-line takes about four to six weeks. (See the Appendix, p. 193, for the essay prompt.)

### Teaching Writing

At Whittier College, our curriculum is designed to nurture the students' writing over the course of their entire undergraduate career. Upon completing CWS, the students are required to take a follow-up, Writing-Intensive Course (WIC), taught by trained specialists in English composition and rhetoric, where students will formally and rigorously learn how to write a college-level research paper that supports a thesis statement. Then students take liberal education courses that have the WIC as a pre-requisite in which they write research papers in various contexts. The students' writing journey culminates with a required senior capstone paper.

My best insight from the first time I taught CWS was realizing that although CWS has a considerable writing component, as the instructor my goal is to help my students improve as writers, and transition from high school writing to college writing. From teaching various interdisciplinary courses over the years, I have read enough papers to have observed some of the most common writing "bad habits" students have. To help them avoid these, I have developed a set of writing rules that I emphasize throughout the course. Although I have this list ready at the start of the semester, it is a fluid list, and I may add or remove items from the list throughout the term. I roll out these rules one at a time, usually after students submit a reflection. The current form of this list is as follows.

1. Use proper grammar and punctuation.
2. Minimize using the verb "to be."
3. Do not feel or believe these statements to be true; justify them with evidence.
4. Cite. Cite. Cite!
5. Do not over-complicate sentences. Keep sentences simple.
6. Do not use the passive voice. Write in the active voice.
7. Avoid/minimize using prepositional phrases (including infinitives).
8. Avoid/minimize using adjective clauses that start with which or that.

I use the students' own work to introduce these rules to them. On a day a reflection is due, I have the students bring two print-outs of their reflections: one for me or the class student assistant to give them feedback, and one for them to do a peer-review exercise during class. Once the students have traded reflections, I introduce the day's rule, and I describe the grammar or style that might be in play. I give a couple of exaggerated examples (that I made up), and I ask the students to highlight every time their classmate broke the day's rule in their reflection. The students get their reflection back at the end of the class, to see "how they did." With the second copy, I give each student more formal feedback on that rule, such as any relevant patterns in their writing, or any customized hints on how to avoid breaking that rule in the future. Once they have received my feedback, I tell the class that starting with the next paper, not following the rule will result in lost points.

## 19.4 Reflections

I have taught *Math in Pop Culture* twice. My students seemed to enjoy this experience, as reflected in positive course and instructor evaluations. I was pleased to learn that both times my CWS was one of the first sections to fill up. One reason might be that the words "Pop Culture" in the title drew the attention away from the word "Math." (Often, Directors of Writing Programs advise against using the word "math" in the title.) Partnering my CWS with a Calculus class might also have attracted students who are receptive to mathematics to my course.

### 19.4.1 Films

Originally, I did not intend to have students watch many films/documentaries. However, in my interdisciplinary course on mathematics in literature and cinema [3], we observed that our students were more likely to watch an assigned movie than read an assigned book, even if we gave them significantly longer to read the book. So by keeping the reading assignments shorter and by assigning more films/documentaries, more students complete the assignments, which in turn leads to more informed (and livelier) discussions.

### 19.4.2 Readings

One of the challenges of the first iteration of this course was finding representative examples of scholarly writing about mathematics in popular culture that would be easily understood (mathematically) by a student in CWS, and, more importantly, that the students could use to model college-level writing. This was resolved with the publication of *Mathematics in Popular Culture: Essays on Appearances in Film, Fiction, Games, Television and other Media*, edited by Sklar and Sklar [26]. It includes scholarly essays on practically every well-known mathematical appearance in popular culture prior to 2012 and has been an invaluable resource for the writing component of this course.[5]

### 19.4.3 Gender and Diversity of Mathematical Characters

Another challenge with the first iteration of this course was the dearth of material where the main (mathematical) character was not a white male who was socially awkward or suffered from mental illness. Things have gotten slightly better since then thanks in no small part to the book *Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race* [21], its film adaptation [14], and other non-fiction books about women in technology.[6]

Recently, STEM is featured more prominently in popular culture, and mathematical characters are portrayed more broadly, though there remains room for improvement. Some mainstream examples include the films *The Imitation Game* [27] whose main character is gay, *The Man Who Knew Infinity* [2] whose main character is Indian, *A Brilliant Young Mind* [13] that is set at the International Mathematics Olympiad, and *Gifted* [28] whose title character is a female mathematics prodigy; the television series *The Bletchley Circle* (PBS/BBC 2012-2014) about women who worked at Bletchley Park, *Halt and Catch Fire* (AMC, 2014–2017) set during the rise of the personal computer and the birth of the internet, and *Mr. Robot* (USA, 2015–2019) about hacking and the cybersecurity industry, to name a few. While there has been a pleasant uptick in STEM and gender representation, I am always on the lookout for pieces in mainstream media or popular culture that present more diverse participants in STEM.

[5]Instructors interested in more advanced scholarly essays on mathematics (but not necessarily popular culture) might consider the yearly series *The Best Writing on Mathematics* edited by Mircea Pitici that began in 2010 [19].

[6]For example, *Grace Hopper and the Invention of the Information Age* [1], *Code Girls: The Untold Story of the American Women Code Breakers of World War II* [16], and *Broad Band: The Untold Story of Women who Made The Internet* [8].

### 19.4.4 Writing

Of the writing rules I try to emphasize during the course, my students seem to have the most difficulty with "Cite. Cite. Cite!" If your campus subscribes to any anti-plagiarism software, I recommend that you include this software in the course as soon as possible. It is my standard practice to have students turn in their earliest drafts of the short papers through our anti-plagiarism software. When I provide feedback on their drafts, I will tell them their "similarity score" and let them know that anything above 15% is unacceptable. The reality is, of course, that it depends on what type of similarity—sometimes the anti-plagiarism software flags a submission for being highly similar to a previous draft of that same paper that was uploaded earlier in the course! So, it is important that the faculty be comfortable with whichever software platform is preferred at their institution. Still I often find that my students have more disparate definitions of an "appropriate citation" than I do. Since this is their first writing course in college, it is hard to strike the right balance between helping the students become accustomed quickly to the collegiate urgency and expectations for proper citations without getting too draconian about the penalties for improper citations. That is to say, I want the students to understand why it is important to cite properly rather than cite out of fear of a bad grade or other such consequences. An effective way to achieve this still eludes me.

### 19.4.5 Concluding remarks

Looking ahead to the next time I teach *Math in Pop Culture*, I would like to feature mathematical web-based media or vlogs, such as TED Talks or Numberphile videos, more formally in the course as a fun way to teach the students about specific topics in mathematics. I would also like to teach the students how to give presentations and maybe replace one of their papers with an oral presentation.

## 19.5 Bibliography


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[5] Apostolos Doxiadis and Christos Papadimitriou, *Logicomix: An Epic Search for Truth*, With art by Alecos Papadatos and Annie di Donna, Bloomsbury, New York, 2009.

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[7] LeAnn Erickson, director, *Top Secret Rosies: The Female "Computers" of WWII*, PBS, Arlington, VA, 2010.

[8] Claire Evans, *Broad Band: The Untold Story of Women Who Made the Internet*, Portfolio Penguin, New York, 2018.

[9] Hannah Fry, "The Mathematics of Love," *TEDx Binghamton University*, April 4, 2014. `youtube.com/watch?v=N37x4GgDVBM`

[10] Sarah Flannery, "Part I: Background", *In Code: A Mathematical Journey*, Profile Books Ltd, London, 2000.

[11] G. H. Hardy, "Part XXIX", *A Mathematician's Apology*, Cambridge University Press, Cambridge, UK, 1940.

[12] Paul Lockhart *A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form*, Bellevue Literary Press, New York, 2009.

[13] Morgan Matthews, director, *A Brilliant Young Mind*, Samuel Goldwyn Films, Culver City, CA, 2015.

[14] Theodore Melfi, director, *Hidden Figures*, 20th Century Fox, Century City, CA, 2016.

[15] Ramón Menéndez, director, *Stand and Deliver*, Warner Bros., Burbank, CA, 1988.

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[20] Constance Reid, "The Autobiography of Julia Robinson", *The College Mathematics Journal*, 17:1 (1986) 3–21.

[21] Margot L. Shetterly, *Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race*, William Morrow, New York, 2016.

[22] Simon Singh, *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem*, Walker & Co, London, 1997.

[23] —, director, *The Proof*, BBC TV/PBS-WGBH Boston, 1997.

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[25] Nat Sharman, director, *Calculating Ada: The Countess of Computing*, BBC, London, 2015.

[26] Jessica Sklar and Elizabeth Sklar, editors, *Mathematics in Popular Culture: Essays on Appearances in Film, Fiction, Games, Television and Other Media*, McFarland Press, Jefferson, NC, 2012.

[27] Morten Tyldum, director, *The Imitation Game*, The Weinstein Company, New York, 2014.

[28] Marc Webb, director, *Gifted*, Fox Searchlight Pictures, Century City, CA, 2017.

[29] Alec Wilkinson, "The Pursuit of Beauty: Yitang Zhang Solves a Pure-Math Mystery", *The New Yorker* (February 2, 2015).

## Appendix

### Additional Reading and Films

*Here are some additional readings and viewings for each unit.*

0. Introduction: Math in Pop Culture
   (a) "Math Takes Center Stage" by Robert Osserman [6, pp. 187–194]
   (b) "Rise of Narrative Non-Fiction" by Simon Singh [6, pp. 183–186]

1. Mathematical Biographies
   (a) "Part I: Background" by Sarah Flannery [10, pp. 1–40]
   (b) *A Mathematician's Apology* by G.H. Hardy [11, part 29]
   (c) "The Autobiography of Julia Robinson" by Constance Reid [20]

2. Famous Mathematical Problems
   (a) "The Pursuit of Beauty: Yitang Zhang Solves a Pure-Math Mystery" from *The New Yorker* [29]
   (b) Excerpts from *Fermat's Enigma* by Simon Singh [22]
   (c) *The Proof* (documentary) [23]
   (d) "Manifold Destiny: A Legendary Problem and the Battle Over Who Solved It" from *The New Yorker* [17]

3. Mathematics in World War II
   (a) ‘‘Cracking the Enigma” from *The Code Book* by Simon Singh [24, pp. 143–189]
   (b) “Alan Turing: Reflecting on the Life, Work, and Popular Representations of a Queer Mathematician” by K. G. Valente [26, pp. 291–232]
4. Women in Mathematics
   (a) *Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race* by Margot Lee Shetterly [21]
   (b) “Smart Girls: The Uncanny Daughters of *Arcadia* and *Proof*” by Alker and Davidson [26, pp. 172–186]
   (c) “*Mean Girls*: A Metamorphosis of the Female Math Nerd” by Rowan [26, pp.187–197]
   (d) “Monsieur LeBlanc”, from *Fermat’s Enigma* [22, pp. 97–109]
5. Mathematics Education
   (a) *Gifted* (film) [28]
   (b) “*Stand and Deliver*, Twenty Years Later” by Simic-Muller, Gutiérrez, and Gutiérrez, [26, pp 163–171]
   (c) *A Mathematician’s Lament* by Lockhart [12]
   (d) “Girls Just Want to Have Sums” *The Simpsons*, Season 17, Episode 19, FOX, 2006.

## Essay #1: A Mathematical Autobiography

*The students receive this prompt at the start of week 4, and a draft is due during week 5. Essay #1 is due at the end of week 6.*

Write your mathematical autobiography. In it, you should write about:

1. Your autobiography.
2. Your relationship to mathematics. (This can be either positive or negative.)
3. Your future with mathematics (or not).
4. Something else that brings these together.

The “something else” is what Singh writes about in “The Rise of Narrative Nonfiction.” That is, something else outside of mathematics and to a certain degree outside of the character (the mathematician) that brings everything together and that allows us to care about the mathematician. In *Logicomix*, for example, the authors write about Bertrand Russell, about the foundations of modern logic, and one of the many something elses might be the idea that there is a fine line between genius and insanity.

We have read *A Mathematician’s Apology*, Part 29, by Hardy and the introduction to *In Code*, by Flannery, giving you two more examples of mathematical (auto)biographies.

This piece should be about two to three single-spaced pages (skipping spaces between paragraphs), so maybe 1200-1500 words. Please pay particular attention to citations. You have had feedback about when to cite with some of your previous in-class writing exercises.

Please recall the class’s 11 characteristics of good writing and use all that apply to writing this type of piece (a memoir that is also narrative non-fiction). Also, please consider the rules about what not to do.

## Essay #2: Mathematical Film Review Essay

*The students receive this prompt at the start of week 7, and a draft is due during week 8. Essay #2 is due at the end of week 9.*

Since our course is subtitled “Mathematics in Popular Culture,” the film review essay, at its core, should reflect this. That is to say, the film review essay should consider both the mathematical and the cinematic merits in the film you chose to watch (which you should watch at least two or three times).

Note: I use the term "film" loosely. You may use approximately two hours of a TV series as well. I also use the term "mathematical" broadly (computer science, physics, technology or philosophy might be considered acceptable).

GUIDELINES: Recall, you are writing a film review essay. It should be well-organized, well-written, cited properly, and include both descriptions and analyses of the film, where appropriate.

1. The body of the film review essay should be two to three pages, single-spaced in 10-12 point font. So about 1200-1500 words. I will do a word count. No formatting shenanigans, please.
2. Begin your film review essay by providing your mathematical film's credit/data. So: title, director, screenwriter, year, studio, etc.
3. Do not begin your film review essay by telling us whether you liked the piece or not. In fact, you will never write whether you liked the piece or not.
4. Near the beginning of the film review essay, please provide a brief synopsis of the plot, without any spoilers, so that the reader can get a general sense of what happens, of the setting, and of who the major characters are.
5. Describe and comment on the film's mathematical content.
   (a) What is it?
   (b) How much is there?
   (c) How is the math used to support the major filmmaking devices (plot, characters, setting, etc.)?
   (d) How does the author handle dramatic conflict vs. mathematical accuracy?
6. Describe the film's strengths and weaknesses, both in terms of filmmaking/storytelling and in terms of mathematics. Provide supporting evidence where appropriate. Each strength or weakness should be demonstrated by three to five examples in the film. When you cite something that happened in your film, please include time. And you should describe a total of at least three strengths or weaknesses (so for example, describe two strengths and one weakness).
7. Describe the ideal target audience for your film, both in general and in terms of their mathematical preparation, i.e., give your recommendation.
8. You must use formal and proper quotations or citations. When in doubt, cite.

## Final Essay: Embedding Math in the Soul

*We first discuss this assignment during week 8, and the students select their book during week 9. The students receive this prompt at the start of week 10. An outline is due during week 11, and a draft during week 12. The Final Essay is due during final exam week, week 14.*

Your motivation for this assignment is to read a book with a "mathematical main character" (this book could be fiction, fiction based on a real mathematician, or non-fiction) and answer the question, using evidence from the text and not your personal opinion, whether or not this book embeds mathematics in the soul (based on Doxiadis's lecture). If you wonder what I mean by "to embed math in the soul," please read this lecture carefully. Doxiadis describes it very clearly though he calls it something different. (Hint: he proposes four characteristics.) But he also gives a greater context, or mission, for embedding math in the soul. As you write your paper, think about whether or not your book satisfies these four characteristics, and whether or not it can be used for the greater context or mission. Again, the main thing is that you are using examples from the text (both your book and Doxiadis's lecture) to justify your claim. And I reiterate, this is not an opinion essay. You are looking for tangible evidence in your book to either meet the requirements Doxiadis spells out or not.

GUIDELINES: Points will be assigned to the essay as follows.

1. Opening paragraph (10 pts.)
   Do you have a good title? How quickly and efficiently do you tell me what you are going to write about? Introduce your book. Introduce "Embedding Math in the Soul." Transition into the rest of the essay.

2. Summary of the important points in Doxiadis (10 pts.)
   In about one page you will summarize the main points, i.e., the four characteristics and the greater context from Doxiadis's lecture. This should be primarily paraphrased in your own words; however, it will also need to be properly cited.
3. An analysis of the four points (20 pts. total)
   In essence, using evidence from your book, you find examples of each point. When you cite these, include the page number. For example, [2, pp. 77]. If you cannot find an example of the point, then you should include examples of the opposite of Doxiadis's point and describe another piece from our course that does have an example of this point.
4. Can your book be used towards the greater context? (10 pts.)
   Based on the evidence you provided above, show in what ways (if any) your book can be used to accomplish the greater goal Doxiadis describes in his lecture.
5. Citations (10 pts.)
   Use endnotes with authors listed alphabetically (not in order of appearance). Cite where necessary. In essence I will take off one point for every place I think there should have been a citation. You can earn negative points for this section and remember what I wrote about having multiple sentences from the same source. Find an efficient way to cite it. Wikipedia is *not* allowed as a source for this paper.
6. Structure (10 pts.)
   Does this paper have a cohesive flow? There should be a beginning, a middle, an end. The beginning should get you to the middle, the middle to the end. They should all be related.
7. Grammar/style (10 pts.)
   In essence, I will take one point off for every infraction of the Eight Rules. It is possible to lose multiple points on the same sentence, and it is possible to earn negative points for this section. You will lose one point for every sentence I delete for being unnecessary to the assignment.
8. Other/Instructor's discretion (20 pts.)

# 20

# A First-Year Seminar on STEM Breakthroughs and Controversies

Sarah J. Greenwald

**Abstract** A first-year seminar on STEM breakthroughs and controversies offers opportunities to help students appreciate the interconnectedness of the disciplines as they develop critical thinking, communication and other skills. I provide examples of how to intentionally infuse mathematical elements into such a course and reflect on the benefits and challenges.

## 20.1 Background and Context

Appalachian State University, with over 20,000 students, is a comprehensive master's institution and first-year seminar is a university-wide program with sections typically consisting of 20–22 students. The three credit-hour course meets three hours weekly for 15 weeks. First-year seminar is required of all students who enter Appalachian State University within one year of high school graduation or who transfer in with less than 30 credit hours. The course is housed within University College, which was founded to coordinate programs like first-year seminar. The goals of the course are broad and center around practicing critical and creative thinking as well as effective communication. The course also introduces students to local and global connections and the responsibilities of community membership. Students must examine issues from multiple perspectives, learn to conduct quality research, and analyze the arguments of others as they make connections with students, faculty, and the university.

Faculty have great freedom in selecting themes for their classes to satisfy the overall goals, although the topics and courses undergo a rigorous review and approval process by a faculty coordinating committee. It is unfortunate that the seminar proposal process seems to dissuade STEM faculty from teaching it. In Fall of 2009, when the university first offered the course with its present goals, only two out of 93 total sections were connected to STEM themes. I and some other STEM faculty I have spoken to, both those who teach the course and those who do not plan to apply, have found the extensive application unfamiliar and somewhat formidable because it requires us to plan and justify how we will meet and assess the broad general education goals rather than any specific content goals. It helps to share our models although even today it is still the case that most first-year seminar faculty are from the humanities or social sciences and the themes are heavily influenced by those perspectives.

## 20.2 Mathematical Theme

In order to increase the number of STEM-related first-year seminar offerings on campus, I created and have repeatedly offered a whole course on STEM breakthroughs and controversies. One common university expectation about the topics for these seminars is that they cannot be focused on a single discipline. The theme of STEM breakthroughs and controversies allows students to engage with various disciplines, including mathematics, in a fun, creative, and intellectually stimulating environment. In this context the students explore what science and mathematics is, strategies for success in these fields, ethical considerations, humanistic perspectives, public perceptions, and applications to daily tasks. The course also focuses on breakthroughs and the process of discovery as well as the implications of historical and recent developments. We do make connections to mathematical people and ideas regularly (in at least 50% of the classes), but there is no specific content required, only broad general education goals (see the Appendix, p. 204). I have designed the seminar so that the students select many of the topics we cover and these may be more (or less) connected to mathematics, depending on their interests. Ultimately, regardless of what precise topics they select, we make a variety of connections to mathematical topics such as comparing infinite quantities, equations, game theory, geometry of the earth, confidence intervals, correlation, mathematicians, proofs, random numbers, regression, and sabermetrics, just to name a few (see sections 20.3.1 and 20.3.2).

The theme has always attracted a wide variety of students including majors from biology, chemistry, creative writing, building sciences, exercise science, management, mathematics, mathematics education, computer science, fermentation science, finance and banking, journalism, marketing, music industry studies, political science, and physics, as well as undecided students. The gender distribution is evenly split among students that identify as women or men, approximately 95% of the class. The remaining students report as genderqueer. The vast majority of students select this particular theme for their first-year seminar because they are interested in controversy, mathematics, or science. For instance, one year 15 out of 21 students gave science as a reason they selected the course, 14 out of 21 gave controversy, and 10 out of 21 gave mathematics. These numbers are based on textual analysis of survey responses at the start of the semester. It has been common to see more interest in science or controversy than in mathematics in offerings of this course. That same year, no one indicated any other reasons for registering, but in prior courses a few students divulged that it was their advisor or parent who selected the class on their behalf or that they selected it because the time worked well with their schedule.

## 20.3 Course Structure

My overall course structure (see the Appendix, p. 203, for a course calendar) is shaped by the book *Make it Stick: The Science of Successful Learning* [4], which emphasizes that active and varied retrieval is crucial for deep learning. Students first encounter a topic outside of class. They answer homework questions on readings or videos in preparation for engagement during small group and whole class activities. Afterwards they complete a reflection assignment. The students also complete and present two major research projects during the semester.

A standards-based grading rubric connects assignments and classroom activities to the learning goals of the seminar and provides multiple chances to meet each goal (see the Appendix, p. 204). The rubric is quite general and could almost apply to any first-year seminar at the school, with one exception. A main university goal for the course is to analyze the arguments of others in the context of personal perspectives. I purposely modified the goal as follows: "I can analyze the arguments of others and connect them to me including current mathematical/scientific consensus." The additions help students move beyond only the controversies and their personal beliefs to mathematical and scientific research and evidenced-based debate. In designing specific assignments and activities, I look to curricular recommendations from professional associations, including the National Council of Teachers of Mathematics, which recommends that "Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society" [5, p. 5]. Similarly, the Mathematical Association of America (MAA) advocates that instructors and programs should "Guide students to learn mathematics in a way that helps them to better understand its place in society: its meaning, its history, and its uses" [9, p. 5]. Given the broad nature of the goals from the university and the recommendations for the professional associations, I discuss how to infuse mathematical connections into such a structure and describe assignments and classroom activities that have worked particularly well.

### 20.3.1 Assignments

Assignments consist of homework in advance of a class discussion or activity, where students respond to targeted questions that relate to the course topics and goals, reflection pieces afterwards, and research projects. The students select many of the topics we cover as a class as well as what they focus on in reflection assignments and research projects, so they feel a sense of shared ownership in the material as they work toward meeting the course learning goals. This flexibility does mean that the amount of mathematical connections depends on the individual, the type of assignment, and the topic.

### Homework

Mathematical ideas and people permeate the homework or the classroom activities that the homework is a precursor for. For example, students indicate their level of interest in the chapters of the textbook *Taking Sides: Clashing Views in Science, Technology, and Society* [14]. I use this as a way to let them select the chapters we will read and also as an opportunity to connect to voting theory. One popular topic that is almost always selected is "Will the Search for Extraterrestrial Life Ever Succeed?" and this chapter includes the Drake equation, an equation named for Frank Drake that quantifies the probability of extraterrestrial life [14, pp. 213–224]. Another mathematical connection is through Hans Freudenthal, a mathematician who wrote *Lincos: Design of a Language for Cosmic Intercourse* [15]. It is easy to have the students research the role mathematics and mathematicians have played in the various topics we cover, or to bring these linkages into the course myself. I do not infuse mathematics into every topic just as I do not do so with the other STEM disciplines. I select the most interesting connections to STEM people and concepts that relate to the theme and goals of the course. Drake's equation itself is controversial and results in lively discussion, but what I really like is the linkage of both Drake and Freudenthal to the theme of communicating effectively, a goal of the class, as Drake's original motivation was to focus discussion on this topic at a meeting of scientists rather than give a realistic probability estimate [13].

### Reflection Assignments

While the homework assignments and follow-up activities regularly contain mathematical ideas, within reflections they are spotty and highly dependent on the individual. Aside from the title of a reflection and the learning goals that must be satisfied within them (see the Appendix, p. 204), the focus of a reflection is flexible so that students can make connections in a way that most interests them. In the first reflection, students select from among four topics: the geometry of the earth, the sixth extinction, the university common reading book, and successful learning. Many students select the geometry of the earth as a subject. Many of the remaining reflections come from the *Taking Sides* chapters. Students also write a reflection on an interview they conduct with a faculty member on that person's scholarly breakthroughs and controversies in order to help the students make connections to other faculty and the university. The final reflection is on university sponsored activities which includes a requirement to attend and reflect on at least one STEM-related event. The Appendix shows how the reflections are spread throughout the course (p. 203) and it contains the Reflection Assignments Prompt and Rubric (p. 205). Students bring in current mathematical/scientific consensus as part of a learning goal. In-class peer review of reflections, where students swap papers with each other, further helps students engage with the learning goals. Even though mathematical perspectives are not necessarily a large part of these reflection assignments, the other aspects of the theme do play a significant role, with many interesting reflections related to controversies, breakthroughs, and science. Since I am not teaching a mathematics class, I do not view this as a problem, especially because mathematics is well represented in other aspects of the class.

### Research Projects

The publication-quality research projects articulate STEM research and analysis. In the first research project, students create a historical timeline and annotated bibliography that explores the interesting and important scientific and mathematical breakthroughs and controversies of a topic they select (see the Appendix, p. 206). They typically select a person, theory, invention, or health issue. A few sample topics that students have selected in the past include Leonardo da Vinci, Stephen Hawking, game theory, general relativity, cloning, and concussions. Since the contributions and mathematical breakthroughs of mathematicians and diverse cultures is specifically a component of the project all students pull in some mathematical concepts, some more than others. In the second research project, students research an unsolved STEM question or problem of interest. They conduct a literature review and create a list of references, summarize conflicting viewpoints, and explore mathematical or scientific consensus as well as their own ideas. Here is a small portion of a student reflection on the question of whether odd perfect numbers exist:

> After researching the topic I have found that the chances of perfect odd numbers existing are considered very slim by most. Most researchers that even tried to find odd perfect numbers always end up restricting the chances of there actually being one. Since there are an endless supply of numbers I believe that there could be a day where we find one. The only thing I see preventing this is if someone finds a limit to how high the number can be.

Students present the research projects in poster sessions, where the class conducts peer review, so even if their own project is light on mathematicians or mathematics, they will peer review other projects that are more mathematical.

## 20.3.2 Classroom Engagement

One of the goals of the course is to make connections within our classroom community through a shared process of inquiry, so each class is full of diverse engagement techniques. I plan classroom activities to complement the assignments and ensure a breadth of mathematical topics. Effective engagement includes active listening as well as respectful and meaningful participation and a willingness to complete activities. It forms a portion of the course grade as in the Appendix, p. 204. Some classes begin with a worksheet that requires students to analyze content and work on creative solutions to real-life problems. We form groups using icebreakers, some of which are mathematical as described in the next section. Sometimes we watch a video and discuss it or conduct library research online. We also use a classroom response system. The anonymity allows us to gauge class attitudes about controversial issues and check in to see whether the class understood the material. I often use a think-pair-share model where individuals answer questions on their own and discuss material in small groups before we come back together to have a larger class discussion. To give a sense of how mathematics is regularly a part of the classroom, the following are examples of some of the mathematical activities.

### Mathematical Icebreakers and Games

Byrne discusses the challenge of getting students to actively engage in mathematical inquiry and uses games "as a vehicle for generating deep discovery" [5, p. 272]. In our class, using a random sequence generator is an easy way to form groups as we discuss probability and randomness. Another method is to have students cross their arms and break up into groups by whichever arm is on top. We follow up by examining research articles on lateral preferences for arm folding and the regression analysis that is a part of these articles. One icebreaker that I use more than once in a semester is to cut up mathematical cartoons or images from research articles into the number of puzzle pieces that I want in each group. The students must match the pieces, discuss the full image, and then present the mathematics to the rest of the class. Another icebreaker requires students to initially form a baseball-related team consisting of a catcher, hitter, hot dog seller, pitcher, and sabermetric statistician. Since these groups tend to be composed of people who sit near each other, we reform into what I call "specialty conferences," such as for the catchers or for the sabermetric statisticians. Next these reformed groups discuss the role of sabermetrics in sports. Other icebreakers during the semester connect to science. My icebreakers are not as mathematically deep as Byrne's games, but they are successful in getting students to engage with mathematics and statistics as they get to know each other better.

A segment on games and game theory was modeled after those used in the Consortium of Mathematics and Its Applications (COMAP) Mathematical Contest in Modeling (MCM) (`http://www.comap.com`). Each group is assigned a different game related to Hex, a Knight's Tour, Nim, Texas Hold-em, or the Tower of Hanoi and plays the game as they work to answer questions. Groups present their breakthroughs to the rest of the class as well as a reflection on the process of discovery. They follow up outside of class by continuing to work on the problems or by researching information. In a future class we discuss their homework responses as well as the history of these games and connections to recent research in mathematics, computer science, and other fields. A number of students report that this is their favorite classroom segment.

### Controversy in Truth and Reality

One theme that we explore over the entire semester relates to truth and reality, including how and when we are convinced that something is true and the role of chance and probability. We begin this on the first day of class during a discussion of how we can tell the earth is round, where there is undeniable evidence and yet flat earthers are in the current headlines, and we continue this theme in almost every topic we cover. The American Statistical Association (ASA) advises that "Students require exposure to and practice with a variety of predictive and explanatory models in addition to methods for model building and assessment. They must be able to understand issues of design, confounding, and bias" [3, p. 4]. We do not cover these issues in the depth that a statistical class would but we do compare and contrast proofs and statistical significance, including correlation versus causation and reproducibility of results in the *Taking Sides* chapters as well as Bradford Hill's criteria to determine causation in the relationship between smoking and cancer [19]. Predictions, confidence intervals and the 2016 election also work well to generate discussion. Proofs themselves do not escape controversy as we explore the history of comparing infinite quantities from Galileo Galilei, who said we could not compare infinite sets, to Georg Cantor's work, which was challenged by mathematicians and theologians of his time. We also discuss Gödel's incompleteness theorems, questions about computer proofs, and whether the proof is in the picture. The precision of $\frac{385}{23}$ versus "about 16.7" and controversies in representing reality and how and what we teach originate in the video "Down with Fractions" [12] and leads to lively debate.

### Ethical Considerations

The American Chemical Society specifies that professional ethics should be included in the curriculum so that students understand the relationship between chemistry and society and the responsibilities of a chemist to society [2, p. 1], and the ASA recommends that students "must have a clear understanding of ethical standards" [3, p. 4]. A number of activities towards the end of the semester are focused on ethical considerations and standards.

Each small group debates a different case study adapted from Goldfarb and Pritchard [16], which they present to the rest of the class. In one scenario, a student in a lab suspects that the equipment is malfunctioning, but her instructor is occupied. So she performs the mathematical calculations to compute what a correct data set would be and modifies the data to match the computations. We further explore the ethics of data by engaging in discussions about whether it is appropriate for scientists and mathematicians today to use data obtained under what we agree were morally reprehensible conditions, as in Nazi Germany or the Tuskegee syphilis experiments. The students write down and share what they think are the strongest argument(s) from both sides. Next we examine a related article [8] as well as responses from professional and governmental associations. The students also discuss the ethics of transplanting a murdered person's heart without consent or using forensic evidence obtained during illegal search and seizure.

As we investigate careers in STEM, we discuss quotations like the following from mathematician David Blackwell: "So, when I saw that this led to an armaments race, so to speak, I realized I was not the one to come up with a satisfactory theory for non-zero-sum games" [1, p. 21]. We play *Friend or Foe* based on the game show whose use in the mathematics classroom can be found in the article "Prisoner's Dilemma Applied and in the Classroom: The TV Game Show *Friend or Foe*" [7]. We connect this to the National Security Agency, as the stated number one employer of mathematicians, and reasons why mathematicians may or may not want to work for the agency [20] or on specific topics as Blackwell mentioned. We also discuss whether mathematicians should be held responsible for any future unanticipated uses of their work such as bio-weapons, computer viruses, or code-breaking algorithms. Students consider the statement "The Manhattan Project exemplifies many of the moral dilemmas faced by mathematical scientists" [17, p. 357] to identify the ethical conflicts.

### The Contributions of Mathematicians and Statisticians

Stories of mathematicians and statisticians and details of their contributions can help students connect to mathematics and inspire them [18]. The MAA recommends that we "introduce historical and contemporary topics and applications, highlighting the vitality and importance of modern mathematics, and the contributions of diverse cultures" [10, p. 11]. Diverse people and cultures and their breakthroughs are woven throughout the seminar, as some of the prior examples illustrate. Typically I present or have the students research the educational backgrounds of any names we connect to course content in order to showcase a variety of career paths. We also look at quotations, audio, or videos whenever possible to further humanize the people.

Bringing in mathematicians and scientists with connections to the school can also be a powerful motivator. Students respond well to video of Helen Moore from `WeUseMath.org`. Moore has a PhD in differential geometry as I do. From our somewhat common starting points, our pathways and current specialties demonstrate the flexibility that mathematicians have in their professional life. Moore is a research mathematician who uses mathematical models to optimize therapies for patients and the drug development process [11], while I have ventured from differential geometry into mathematical connections to society. To further help the students connect to me, which is one of the expectations of the course, I discuss controversies and breakthroughs in my own scholarship. Another link to our school is alumnus Michael Coble, who earned his undergraduate degree in biology and is the lead author on a paper we examine to showcase how statistical methods are applied in identifying the remains of the Romanov children via "virtually irrefutable evidence" [6, p. 1]. Coble recorded audio advice to the students in my seminar as he relayed his own experiences as a first-year student, which we discuss. We also discuss the diverse views of nonspecialists today about whether the children were able to flee Russia and connect this to the distrust of scientists and scientific consensus.

## 20.4 Reflections

I have taught the class six times, using feedback from students to improve the course each time, and while the course is working well, I will make more changes in the future. In official evaluations and surveys most students say they would not change anything about the course. However, a few students were confused about standards-based grading initially. I did provide sample pathways at the beginning of the semester and very regular feedback throughout about their standing in the course, which alleviated the confusion eventually, but it did take some time for students to get used to the course rubric (see the Appendix, p. 204). One idea I have to help more students understand the grading system even earlier would be for small groups to analyze the rubric using different scenarios, which they would then present to the others. One student thought that an A should be easier to obtain, but I disagree and instead I plan to bring grade expectations into the discussion on successful learning. In terms of continuing to help the students make connections to STEM, I will try a concluding icebreaker in the future. Each student in the class would have a different strip of paper, either a concept or course topic. Matching these would form groups of two and I think this will work well to review and further help the students see just how important mathematics and science was in the course and in real-life applications.

Overall, the students and I concur that they are meeting the learning goals but I have improvements to make in the future. The most recent course evaluation form specifically asks about this. In this official course evaluation 100% of the class agreed that the course had them practicing critical thinking, communication effectively, and encouraging them to analyze a single issue from multiple perspectives. These were separate questions on the evaluation form. The students also unanimously agreed that they were required to complete library research projects. The results about other learning goals were less positive, but still higher than the combined percentages of all other first-year seminars. When asked about whether the course introduced concepts related to the responsibilities of community membership, 75% agreed, 10% disagreed, and 15% were unsure, compared with 71%, 15%, and 14% from all other sections. In response to asking about whether the course introduced concepts related to making local to global connections, 90% agreed, 0% disagreed, and 10% were unsure, compared with 78%, 11%, and 10% from all other sections. While I am only required to introduce these last two goals, I still want the students to understand that we did indeed do that, so I will work to clarify those in the future. While a stated expectation in the catalog description of the seminar is that the course facilitates engagement with fellow students and the university, this is only partially assessed on evaluations, by asking whether the course helps "make co-curricular connections within the university." This is an area where I and the other first-year seminar sections will have the most improvements to make as 57% agreed, 24% disagreed, and 19% were unsure in our section, compared with 55%, 21%, and 24% from all other sections. Even though it was not assessed specifically, evaluation comments show that students saw that the course helped students engage with fellow students and faculty.

There are great rewards in teaching this course. The students responded extremely positively as evidenced by their engagement in the class and the results of course evaluations. Typical comments over the six times I have taught the class include 'This class was extremely interesting and I would recommend more sections of this course," "IT WAS GREATTTT," and "I had a wonderful experience." When I surveyed students anonymously about their favorite activities, the most popular responses were the geometry of the earth, game theory, and the research projects. A representative comment is "I loved all the topics and my favorite part was the research projects because we had so much freedom on what to focus on." Individual students commented that they appreciated "having in-depth conversations with classmates based on controversies occurring in math and science" and valued how the seminar "connects math with science and many other courses that interconnect." In addition, 100% of the students from the most recent course agreed that they gave the course their best effort. Teaching the course provides a fun way to get to know students well and see STEM and non-STEM majors develop over the semester. I and they really enjoy thinking deeply about societal connections to mathematics and science and engaging with each other around these issues. A peer evaluation was very positive and there were no suggestions for improvement. Among other comments, the peer wrote that "The structure of this class period was impressive. Sarah engaged the students in the debate in a carefully crafted process that allowed students to think about and discuss the issues of this problem (informed by their reading & homework before class). She gradually added more scholarly voices (via websites & slides) to the conversation, which allowed the students to deepen their small group conversations." I could not put it better than this comment from student evaluations: "Having to consider which side of a controversy or argument you agree with while arguing for it in a reasonable and mature way is essential."

## 20.5 Bibliography


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# Appendix

## Course Calendar for Two-Day a Week Class

*This is the course calendar for a two-day a week class, where each row is a week. The first and third columns show what assignments students completed between classes. The second and fourth columns are the activities that occurred during the one hour and fifteen minute classes on Tuesday and Thursday, respectively. The course calendar was provided to students at the beginning of the semester and referred to regularly throughout the semester.*

| Assignments | Class Session | Assignments | Class Session |
|---|---|---|---|
| | geometry of the earth<br>academic reading<br>sixth extinction | common reading book | common reading book<br>current events in STEM |
| *Make it Stick* [4]<br>syllabus<br>survey | successful learning<br>good writing<br>advice from prior students<br>voting theory<br>STEM consensus | reflection 1 (choose one)<br>geometry of the earth<br>sixth extinction<br>common reading book<br>successful learning | reflection 1 peer review<br>general education<br>classroom etiquette |
| homework on<br>*Taking Sides* topic 1 | sample learning goals<br>*Taking Sides* discussion | reflection 1 revision | faculty scholarship<br>statistical significance |
| reflection 2 on<br>*Taking Sides* | reflection 2 peer review | homework on<br>current events in STEM | current events in STEM |
| reflection 2 revision | my scholarship | reflection 3 on faculty interview | faculty interview |
| homework on<br>*Taking Sides* topic 2 | *Taking Sides* discussion | reflection 4 on *Taking Sides* | proof<br>chance and probability |
| library module 1 | scientific method<br>breakthroughs amidst the controversies | library module 2<br>choose a topic for project | research for project 1 |
| *Radium Girls*<br>performance | *Radium Girls* | fall break | fall break |
| communicating project 1 | project 1 presentations | begin working on reflection 7 on university sponsored activities | project 1 presentations continued |
| homework on<br>*Taking Sides* topic 3 | *Taking Sides* discussion | project 1 revisions<br>reflection 5 on*Taking Sides* | Mental Health Ambassadors |
| reflection 5 on<br>*Taking Sides* | generating knowledge | project 1 revisions | research on games |
| homework on placebos | placebos | homework on games | ethics |
| homework on<br>*Taking Sides* topic 4 | *Taking Sides* discussion | reflection 6 on *Taking Sides* | ethics continued |
| library module 3 | author credibility | Thanksgiving break | Thanksgiving break |
| library module 4 | unsolved questions | research for project 2 | research and evaluations |
| reflection 7 on university sponsored activities | concluding activities<br>STEM careers | research for project 2 | reading day |
| communicating project 2 | project 2 presentations | | |

## Course Rubric

*This is the grading rubric for the course grade provided to students at the beginning of the semester. Students regularly received updates on their progress on this rubric throughout the semester, with each reflection assignment and research project, so that they could see what targets they had already satisfied and what remained for them to complete.*

| | C | A |
|---|---|---|
| I can make a good faith effort in homework questions. | 5 | 9 |
| I can produce thoughtful reflections. | 2 | 5 |
| I can produce quality projects. | 2 attempts<br>1 proficient | 1 proficient<br>1 outstanding |
| I can examine a single issue from multiple perspectives, e.g. local and global connections. | 1 reflection<br>1 project | 2 reflections<br>2 projects |
| I can conduct quality research and summarize what I found. | 1 project | 1 reflection<br>2 projects |
| I can analyze the arguments of others and connect them to me including current mathematical/scientific consensus. | 1 reflection<br>attempt project | 2 reflections<br>1 project |
| I can communicate effectively to produce publication-quality written, verbal and visual work in a logical, organized manner that demonstrates consideration of context, audience, and purpose. | 1 reflection<br>1 project | 2 reflections<br>1 project |
| I can make connections with others and the university through a shared process of inquiry via regular attendance and effective engagement in our classroom community. | $\leq$ 8 absences and effective engagement | regular attendance and effective engagement |

Turning in all work due and effectively engaging in class activities distinguishes between -, regular, or + in the grade. To earn a grade in the B range complete C requirements plus half way to an A. There are two pathways to achieve such a B. One is to satisfy at least 50% of the A requirements (four of the eight requirements) with the others at the C level. The other is to surpass every one of the C requirements at the half way mark between A and C. The D range is half way to C.

## Reflection Assignments Prompt and Rubric

*This is the text that was provided to students as a guideline and grading rubric for the seven reflection assignments assigned throughout the semester, one approximately every two weeks.*

Reflections are typed or spoken expositions, typically the equivalent of one page of single-spaced text. They are due after readings, class discussions and activities on the topic. Aside from the topic, the focus is flexible so that you can follow up on and make connections to your own interests. For each reflection, list any learning goals that you want me to assess, if there are any. Each individual reflection may address any, all, or none of the course learning goals. You will receive feedback from peers and myself and will have a chance to revise the first reflection. A successful reflection contains creative reflection and personalization as well as supporting arguments:

| | Needs Improvement | A Successful Reflection |
|---|---|---|
| Reflection and Personalization | Minimal reflection or personalization. | Creative reflection on, and personalization of theories, concepts, and/or strategies related to the topic, typically the equivalent of one page of single-spaced text. |
| Support of Arguments | Viewpoints and interpretations are unsupported or supported with flawed arguments. | Viewpoints and interpretations are supported appropriately. Suitable examples are provided, as applicable. |

For instance, to personalize, you could include a discussion of what you agree and disagree with and why, or what you found most compelling and why. A successful learning goal addresses all components of the learning goal:

| | Needs Improvement | A Successful Learning Goal |
|---|---|---|
| | | If there are any learning goals you want me to assess then they are identified. |
| Learning Goal 1 | Multiple perspectives are unclear or unsuccessful. | Examines a single issue from multiple perspectives, e.g., diverse and opposing viewpoints, historical and recent perspectives, local and global connections. |
| Learning Goal 2 | Missing research or a summary, or the research is not of good quality. | Demonstrates quality research and summarizes it. |
| Learning Goal 3 | Missing scientific consensus, analysis, or personalization. | Analyzes and personalizes the arguments of others, including scientific consensus. |
| Learning Goal 4 | The writing, speaking, or visual work could use improvement. | Communicates effectively to produce publication-quality written, verbal and visual work in a logical, organized manner that demonstrates consideration of context, audience, and purpose. |

## First Research Project Prompt and Rubric

*This is the text that was provided to students as a guideline and grading rubric for the first research project, which the students presented midway through the semester.*

Research and then create an attractive and publication-quality historical timeline that explores the interesting and important breakthroughs and controversies that relate to science and/or mathematics—not the entire history. Be sure that the timeline is in your own words and includes important contributions from diverse scientists or mathematicians around the world, noting what kind of scientist and where they are from, as well as interesting pictures, and that the scientific and/or mathematical connections are clear. Approximate dates can be noted as ∼1762 or by a range of dates, such as 1700–1800. A maximum of two-pages will be allowed (not including references). If you can, also include some modern connections or future work.

Use many different types of sources, including scholarly references and library sources. Submit a separate annotated bibliography of all of the sources you used in the timeline, with annotations explaining how you used each reference in your timeline, where the pictures came from, etc. Use as many pages as you need for the annotated bibliography.

The presentation sessions are similar to poster presentations at research conferences or science fairs. Bring a printed version of your timeline and annotated bibliography to class to post on the wall. We will divide the class into two sessions. Half the class will stand next to the timeline as the other half examines them, and then we will switch roles. During your session, you must stand by your timeline to discuss your topic and answer questions. If you work with another person, they will be in the other session so you should be prepared to present the entire project. The presentation component typically involves a group of one or two students at a time listening to your presentation and looking at your project so they can take notes for peer review.

| Project 1 Rubric | Needs Improvement<br><br>missing numerous | Proficient<br><br>generally demonstrates | Outstanding<br><br>thoroughly demonstrates |
|---|---|---|---|
| I can examine a single issue from multiple perspectives.<br>• Interesting and important controversies<br>• Interesting and important scientific breakthroughs and their timeframes (can use ∼ or range of dates), not the entire history of a topic<br>• Contributions from diverse scientist/mathematicians from around the world<br>• Scientific/mathematical pictures | | | |
| I can conduct and summarize quality research.<br>• Contains quality research, citations and annotations, and sources of pictures<br>• Items are firmly grounded in science/mathematics<br>• Researches what kind of scientist and where from<br>• Researches some modern connections, if possible | | | |
| I can communicate effectively to produce publication-quality written, verbal and visual work in a logical, organized manner that demonstrates consideration of context, audience, and purpose.<br>• Publication-quality exposition, consistent format, own words<br>• Attractive display, timeline portion is two pages<br>• Quality peer review | | | |

# 21

# Uncovering the Hidden Figures

Cynthia Farthing

**Abstract** This article describes a first-year seminar based upon the book *Hidden Figures* written by Margot Lee Shetterly. This seminar explores women's contributions to the development of mathematics and technology starting in World War II and continuing through the race to send an astronaut to the moon. This seminar identifies historical and political factors that have influenced changes in science and technology since World War II. It also examines reasons why certain groups—including African-Americans, Latinos, and women—are not participating in math at higher rates. This article includes key readings, films, and discussion topics that resulted in an engaging seminar.

## 21.1 Background and Context

The University of Iowa is a public institution with approximately 24,000 undergraduate students and 8,000 graduate and professional students [17]. First-year seminars (FYS) are available to traditional students during their first fall semester. Students are not required to take an FYS, but they are encouraged to do so as students who take FYS report positive experiences and get to know faculty and peers in a smaller classroom setting. On average, about 120 FYS are offered each year, and approximately 30% to 40% of first-year students enroll in an FYS.

Instructors who wish to teach an FYS submit a proposal to the Associate Provost for Undergraduate Education. If a proposal is accepted, the FYS is placed on the schedule for the following fall and runs if enrollment in the seminar is high enough. First-year seminars at the University of Iowa usually enroll 15 to 20 students and are worth one credit hour, which means they meet for the equivalent of 50 minutes each week during a 15-week semester.

The topics of FYS vary greatly, but they are designed to introduce students to the "intellectual life and academics" at the university. Courses must develop students' skills in critical thinking and effective communication in an environment that encourages personal growth by allowing students to reflect on their own values and beliefs [18]. Some FYS are designed for students in certain majors and introduce them to their field of study (for example, *Accounting: The Language of Business That Leads to Many Different Career Options* and *Journey to Medical School*). Others are based on topics of interest to the instructors who want to interact with first-year students in a classroom environment different than they usually work in (such as *Tsunamis: Deadly Waves of Destruction*, an earth and environmental sciences course; *Bicycles, Kayaks, and Snowshoes: Writing About Travel*, a rhetoric class; *Medieval Health Coach*, a history course about medicine in the middle ages; *The Politics of Sin*, a political science class exploring the politics behind topics like gambling, drugs, and alcohol). For the FYS described here, the topic is related to mathematics, but the focus of the course is on the people doing mathematics and the environment in which the research is done. Students who take this FYS come away with an appreciation for mathematicians and the power of mathematics more than specific mathematical skills that a student would acquire in a typical math course.

## 21.2 Mathematical Theme

The full title of this FYS is *Uncovering the "Hidden Figures": Women's Role in Mathematics and Science from World War II to Today*. As the title suggests, the seminar examines the significant role that women, and African-American women in particular, have played in mathematical research since the 1940s. It is inspired by the book *Hidden Figures: the American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race* by Margot Lee Shetterly [11]. Shetterly tells the story of three African-American mathematicians who began working at the National Advisory Committee for Aeronautics (also known as the NACA and has since become NASA) during and after World War II: Dorothy Vaughan, Mary Jackson, and Katherine Johnson. Dorothy Vaughan was one of the first African-American women hired into the West Computing group, the segregated office of women computers employed by the NACA to help perform calculations required to develop aircraft and eventually send a person to the moon. Dorothy Vaughan became the first African-American supervisor at the NACA and eventually retrained herself as a computer programmer once the West Computing group was disbanded [20, p. 68]. Mary Jackson was the first woman African-American engineer employed at the NACA; she worked in aeronautical research analyzing wind tunnel and flight experiments [20, p. 80]. Katherine Johnson's work was key in calculating the trajectory of Alan Shephard's first flight into space, John Glenn's orbit around the Earth, and the lunar landing mission of Apollo 11 [20, pp. 73–75].

The women profiled in *Hidden Figures* are one of several groups of women that worked in STEM fields and made important contributions to scientific and technological innovations during this time. Therefore this seminar investigates other examples of women in science who have been overlooked. Students also give research presentations on female scientists related to the course material. Approximately one half of the semester is devoted to learning about these scientists. During the other half of the semester, we cover a variety of topics that relate to the issues discussed in *Hidden Figures*. The fact that the word "computer" used to refer to a job title rather than a machine points to the huge impact that the development of computers has had on science. In addition, students are asked to explore how mathematics has been influenced by historical events and political factors, including WWII, the Cold War, segregation, and the civil rights movement. Why are African-Americans, Latinos, Native Americans, and women underrepresented in STEM fields [21]? What can we do to make STEM fields more diverse? Asking these questions leads to a discussion of how gender roles, implicit bias, and university and public policies affect participation in STEM.

This FYS requires students to have very little, if any, prerequisite knowledge of mathematics. Most of the content focuses on historical events and issues in STEM fields. The activity showing students how to use slide rules outlined in the Appendix (p. 217) is the only activity that requires students to do any calculations. This accounts for about 5% of the course and could be eliminated. Although this class is designed for a general student audience, there are many ways the mathematical content could be increased; some ideas are given in Section 21.4.

## 21.3 Course Structure

This course meets for 50 minutes once each week during a 15-week semester. The final grade is based on student participation (25%), a group presentation (25%), four response papers (25%), smaller periodic assignments (10%), attendance (10%), and an office visit with the instructor during the first three weeks of the semester (5%). Many of the class meetings involve small group and class discussions of the readings from the previous week, which is why class participation and attendance account for so much of the final grade.

The main learning objectives for the seminar help to guide the assignments and discussion. They include:

- Identify events and political pressures during and after WWII that provided women with new opportunities in scientific research.
- Describe the challenges resulting from segregation that African-American women faced while working at NASA and other institutions.
- Identify factors that may prevent women from participating in STEM careers and factors that may cause women to leave STEM careers.
- Describe how computers have changed the way mathematics and science is done.
- Investigate and research a woman in science or a current area of research in science and technology.

The rest of this section identifies key readings, films, and discussion topics that help to achieve the above learning objectives. Section 21.4 discusses some ideas to adjust the course and extend the topic to different areas of study.

### 21.3.1 *Hidden Figures*

As the key inspiration for the course, the themes in *Hidden Figures* contribute to many of the learning objectives. Dorothy Vaughan began working at the NACA (now NASA) during WWII when the need for human computers grew considerably. After WWII ended, the race with the Soviet Union to be the first country to send a person to space continued to increase opportunities for African-American women to work with research groups at NASA.

*Hidden Figures* gives great insight into the current events that influenced the women in the book. Key events in the civil rights movement, such as *Brown v. Board of Education* and the desegregation of schools in the southern United States, are discussed alongside the personal experiences of the African-Americans working at NASA. Other events, like the launching of Sputnik, highlight how outside factors can affect the development of the entire field of mathematics. Tragedies, including the assassinations of President John Kennedy and Dr. Martin Luther King, Jr., or victories, like Neil Armstrong taking the first step on the moon, are experiences shared by entire populations. One interesting class discussion involves asking students to find events similar to these that happened during their lifetimes. It helps to connect the themes of the book to what is going on in the world today as well as providing an opportunity to discover experiences shared among the students.

Another theme in *Hidden Figures* is how computers changed the way science and mathematics were done. The invention of computers allowed for the complex calcuations done by the human computers to be completed by machines. Students spend one day of class learning how to use a slide rule which makes it abundantly clear how much computers have changed how we perform calculations. More details about this assignment are provided in the Appendix (p. 217). As a result of technological advances, the human computer groups at NASA were eventually disbanded, and women working in these groups needed to grow and adapt to remain relevant at NASA. A discussion topic or response paper related to this asks students how changing technology affects skills that workers have. It also provides a chance to discuss how a lack of understanding of science, mathematics, and technology can hamper a person's employment opportunities. This is related to equality in STEM issues as discussed in Section 21.3.3 and is important for first-year university students to think about.

*Hidden Figures* concludes with a look into how women's status in the workplace has changed since WWII. When Mary Jackson and other women were hired at the NACA, women could not be hired as engineers (a rank higher than a mathematician). Mary Jackson had to work very hard to earn her rank as an engineer, and promotions for women at NASA Langley were more difficult for women to obtain than men [11, p.255]. By the 1970s, Mary Jackson held the position titled Federal Women's Program Manager in the Human Resources Division, and in this role, she worked to help women advance at NASA Langley. Women began questioning the status quo, asking administrators why men were often hired to work in engineering groups, while women were initially hired into computing groups [11, p. 261]. Women were becoming less willing to accept inequality and wanted their work to be judged fairly regardless of gender. Women are still striving to be promoted to higher ranked positions at the same rate as men, and women on average earn less than men doing the same job. STEM fields are certainly not immune to this phenomenon, and this provides another way to connect current events with the historical perspective given in *Hidden Figures*.

### 21.3.2 *Top Secret Rosies*

In addition to the women computers working at the NACA during World War II, there were several other examples of women who used their mathematical skills to help the war effort [5, 7, 9], including a group of women who worked at the University of Pennsylvania Ballistic Computing Lab profiled in the film *Top Secret Rosies*. The film details the development of one of the first computers, the ENIAC (Electronic Numerical Integrator and Computer). It also highlights the contributions women made programming the ENIAC and discusses why these contributions were not recognized for many years. Many of the women working at Penn were white, single, and for the most part left the workplace after WWII ended while the women in *Hidden Figures* were married with children and needed to work to help support their families. These differences make for a good written response paper that allow students to compare and contrast the experiences of the two groups and reflect on the sociological reasons behind them.

*Top Secret Rosies* also discusses how the use of technology affected the way war was fought (such as airplane bombers and nuclear weapons). This film provides an opportunity to discuss the ethical considerations that go into fighting a war, and in particular the ethical issues scientists grapple with if the discoveries and technology they develop can be used as a weapon. Students are asked to find an example of a scientific discovery or technological development that came out of WWII. Examples include radar, penicillin, and nuclear technology, demonstrating that war can lead to both helpful and destructive innovations.

### 21.3.3 Equality in STEM

When you look at the data compiled by the National Science Foundation, the fact that there are groups underrepresented in STEM is abundantly clear [21]. There are many factors that contribute to this. In fact, entire courses could be devoted to this topic alone. In this course, we discuss why it matters if we have equal representation in STEM fields. We also begin to discuss some of the factors contributing to it, including implicit bias, gender schema, and lack of mentors.

Students examine the NSF data on their own [21] and answer a variety of questions about the data. What statistic stood out to them the most? Was any data surprising to them? What are some possible reasons contributing to the data? Students then explore some data collected by the National Department of Labor that shows the highest paying jobs [15]. Many of these jobs are in engineering and computer science—two of the STEM fields with the lowest participation rates of women, African-Americans, and Latinos. The class thinks about the experiences of Dorothy Vaughan and the women who programmed the ENIAC and compares the participation rates in computer science when the field was just beginning to those today. Students share why this matters and brainstorm how to change these statistics. Students are usually very engaged in this discussion because it highlights what is going on in today's workforce. It is often inspiring to hear the ideas students offer and see the passion students have for making changes.

To start a discussion of implicit bias, students are asked to list the top five most common professions for women and for men, and compare it with data from the Department of Labor [16]. Students usually list jobs like teacher, secretary, nurse, and dental hygienist for women. The top careers for men typically include doctor, engineer, businessman, lawyer, and truck driver. In my experience, students are generally more accurate with the women's professions than the men's, and often the men's professions are associated with a higher level of status than the women's. This provides a great introduction into implicit bias and how it can affect our view of the world. Students read a portion of the book *Who's Afraid of Marie Curie?* [4, pp. 81–106] which gives an overview of bias toward women in science. Students then investigate the idea of gender schema: "a set of implicit, or nonconscious hypotheses about sex differences" that influence how we evaluate men and women [19, p.2]. One example is that a man who takes charge and gives others directions might be called a good leader while a woman who behaves the same way might be called bossy. Gender schema affect how boys and girls act as they become men and women. Students discuss how schema can affect what toys boys and girls play with, what chores they are asked to do around the house, and how schema influence career choices. Schema also exist about race and can affect the opportunities people from minority groups have and how hard they must work to prove themselves in careers where they are underrepresented. Students are encouraged to come up with possibilities of how to make representation in STEM fields more equal.

Students also examine the importance of role models and mentors in choosing their career paths. Students either write a response paper or a short response in class about a time when they wish they had a role model to look up to or a mentor to offer advice. This topic relates nicely to *Hidden Figures* because Shetterly discusses how the women of the West Computing group had no one to offer advice on their career paths; they had to blaze the trail in the best way they knew how [11, Chapter 9]. The documentary *Mercury 13* also discusses how important role models can be while it introduces us to another group of women many students have never heard of. The film profiles 13 women pilots who went through the same battery of tests as the Mercury 7 astronauts (the group of men from which the first astronauts were chosen, including Alan Shephard and John Glenn). These women petitioned the Senate for the opportunity to compete for their chance in space, but sexism and gender roles of the time kept them from pursuing these dreams. The film asks what would have happened if the first person on the moon had been a woman. Of course, we cannot know how rewriting history would change our current world, but these sorts of "What if ... ?" questions allow for good discussions that students enjoy.

### 21.3.4 Modern Women in Science

Building on the idea that we need to celebrate more women in science, this seminar aims to highlight the careers of numerous female scientists. Students learn about scientists of the past that are related to computer science and flight technology: Grace Hopper (computer science), Dorothy Vaughan, Mary Jackson, Annie Easley (aeronautics research), Katherine Johnson, and Margaret Hamilton (computer scientist who helped develop the guidance and navigation system for Apollo program [20, p. 94]). Moreover, they learn about modern female mathematicians and scientists. Excellent resources that highlight a diverse group of women include: the books *Power In Numbers: The Rebel Women of Mathematics* [20] and *Headstrong: 52 Women Who Changed Science - and the World* [13], the *Time* article, “Firsts: Women Who Are Changing the World,” and the websites “Mathematically Gifted & Black” [10] and “Latinxs and Hispanics in Mathematical Sciences” [8]. The goal is that students will be able to name women of all backgrounds who are doing science so that they can learn from the paths these women have taken and perhaps find inspiration for their futures.

In addition to readings about particular scientists, students are encouraged to do their final group presentation on a woman in science. Students can do a traditional research project, or they can interview a scientist about her experiences in graduate school and research areas. One class is spent talking with research librarians about where to find ideas for presentations and how to evaluate the quality of a source of information. At the end of the semester, students present their research to the entire class to further expand their knowledge of current scientists. Details about the group presentation are provided in the Appendix (p. 214).

## 21.4 Reflections

This course has been taught twice. For the most part, the student evaluations were positive. Many students enjoyed finding connections between the experiences of the women in *Hidden Figures* and people today. Some students were inspired to think more broadly about their career choices, and others were motivated to promote equity and diversity. The hope and optimism of many of these students was very encouraging to see. The energy of the students taking this first-year seminar is different than what I usually experience in my usual math courses. Teaching this first-year seminar was a nice change of pace from my typical teaching assignments, and overall, I had a lot of fun.

I have made adjustments to the course based on student feedback. Discussion of the course material makes up a large portion of the class time. The first time the course was offered, students usually spent time discussing their opinions and responses to questions in small groups before discussing the topic with the entire class. By the end of the semester, this process became a bit boring for me and the students. A variety of discussion techniques are used now to solicit students’ views of the reading assignments, many of which are described in [1]. Students responded postively to these changes. For the next course offering, I will ask students to work in groups to lead different classroom discussions; this will help to assess the key points students think are important. When the course was first offered, students wrote a weekly response paper on the assignment or class discussion. Students now write four response papers throughout the semester. In addition, I provide writing prompts (examples provided in the Appendix, p. 213) to help students focus their ideas and help assess what students are getting from the readings and discussions. When I reduced the number of response papers required, I added “smaller periodic assignments” as a grade category in the syllabus. The purpose of these smaller assignments is to hold students accountable on weeks when there is no response paper due. Examples of these smaller assignments include submitting discussion questions, finding an invention developed during WWII, and finding a current events story that relates to the reading material. In the future, I plan to ask students to interview women on campus working in STEM; this will provide another opportunity for students to connect with other faculty and researchers as well as give the class some insights on what sort of science is done in their neighborhood. In general, I found that the smaller assignments helped to make the classroom discussion more lively because students knew what to prepare ahead of time, and I think they enjoyed sharing what they had learned.

As mentioned before, there are a variety of topics in *Hidden Figures* that one could focus on, and each one could create a different first-year seminar course. The way Shetterly weaves together science, technology, history, and individual stories lends itself to the creation of an interdisciplinary course that could be co-taught with another faculty member. At many schools, first-year seminars provide opportunities for faculty to work together to create fun courses for themselves and the students. Possibilities include:

- Women and Gender Studies: Explore how events from the middle of the 20th century changed women’s role in society. Compare events of the past with current events.
- African-American Studies: Investigate key moments in the civil rights movement that are emphasized in *Hidden Figures*. Compare events of the past with current events.

- History: Examine WWII and the space race from a historical perspective.
- Political Science: What were political influences that brought about the civil rights movement and women's movement in the 20th century? How are current policies impacting equality in STEM fields?
- Computer Science: How has computer programming transitioned from a field with many women pioneers to a field where the percentage of women working in computer science is currently decreasing?
- Physics and Engineering: Examine in more depth the physics and engineering involved in sending rockets and astronauts into space.

First-year seminars at different institutions often require significant writing assignments, and this seminar topic has many possibilities for this. If this FYS was worth more than a single credit hour, I would require students to write an in-depth analysis of themes in *Hidden Figures* and a research paper on a topic related to groups underrepresented in STEM fields.

Teaching this FYS has been a wonderful experience for me personally and professionally. The course provided good motivation to update some of my knowledge and views related to equality in STEM. I have also appreciated the chance to work with first-year students in a learning environment considerably different from the math courses I teach. I feel as though I have a better understanding of how first-year students think. I have benefited from the excitement and energy many of them have as they start their university careers and charge toward the possibilities that lie ahead.

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## Appendix

### Response Papers

*Students are assigned four response papers (one to two pages in length) throughout the semester. These papers are designed to give students a chance to reflect on the readings and class discussions and the instructor a chance to gain additional insight into what students are taking away from the class.*

**Assignment:** Students are asked to think about the given writing prompt (some examples are provided below) and submit a one to two page response. The tone of the papers can be fairly informal or conversational (as if they were talking to the instructor), but students should still use good grammar. Each response paper is graded on a five point scale: 5, demonstrates excellent understanding of the material and/or includes several personal insights about the material: 3, demonstrates good understanding of the material and/or includes some personal insights about the material, more summary than personal reflection and response: 1, demonstrates poor understanding of the material and no or few personal insights, off-topic response.

**Sample Writing Prompts:** The following are examples of prompts given for the response papers.

1. Have you ever experienced a time when you felt hidden or invisible? When was it, what made you feel that way, and how did you react?
2. In what ways did current events influence the opportunities available to the women profiled in *Hidden Figures* and *Top Secret Rosies*? Discuss the events and the impact of the the events on the women. Consider whether the impact was positive or negative as well as how race and gender created different experiences for different individuals.
3. When Neil Armstrong landed on the moon, many Americans were glued to their television or radio and will never forget such a wonderful and exciting moment. For other events, like the assassination of President Kennedy, the assassination of Dr. Martin Luther King, Jr., or the September 11 attacks, people remember exactly where they were when they heard the horrible news. What are the events in your lifetime that have created shared experiences with others in your community (whether that community is a school, a town, or the entire country)?
4. Take some time to think about your mentors or role models. Is there a difference between a mentor and a role model to you? If so, what is it? Do you currently have any mentors? Who are they, and how are they helping you? Have you ever tried to do something without having an experienced person guide you? What was that experience like, and how might the experience have been different if you did have a mentor?
5. In what ways do your race and gender influence how others treat you and how you treat others?

6. (End of semester response paper.) Reflect on the entire course before writing this final response paper. What are the two or three parts that have had the greatest impact on you this semester? Why? What questions has this course raised for you, and what do you think you might do to find the answers? What was your favorite part of this class? What other thoughts do you have about the class that you would like to share with me?

## Group Presentation

*At the end of the semester, students are asked to give a group presentation on research they did about a woman in science or an area of scientific research stemming from the space race. The presentations are given over the last one or two class periods. Details of the assignment, rubrics, and student evaluations of their group's work are provided below.*

**Assignment:** Students work in groups of four to give a presentation to the class about a female scientist or a discovery, invention, or area of research that can be linked to the space program. The topic is open-ended on purpose; each group will have their own interests and perspectives and will choose a person or topic based on those interests. The group presentation is 15 to 20 minutes, and the presentation is graded on

- Information and Content
- Presentation Organization
- Presentation Delivery
- Presentation "Props" (slides, videos, demonstrations, etc.)
- Teamwork

In addition, each group is expected to provide a one-page informational handout to the class. The handout should provide the references used for the presentation. The handout can provide a summary of the presentation, key facts the group would like the class to remember, or suggestions for additional readings. The layout and format is up to each group as long as it is organized and easy to read.

Each group member is expected to complete a similar amount of work and to contribute equally to the final project. To ensure this, the group completes a group evaluation and each student completes a self-evaluation describing the equality—or the lack of it—of the group's work.

**Grading:** This assignment is worth 50 points, allocated as follows.

| | |
|---|---|
| Student Evaluations of Presentation | 20 points |
| Instructor Evaluation of Presentation | 10 points |
| Quality of Handout Provided to Class | 10 points |
| Evaluation of Group Process and Experience | 5 points |
| Self-evaluation of Contributions to Group | 5 points |

Students complete evaluations of other groups' presentations, helping to keep students engaged in the presentations. While each student is given a copy of the class handout, the grade for the handout quality is determined only by the instructor.

**Group Presentation Rubric**: Each student in the audience and the instructor evaluate the presentation in five categories according to the criteria listed below. The average of the student scores for each category and any student comments are provided in the group's grade report.

*Information & Content* (4 points)

4 points
- Presenters are knowledgeable of the topic.
- Includes interesting and relevant details that support the theme of the talk.
- Sufficient facts and examples are provided to support claims or statements.
- Information is taken from reliable sources.

3 points
- Presenters are knowledgeable of the topic.
- Mostly includes interesting and relevant details that support the theme of the talk. Some details may be off-topic or not clearly linked.

- Sufficient facts and examples are provided to support most claims or statements.
- Information is taken from reliable sources.

2 points
- Presenters are unsure about some areas of the topic.
- Includes several details or facts that are off-topic or not clearly linked.
- Several claims or statements are made without supporting facts or examples.
- Source of information is unclear or not from reliable sources.

1 point
- Presenters do not appear to be knowledgeable of the topic.
- Lacking interesting and relevant details to support the theme of the talk.
- Facts and examples to support claims or statements are missing.
- Source of information is unclear or not from reliable sources.

*Presentation Organization* (4 points)

4 points
- Presentation had clear introduction and conclusion.
- Material was presented in an order that made sense.
- Transitions between topics were evident and smooth.
- Presentation was completed within time limits.

3 points
- Presentation had clear introduction and conclusion.
- Material was presented in an order that made sense.
- Transitions between topics were evident, although some transitions were rough or abrupt.
- Presentation was completed within 1-2 minutes of time limit.

2 points
- Presentation was missing introduction or conclusion.
- Material was presented in an order that made some sense; some topics seemed out of place.
- Transitions between topics were evident, but many transitions were rough or abrupt.
- Presentation was completed within 1-2 minutes of time limit.

1 point
- Presentation was missing introduction and conclusion.
- Material was presented in a disorderly way.
- Transitions between topics were not present or did not make sense.
- Presentation was much too long or much too short.

*Presentation Delivery* (4 points)

4 points
- Presenters spoke clearly and loudly.
- Presenters maintained good eye contact with the audience.
- Presenters only glanced occasionally at notes or slides.

3 points
- Most presenters spoke clearly and loudly.
- Most presenters maintained good eye contact with the audience.
- Most presenters only glanced occasionally at notes or slides.

2 points
- Some presenters spoke clearly and loudly.
- Some presenters maintained good eye contact with the audience.
- Some presenters only glanced occasionally at notes or slides.

1 point
- Difficult to understand presenters.
- Presenters made little eye contact with the audience.
- Presenters most of the time read from notes or slides.

*Presentation "Props" (Slides, videos, models, etc.)* (4 points)

4 points
- Props (audio-visual aids/slides) helped to illustrate the main ideas and theme of the presentation.
- Props were easy to read/hear.

3 points
- Props helped to illustrate the main ideas and theme of the presentation.
- Props were sometimes distracting or difficult to read/hear.

2 points
- Props did not add to the main ideas and theme of the presentation.
- Props were sometimes distracting or difficult to read/hear.

1 point
- Props were not used.

*Teamwork* (4 points)

4 points
- All group members participate in the presentation equally.

3 points
- All group members participate in presentation.

2 points
- Two group members participate in presentation.

1 point
- Only one group member participates in presentation.

**Handout Grading Scale:** The handout each group creates is evaluated in three categories: references, content, and presentation according to the following criteria.

*References* (3 points)

- Several references (at least five) were used.
- References are from reliable sources.
- A variety of references (books, articles, webpages, etc.) are used.
- References are included on the informational handout using APA style.

*Content* (4 points)

- Information included summarizes the main ideas of the presentation or provides other interesting and useful information relevant to the topic.
- It is clear why information is included (relevant to the topic).

*Presentation* (3 points)

- Information is presented in an organized way.
- Information is easy to read.
- Formatting does not detract from the overall impact of the information.

**Group Process Evaluation:** Each group is asked to complete a document with the information below and submits that document after their group presents. Every group member must sign the document to verify that they contributed to the group evaluation.

1. List the name of each person in the group, and describe the tasks this person completed.
2. On a scale of 1 to 5 (1: "Not very well" and 5: "Very well"), rate how well your group functioned in each of the areas below.
   - Our group developed a goal for this project and a plan to achieve that goal.
   - Our group showed respect for each group member and their opinions.
   - Our group was able to settle disagreements in a way that was satisfactory to everyone.
   - The final presentation accurately reflects the opinions and thoughts of each member of the group.
3. Describe the most positive aspect of your group's interactions.
4. If your group was going to work together on another project, what would you need to improve?

**Self-Evaluation of Group:** Each student submits answers to the following questions about how they and the other members of their group contributed to the project. This provides a way for a group member to tell me privately of any issues with their group I might need to address. It also provides a chance for students to give feedback on the project and how to improve it in the future.

1. Reflect on your contributions to the group. Did you complete all of the tasks that you agreed to complete? Were you prepared for your portion of the presentation? Do you feel that your contributions to the group were equal to the other members contributions?
2. Reflect on the other members of the group. Did they complete all of the tasks they agreed to complete? Were they prepared for their portions of the presentation? Did they each contribute equally to the group? Please be specific.
3. Describe the dynamics of your group. Were you all equal partners, or did one person take on more of a leadership role?
4. Did you enjoy working with this group? Why or why not?
5. Did you enjoy this project? Why or why not?
6. What suggestions do you have for improving this group project in the future?

## Using Slide Rules

*The birth of computers led to the development of hand-held calculators which ultimately led to the death of slide rules. Most students today have never seen a slide rule, let alone used one. One day of class is spent performing calculations with slide rules.*

**Activity:** In preparation for the slide rule activity, students are asked to read an article giving a history of slide rules [12]. After a quick review of scientific notation, students are provided with instructions for doing different types of calculations with slide rules and some computations to complete. Students work on basic calculations in groups using a classroom set of slide rules borrowed from the International Slide Rule Museum [6]. Then the class discusses how estimation and number sense, as well as performing algebraic simplification before calculating, were key skills required to use a slide rule. (For example, after a calculation is done with a slide rule, the correct placement of the decimal point needs to be determined.)

Many students cite this activity as one of their favorites in the entire semester! While students only perform multiplication, division, squaring, and taking square roots of numbers, these exercises are enough for students to gain a new appreciation for their calculators and for the skills of the human computers.

# 22

# Culture, Science, and Mathematics in the pre-Columbian Americas

Ximena Catepillán

**Abstract** Our student population is growing more culturally diverse and with this, the need to offer courses that emphasize the importance of cultural diversity and inclusion in mathematics is of paramount importance. The limited way in which pre-Columbian Americas perspectives are introduced in American public schools is disturbing. This chapter describes a First-Year Experience seminar—part of the broad interdisciplinary field of Native American Studies—in which students study the history, mathematics, and scientific knowledge of indigenous peoples of the pre-Columbian Americas and how to formulate problems from these civilizations in the language of mathematics.

## 22.1 Background and Context

Millersville University of Pennsylvania is a regional public institution with approximately 7,900 students. The institution encourages faculty members to create first-year seminars on subjects about which they feel passionate. For me, this presented an ideal opportunity to propose a seminar on the mathematics and science of the civilizations of the pre-Columbian Americas, which allowed me to combine two of my intellectual passions: mathematics and pre-Columbian studies. The seminars, which are overseen by the Office of Academic Affairs with offerings through a campus-wide First-Year Experience (FYE) program, consist of 20 to 25 students in each section, and are required of all first-year students. The student minority population in the seminars is about 24%, as it is at the university. For students that have selected a major, the FYE seminar they enroll in is typically offered by the major department. For students who are undeclared/undecided, the Millersville University Exploratory Program offers the 3-credit seminars like the one described in this chapter, that meet three times per week for 50-minute sessions. The Exploratory Program students are part of a learning community who share a designated residence hall and have special programming to support students' transition into the college experience academically, socially, and personally and during their search for a major.

The Exploratory Program seminars are also linked to a foundations course—either an English or a Communications course. The seminars' objectives—given by Millersville University General Education Governance and Policies—are to introduce and support the development of critical inquiry skills and the exchange of ideas; to provide intellectual richness; to strengthen students' information literacy; and have meaningful oral and written communication components. In addition, there are social and cultural objectives in support of the students' successful transition into college life. To support these objectives, the FYE program includes peer mentors. The mentors are upper-class students trained to provide support and advice to first-year students in the seminars. Mentors attend at least one class session each week and meet with students from the class on a regular basis throughout the semester. They highlight programs, activities and events that seminar students should consider attending, and they direct students to campus resources as needed.

## 22.2 Mathematical Theme

The seminar I designed is called *Culture, Science, and Mathematics in the pre-Columbian Americas*. The seminar is an introduction to the study of the mathematics and science of the civilizations of the pre-Columbian Americas, a part of the broad interdisciplinary field of Native American Studies. The emphasis is on the role that science and mathematics played in the cultures of these indigenous groups. The seminar explores the pre-Columbian world through the eyes of ancestors, as well as those of contemporary students. An important aspect of my mathematical instruction is the use of pre-Columbian sources to engage students with topics in mathematics. A historical overview is required at the beginning of every topic and project. The content-related objectives of the seminar I designed are to:

- demonstrate the mathematical and scientific knowledge of the people of the pre-Columbian civilizations
- formulate problems from these civilizations in symbolic mathematics language
- increase awareness of the cultural diversity found within mathematics
- understand and be able to clearly express the mathematical and scientific ideas among the people of the pre-Columbian civilizations.

One of the reasons why I think it is important for mathematics faculty to create interdisciplinary first-year seminars involving pre-Columbian history is summarized in the disturbing statement below from one of my former students—now a high school history teacher [1, p. 281]:

> American History that is pre-1776 like the Ancient Maya is typically (and frustratingly) compacted into a single unit in American public schools, whereby teachers and textbooks fly through the entirety of pre-Columbian America as if it's something of a boring prequel to "the real story."

This seminar allows ample time for students to explore the rich history of mathematical and scientific thought in the pre-Columbian Americas.

## 22.3 Course Structure

The seminar theme is roughly divided into four major topics related to the mathematics/science of the pre-Columbian Americas: (1) Number Systems; (2) Calendars; (3) Archaeoastronomy; and (4) Patterns (see the course outline in the Appendix, p. 224). Within each topic there are lectures, discussions, assignments, and projects that the students work on in teams. For each group activity, students are asked to partner with different teams. In addition, there are several extracurricular activities as part of the seminar. Students enroll in the course platform system where they are able to communicate with their classmates, their mentor, and the instructor via email and discussion board forums. The site provides online access to the course syllabus, assignments, rubrics, external links, videos, photos, and other course resources such as the Mathematical Association of America (MAA) Career Resources. Additionally, the course platform is used extensively as a place to connect with the mentor, ask questions, share reactions to readings and extracurricular activities, and post assignments and projects.

In the subsections that follow, I highlight a sample lesson, an assignment on numbering systems, two projects, and extracurricular activities from my course that I have found especially valuable.

### 22.3.1 A Sample Lesson Plan

One of my favorite lessons is on the study of a glyphic text depicting a royal Maya family, which is part of Calendars topic (see the full sample lesson in the Appendix, p. 225). This lesson is preceded by an introduction to the Maya civilization and their vigesimal and quasi-vigesimal number systems. In their first project presentations (see Section 22.3.3), students learn at least ten Native American number systems. The study of this particular stela is a favorite of mine since the depiction of women among Maya monuments is quite unusual, whereas this stela depicts not only a woman but her daughter too. The sides and front of the stela have additional royal information. Long Count dates and distance numbers inscribed on Maya monuments, stelae, buildings, dintels, codices, pottery, etc. are ubiquitous, making the operations among them quite useful and a great source of exercises and projects for students. The Tzolkin and Haab Calendar dates usually follow the hieroglyphic inscriptions, generating a well-liked list of exercises while studying Maya inscriptions. Students read the Maya calendar dates, work on the conversions and check their results against the hieroglyphic data. The NOVA documentary, *Cracking the Maya Code* [3]—where linguists, mathematicians, archaeologists, and artists worked together to decipher Maya documents—is well received after this lesson.

### 22.3.2 The Family Number System Assignment

Students are asked to create a number system in which all spoken numbers and symbols must be related to their ancestry and/or an indigenous group connected to the area in which they grew up. The objective of the assignment is for students to learn to appreciate their ancestral connections, in this case with mathematics. More specifically, the students must

1. Create a name for their family number system.
2. Choose a base. Their system can't be base 2 (binary), 10 (decimal), 20 (vigesimal), or 60 (sexagesimal).
3. Create a list of at least 4 x (base) spoken numbers.
4. Describe the rules used to create the spoken numbers.
5. Design symbols for their numbers.
6. Describe the rules used for the number symbols.
7. Write a family history associated with their number system.

Most of our students come from rural Pennsylvania areas, and it is interesting to observe their amazement when they find tribal connections with their hometowns. A student from Carlisle, PA, stated in the conclusion of her project, "I drove everyday by a building unaware it had been the Carlisle Indian Industrial School (1879-1918), a boarding school dedicated to "Americanize" the tribal children—many children perished due to poor conditions at the school." Another student, who grew up in the eastern Pennsylvania town of Jim Thorpe, used Algonquian words and symbols from James Thorpe's tribe. James Thorpe was the first Native American to win an Olympic gold medal for the United States in 1912. The assignment is given after the students have had a number systems introductory lecture (see [1, p. 285] for a number system sample lesson) and worked with a reasonable sample of indigenous number systems on their first project (see Section 22.3.3). Students learn how to break down the language and mathematics of several number systems of pre-Columbian tribes. In some cases we initially have to dissect the number system to be able to create composite numbers. We examine how the languages of the number systems of different tribes have influenced each other and have been influenced by outsiders. (It is worth mentioning that in North America there are about 56 classified indigenous languages, not counting the unclassified and extinct.)

Their assignment must be typewritten (symbols may be handwritten), and the use of family names and symbols that have a special meaning to the students and their families is encouraged. The initial skepticism regarding the feasibility of the assignment vanishes soon after they begin working on it and their motivation spikes when they become aware that they can use their own sociocultural history and/or ethnicity to develop a mathematical activity. A collection of journals, magazines, newsletters, regional publications, special reports, and conference proceedings devoted to number systems is used with the help of the university librarian who specializes in mathematics/science resources.

### 22.3.3 Project Presentations

During the semester students present two team projects to their classmates. In the subsections that follow, I describe these projects. Each project is assessed using a rubric that students receive upon selecting their projects.

#### The First Project

For the first project, students are asked to choose a Tribal College of the American Indian Higher Education Consortium (AIHEC), study the history of a tribe served by the college, and present it together with a summary of a tribal related mathematics/science course offered by the college and/or a number system of one of the tribes that the college serves.

The Tribal Colleges offer a myriad of these types of courses, like the Native American Environmental Issues and Archaeoastronomy courses offered at Saginaw Chippewa College in Michigan, the Climate Change for Tribal Peoples course offered by Diné College that serves the Navajo Nation, and a sample of courses from the Indigenous Science Program offered by Little Priest College that serves the Winnebago tribes. The Tribal Colleges are an abundant source of information regarding Native Americans, a perfect fit for the seminar. Students discover early as they work on their projects why it is important for the tribes to have such colleges, and the struggles the tribal nations have endured and are still facing. We conclude the presentations with a discussion of the strengths and drawbacks of Tribal Colleges compared with Millersville University.

#### The Second Project

For the second project, students select a project topic for presentation in teams. A variety of topics is available for selection. The following is a short list of the most popular projects from my recent iterations of the course (see [1, p. 283] for additional project topics).

- The concept of zero in pre-Columbian Americas
- Patterns in Native American blankets
- Calendar conversions among Maya dates
- Extinction of number systems of the Patagonia
- Reckoning artifacts of pre-Columbian groups
- Eclipse calculations in indigenous cultures (included when we have a lunar/solar eclipse in the area)

Students are strongly encouraged to suggest their own topics, and often do so when they have indigenous ancestry of the Americas and/or grew up in an area connected to an indigenous tribe. At this point I usually share my indigenous roots from South America with my students. My last name, Catepillán, is Huilliche, a sub-tribe of the Mapuche. Students are eager to learn about my ancestors and ask multiple questions about the mathematics of the indigenous group. (For the Mapuche number system see [1, p. 285].) Students are also encouraged to create short videos; family video participation is a common occurrence here, especially when they have a pre-Columbian connection.

The second project's grade is based on various phases of student work: topic history, mathematics/science research, meetings with instructor, research outline, initial and final drafts, oral presentation, conclusion, and two or three created homework problems. As an example, for the project "The concept of zero in the pre-Columbian Americas," a team could begin with a survey of the Maya zero, its origins, significance, symbols (the Maya had a variety of symbols for zero), and usages and then create a comparative timeline to its conception in India and Babylonia. Meetings with the instructor are essential for a successful completion and presentation of the project.

### 22.3.4 Extracurricular Activities

A regular guest presenter is an astronomy professor from the Physics Department at Millersville University. On semesters in which we have a lunar/solar eclipse the astronomer—in addition to giving an introduction to astronomy—installs various telescopes on campus to help students gain an experience of watching an eclipse. Observing an eclipse, taking and sharing pictures, and a lecture on how the pre-Columbian groups—known for accurately predicting eclipses—used these phenomena to enhance the power of their rulers, is a favorite theme of the seminar.

Attending a minimum of two university sponsored Native American/Latino Studies cultural events is required. A reaction/reflection summary is required after each activity, followed by a class discussion. These presentations—both informational and inspiring—help students to immerse in the pre-Columbian world.

These extracurricular activities are ongoing events throughout the semester and include outdoor activities like the Ropes Program, an event sponsored by the Wellness Department, in which students have to complete challenging outdoor tasks that can only be successfully done if they work as a team. As a larger campus community, first-year seminar students, their peer mentors, and faculty members participate in the annual United Way Day of Caring. When accessible—during the Day of Caring—we do volunteer work at a local museum where they have the opportunity to see a vast collection of mostly untapped Native American and Mesoamerican artifacts.

It is important to note that the FYE program at Millersville University has funds for extracurricular activities. In the past, I have utilized those funds to take my students to the Eastern Pennsylvania and Delaware Section of the MAA Careers in Mathematics Conference. The highlight of the conference is a panel discussion comprised of successful individuals in actuarial science, finance, statistics, government, and education, among others. This activity is popular among first-year seminar students interested in majoring in STEM careers.

## 22.4 Reflections

I have taught the seminar twelve times during fall semesters, and I soon learned that spending time doing extracurricular and volunteer activities allows first-year seminar students to enhance and build skills such as leadership, innovation, creativity, organization, and teamwork. On the other hand, when working on assignments like the family number system, students decide what aspects of their culture to include and share. They experience ownership of their mathematical work, something new to most of them. A student from a Puerto Rican family was delighted to learn about her Taino roots when working on the tribe's number system. She used the opportunity to reconnect with her estranged "abuela" (grandmother) who lives in Puerto Rico. The abuela was thrilled to hear from her granddaughter and, especially, the interest in the family's Taino roots. There is a strong sense of mathematical belonging with these type of assignments.

Undoubtedly, one of the most rewarding pieces of the seminar has been to witness students leaving their comfort zones to immerse themselves in different cultures by learning the mathematics of our pre-Columbian ancestors. They enjoy exploring and breaking down the linguistics and mathematics of an array of number systems of pre-Columbian tribes which enables them to count in an assortment of languages and dialects such as Chumashan of the Chumash of California, Nahuatl of the Aztec, Cherokee and Comanche of North America, and Mapudungún of the Mapuche of Chile and Argentina, to name a few. Students who take a culturally inclusive seminar will learn tolerance and appreciation of other cultures and view the world from different cultural viewpoints.

An area of improvement is the better utilization of the university's seminar funds. I would like to include a day trip to the Penn Museum in Philadelphia, which is a short 70-minute train ride away. In preparation for the visit, the museum could send an Artifact Loan Box. The box is comprised of replica objects—of my choosing—that come from the Penn Museum's Teaching Collection. The American collections number approximately 300,000 archaeological and ethnographic specimens. At the museum students would have the opportunity to read glyphic information on Stela 14, a monument about a Maya ruler from Piedras Negras, Guatemala. Stela 14 is from the same period as the stela depicted in the sample lesson in the Appendix (p. 225).

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# Appendix

## Course Outline

*The following is the list of seminar topics for Culture, Science, and Mathematics in the pre-Columbian Americas.*

1. Astronomy in Ancient American Cultures
   The focus is on the observatories, astronomical sites, and ecliptic structure alignments built by the pre-Columbians. A naked-eye study of the planets and eclipses as seen by the ancients is also included.
   - Pre-Columbian observatories and astronomical sites: Exploration of sites located in the American Southwest, Mesoamerica, and Inca territory among others.
   - Building alignments: Structures, temples, tombs, and pyramids that were aligned with the precession of planets and the solar equinoxes.
   - Planets: The planets that the ancients could see with the naked eye. Indigenous religious observances based on phases of the moon and solar cycles.
   - Eclipses: Lunar and solar eclipses, and how celestial events empowered leaders.
   - Navigation: Some indigenous groups have navigated for centuries using a combination of geometry, the stars, and constellations.
2. Pre-Columbian Number Systems
   Study of the written numbers that can be divided into grouping, partially positional, positional, and spoken number systems, which in most indigenous groups are closely connected with their language structure.
   - Written numbers: Including the grouping system used by the Aztecs, the partially positional system of the Aymara of the Andean region of Bolivia, and positional systems like the Maya.
   - Spoken numbers: How the methods of counting used by most indigenous groups are closely connected with their language structure.
3. Counting and Recording Devices
   Several indigenous groups developed counting devices to make computations before recording them on special artifacts. We find many variations of finger and body counting across the Americas.
   - Reckoning and game boards: Devices like the yupana used by the Inca to perform computations before the results were recorded. Games of chance and strategy played with pebbles, bones, or other small objects. Patolli, one of the oldest of games played in Mesoamerica, used a board.
   - Recording devices: Objects that were instrumental prior to the invention of paper such as the Inca quipu, the chimpu of Peru and Bolivia, and wooden and bone tally sticks.
   - Finger counting: Variations of finger counting as known in the Americas and an exploration of how finger counting differs according to region, ethnicity, and historical period.
   - Body counting: Usage of the body as a counting tool in the pre-Columbian world.
4. Calendrical Systems
   Many different calendars—solar, lunar, sacred, and others—were developed by the indigenous peoples. Calendrical forms are universal. Students make conversions between a variety of calendars.
   - Celestial calendars: The Sun Dagger site and a celestial calendar of the Pueblo of the American Southwest.
   - The Maya system of calendars: Conversions of dates among the calendars, correlations amid the calendars, and the Julian and Gregorian calendars. See the Appendix (p. 225) for a sample lesson.
   - Other calendric forms—albeit not as complex as the Maya—are ubiquitous in the Americas.
5. Numerical Representations
   Study of the hieroglyphic inscriptions, pictographs, and petroglyphs left by our ancestors of the pre-Columbian Americas.

- Hieroglyphic inscriptions: Exploration of solar and lunar calculations on ancient stelae and pre-Columbian inscriptions on buildings, stairs, lintels, tombs, pottery, fabrics, etc.
- Pictographs and petroglyphs: Study of sites in the Southwest, the Great Basin, the Checta of Peru, and the Cave of the Hands in Argentina, among others.

6. Strip and Planar Symmetries
   The linear and planar patterns created by the indigenous peoples of the pre-Columbian world for their daily life can be identified with symmetry groups. Cultures have their own identifiable pattern styles and meanings.
   - Strip symmetries: The seven groups of symmetries of strip patterns are studied using pre-Columbian artifacts like ceramics, fabrics, jewelry, and basketry.
   - Planar symmetries: In two dimensional symmetry patterns—tessellations—most of the 17 wallpaper symmetry groups are present in pre-Columbian cultures.

## Maya Long Count Dates Computations

*In what follows, a sample lesson of the study of a glyphic text depicting a royal Maya family is presented. The lesson is preceded by an introduction to the Maya civilization and their vigesimal and quasi-vigesimal number systems.*

The civilization of the Maya, which began ca. 1200 BC, was one of the five cultures that developed a writing system—the others were the Chinese, Egyptian, Harappan, and Sumerian. Additionally, they created a vigesimal number system for arithmetical computations and a quasi-vigesimal system for calendrical calculations. Several tables of multiples of various integers can be found in Maya writings in connection with almanacs describing mainly weather forecasting, crop predictions, and astronomical computations, among other items.

Long Count dates, a sequence of five numbers, were used by the Maya to mark events that occurred over long periods of time. The shortest period was a kin (a day for us), with 20 kins referred to as a uinal, and then a tun was 18 uinals, followed by a katun that consisted of 20 tuns, and lastly a baktun corresponding to 20 katuns.

$$\begin{aligned}
\text{1 uinal} &= \text{20 kins} \\
\text{1 tun} &= \text{18 uinals} = \text{360 kins} \\
\text{1 katun} &= \text{20 tuns} = \text{7,200 kins} \\
\text{1 baktun} &= \text{20 katuns} = \text{144,000 kins}
\end{aligned}$$

The stela—a carved, freestanding stone slab—described features several Long Count dates carved on the stone, a marvelous testimonial of an unusual portrait of a royal Maya family, Fig. 22.1 left. Found in Piedras Negras, Guatemala, and named Stela 3, it focuses on Lady K'atun-Ajaw, her husband, Yo'nal Ahk, and their daughter Lady Huntan Ak. The face of the stela shows mother and daughter seated cross-legged on a throne [4, p. 85].

One of the Long Count dates is 9. 12. 2. 0. 16, (9 baktuns, 12 katuns, 2 tuns, 0 uinals, 16 kins). The date in which Lady K'atun-Ajaw was born in a place called Namaan, possibly lying between Piedras Negras, Guatemala and Yaxchilán, Mexico. Another date is 9. 13. 16. 4. 6 in which she gave birth to a daughter, Lady Huntan Ak, in the Turtle lineage of Piedras Negras [2, p. 266]. The Gregorian calendar months and years corresponding to the Long Count dates are July, 674 AD and March, 708 AD respectively.

The top left portion of the stela, describing the Long Count date of the birth of Lady K'atun-Ajaw, is depicted in Fig. 22.1 right. The six hieroglyphs are read in rows top to bottom from left to right, a bar represents five units, and each unfilled dot represents a unit. In Maya monuments, the first Long Count date is preceded by an Introductory Glyph, we can see the glyph on the first row to the left of the Maya number nine, represented by a bar and four dots. On the second row we have the numbers twelve and two, and on the bottom row, zero is represented by a half-flower and the last Long Count number is sixteen. These numbers are depicted with an animal head on the right, representing the periods of time: baktuns, katuns, tuns, uinals, and kins.

Using these dates, we could subtract the mother's birth date from the daughter's to determine how old Lady K'atun-Ajaw was when she gave birth to her daughter.

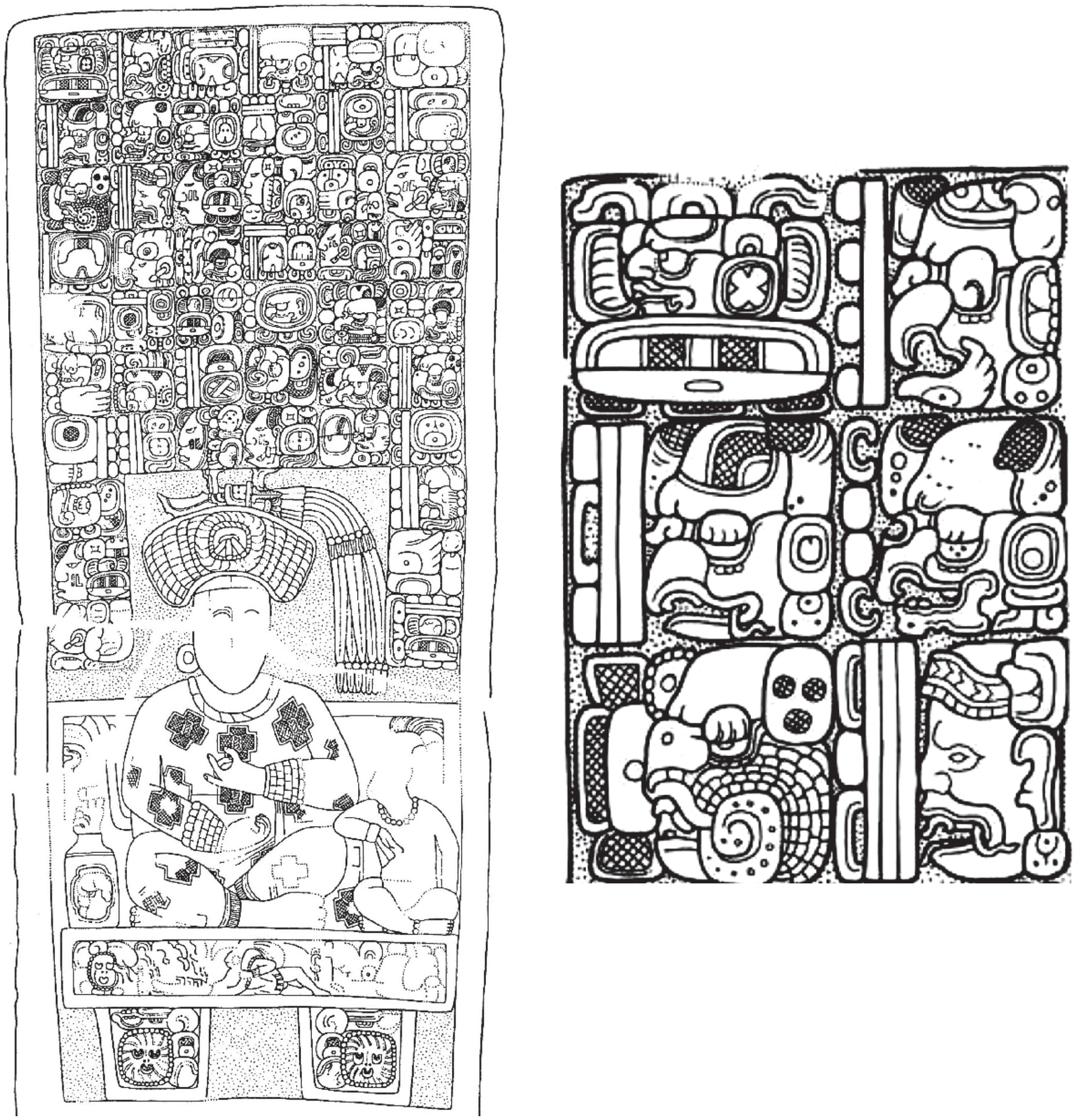

**Figure 22.1.** Stela 3 at Piedras Negras [4]

| | | | | | |
|---|---|---|---|---|---|
| | 9. | 13. | 16. | 4. | 6 |
| − | 9. | 12. | 2. | 0. | 16 |
| | 0. | 1. | 14. | 4. | −10 |

Since 1 uinal = 20 kins, we can rewrite the result as 0. 1. 14. 3. 20. -10 that corresponds to the five-number sequence 0. 1. 14. 3. 10 and is called a distance number, a time period between dates, not a Long Count date.

Now, if we convert the distance number to kins we can estimate the number of days between the dates: $10 + 3 \times 20 + 14 \times 360 + 1 \times 7200 + 0 \times 144000 = 12{,}310$ kins.

Consequently, Lady K'atun Ajaw was 12,310 days old at the time she gave birth to her daughter, and to estimate her age we divide the number by 365.2425, the average number of days in a Gregorian calendar, obtaining approximately 33.7 years. Thus, she became a mother at age 33.

# Part VI

# Mathematics, Art, and the Natural World

When we ask students to find patterns, symmetry, and chaos in the world around us, students can begin to see that “mathematics” and “beauty” are inextricably linked. The first-year seminars in this section highlight the interplay between mathematics, art, and the natural world, attract a variety of students, and help students begin to build strong interdisciplinary connections between seemingly disparate areas of study.

# 23

# *The Art of Mathematics* as a First-Year Seminar

Debra L. Hydorn

**Abstract** The University of Mary Washington requires all incoming first-year students to take a first-year seminar designed to build research and communication skills. This article will describe the first-year seminar titled *The Art of Mathematics*, which explores topics in mathematical art, including symmetry, geometry, fractals, and perspective. Several successful student activities will be described.

## 23.1 Background and Context

The University of Mary Washington (UMW) is a liberal arts institution with an enrollment of approximately 4,000 undergraduate students. A first-year seminar is a requirement of the General Education program with the specific goals of improving students' skills in (1) information literacy/research, (2) written arguments, and (3) effective oral communication. The development of writing and speaking skills in a first-year seminar prepares students to complete two Speaking Intensive and four Writing Intensive courses from across the disciplines, which are also General Education requirements. Incoming first-year students are given a list of scheduled first-year seminars with short descriptions during their summer orientation and are asked to indicate several seminars that are of interest to them. Faculty members are encouraged to create first-year seminars on any topic of interest that can support the learning goals of the First-Year Seminar program. The *Art of Mathematics* is the first seminar I created for this program and it is based on an online Mathematical Association of America Professional Enhancement Program (MAA PREP) course I took a few years ago [1] [7]. I have created two other first-year seminars, one on infographics and the other on mathematical modelling, and I am considering developing another one, on data analytics.

First-year seminars are typically three-credit courses taught in the fall 15-week semester and enroll 15 students, with the faculty member serving as the students' academic advisor until the student declares a major. In addition to the seminar topic and the three goals mentioned above, faculty are asked to address issues related to supporting student success, such as developing time management skills and good study habits, and to let students know about upcoming workshops offered by the library and the Writing and Speaking Centers on campus. The university creates a course management site for each seminar where faculty can post course materials. Each site also includes Modules created by the First-Year Seminar Program to further help students to develop effective research, writing, and speaking skills. Faculty must assign nine modules for their seminar, three for each of the three overarching course goals.

## 23.2 Mathematical Theme

In this course, students identify, describe, and explore the mathematical components in the art of M.C. Escher and other artists. They also use mathematical properties and relationships to create their own works of art. The course starts with exploring the operations of reflection, rotation, and glide reflection to create point, line, and plane symmetry groups using an asymmetric motif. Plane groups are further explored through tessellations. The course then continues with an exploration of Euclidean and non-Euclidean geometries, iteration, fractals, and perspective. Depending on time and student interest, random and computer generated art may also be included. Students explore additional topics related to mathematical art, such as spirals, polyominos, and aperiodic tiling, in a group project.

The course is structured around developing students' knowledge of different ways that mathematics can be used to create and describe art. Each topic is explored through in-class activities where students identify and describe the mathematics involved and is supported through readings in an assigned text. As a seminar course, class time is not spent on teaching mathematical content. Rather, students explore and discover the mathematics involved through readings and the in-class activities. The mathematical content is then summarized in class and supported with handouts and additional examples. I have used a variety of different texts for this course, including *M.C. Escher: Visions of Symmetry*, by Doris Schattschneider [8], *Math and Art: An Introduction to Visual Mathematics*, by Sasho Kalajdzievski [5], and *Escher on Escher: Exploring the Infinite*, by M.C. Escher [4]. Each of these books provides a different approach to describing mathematical art, and supports different kinds of discussion and writing assignments. All three books include examples of Escher's work, but the book by Kalajdzievski [5] does not include information about Escher. The book by Schattschneider [8] contains the greatest number and variety of Escher's work, while Escher's book [4] includes more details about his life and inspiration for his work. The book by Kalajdzievski [5] was written for a course on mathematics and art and includes more mathematics than the other two, plus exercises at the end of each section. Another book I have considered using is *The Magic Mirror of M.C. Escher*, by Bruno Ernst [3]. Like the Schattschneider [8] and Escher [4] books, this book includes a wide variety of works by Escher along with a discussion of Escher's life and inspirations. The MAA PREP course has a website of materials *Math and the Art of M.C. Escher* [1], including assignments and a wealth of other information about Escher and his work. The variety of different texts and other resources that are available to support this course provides faculty with alternatives for the structure of their course and the topics they wish to include.

## 23.3 Course Structure

The course is organized into two main sections, the mathematics of point, line, and plane groups and Euclidean and non-Euclidean geometries. For each section, students are introduced to the mathematics through handouts and activities that demonstrate and describe the mathematics involved. Working in small groups, students are given examples of artworks that employ each of the mathematical topics and are asked to identify the mathematics involved and how it is used to create art. Students complete homework assignments, tests, and projects to practice and demonstrate their understanding of the mathematics. Depending on available time and student interest, other topics can be included, such as perspective, fractals, and the mathematics of higher dimensions.

### 23.3.1 Symmetry and Tessellation

For the first part of the course students are given examples of artworks or objects that possess symmetries or translation properties, and are asked to identify and describe the patterns they observe. I typically choose ten different tessellations by M.C. Escher for this assignment as these incorporate a variety of symmetry and translation operations that most students can identify. A large variety of different works by Escher is available from online sites and his use of animals and other objects to create tessellations provides a relatively easy structure for students to identify the operations. This activity is then followed with a handout that introduces the mathematics of patterns, following the notation used in *Handbook of Regular Patterns* by Peter S. Stevens [10]. The handout starts with point groups, then uses the point groups to develop the line groups. Line groups are then used to develop the plane groups. At each stage of pattern development, students are presented with examples that they are asked to describe, including identifying the fundamental unit used to create the pattern and the notation to classify it. Students complete a series of homework assignments to demonstrate their mastery of the mathematical operations and the notation used to identify patterns. Once students gain confidence with identifying point, line, and plane groups, they start to recognize them around campus, so I sometimes include a mathematical patterns scavenger hunt in a homework assignment.

This section of the course culminates in the first individual project, for which students are asked to create their own Escher-style tessellation. After I demonstrate the process for creating a tessellation starting with a square, regular triangle, and regular hexagon, students are asked to create two possible tessellations for each shape, using at least one

**Figure 23.1.** Examples of Student Tessellations

translation and at least one rotation. Students then choose one tessellation to develop further. They are required to create their tessellation using a template and then to add color and other features to complete the work. I present them with many examples to supplement those created by Escher and other artists, including ones that I have created, to help develop their creativity. I encourage them to make the tessellation and then try to decide what it looks like, rather than to try to create a tessellation with a particular theme. Students are required to present their tessellation to the class and to write a blog post to describe the process they used and their inspiration for how they completed it. Some examples of students' tessellations are shown in Figure 23.1. The assignment and grading rubric are in the Appendix (p. 235).

## 23.3.2 Geometries

In the second part of the course, students explore both Euclidean and Non-Euclidean geometries, as well as dimension, perspective, and optical illusions. This section follows the same structure as the first part of the course, with topics introduced using examples to develop students' familiarity with geometric concepts, followed by readings and homework assignments. In addition to Escher's works that explore dimension and the infinite, I have used examples from *Masters of Deception: Escher, Dali & the Artists of Optical Illusion*, by Al Seckel [9] and *Visual Illusions*, by James Kingston [6] as well as examples of impossible objects and perspective sidewalk chalk drawings. For this part of the course, students are asked to create an artwork using *Geometer's Sketchpad*. One class session is held in the Mathematics Department's computer lab where students are introduced to *Geometer's Sketchpad*. I created a series of videos to provide additional demonstrations on how to use this software. Students are asked to create five preliminary artworks using any topic covered in the course, or another topic that they choose, and then select one to further develop to present to the class and write about on the class blog. Some example student projects are shown in Figure 23.2. The assignment and grading rubric are provided in the Appendix (p. 235).

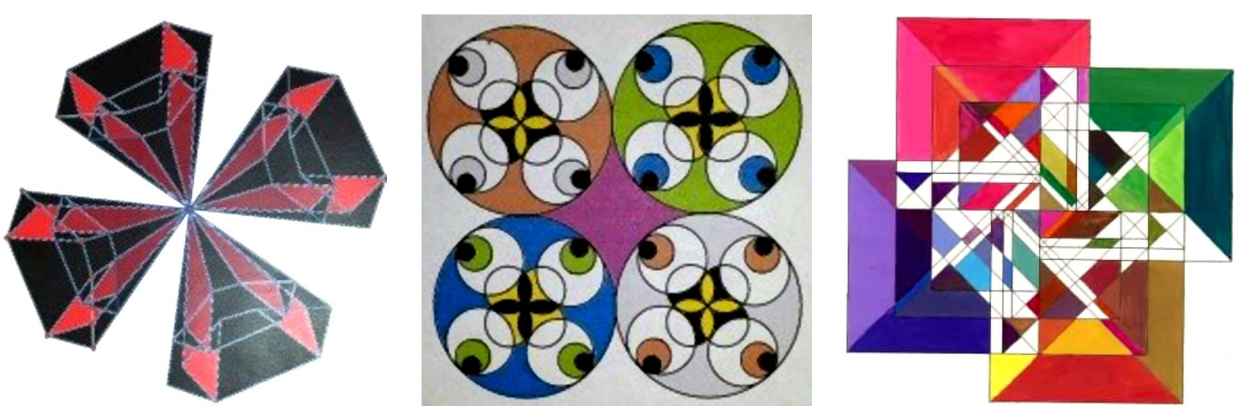

**Figure 23.2.** Examples of Geometry Projects

### 23.3.3 Additional Topics

In addition to the two art projects, students also complete a group project for which they conduct research on a topic that is not covered in class. Some example topics include spirals, knots, and aperiodic tessellations. The groups give a presentation to the class that includes a description of the mathematical topic and example artworks that incorporate the mathematics. Although students give a presentation to the class for each of their two artworks, I like to include at least one group project so that students learn how to collaborate and manage working with others. We spend some class time discussing expectations for effective group work and I ask them to help develop the criteria they will use to evaluate their partners.

Depending on the semester, I also ask students to complete one or two additional papers. For one paper they write a biography of a mathematical artist for which they conduct research to learn about the artist's mathematical and artistic preparation. I provide them with a list of mathematical artists, some who are mathematicians who create art and others who are artists whose works include mathematical influences, but students are free to choose another artist with my approval. They must provide some background information about the artist and choose three of the artist's works to describe in their paper. This assignment provides a good opportunity to introduce the students to the campus library and to have a librarian demonstrate how to conduct research using the library's resources. Another writing assignment I usually include asks students to choose three artworks on display at a campus gallery and to describe the mathematics the artists used to create their work. If the campus gallery exhibitions do not include artwork with sufficient mathematical content, I create a virtual mathematical art gallery using artworks selected from the online catalogs of the Mathematical Art Exhibition at the Joint Mathematics Meetings. I have also contacted our campus gallery about having a mathematical art exhibition for my seminar. In the fall of 2014 the university held the exhibition "The [Visual] Poetry of Logical Ideas" [11], for which I was asked to contribute some of my mathematical artworks.

I have also included class projects on perspective or on using probability to generate random art. The perspective project is based on the work of artist Jan Dibbetts who created *6 Hours Tide Object with Correction for Perspective*

**Figure 23.3.** Examples of Perspective Projects

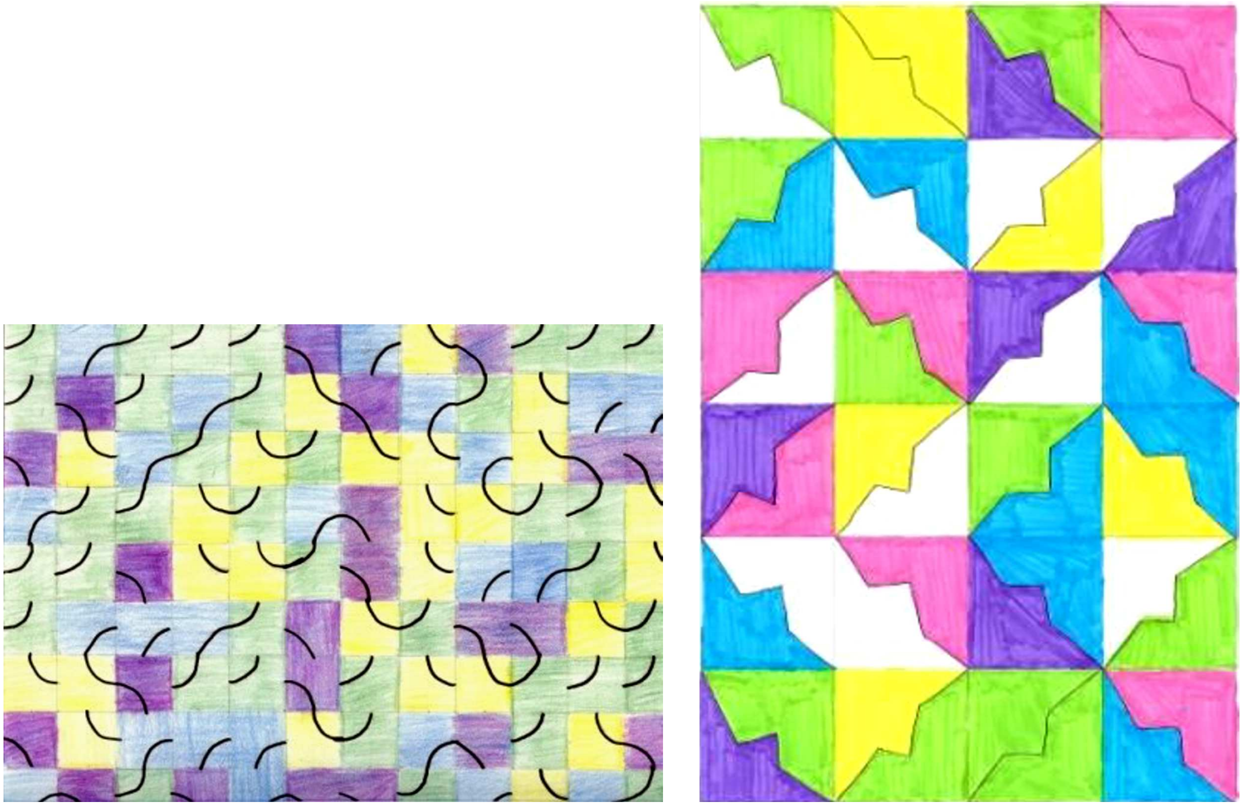

**Figure 23.4.** Examples of Random Art Projects

[2], a land-art project that was photographed from different vantage points to reveal how perspective impacts how art can be viewed. For this project, students work in groups to create an artwork on campus that can be observed from different angles or vantage points, or an anamorphic artwork using examples created by sidewalk chark artists. Two example perspective projects are shown in Figure 23.3. Another project I sometimes include is based on my interest in probability and randomness. For this project I asked students to generate a process for randomly creating a tesselation using geometric shapes. The students design several versions of their chosen shape by varying the color or other details, and then use a randomization scheme to create the tessellation. Two example random art projects are shown in Figure 23.4.

## 23.4 Reflections

I have taught this course eight times and have found that students enjoy the opportunity to explore mathematics from a visual approach and to create art using a variety of tools. Through the First-Year Seminar program, the university provides much support for faculty to meet the research and communication goals that are required components of a first-year seminar. I include writing and speaking assignments in some of my other classes and, since every incoming first-year student must take a first-year seminar, I know that my students have been prepared for these assignments because of the writing, speaking, and research they completed for their first-year seminar. Because I have chosen a topic that I am passionate about, most students become engaged in the course and come to appreciate the mathematical topics. They also appreciate the beauty of mathematics and the opportunity to be creative. I sometimes get emails from students who have had this seminar with me to share some mathematical art website or other resource that they have come across. I rarely get these kinds of emails from students who have taken a mathematics or statistics course with me.

Most of the students who take this seminar are not mathematics or art majors. Although many of them will have seen works by Escher before taking this seminar, they might not have observed the patterns in these works and thought about how to describe them. It can be a challenge to get these students to explore mathematics in a visual context and to trust their ability to identify patterns. Some students may lack confidence in their artistic ability or in their mathematical skills. I show them example artworks that I have created using the processes I ask them to use and encourage them to be creative. My examples show them that although I cannot draw very well, I can create interesting tessellations through the use of a template and geometric art using *Geometer's Sketchpad*. Projects are not graded based on artistic ability, but rather on whether or not the student has demonstrated an understanding of the mathematics they are to use and have followed the project guidelines. They are also graded on their presentations and blog posts. Regardless of artistic ability, most students have been pleased with the artworks they create for this course. Our First-Year Seminar program offers funds for faculty to purchase resources for their course or to support travel to a museum or other site of interest related to course content. I have asked for funds to publish a calendar where the picture for each month is one or more of the students' work, and most students contribute an artwork for this project.

Much of the content of this seminar also provides a foundation for supporting a multicultural mathematics approach. Because there are only seven different line groups, for example, students will be able to identify similarities between examples from different cultures around the world. I created a handout with six different Maori rafter designs which required students to identify the line group and fundmental unit. I could not find an example that used the seventh line group so I asked students to create their own Maori rafter design for that group using a fundamental unit they created. Japanese family crests provide examples of many different point groups and students can compare these to company logos or other examples of point groups. The Alhambra and other structures provide a wide array of different plane groups and Stevens' book [10] includes examples of point, line, and plane groups from many different cultures and countries. Although not all of the content of this *Art of Mathematics* first-year seminar possesses a multicultural aspect, there is enough mathematical and art content to support a version of the seminar with a multicultural approach. For example, the course could include a comparison of the different art media used by different cultures, such as print, paint, ceramics, or wood carvings. Faculty with more artistic abilities and resources could explore the use of different printmaking methods and ask students to create prints following the style of a country or culture. One example I have considered exploring is the different patterns that can be found in African fabric prints.

This seminar is fun to teach and offers a wide variety of different mathematical topics and artists to explore. One semester, I had a student who was an expert at creating fractals using some free software. He taught me and the class how to use the software and the class was allowed to use this software for the second class project instead of *Geometer's Sketchpad*. I like to include at least one project or activity where students will use a digital tool to create art because the digital process does not rely on the ability to draw. I have experimented with some free online tools for creating digital art but have not yet found a tool that is as easy to use as *Geometer's Sketchpad* with the same functionality that this software provides. Our Art and Art History department offers a course on *Digital Approaches to Fine Art* and partnering with the faculty who teach this course to include more digital options for creating art would be a good way to further develop this seminar. Future offerings of the seminar could explore additional tools for creating digital or computer generated art as well as additional topics, such as topology and architecture. Every time I have taught this seminar I have learned about other types of mathematical art that are accessible to an undergraduate audience. As a faculty member in a mathematics department my approach to the topics in this course starts with the mathematics involved and my experience with design principles and processes for creating art is limited. UMW has recently added peer mentors to work with faculty who teach first-year seminars. Having a peer mentor with a strong art background to work with students on their projects would complement the mathematical approach I bring to the course.

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[10] Peter S. Stevens, *Handbook of Regular Patterns*, MIT Press, Cambridge, 1981.

[11] *The [Visual] Poetry of Logical Ideas.* 4 Sept.–5 Oct. 2014, University of Mary Washington Ridderhof Martin Gallery, Fredericksburg, Virginia.

# Appendix

## Project 1 Instructions

*This is the assignment document that is provided to students to complete their first mathematical artwork, an Escher-style tessellation. The grading rubric used for reviewing student work is also provided.*

Part I. Create at least two draft tessellations using each one of the square, regular triangle and regular hexagon, a non-regular quadrilateral, and a non-regular triangle. (So, you should have at least ten different draft tessellations.) At least one of your tessellations for the square and the regular hexagon should include a combination of translation(s) and rotation(s). Make sure to do something to each side of each polygon. Don't leave any sides flat.

For each tessellation, indicate what transformations you used to create the shape to tessellate (your fundamental unit) and prepare a cut-out template to verify that the shape will tessellate. The more draft tessellations you try the more likely you will find a motif that will produce an interesting tessellation and one that will motivate your choice of design for Part II of this assignment. Notes: Make sure that you establish that your tessellation will cover the page—do more than just one or two copies of your template. Also, you do not need to determine a possible decorative design for each of your draft tessellations, but it will be useful if you try to determine what sort of figure your fundamental unit might represent. I have cardboard that you can use to create your templates if you need it.

Due date for Part I: Friday September 26. Turn in at least ten draft tessellations with (i) a list of the transformations used, (ii) the template you created, and (iii) a verification that the motif will tessellate.

Part II. Select one of your tessellations to complete, including coloring, shading, or other methods to define what the tessellated shape represents, or you may design a new tessellation. Refine your motif to improve the layout and design. Your final tessellation must be a "high quality" piece of artwork. Your tessellation will be graded using the following criteria:

1. Accuracy: The underlying lattice (grid) must be precise. There should be no gaps in your design and you may not use a border around the motif to fill gaps. Any underlying grid that you used to create the design should not be visible.

2. Mathematics used: Choose a tessellation with a sufficient degree of difficulty

3. Details: Use color and pattern on your motif to add interest and help define the motif. A simple motif can be used to create an interesting tessellation by the artist's use of details. You will need to practice that so that you can recreate the exact color and details each time the motif is repeated. To improve your accuracy in repeating the same design you might consider creating a template to place over your fundamental unit shape.

4. Materials used: Pencil and ballpoint pen are not acceptable drawing implements. Use paint, ink, markers, pastels, colored pencils, or other art media. Typing, copier, or loose leaf paper are not acceptable surfaces to produce your tessellation. Use heavy artist's paper or poster board or better.

5. Artistry (degree of polish): Your finished tessellation should not be the first time you draw your tessellation (and it shouldn't look like it). It should look as though you took time to create it and paid attention to the design details. You need to practice so that your finished tessellation is accurate and well designed. Refine your sketches until you are satisfied with the design. Erase any light pencil marks you make to lay out the pattern.

Above all, make something you are proud of. Give your finished tessellation a title and sign your work. Example student tessellations can be found at the Spring 2009, Fall 2009, Fall 2010 and Spring 2012 class blogs.

Presenting your work: You will present your final tessellation to the class both orally and as a blog posting. In these written and oral presentations you will describe the process you went through to create the fundamental unit motif (e.g., the transformations you used), the methods you used to create the tessellation (e.g., using a physical template by hand or using a computer program) and your inspiration for what the motif represents. Describe the materials you used. Identify the underlying geometric tessellation (which polygon you used) and the wallpaper group of your pattern. If you used a new design (not one from Part I) turn in the template of your motif.

Your in-class presentation should be no more than four minutes and it should be practiced at least once before class. Your blog posting is a written account of your in-class presentation. It should be well organized and well written.

Grading:

- Tessellation 75%
- Blog Posting 10%
- Oral Presentation 15%

### Grading Rubric for Project 1

| Presentation Component | Included? | Comments |
|---|---|---|
| Introduction | | |
| Process (Polygon, Transformations, Motif) | | |
| Methods Used | | |
| Inspiration for Design | | |
| Materials Used | | |
| Symmetries and Symmetry Group | | |
| Turned in Tessellation and Motif | | |

## Project 2 Instructions

*This is the assigment document that is provided to students to complete their second mathematical artwork, a geometric artwork using Geometer's Sketchpad. The grading rubric used for reviewing student work is also provided.*

For this project you will create a work of art using Geometer's Sketchpad. Similar to Project I you will create five preliminary artworks with an explanation of how you created each and possible modifications you could make for the finished project. Then you will pick one of these artworks (or create another if you prefer) to complete and present to the class for Part II. You may base your artworks on any topic from this semester, with the exception of the topic of periodic tessellations, or some area or topic in mathematics not covered in this course. Some possible topics are:

1. The four isometries (reflection, rotation, translation, and glide reflection)
2. Similarity transformations and dilations
3. Iterations and fractals
4. Aperiodic tessellations
5. Spirals

You must explore at least two different topics for your five experimental works. You can combine different topics if you choose. For example you can try creating tessellations using spirals or iterations. The only rule concerning tessellations is that they must be different from the kinds of tessellations we did for Project 1. Check with me if you're not sure about this.

If you have trouble coming up with some ideas you can start by just experimenting with Geometer's Sketchpad (GS) to see what is possible. You might also look at works by different artists for inspiration. See if you can figure out how to do what they did using GS and then add some features to make the work your own. I also often find inspiration in patterns or designs from nature—just open your eyes and look around!

Links to some videos to help you learn how to use Geometer's Sketchpad are posted in the class website. They include:

1. Getting started: the GS tools
2. Using GS menus: creating line segments and polygons, doing reflections and rotations, adding color
3. Using GS menus: making polygons
4. Using GS menus: translations and dilations
5. Active feature

Deadline for Part I: Monday November 3: Five artworks with a written explanation of how you made them; describe your inspiration for these projects, if any.

Deadline for Part II: Monday November 10: Completed artwork with description and explanation of how you made it.

Grading of the presentation and blog posting is based on how well you describe your project and the mathematics you used and how well you prepared for and executed this part of the project. This means having a well thought out (and practiced) presentation and a blog posting that is well written and complete. You should include any early versions of the artwork and describe the changes you made to result in the final version.

Grading:

- Part I 20%
- Part II Artwork 50%
- Blog Posting 15%
- Oral Presentation 15%

### Grading Rubric for Project 2 Presentation

| Presentation Component | Included? (Y/N) | Comments |
|---|---|---|
| Introduction | | |
| Description of artwork | | |
| Mathematics involved and how incorporated | | |
| Methods used | | |
| Materials used | | |
| Inspiration for project | | |
| Turned in preliminary work | | |
| Turned in paper | | |
| Turned in artwork | | |

# 24

# The Magic of Chaos

Roland Minton

**Abstract** One third of the first-year seminar *Science*, *Myths*, *Magic*, *and Chaos* at Roanoke College, a small liberal arts college, is devoted to chaos theory and fractals. The objectives are more conceptual than technical. Students write a research paper analyzing a historical chain of events in terms of butterfly effects and attractors. Other small writing assignments are to find chaos and fractals on campus, and to write a personal butterfly effect story. Students have consistently overcome a fear of the technical nature of the material and taken pride in developing a new outlook on the world.

## 24.1 Background and context

*Science*, *Myths*, *Magic*, *and Chaos* (SMMC) is a first-year seminar (FYS) at Roanoke College, a liberal arts college of 2,000 students. The course is part of the college's Intellectual Inquiry general education curriculum. The General Education Committee establishes broad requirements (three drafted papers, one research paper, reading- and writing-intensive) while allowing instructors free choice of topics and readings, subject to initial approval and some monitoring.

Faculty from all disciplines are encouraged to develop an FYS. No individual is required to teach an FYS, although each department has a quota to meet. Class size is capped at 15. The class meets for 180 minutes per week for thirteen weeks, and is a full one-unit course (Roanoke does not assign credit hours, but transfers generally receive four hours credit for one unit.) The students are first-semester students coming from a variety of intended majors. Ideally, they sign up for the SMMC section of FYS because they enjoy science and math, but in many cases they choose the section that best fits their time schedule.

The SMMC course explores challenges to knowledge of things past, present, and future. Required books include Stephen Jay Gould's *Bully for Brontosaurus* [2], Macknik and Martinez-Conde's *Sleights of Mind* [4], and James Gleick's *Chaos* [1]. Many of Gould's essays explore modern misinterpretations of episodes in the history of science. These and other "myths" constitute slightly more than one-third of the course. Macknik and Martinez-Conde are neuroscientists and magicians, who use magic tricks to illuminate biases in the human brain. This work is enjoyable, memorable, and important, and accounts for slightly less than one-third of the course. Gleick gives excellent case studies in the development of a new scientific field, a field which challenges our understanding of what is simple and what is complex.

## 24.2 Mathematical Theme

Chaos theory can be appreciated on many different levels. A full mathematical treatment can be quite technical for a graduate student, but many of the basic concepts are accessible to everyone. There are ramifications of chaos theory in everyday life, and fractals provide attractive images. These make chaos theory an excellent topic for a first-year seminar.

Chaos theory and fractals constitute a third of the SMMC course. We focus on the butterfly effect, as a challenge to knowing the future. The butterfly effect is explored both numerically and physically, as are system attractors and bifurcations. Fractals are introduced as complex geometric objects and as beautiful representations of the complexity of dynamics.

For the purposes of this seminar, we work with the following brief and imprecise explanations of the basic concepts in chaos theory. The butterfly effect is "sensitive dependence on initial conditions" in which a small change in the state of a system can drastically change the future evolution of the system. An attractor is a stable equilibrium, a state that the system converges to from a significant set of initial conditions. A bifurcation point is a value of a parameter in a system at which the system attractor changes. Fractals are sets with non-integer dimension. They are geometric objects that do not fit comfortably into standard geometric classifications, typically due to the presence of details on an infinite number of size scales.

## 24.3 Course Structure

### 24.3.1 Overview

The chaos theory portion of SMMC begins right after fall break. The material can be scary to some students, so beginning when everyone is relaxed and fresh is helpful. I mix numerical demonstrations of the properties of chaos with physical demonstrations (e.g., a double pendulum whose motion is dramatically chaotic) and videos to introduce the main concepts. The students read Gleick's book [1] and we discuss the development of chaos theory and the initial resistance to the field in the scientific community.

This portion of the course is more professor-dominated than the rest of the course, since the ideas of chaos theory are new to most students. Lecture is mixed with demonstrations and videos to give a multifaceted introduction to the concepts, and each idea is personalized with examples and student response. For example, after introducing the butterfly effect we talk about the importance of individuals in history. What if Steve Jobs or The Beatles had never existed? Discussions of several movies, my favorite being *Back to the Future* [5], help the students relate to the butterfly effect.

There are two short papers (1–2 pages each), a research paper (4–6 pages), and a test on chaos theory. The prompt and the grading rubric for the research paper are in the Appendix (pp. 243 and 244). Questions from which the test is constructed are also in the Appendix (p. 245). Prompts for the two short papers are in the Appendix on p. 246.

### 24.3.2 Short Paper: Personal Butterfly Effect

In one of the short (1–2 pages) papers, the students describe a butterfly effect in their lives. I tell them in advance that most students write good papers on one of two topics. One topic comes from family history: how their parents met, for example. These are always enjoyable for me to read. Family stories have usually been polished over years of telling, and I like the idea of helping young students think about what they owe to their families.

The second topic that is commonly used is more personal: how they came to Roanoke College. These are typically less well developed and less enjoyable to read. However, this topic prompts the students to think about the consequences of their decisions. This goes deeper than whether a decision is good or bad. The choices they make affect their lives in a dizzying variety of ways.

There are two aspects of the butterfly effect that I want students to understand. The starting point for a butterfly effect should be small and seemingly insignificant, and the ending point should be unpredictable in some way. For this paper, I emphasize the small starting point. A story of choosing Roanoke College should not start with coming to campus for a sports try-out camp. That is clearly an important event that is likely to have significant consequences. Finding out about the camp by finding a flyer that somebody else had dropped on the floor is better. There needs to be an element of "there's no way that *that* could matter" to the beginning point of the story.

The unpredictability of the outcome would be challenging for the two common topics mentioned above, since the outcome of marriage or college choice is already known. The aspect of unpredictability is emphasized in the major research paper, discussed next.

### 24.3.3 Major Paper: Historical Chaos

Two course readings provide the model for the chaos paper. Both are from *Bully for Brontosaurus* with *George Canning's Left Buttock* [2, pp.21-31] and *The Panda's Thumb of Technology* [2, pp.83-118] giving examples of what Gould calls historical contingency. This is the idea that the unfolding of history is often not an inevitable destiny but instead the result of some quirky event(s). Historical contingency has much in common with the butterfly effect.

In one essay, Gould traces consequences of a duel in which George Canning is injured, but not killed, to Andrew Jackson's Presidency on one end and Charles Darwin's *Beagle* voyage and subsequent development of the theory of natural selection on a different end [2, pp.21-31]. In another essay, Gould traces the unlikely events that gave the QWERTY keyboard its monopoly, in spite of being designed specifically to slow down typists [2, pp.83-118).

The presence of these two essays in Gould's book reinforces one of my themes for this section of the course: chaos theory can be applied in many different contexts. The concepts of butterfly effects and attractors are new to the students and somewhat technical. To help them successfully assimilate these ideas, I provide many examples, both mathematical and practical, and have them write about their own examples.

Each student is then tasked with researching and writing about a historical contingency of their own choosing. I require a chain of events at least three events long, with solid research for each link. At least one of the links should be surprising and small. Examples that have been used include the wrong turn leading to the assassination of Archduke Ferdinand, and the accidental discovery of saccharine.

In the larger context of the college, an important aspect of the paper is sound research with correct citations. A subtler goal of the paper is to activate intellectual curiosity. We talk about how to find a chain, starting with a topic of interest and then following clues from the research. Using examples from Gleick's book [1, pp.91-92], a student could start with the survival of a Jew named Benoit Mandelbrot in Nazi-occupied France, find a reference to Mandelbrot's work with data transmission at IBM, and finish with the internet and smart phones. A different path could connect Mandelbrot to Michael Barnsley and the efficient storage of fractals, then to computer graphics in George Lucas movies.

Having constructed a chain of events, the students then analyze each link in terms of chaos theory. Specifically, I ask them to characterize each part of the chain as a butterfly effect or an attractor. That is, which events were inevitable and which were contingent on a specific happenstance? For example, a driver's wrong turn is likely the only reason the assassination of Archduke Ferdinand occurred that day; this is a nice butterfly effect. If he had survived that day, would he have eventually been assassinated anyway? Would World War I have occurred regardless? With all of the tensions in Europe, an argument can be made that widespread war was an attractor, an outcome that was essentially inevitable. Obviously, there are not right or wrong answers to these questions, but hypothesizing answers pushes the students deeper into their research and helps them think of events in terms of chaos theory.

### 24.3.4 Class Activity: Chaos and Fractals Everywhere

As the end of the chaos theory module approaches, most students have a grasp of the general ideas of chaos theory, although that grasp feels a little tenuous for many. An exercise that helps almost everyone is an excursion outside the classroom. The instructions are minimal: find three examples each of chaos and fractals. At first, some students think the assignment is unreasonable. They have seen chaos and fractals in books, videos, and demonstration, but can they find multiple examples on campus, on their own?

After some grumbling and looking to others for guidance, the students start asking good questions. Does the large tree over there count? My usual answer is that it could work if they provide a good explanation. Why do they think it is a fractal? If someone replies with some thoughts on the self-similarity of branches, that sparks another student to see self-similarity in leaves and the cracks in the sidewalk. Looking overhead, on most days there are some excellent fractal clouds. One year the unfortunate hair style of a student walking by was a popular fractal.

With fractals under control, the panic is now about finding examples of chaos. Usually somebody wants to use leaves on the ground. Explain how this represents the *process* of chaos, I prompt. The path that a falling leaf follows is unpredictable and dependent on minor variations in wind currents. As one student gleefully pointed out, the path of the leaf could literally illustrate a butterfly effect. Having read about Edward Lorenz, good students quickly think of various aspects of the weather. Students who did not think of this on their own listen to other conversations and often master in this assignment what had confused them in class or in their reading.

The students' success finding, on their own, numerous examples helps them take ownership of chaos theory. Some students report that they start seeing chaos and fractals everywhere and enjoy their new perspective on the world.

### 24.3.5 Class Activity: Hunting the Best Videos

The early study of chaos and fractals relied heavily on computer technology to perform calculations and to create the graphics necessary to understand the complicated underlying behavior of certain systems. The challenge in an FYS is to convey some of the details without asking the students to do any programming. My solution is to slip in brief computer demonstrations and video clips.

The NOVA video *Hunting the Hidden Dimension* [3] shows a variety of applications and brief interviews, making it ideal to show 10-minute segments at appropriate times. An interview with Benoit Mandelbrot gives a face and voice to the complex man we read about, and allows me to tell about meeting him at a mathematics meeting. The sophisticated graphics that accompany the IBM transmission noise problem in the video add tremendously to the description in Gleick's book [2, pp.91-92]. Applications of fractals in fashion, computer graphics, antenna development, and other areas convey images of relevant and modern breakthroughs from the material the students are learning.

The video clips further serve the function of breaking up the class, supplementing what might otherwise be too much of a talking head professor. The students always welcome a suggestion to watch a video, and turning the lights back on jars the sleepier students awake. Especially when working with technical material, frequent changes of pace are important.

## 24.4 Reflections

Chaos theory is an important part of SMMC, being able to satisfy a wide variety of course objectives. At the college level, there are writing skills and critical thinking skills to develop. The writing assignments are interesting to the students and promote their best efforts. The study of simple rules giving rise to complex behaviors and complex rules giving rise to simple behaviors engages the students in deep critical thinking. With respect to my personal teaching goals, the course introduces fascinating material that pushes students to view the world in a different and more mathematical way.

While I am happy with how the learning objectives of SMMC align with college-level FYS objectives, I am always tweaking the details of the course to update examples and clarify assignments. I have taught SMMC six times, always in the fall semester. The only significant change to SMMC has been to the major chaos paper. The original format was a paper that mashed together a personal butterfly effect with a research paper in their intended major. The separation of the personal butterfly effect into a short paper has improved the research quality of the students' major papers. Many students found it hard to find a good topic in their major. The change to a historical chain of events has produced much better work, and students often find the chain of events from within their major field.

The chaos portion of SMMC produces less discussion than the myths and magic portions, but I do not find this to be a negative. Since few students have seen these ideas before, discussion slowly develops as we repeat the concepts from different perspectives. Eventually, some students share encounters with chaos and fractals in life or online. An example is one student who was late to class because of not finding a parking place after stopping to let a pedestrian cross the road. This resulted in the last parking spot being taken by someone else. How often do you get to be excused for being late by invoking class material? I am happy with student engagement in the sense that most students sincerely work to understand the concepts.

The numerous demonstrations (e.g., the Chaos Game in which a sequence of random choices always produces the same picture, a fractal known as the Sierpinski triangle) keep the class entertained and engaged even when the underlying ideas remain a little mysterious. Video clips provide further enrichment for those who are following along, and distraction for those who have not quite figured it out.

The general reaction to this third of the course has been consistently positive. I think that it is important that this portion of the course starts right after fall break. This fresh start buys some time with the students, who are generally a little confused at the beginning. The interleaved learning approach of repeating the same concepts from many different perspectives helps them adjust to the concepts and find an entry point using their preferred learning style (visual, aural, hands on, and so on). By the end of the excursion outside, most students are genuinely pleased to have learned something new, technical, and cool. There is a predictable class trajectory moving from fear and confusion to basic understanding to embracement of the concepts.

The first word in the SMMC title is “Science” to discourage anti-science students from being in the class, but that does not always work. My best writer last year enjoys science but hates math (I never found out why, but it is non-negotiable with her), so she checked out at the beginning of the chaos section. She eventually admitted to liking fractals, as long as I allowed her to say that they are not mathematical. She is one example of why chaos theory is a good mathematical topic for an FYS. Some students love the pretty pictures (fractals), some find the what-ifs of historical contingency fascinating, and some enjoy knowing a little about how seemingly random processes work. Very few students have been unable to find something to like. The technical nature of the course is very much a concern at a liberal arts college that does not graduate many mathematics or physics majors. Non-scientists have enjoyed chaos.

In student evaluations at the end of the semester, the chaos theory section is most frequently cited as the “best” part of the course. The fact that students find the most complicated material to be the best speaks to a high interest level in the topics. The pictures and applications are great hooks. The final writing assignment of the course, included in Appendix 24.4, is a reflection on the course. It is, admittedly, an invitation to write about why SMMC is a great course. Even allowing for the fact that few students will give a negative review when they know the paper will be graded, the comments on chaos theory are often very insightful and affirming. Many students see the world as being more complex and more interesting after learning about chaos.

In summary, I have enjoyed the topics in SMMC and am happy with student appreciation of the chaos material. Chaos theory has good mathematical content as well as excellent pictures and applications to interest students who do not enjoy the technical side of mathematics. Gleick’s book is well written and its historical details provide an important framework for the development of chaos theory in class. The writing assignments, especially the two short papers, work well in terms of engaging the students and helping them understand the basic concepts. The major paper is now in good shape and I do not have plans to revise it further.

One of the most magical moments in teaching is seeing students understand and enjoy something they thought was going to be too difficult for them to learn. This happens regularly with chaos theory.

## 24.5 Bibliography


[1] Gleick, James. *Chaos: Making a New Science*. New York: Penguin, 1988.

[2] Gould, Stephen Jay. *Bully for Brontosaurus*. New York: W.W. Norton and Company, 1991.

[3] Schwarz, Michael and Bill Jersey. *Hunting the Hidden Dimension*. Boston: Quest Productions for NOVA, 2008.

[4] Macknik, Stephen and Susana Martinez-Conde. *Sleights of Mind*. Picador, New York. 2010.

[5] Zemeckis, Robert, et al. *Back to the Future*. Los Angeles: Universal Studios, 1985.


## Appendix

### Paper #3: It’s a Chaotic Life

*This is handed out at the end of the first of four weeks of chaos/fractal activities, a month before the final paper is due. They have seen butterfly effect demonstrations, read two chapters in Gleick’s book and read and discussed the two chapters referenced below in Gould’s book. The students are at this point finishing their second paper, but since I am asking them to create their own topics, I want them to have time to mull over possible topics and talk with me about what is likely to work the best.*

We have read about chaos theory in different contexts. Stephen Jay Gould gave us an elaborate example of a butterfly effect in the George Canning essay, and a simpler example with the QWERTY keyboard. The opposite of the butterfly effect, in some ways, is the attractor, where you get the same result no matter where you start. We saw this with the Chaos Game, where the three rules produce the Sierpinski triangle whether we start with a point or a square or whatever. A fantastic fact is that opposites can coexist, as we have seen with the Lorenz attractor. The weather equations always produce that butterfly-like attractor, but the attractor itself exhibits the butterfly effect. Within order, there is chaos.

There are two parts to the paper, although the paper should read as a unified whole. The structure of the paper may well be the first part followed by the second part, but different structures are fine.

In the first part of this paper, create a historical chain of people and events which affected each other. The Canning and QWERTY essays are excellent models for this, where one person (Lord Castlereagh) had connections that extended to Charles Darwin in one direction and Andrew Jackson in a different direction, and one seemingly unimportant typing contest made the QWERTY arrangement universal. Find information about a historical figure/event (at least three good sources) and how his/her/its work influenced others. You should have a chain of connections with at least three people or events involved. As with the Canning and QWERTY essays, it is most impressive if at least one of the figures or events in your chain is relatively obscure or seemingly out of place.

In the second part of the paper, discuss your chain of events in terms of the butterfly effect and attractors. In particular, if one person or event were removed from the chain, would the world be much different? If there is a butterfly effect in your chain (like the bullets that narrowly missed Castlereagh and vital parts of Canning), identify it. If there is evidence that somebody else would have done the same tasks (as Gould points out, Darwin was not the only one working on the theory of natural selection), then perhaps parts of this chain are more accurately described as an attractor that was going to happen regardless. Speculate as to how things might be different if aspects of your chain were different. (Perhaps we could all type faster if we had a better keyboard!)

**Requirements:**

Approximately 4–6 pages in length.
At least three good sources, plus other sources.
The first draft is due Monday 11/27.
Peer review will be Wednesday, 11/29 (graded).
Final paper due Friday 12/8.

**Other thoughts:** The historical chain can be from any field (math or biology or literature or political science or baseball, and so on). A timeframe of 50–200 years ago works well. We have read about several people who could be good starting points for your research. Start with a person or event of interest, and do your research with a chain in mind. (For example, in reading about Benoit Mandelbrot you find the name Michael Barnsley, and part of Barnsley's work leads to computer graphics and you now have Star Wars on one end of your chain.) Inventors are especially good people to look at.

The primary criteria for the grade will be the quality of your research and historical chain, the quality of your analysis of the chain in terms of chaos theory, and the quality of your writing.

## Paper #3 Rubric

*This is handed out at the same time as the paper assignment above, a month before the final paper is due. It is discussed briefly at that point, then more thoroughly two weeks before the first draft is due.*

**History Chain.**
*Excellent*: Strong connections are described clearly. Good details are provided for each piece of link. At least one connection is surprising.
*Above Average*: One connection is weak or not well developed. Connections are important but not surprising.
*Average*: Only two connections are made well. The connections all involve well-known events.
*Below Average*: The chain is a string of loose connections, with no interesting details from research.

**Research.**
*Excellent*: Three good sources are used extensively. Good insight into the life and times of the historical events are used in the analysis.
*Above Average*: Sources are good, but the insight into the events adds little to the analysis.
*Average*: Sources are good, but the information used is basic biography. Little information about the era.
*Below Average*: Two sources are used a little, but nothing beyond basic biographical information is obtained.

**Analysis.**
*Excellent*: Thoroughly explains how the butterfly effect and attractor concept apply. The speculation about how events would change is insightful.
*Above Average*: Explains the butterfly effect and attractor with good argument for how events would change.
*Average*: Explains the butterfly effect and attractor, but little insight into relation to history chain.
*Below Average*: Minimal explanations of the butterfly effect and attractor. No alternatives explored.

**Style.**
*Excellent*: Tone is consistent and in the author's unique voice. The two parts of the paper are tied together in a very meaningful way.
*Above Average*: Some changes in voice mildly disrupt flow. The vocabulary is appropriate.
*Average*: Some disruptions of flow. Poor word choices cause confusion. Two parts of the paper are not connected.
*Below Average*: Changes in tone create distractions. Poor word choices cause ideas to be lost.

**Mechanics.**
*Excellent*: Sentences are well structured, with a variety of forms and lengths. No major grammatical errors. Excellent word choices throughout.
*Above Average*: Varied sentences. Few errors in sentence construction or grammar.
*Average*: Sentences show some errors of structure and grammar, and have the same form.
*Below Average*: Errors of sentence structure and grammar disrupt communication of ideas.

Note that the ratings do not correspond to A/B/C/D/F. Also, these paper traits will not be equally weighted and I do not plug my evaluation of traits into a magic formula. As a mathematician, I need to know when formulas apply and when they don't; grading a paper is definitely a time when formulas do not apply! However, do not expect to get an above average grade if your paper is average in every way.

## Review for Chaos Test

*This is handed out at the end of class two class periods before the test day. Students are instructed to bring questions to class the next period, which is devoted to reviewing for the test.*

Test questions will be selected from these. Be prepared to answer them all.

**Vocabulary:**
(1) fixed point (2) cycle (3) chaos (4) butterfly effect (5) period-doubling (6) attractor (7) bifurcation point (8) chaos game (9) Sierpinski triangle (10) fibrillation (11) signal and noise (12) fractal (13) self-similarity (14) Mandelbrot set (15) Lorenz attractor (16) Edward Lorenz (17) Benoit Mandelbrot

**Short Answer:**
How did Lorenz discover the butterfly effect?
Given a function, compute some iterates by hand.
Identify fixed point, cycle from iterates.
A basin of attraction is often a fractal. Explain how this illustrates the butterfly effect.
Why were ecologists better prepared than mathematicians to see chaos?
Lemming populations rise and fall unpredictably. How would chaos theory explain this?
Explain why chaos theorists often use overly simple equations to explore processes.
How long is the coastline of England? Explain.
Describe the bifurcation point in Elvis the dog's fetching strategy.
The Sierpinski triangle has dimension between 1 and 2. Explain what this means.
Describe the fractal that Mandelbrot found in the IBM transmission data.
How does chaos theory explain atrial fibrillation in patients with no previous heart disease?
Are bad political analysts an attractor of the television system?

**Examples:**
Give two specific examples of phenomena that chaos theory explains.
Describe two problems that led to chaos theory.

Give an examples of bifurcation points in the physical world.
Give two examples of (human-made) applications of fractals.
Give two examples where simple rules produce complex behavior.
Give two examples where nature is playing the chaos game.
Give two examples where period-doubling occurs.
Give two examples where issues involving human health have been explained by chaos theory.
Give two examples where chaos research was initially criticized by mainstream science.

**Essays:**

In time travel, describe how the butterfly effect applies. In a movie, why would writers need to oversimplify the butterfly effect? Use *The Simpsons* and one other movie as examples.

An underlying assumption of modern science is that if we can understand a process completely, we can make accurate predictions. Explain how chaos theory undermines this assumption. Give an example. Explain how *The Signal and the Noise* characterizes good predictions.

A summary of the chaos theory view of life is that it is both orderly and complex. Use the Lorenz attractor and one other example to discuss how order and chaos can combine.

## Short Paper, Chaos and Fractal Examples

*This is done the third week of chaos/fractal activities, after students have seen fractal demonstrations and videos, and read about fractals in nature in Gleick's book. The second half of that class period is spent outdoors, and the paper is due before the next class.*

For each, write 2–3 clear sentences describing the example and explaining why it illustrates the property.

(1) Find three examples of chaos.
(2) Find three (different from the first three) examples of fractals.

## Short Paper, Personal Butterfly Effect

*This is given out on the second day of chaos activities, after seeing demonstrations and reading about Lorenz and the butterfly effect. Students had been told before fall break about the assignment in general terms, so that they could ask for family stories over break.*

We have discussed the butterfly effect in class: a small, seemingly insignificant change can have large effects on the future. Write a personal butterfly effect—examples used in the past are how your parents met or how you ended up at Roanoke College, but be creative! Remember that the butterfly effect starts with a small event that you wouldn't normally think would matter.

## Short Paper, Final Reflection

*This is given out on the last day of class, and is due at final exam time. The course does not have a regular final exam otherwise.*

Due by email. Each of these short papers should be 1-2 pages in length. Neither requires a Works Cited list unless you use sources other than the three course textbooks and the Freedom with Purpose statement.

Paper 1: We each have a personal mythology, a way of viewing and making sense of the world. Identify one chapter from *Bully for Brontosaurus,* one chapter from *Sleights of Mind* and one chapter from *Chaos* that was meaningful to you and altered or resonated with your personal mythology, and describe what you have learned from each.

Paper 2: Roanoke College has an excellent statement of what we think a liberal arts education is about. Titled "Freedom with Purpose," it summarizes our goals for your education. It opens with

(A liberal arts education) leads us out from small, safe worlds into larger, more interesting ones by training in us a dissatisfaction with partial knowledge, with sloganeering, and with fixed ideologies. It instills in us instead an appreciation for the true complexity of things and a lifelong commitment to learning. A mind so trained respects facts, employs apt methods, and engages in creative problem solving. It examines alternatives; it does not fear tension or paradox. It welcomes the stubborn “misfit” fact that cracks open a too-small view and releases us into a wider play of thought. And it encounters this liberating openness in the vision of artists; in the venturesome thought of philosophers, theologians, and mathematicians; in the observation and experimentation of scientists; in the insights of social scientists; and in the experience of living in community.

Describe some ways in which this course explores partial knowledge, the true complexity of things, paradox, and misfit facts. In *Sleights of Mind,* the authors write that “increased humility deepens the mystery” of life. Does learning the secrets of magic and the human mind decrease or increase the wonder and the mystery for you? Do you feel like you have been led out into a larger, more interesting world?

# 25

# A First-Year Seminar on Symmetry, from a Mathematical and Interdisciplinary Point of View

Tamara J. Lakins

**Abstract** Allegheny College requires each of its students to complete a sequence of three First-Year/Sophomore seminars that emphasize writing and speaking. My First Seminar, *Symmetry through the Eyes of Mathematics*, investigates the interdisciplinary nature of symmetry: what it means in mathematics, how it occurs in other disciplines, and how to apply the mathematical ideas to symmetry in art and architecture. This article focuses on the major assignments that emphasize the interdisciplinary nature of symmetry.

## 25.1 Background and Context

Allegheny College, a selective liberal arts college of approximately 2,100 students located in northwest Pennsylvania, has required its students to complete a sequence of three First-Year/Sophomore seminars since 1999. Among the learning outcomes of all three courses in Allegheny's First-Year/Sophomore Program are the goals that students learn to think critically and creatively, and that students learn to communicate clearly as writers and speakers. Students take the First Seminar (FS), whose focus is on descriptive writing and speaking, in the fall of their first year. The Second Seminar, which is taken in the spring of the first year, emphasizes argument. The Sophomore Seminar, which is typically taken in fall or spring of the sophomore year, is discipline-specific and focuses on the conventions of writing and speaking within a specific discipline. Each of the First Year/Sophomore seminars is a four credit course, meeting for 150 minutes per week (which is typical of most courses at Allegheny). The semester is 14 weeks long, not including the final examination period.

All faculty at Allegheny are expected to regularly teach in the First-Year/Sophomore program and may choose their own topic for the first-year courses. Faculty in the mathematics department typically choose topics of interest to them that are related to mathematics in some way, out of comfort, but this is not a requirement.

FS instructors are encouraged to assign

> regular opportunities to speak in class and short writing assignments, … at least two formal writing assignments of 3–5 pages that include the possibility of revision, at least one formal speech assignment (4–5 minutes) that includes preparatory work, [and] library research as a part of at least one of the writing or speech assignments. [6]

The FS instructor also serves as academic advisor for each student in the class, which is capped at 15 students, until the student declares a major and chooses an academic advisor in that major some time in the sophomore year. This advising component leads to some FS class time spent on issues other than the mathematical topic of the seminar, such as discussing the elements of effective writing and speaking; plagiarism and academic honesty (including Allegheny's Honor Code); speaker, audience, and peer reviewer responsibilities; library skills; writing and speaking resources at Allegheny; and academic planning. In addition, the FS instructor is expected to meet with each individual FS student at least twice outside of class during that fall semester.

## 25.2 Mathematical Theme

The topic of my FS course is *Symmetry through the Eyes of Mathematics*, and it investigates the interdisciplinary nature of symmetry: what it means in mathematics, how it occurs in other disciplines, and how to apply the mathematical ideas to symmetry in art and architecture. The major mathematical goals are for students to understand how rigid motions in the plane are used to mathematically define symmetry, how rigid motions are combined, the concept of a mathematical group, and how we can describe (i.e., classify) the symmetry type of a finite figure, frieze (strip) pattern, or wallpaper pattern in the plane. The mathematical topic, and its connections to other disciplines, is woven into the entire course. I spend approximately 50% of the total class time teaching mathematics.

Because the FS is required of all first-semester, first year students at Allegheny, prerequisites are not permitted. Students in the seminar typically have a wide range of math skills, and there may be only a few intended mathematics majors. The word "mathematics" is deliberately used in the title of the seminar, so that students know what to expect. The college's process of assigning first year students to an FS section ensures that all students enrolled in the seminar had expressed interest in the topic.

## 25.3 Course Structure

Classroom activities, readings, class discussions, and assignments related to the mathematical theme focus on defining the mathematical language of symmetry and applying these ideas to other disciplines. The course begins with a short introduction to symmetry that culminates in class discussion of a reading that foretells the mathematical and interdisciplinary ideas discussed in the course. This introduction to the main ideas of the course leads naturally to the first major writing and speaking assignment, which focuses on symmetry around us in the "real world." Next follows a period of seven to eight weeks of in-class discovery activities that focuses on the mathematical language and tools needed to analyze symmetry and prepares students for the second major writing and speaking assignment. This assignment, which returns to the idea of symmetry in the "real world," focuses on symmetry in art and architecture. The course ends with a common reading and assignment focused on how two specific cultures use symmetry in their art and architecture.

Several elements go into the computation of final grades in the course. The major writing assignments are each worth 15% of the grade, and the major speaking assignments are each worth 10% of the grade. The grades on the major writing assignments are based not only on the final product, but also on effective participation in the writing process (i.e., response to feedback). Other graded aspects of the course include participation and informal speaking opportunities (10%), other written work (15%), and a take-home final exam (5%). Peer review of writing and speaking counts 10%, and academic planning assignments count 10%; each of these items is graded on a $-$, $\surd-$, $\surd$, $\surd+$ scale.

### 25.3.1 An Interdisciplinary Introduction to Symmetry

A short (three to four class period) and comprehensive introduction to symmetry culminates with class discussion of a unique reading, the introduction "Patterns of Symmetry: Seek and Ye Shall Find" to *Patterns of Symmetry*, the proceedings of a 1973 interdisciplinary "Symmetry Festival" held at Smith College [7, pp. 3-19]. The reading is fairly advanced, as its audience is actually university faculty. However, it gets the ideas of the course out fast: rigid motions, mathematical groups, the idea of analyzing patterns, and symmetry in art, music, dance, poetry, and physics.

The assignment (see the Appendix, p. 254) directs students to read the introduction to *Patterns of Symmetry* using the "guidelines for active reading" described in Hacker's *A Writer's Reference* [4, pp. 57–58]. Included in the assignment is a list of discussion questions; students must bring two copies of their answers to class (one copy for me and one for them) and be prepared to contribute their answers in class. The discussion questions are designed to cover the main ideas in the reading, from the mathematical concepts, to the universal nature of symmetry, to some specific examples of symmetry in the world.

Finding this source was a lucky event. While preparing for the first semester this FS was offered, I had been looking for material to support the interdisciplinary aspect of the course. What I found was even better: a reading that reinforced all the ideas I hoped to cover in the course.

### 25.3.2 The First Writing and Speaking Assignment: Symmetry in Many Disciplines

The discussion of symmetry provided by the introduction to *Patterns of Symmetry* [7] leads naturally to the first major writing and speaking assignment, on a topic related to how symmetry occurs in the "real world" (see the Appendix, p. 254). The assignment consists of a three page paper and a three to five minute speech on a discipline in which symmetry occurs; art and architecture are not allowed in this first assignment as they are the focus of the second major writing and speaking assignment. In this assignment, students are asked to address the following questions:

- What does the concept of "symmetry" or "symmetrical" mean in this field?
- Where and how does symmetry occur in this field? Describe at least two specific examples.
- What is the significance of symmetry in this field? How is it interpreted, or what is its purpose?
- How and why is symmetry broken, if applicable?

Students must consult two sources: one published source, and an interview with an Allegheny faculty member. A list of possible topics is provided to the students, with a list of potential faculty members. These faculty members are "hand-picked"; the assignment is discussed with them, and permission to include them is obtained, ahead of time. Suggested subject areas are biology, chemistry, dance studies, geology, literature or poetry, music, neuroscience, physics, psychology, and theatre performance. I restrict the number of students who may choose any given field to two, and at most one student may contact any given faculty member. Examples of topics that students have chosen include symmetry in contra dance, symmetry in plants, and symmetrical versus asymmetrical ripples in shoreline environments.

I require the interview with an Allegheny faculty member for several reasons: to get students used to consulting with faculty outside of class, to enable students to meet other faculty, and to ensure that someone knowledgeable about the student's chosen discipline guides them in narrowing down the topic and finding an appropriate second source for the paper. While I allow students to choose a field and/or faculty member on their own, they must discuss it with me and receive my approval in advance; this enables me to approach the relevant faculty member in advance.

Students are required to make use of either a peer writing consultant or a peer speech consultant in Allegheny's Learning Commons for this assignment. This requirement serves as an introduction to the resources found in the Learning Commons and demonstrates that seeking feedback is part of the writing and speaking process, regardless of whether or not a student feels they "need help."

The two major writing and speaking assignments are graded according to specified rubrics, copies of which are distributed to students with the assignment. Class time is devoted to discussing the elements of effective writing and speaking, plagiarism and academic honesty, library skills, and writing and speaking resources in Allegheny's Learning Commons. Hacker's *A Writer's Reference* [4] and Sprague and Stuart's *The Speaker's Compact Handbook* [8], which are explicitly referred to in class when discussing the assignments, are required textbooks. Students are required to submit a "formal draft" of their paper for feedback and a non-binding "current grade"; the phrase "formal draft" is used, rather than "rough draft," in order to communicate to students that they should be handing in their best work from the start. As noted earlier, the final grades on writing assignments are based not only on the final product, but also on response to feedback. In addition, students participate in a peer review of the speech assignments. Finally, to help students prepare for the first major assignment, one class session with a reference librarian is held in the college library.

### 25.3.3 The Second Writing and Speaking Assignment: Symmetry in Art and Architecture

Once the first major writing and speaking assignment is distributed, the class settles into a seven to eight week discussion of the mathematical ideas and language of symmetry, as preparation for the second major writing and speaking assignment. The primary instructor resources used for this material are David Farmer's text *Groups and Symmetry: A Guide to Discovering Mathematics* [3] and Clayton Dodge's text *Euclidean Geometry and Transformations* [2]. Mathematical concepts covered during this period include:

- the idea of combining rigid motions, leading to the idea of a mathematical group
- describing the symmetries of a finite figure in the plane as type $C_N$ (exactly $N$ rotation symmetries) or $D_N$ (exactly $N$ rotation symmetries and $N$ reflection symmetries)
- discovering the seven possible symmetry types for a strip (frieze) pattern (a pattern with nontrivial translation symmetry in one independent direction) and identifying the symmetry type of such a pattern using the four character IUC notation
- learning to use a flowchart [3, p. 58] to identify the 17 possible symmetry types for "wallpatterns" (patterns with translation symmetry in two independent directions; the terminology is due to Farmer [3, p. 43]) in the crystallographic notation.

The second major writing and speaking assignment asks students to investigate an example of how symmetry and patterns occur in art or architecture (see the Appendix, p. 256). The assignment consists of a five page paper and a six to eight minute speech. The students are asked to provide background material on a particular culture, artist, or architect, and a description of where their chosen type of symmetry occurs. Students must also provide at least six symmetrically different patterns in their paper, including information on where the patterns can be found, and explanations of how to mathematically analyze those patterns. The explanations must include identifying the rotations, mirror reflections, translations, and glide reflections present in the patterns, and indicating relevant rotocenters, mirror lines, translations, and glide reflections in the illustrations. In the associated speech, students must provide an active analysis of four of the patterns in their paper, using projected images of these patterns. Students are given a warm-up exercise for this requirement before the formal presentations begin.

Students must consult at least two published sources. They are given a list of possible topics, most of which come from Section 8.2 of Farmer's *Groups and Symmetry* [3]. However, some additional topics have been added over time, as students may also suggest their own topic in consultation with me. Examples of possible topics include the art of M. C. Esher, the art of William Morris, the art of Robert Adam, the Alhambra, African art or textiles, Native American pottery, rugs, or beadwork, Amish quilts, Turkish art or architecture, Mayan art or architecture, and Aztec art. One particularly creative student chose to analyze the symmetry in the stained glass windows in Allegheny's Ford Chapel.

As with the first major writing and speaking assignment, this assignment is graded according to specified rubrics, and students are required to submit a "formal draft" of their paper for feedback. Students are strongly encouraged, but not required, to work with a writing consultant or speech consultant from the Learning Commons, or with a reference librarian. It is nice to see that some students learn the benefits of using the resources in the Learning Commons from the first writing and speaking assignment and choose to do so again for this assignment.

### 25.3.4 Symmetry in the World

The semester ends with a common reading in the spirit of the second major writing and speaking assignment, which again connects the mathematics of the course to "real world" examples. While students are working on the second major writing and speaking assignment, they are assigned for reading and discussion Sections 3 and 4 from Chapter 6 (Symmetric Strip Decorations) of Ascher's *Ethnomathematics* [1]. The goal of this assignment is to reinforce the ideas of the course, as well as to expose students to other cultures. Section 3 of the reading describes the symmetric strip patterns found on the rafters of *marae*, the community and spiritual meeting places of the Maori, the indigenous people of New Zealand. Section 4 describes the patterns found on Inca pottery. As part of the assignment, students submit to me two discussion topics related to the reading, and they must be prepared to lead discussion on their topics during the class discussion of the reading. Examples of discussion topics chosen by students include the comparison of the curvilinear patterns of the Maori versus the rectilinear patterns of the Inca, and the comparison of the natural themes used by the Maori versus the decorative themes used by the Inca.

## 25.4 Reflections

I have taught this particular FS four times over the last 15 years, as the instructors for the First Seminars taught in our math department rotate among several faculty members. I chose the topic because of my interest in abstract algebra and my wish to try an interdisciplinary topic. I was aware of, and interested in using, the Farmer text [3], but many of the details of the geometry were new to me. I spent a summer learning the mathematics, primarily from Dodge [2].

I have been pleased at how well the mathematical and interdisciplinary ideas of the course have been received by students, as documented by their feedback on a questionnaire I distribute at the end of the semester. Students reported that they enjoyed analyzing patterns, and they enjoyed the readings.

The many goals of the First Seminar (critical thinking, writing, speaking, advising) limit the amount of time one can spend on the mathematical and interdisciplinary content of the course, making it a challenge to discuss the mathematical ideas needed in time for students to successfully complete the second major writing and speaking assignment. Over time, I have decreased the number of mathematical tasks in Farmer's text [3] that I include in the course, in response to some student difficulty with some topics and a desire to focus more on the essential ideas.

Learning (as a mathematician) how to help students improve their writing and speaking skills is another challenge. Allegheny regularly schedules workshops on teaching writing and speaking for faculty teaching an FS course, which were a big help to me when I started teaching the course. My approach has been "teach them to write better," rather than "teach them to write." However, based on my experience grading writing and speaking in this course, and on student feedback on the end of semester questionnaire, I hope to devote more class time to helping students become better writers and speakers the next time I teach the course. Possible ways of doing this include discussing a "top 10" list of writing errors (such as the comma splice and subject-pronoun agreement), assigning a paragraph to "correct" for class discussion, assigning an exercise on proper paraphrasing, and allowing time in class to practice speeches, with the goal of providing feedback to help the students be ready for the graded speeches. I had previously included some assignments on summarizing the two readings in the course, but I discontinued them when they didn't go well (writing a summary is surprisingly difficult!). The next time I teach the course, I hope to learn strategies for improving my teaching of good summary writing, so that I can reinstate those assignments.

One of my personal goals in offering this FS course is that I hope students begin to see the universal nature of symmetry. The final assignment in my course, which is actually a compilation of three assignments given throughout the semester, brings the topic home. Students are asked to find two examples each of finite figures, frieze (strip) patterns, and wallpatterns on the Allegheny College campus and to write a short essay describing them and analyzing them mathematically. At the end of the semester, the class takes a walking tour of campus to see the various patterns. This final assignment brings the course full circle. In the opening sentence of the introduction to *Patterns of Symmetry*, the editors note that "Symmetry can be seen just about everywhere when you know how to look for it" ([7], p. 3). After completing the semester, students in the seminar report that they now actively look for symmetry in the world around them.

## 25.5 Bibliography


[1] Marcia Ascher, *Ethnomathematics: A Multicultural View of Mathematical Ideas*, Brooks/Cole Publishing Company, Belmont, 1991.

[2] Clayton W. Dodge, *Euclidean Geometry and Transformations*, Dover Publications, Inc., Mineola, 1972.

[3] David W. Farmer, *Groups and Symmetry: A Guide to Discovering Mathematics*, The American Mathematical Society, Providence, 1991.

[4] Diana Hacker, *A Writer's Reference*, 6th edition, Bedford/St. Martin's, Boston, 2007.

[5] Tamara J. Lakins, "An Interdisciplinary First Seminar on Symmetry," in *Bridges Banff Proceedings 2009*, Craig S. Kaplan and Reza Sarhangi, eds., MATHARTFUN.COM and Tarquin Books, 2009.

[6] "Learning Outcomes Specific to FS 101," in *First-Year/Sophomore* (*FS*) *Program*, `sites.allegheny.edu/facultyresources/fsprogram/`, accessed on September 6, 2018.

[7] Marjorie Senechal and George Fleck, "Patterns of Symmetry: Seek and Ye Shall Find," in *Patterns of Symmetry*, Marjorie Senechal and George Fleck, eds., University of Massachusetts Press, Amherst, 1977.

[8] Jo Sprague and Douglas Stuart, *The Speaker's Compact Handbook*, Thompson Wadsworth, Belmont, 2006.

# Appendix

## Patterns of Symmetry: Seek and Ye Shall Find

*This assignment is the first major reading and discussion assignment. It is given to students in the second week of classes, immediately after the class period in which we discuss the four rigid motions of the plane. The discussion typically occurs during the next class period.*

**The assignment:** Read pages 57–58 in Hacker's *A Writer's Reference, 6th edition*. Then read *Patterns of Symmetry: Seek and Ye Shall Find*, using the "guidelines for active reading" described by Hacker. Bring two copies of your (word-processed) answers to the following questions for discussion to class, and be prepared to contribute your answers in class.

**Questions for discussion.**

1. Name as many examples as you can of subjects where the notion of symmetry occurs which are mentioned in the article.
2. The authors claim that "Artists, musicians, and scientists deal with similar concepts . . . ." What evidence is given in the article for this?
3. What evidence is given in the article that humans have been fascinated with the concept of symmetry over time? Why do you think this is so?
4. What are some of the different kinds of patterns that can be created using mirrors? How many mirrors, and what configuration of these mirrors, produce these patterns?
5. Do you think that "perfect symmetry" occurs in artistic works (i.e., works of art, or music)? Why or why not?
6. What is the difference between the dual notions of "symmetry as cause" and "symmetry as effect"?
7. What is the name of the mathematical system that corresponds to a pattern of symmetry?
8. What is a canon? What is a "crab canon"? What is a "puzzle canon"?
9. What is the significance of the secondary title of this article: "Seek and Ye Shall Find"?
10. What ideas in the article did you find interesting or confusing?

## First major paper and presentation

*The first major paper and presentation assignment is given to students immediately following the discussion of the assignment (Patterns of Symmetry assignment, above). The formal draft of the paper is due approximately two weeks after the assignment is distributed. The final version of the paper is due approximately two weeks after that, with the presentations occurring an additional two weeks later.*

**First major paper and presentation: Introduction.**

For this paper and presentation, you should describe how symmetry occurs in the real world. One of the sources for the paper should be an interview with an Allegheny faculty member; you should also cite at least one published source.

A possible list of fields to consider and names of faculty who have agreed to be interviewed appears on the next page; no more than two students may choose the same field. If you wish, you may choose a field and/or faculty member on your own, but you must discuss it with me and receive my approval in advance of the September 14 topic deadline. In order to ensure that no more than two students choose the same field, and that two students do not pick the same faculty member to interview, you must discuss with me during office hours who you intend to interview by September 14 and before you contact the faculty member in question.

Your paper should address the following types of questions:

1. What does the concept of "symmetry" or "symmetrical" mean in this field?
2. Where and how does symmetry occur in this field? Describe at least two specific examples.
3. What is the significance of symmetry in this field? How is it interpreted, or what is its purpose?

4. How and why is symmetry broken, if applicable?

Your first presentation will be a three to four minute speech of description based on your paper. The speech should summarize the content of your paper and describe at least one specific example from your paper. More details about the presentation will be given at a later date.

**Policies:**

- Your paper should be three pages long and be typed using Word or another word processor.
- You must correctly cite your sources using the MLA format. Remember that whenever you include information or ideas from an outside source, you must cite it, regardless of whether it is a direct quotation or a paraphrase. Also remember that poor paraphrasing can lead to charges of plagiarism.
- Internet sources are not permitted, although you may use sources from electronic journals or databases on the Pelletier Library website.
- If you have difficulty locating appropriate written sources, then you should contact a reference librarian in Pelletier Libarary for assistance (and discuss it with me).
- Late drafts will not be accepted without documented extenuating circumstances, and you must inform me of any extenuating circumstances as soon as you are aware of them. Failure to turn in a formal draft will result in a one letter grade reduction on the final paper. Late final papers will be accepted up to two days after the due date and will receive a one third letter grade reduction.
- As you prepare your paper and/or presentation, you must make use of either a writing consultant or a speech consultant in the Learning Commons. You must include a receipt from a writing consultant with the formal draft of your paper. Receipts from a speech consultant are due on the day that your presentation is given. Failure to consult a Learning Commons resource will result in a two thirds letter grade reduction on the final paper.

**Due dates:**

1. Topic you have chosen and faculty member you intend to interview (office hour consultation with Prof. Lakins): Friday, September 14.
2. List of potential published sources: Friday, September 21.
3. Formal draft: Friday, September 28.
4. Final version due date: Friday, October 12.
5. Presentations: October 19, 24, 26.

**Possible fields** (names of faculty members have been omitted): biology, chemistry, dance studies, environmental science, geology, literature or poetry, music, neuroscience, philosophy, physics, psychology, theatre performance.

The following sources have been suggested by some of the faculty members above; others may suggest a source during the interview. In any case, be sure to ask the faculty member during the interview what source they recommend! These sources will give you a starting point for your research, but you may need (or want) to search for other sources. If you have difficulty locating appropriate sources, you should consult with a reference librarian or me.

Be sure to start working on your paper early enough to allow time to interview (and perhaps, reinterview) a faculty member, locate and read appropriate sources, and use the Learning Commons.

1. biology
   (a) Coen, "The making of a blossom," *Natural History* **111** (May, 2002) 48–55.
2. chemistry
   (a) Jaffé and Orchin, *Symmetry in chemistry*.
   (b) McManus, *Right hand, left hand*.
3. music
   (a) Copland, *What to listen for in music*.

   (b) Earhart, *Music to the listening ear*.
   (c) Machlis and Forney, *The enjoyment of music*.
   (d) O'Brien, *The listening experience*.

4. psychology
   (a) "Organizing the environment: perceptual organization," pages 176–191 from Goldstein, *Sensation and Perception*.

5. physics
   (a) "The architecture of crystals," Interlude I from Ohanian, *Physics*, 2nd edition, pages I-1 through I-13.

**First presentation: Speech to Explain**

The first presentation is a three to five minute "Speech to Explain" based on your paper: describe how symmetry occurs in the real world. A speech to explain should focus on the basic skills of summary and explanation; your goal is to transmit information in a clear and concise manner. Your speech should address the same questions as your paper:

1. What does the concept of "symmetry" or "symmetrical" mean in this field?
2. Where and how does symmetry occur in this field? Describe at least two examples.
3. What is the significance of symmetry in this field? How is it interpreted, or what is its purpose?
4. How and why is symmetry broken, if applicable?

Use the extemporaneous mode of delivery (see *The Speaker's Compact Handbook*, 28a, 33b). This involves preparing a full-sentence outline of your speech (see 20b), converting it to a key-phrase outline, and finally organizing your key-phrase outline into speech notes, which you should write on notecards for use when your give your speech.

Your full-sentence outline is due to me when you give your speech.

Your speech will be assessed by me and reviewed by your peers in the class. The assessment criteria, keyed to the appropriate chapters in *The Speaker's Compact Handbook*, are given on the back. An extremely short or lengthy presentation will adversely affect your grade.

If you would like to use an overhead projector, please let me know by October 12.

You are expected to practice your speech before you give it in class. Your audience may be an obliging friend or classmate or, even better, a speech consultant at the Learning Commons. If you choose to consult a speech consultant as part of the original assignment (see the first handout), then you should turn in your "receipt" from the speech consultant on the day of your speech. Failure to consult a Learning Commons resource will result in a two thirds letter grade reduction on your final paper or speech (as appropriate).

You will be assigned a day on which to give your presentation. Failure to give your speech on your assigned day will result in a one letter grade reduction.

## Second major paper and presentation

*The second major paper and presentation assignment is given to students approximately one week after the presentations associated with the first major paper assignment (see p. 254) are given. The formal draft is due approximately two weeks after the assignment is distributed. The presentations occur approximately two weeks later, with the final version of the paper typically due the next class period, on the last day of class.*

**Second major paper and presentation: Introduction.**

For this paper, you should research an example of how symmetry and patterns occur in art or architecture. A list of possible topics appears below. No more than two students may choose the same topic. If you wish, you may choose a topic of your own, but you must first discuss it with me and receive my approval (before the first deadline below of November 7). Read sections 8.1 and 8.2 of our text for a more thorough description of the some of the topics below.

- Your paper should include background information on your topic, such as a summary of the history of the culture, artist, architect, etc., in question, and a description of where your chosen type of symmetry occurs (be as specific as you can; the type of background information that is appropriate will depend on the topic you've chosen).
- You must use at least two published sources.
- Include photocopies, computer graphics, or careful drawings of at least six symmetrically different patterns that are appropriate to your topic and you find interesting; you may include any combination of wallpatterns, strip patterns, or finite figures. Explain where the examples can be found, and carefully analyze their symmetries mathematically. You should not only classify these patterns but also explain in English how you did so, by discussing the rotations, mirror reflections, translations, and glide reflections present in the patterns. You should indicate relevant rotocenters, mirror lines, translations, and glide reflections on your illustrations. Use examples whose symmetries you analyze yourself, rather than ones whose symmetries are described in a source.

Your second presentation will be a six to eight minute speech of description and explanation based on your paper. The speech should not only address the background material but also include a discussion of four of the patterns that you analyzed for the paper. You must use an overhead projector when discussing your examples. More details about the presentation will be given at a later date.

**Policies:**

- Your paper should be five pages long, not including the illustrations, and be typed using Word or another word processor.
- You must correctly cite your sources using the MLA format; this includes the figures in your paper. Remember that whenever you include information or ideas from an outside source, you must cite it, regardless of whether it is a direct quotation, a paraphrase, or a graphic. Also remember that poor paraphrasing can lead to charges of plagiarism.
- Internet sources for the content of your paper are not permitted, although you may use sources from electronic databases on the Pelletier Library website. You may use internet sources for your graphics only with my prior approval. Such approval must be attained before turning in your source list on Monday, November 12.
- Late drafts will not be accepted without extenuating circumstances related to illness, which must be documented. Failure to turn in a formal draft will result in a one letter grade reduction on the final paper. Late final papers will be accepted up to two days after the due date and will receive a one third letter grade reduction.
- While not required, I strongly encourage you to make use of the Learning Commons as you prepare your paper and/or presentation. If you choose to work with a writing consultant or a reference librarian, then you must include a "receipt" with the formal draft or final version of your paper, as appropriate. Receipts from a speech consultant should be turned in on the day that your presentation is given.

**Due dates:**

1. Inform Prof. Lakins of your chosen topic (office hours consultation with Prof. Lakins): Wednesday, November 7 by 5pm.
2. List of proposed sources: Monday, November 12.
3. Formal draft: Monday, November 19.
4. Final due date: Wednesday, December 12.
5. Presentations: December 3, 5, 7, 10.

**Possible topics** (see also pages 89–90 of Farmer): brick patterns; decorative floors and walls; the art of M.C. Escher; the art of William Morris; the art of Robert Adam; Islamic art, not including the Alhambra; the Alhambra; Turkish art or architecture; African art or textiles; Native American pottery, rugs, or beadwork; rugs, carpets, or pottery from countries such as Iran, Turkey, India, or Pakistan; Amish quilts; Mayan art and architecture; Mexican art.

The following sources have been put on reserve in Pelletier Library. These sources will give you a starting point for your research, but you may need (or want) to search for other sources. Please search for sources as soon as possible, and see me if you have any difficulty.

1. Appleton, *American Indian design and decoration*.
2. Aslanapa, *Turkish art and architecture*.
3. Berrin and Pasztory, *Teotihuacan: Art from the City of the Gods*.
4. Brend, *Islamic Art*.
5. Burkhardt, *Art of Islam: language and meaning*.
6. Grabar, *The Alhambra*.
7. Hughes, *Amish: the art of the quilt.*
8. Schattschneider, *Visions of symmetry: notebooks, periodic drawings, and related work of M.C. Escher*.
9. Stillman, *Decorative work of Robert Adam*.
10. Watkinson, *William Morris as designer*.

**Second presentation: Speech to Describe and Explain**

The second presentation is a six to eight minute "Speech to Describe and Explain" based on the topic of your paper describing how symmetry occurs in art or architecture. A speech to describe and explain should focus on the basic skills of summary and explanation; your goal is to transmit information in a clear and concise manner. Your speech should address the same questions as your paper:

1. Provide some background information on your topic, such as a summary of the history of the culture, artist, architect, etc., in question, and a description of where your chosen type of symmetry occurs (be as specific as you can; the type of background information that is appropriate will depend on the topic you've chosen).
2. Present your symmetry analysis of four of the patterns that you analyzed for the paper. This involves explaining how you determined the symmetry type of each pattern (as a finite figure, strip pattern, or wallpattern) by discussing the rotations, mirror reflections, translations, and glide reflections present in the patterns.

You should arrange the timing of your speech so that roughly half the time is spent on each of items 1 and 2 above.

You must use an overhead projector to present your patterns. You can have transparencies made in the mathematics department office for 25 cents each. (The Print Shop may be able to make color transparencies.) You should indicate relevant rotocenters, mirror lines, translations, and glide reflections on your illustrations as you make the presentation, rather than marking the transparencies ahead of time (I will bring transparency pens). See *The Speaker's Compact Handbook*, 27b, 36.

Use the extemporaneous mode of delivery (see *The Speaker's Compact Handbook*, 28a, 33c). This involves preparing a full-sentence outline of your speech, converting it to a key-phrase outline, and finally organizing your key-phrase outline into speech notes, which you should write on notecards for use when your give your speech.

A full-sentence outline and copies of the figures you analyze are due to me when you give your speech.

Your speech will be assessed both by me and by your peers in the class. The speech assessment criteria, keyed to the appropriate chapters in *The Speaker's Compact Handbook*, were given on the first assignment. An extremely short or lengthy presentation will adversely affect your grade.

You are expected to practice your speech before you give it in class; you should particularly practice your presentation using transparencies of the symmetry types of your patterns. Your audience may be an obliging friend or classmate or, even better, a speech consultant at the Learning Commons. If you choose to consult a speech consultant, then you should turn in your "receipt" from the speech consultant on the day of your speech.

You will be assigned a day on which to give your presentation. Failure to give your speech on your assigned day will result in a one letter grade reduction.

# 26

# *The Mathematics of Chaos* as a First-Year Seminar

Suzanne Sumner

**Abstract** This article describes the first-year seminar titled *The Mathematics of Chaos*, which encompasses chaotic dynamics and fractal geometry. This first-year seminar uses primary sources to explain course content and uses computer experiments for inquiry-based learning. Students enhance their research and communication skills through learning modules, a research project, papers, and presentations. Sample assignments and grading guides are provided as well.

## 26.1 Background and Context

The University of Mary Washington (UMW) is a small public liberal arts institution of about 4,000 undergraduate and 350 graduate students. UMW requires all incoming first-year students to take a first-year seminar in the fall semester, with the following learning outcomes: to utilize research techniques for retrieving and synthesizing information and to communicate their results via writing and speaking. This first-year seminar requirement is the cornerstone of UMW's first-year experience and became the basis of our Quality Enhancement Plan for reaccreditation. The intent is to develop better researchers and communicators and to continue the process of improving the students' research and communication abilities all throughout their college experience. Fortunately, the enrollment cap of 15 students helps alleviate the workload associated with grading numerous writing and speaking assignments. This three-credit seminar meets either twice a week for 75 minutes, or three times a week for 50 minutes, during the 15-week semester.

UMW faculty members have great freedom in the seminar topics offered; we can explore engaging topics we might not normally teach, which may or may not be in our primary discipline. (In addition to the first-year seminar *The Mathematics of Chaos*, I also teach a first-year seminar titled *Race and Revolution*.)

At UMW the first-year seminar instructors serve as the students' academic advisors. This alignment of roles allows for excellent coordination between teaching and advising. Because the advisors also teach the students, the advisors can build a closer relationship with the students, and they can catch problems earlier.

## 26.2 Mathematical Theme: *The Mathematics of Chaos*

*The Mathematics of Chaos* first-year seminar was initially created by my former colleague Jeffrey Edmunds. We chose mathematical chaos as a first-year seminar topic after teaching a senior-level version of this material. Knowing that Robert Devaney has had success teaching mathematical chaos to high school students gave us the confidence we could approach the content at an accessible level for first-year students. We were also compelled by Robert May's " ... evangelical plea for the introduction of these difference equations into elementary mathematics courses, so that students' intuition may be enriched by seeing the wild things that simple nonlinear equations can do" [10, p. 459].

Consequently, course topics include the butterfly effect, iterative processes, fractal geometry, chaotic dynamics, and the mathematical definition of chaos. This content has remained fairly constant over time, although new research in chaos theory is added as appropriate [8]. This first-year seminar was originally developed to be taught without a formal textbook, and I have continued that practice. However, my course does loosely follow the excellent books by Robert Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment* [6], and by Denny Gulick, *Encounters with Chaos and Fractals* [7]. This seminar is entirely content oriented, with instruction on research and communication skills provided in service to the course material via learning modules completed outside class, as described below. This course is structured so that 50% of the grade originates from the research and communication assignments and 50% of the grade originates from traditional quizzes and a final exam on the mathematics content.

## 26.3 Course Structure

Instead of reading a textbook, students begin by reading popular articles from primary sources where the mathematics is not too technical, and important ideas are explained in an intuitive manner. Examples are Robert May's article "Simple Mathematical Models With Very Complicated Dynamics" explaining the chaotic dynamics of the logistic map [10, pp. 459-467] and Stephen Smale's paper "Finding a Horseshoe on the Beaches of Rio" describing his inspiration behind the horseshoe map [11, pp. 39-44]. By the end of the semester, the students are reading more technical articles such as James Cushing's "Chaotic Dynamics in an Insect Population" research demonstrating chaos in an actual beetle population [5, pp. 389-391]. Other examples are papers offering different definitions of chaos, for example "Period Three Implies Chaos" by Li and Yorke [9, pp. 985-992], as well as "On Intervals Transitivity = Chaos" by Vellekoop and Berglund [13, pp. 353-355]. In fact, the first homework "quiz" is for the students to practice using the library's website to find and print out these five articles.

This seminar is also more discussion-oriented than a traditional mathematics course. I encourage the class to discuss subjects such as the important factors needed in a population model and why American and Russian mathematicians developed chaos theory separately during the Cold War and Space Race eras. When the first-year seminar program was initially implemented, more emphasis was given to class discussion and use of primary sources, with less emphasis on research, writing, and speaking. As our Quality Enhancement Plan developed, we have added more structure and assignments to support our goals of performing research and effectively communicating the results of students' research.

### 26.3.1 Learning Modules

One way that UMW first-year students strengthen their research and communication skills is through a series of thirteen learning modules developed by our library, Writing Center, and Speaking Center. The library's research modules are *Finding a Topic, Deconstructing Citations*, and *The CRAAP Test* to evaluate sources' currency, relevance, authority, accuracy, and purpose [3, p. 6]. The Writing Center's modules are *The Writing Process, Punctuation: Commas and Semicolons Made Easy*, and *Introductory and Concluding Paragraphs*. The Speaking Center's modules are *Communication Apprehension, Class Discussions, Effective Visual Aids*, and *Growth Mindset*.

These modules require the students to watch a video about the topic and then to complete a quiz indicating their mastery of the information. Some modules include a survey, such as measuring the students' communication apprehension or their inclination towards having a growth mindset. Seminar instructors must select at least ten of these modules, and they can decide whether to count the module quiz scores as part of the grade. My choice is to deduct points if a student does not complete a required module and to offer extra credit for completing the optional modules. These modules are a valuable supplement to the first-year seminar, and the modules release class time that otherwise would be spent covering these topics.

### 26.3.2 Communication Assignments

Communication assignments, either writing or speaking, satisfy the dual purposes of communicating to learn and learning to communicate. Connolly and Vilardi explain that "'Writing to learn' in science or mathematics classes is most basically about developing students' conceptual understanding of these subjects by developing their capacity to use the languages of these fields fluently" [4, p. 4]. Likewise, Smith states, "The use of speaking assignments across the curriculum beginning in the first year of college not only develops the ability to speak coherently and persuasively, but also helps students learn course content" [12, p. 49]. My own experience teaching the last 33 years leads me to agree with Smith: "we learn by doing ... We understand concepts better and retain them longer when we express these concepts in our own works" [12, p. 49]. As a result, my teaching weaves together written and oral communication along with the mathematics. I find it easier to determine if my students really understand the mathematics when they articulate it in writing or speaking assignments, compared to when they memorize rote techniques for test problems [1, p. 8].

Most of my students appreciate having multiple forms of assessment besides tests and quizzes, especially assignments that demonstrate their creativity. Because half of the grade originates from research, writing, and/or speaking assignments and half from mathematics content, this division allows students who are not good test-takers to demonstrate their knowledge through coursework that builds on their strengths [14, p. 85]. Conversely, the writing and speaking assignments require students who have weak communication skills to strengthen these areas through continued practice [2, p. 121].

### 26.3.3 Experiment Writing Assignments

Over the course of the semester, students complete four short writing assignments for 20% of the grade based on Devaney's experiments The Computer May Lie, Windows in the Orbit Diagram, Iterated Function Systems, and Periods of Mandelbrot Set Bulbs [6, pp. 25, 92, 197, 253]. More details about these assignments can be found in the Appendix (p. 264). The students complete these inquiry-based experiments using *A First Course in Chaotic Dynamical Systems Software: Labs 1-6*, the companion software to Devaney's book, purchased and loaded on computers in the lab. Then the students explain their conclusions about whether the dynamics lead to patterns or chaos and about what patterns are hidden within chaos. All the while the students must define the necessary mathematical terminology. I assess the papers on the completeness of the response, along with the correctness of the mathematics and the quality of the writing (see the Appendix, p. 266, for more details). By writing these papers, the students have many opportunities to improve their mathematical understanding and their writing mechanics.

### 26.3.4 The Fractal Design Presentation

For a speaking assignment, students create their own fractal design using matrices and the *Fractal Attraction* software (also purchased and loaded on the lab computers) for 5% of the grade. The students present their design in class, and they must explain why their design is a fractal and how they used mathematics to create it. See the Appendix, p. 266 for the assignment, and p. 266 for a grading guide. This project is one of my favorite assignments because it allows the students' creativity to shine. An example of a student's fractal creation is found in Figure 26.1.

**Figure 26.1.** "UMW Runner" by Holden Vanderveer

### 26.3.5 The Research Project

The seminar concludes with a research project consisting of a paper and a revision for 15% of the grade and a presentation for 5% of the grade (see the Appendix, p. 267, for more details). Students choose a topic related to chaos theory, a biography of a mathematician, a computer program, or a mathematical proof (see the Appendix, p. 267). They submit a paragraph proposal with an annotated bibliography that performs our library's recommended CRAAP Test for 5% of the grade. A grading guide for the different parts of the research project is found in the Appendix (p. 267). After my comments on their sources and corrections on their writing, students submit a draft of their project paper for more feedback. (I never tell students they are submitting a draft; I tell them they are submitting their final paper, and after receiving my feedback, they can submit a revision. That strategy greatly improves the quality of their first submission.)

### 26.3.6 Homework Quizzes and Final Exam

Other assignments include six homework quizzes for 30% of the grade and a final exam for 20% of the grade. Other than the article search quiz described earlier, the remaining quizzes and the final exam cover the mathematics content, allowing the students to practice skills such as finding fixed and periodic points, determining their stability, creating cobweb plots, calculating fractal dimension, etc. I allow the students to complete the homework quizzes as take-home assignments, rather than as in-class quizzes, so they can use their notes as a reference since they do not have a textbook. On the other hand, the final exam is completed in class. This course is the only class where I allow an open-notes final exam because I want the students to explain the technical details of the many definitions of mathematical chaos. An added advantage is that an open-notes final exam encourages students to take excellent notes for a course with no textbook.

## 26.4 Reflections

I have taught eleven sections of this first-year seminar (and eight sections of the seminar about civil rights). I find it important to remember that we cannot possibly teach students everything they need for college success in one course. Instead, I view my role as a first-year seminar instructor is to wind up my students and point them in the right direction.

Unfortunately, two persistent problems emerged from teaching these first-year seminars. The first problem is that I naively expected the students who registered for this first-year seminar to enjoy mathematics or science. While that belief is true for about half of the class, the other half of the class may enroll based on course availability, rather than interest in the material. The other problem comes with the students not doing the course reading, which is crucial to our class time together. True, students are intimidated by reading mathematical articles, but they fail to do the reading for their other classes too. I learned it is important to provide questions on the readings to guide the students through the articles, otherwise the students become overwhelmed. And as a remedy to both persistent problems, I have taken the easy way out and submitted proposals to teach both of my first-year seminars as Honors Program courses, which satisfy higher standards and have higher expectations for the students. Now my seminars are filled with students who take course requirements seriously, do the reading, and are more comfortable engaging in class discussions.

As for other lessons I have learned: much as I enjoy reading student papers, I have found myself struggling with the hours reading and commenting on students' writing, only to have my comments ignored and the same mistakes reappear later. My partial solution follows: I tell students that they will have an essay question on the next test or quiz where they will discuss another student's project topic (of their choosing). To help each other with their respective essays, I ask the class to post their revised papers to the class discussion board. The advantages to this approach are twofold. Students are motivated to read my comments and make my suggested revisions, and they pay better attention to each other's projects knowing they are responsible for the material later. This test essay question often leads to more interesting and higher quality essays because students know the topic ahead of time.

What works best in this course is how the inquiry-based writing assignments strengthen students' abilities to recognize patterns and make conjectures. Students truly light up when they discover a pattern or make a prediction that comes true. In addition, students enjoy the opportunity to be creative with the fractal design assignment, which is engaging for the students, and a delight for me.

In the future, I hope to take advantage of the Makerspace in my building so that students could 3D print fractals like the Sierpiński pyramid or the Menger sponge. In addition, I plan to work with a team of upper-level mathematics and computer science majors to develop software for the first-year seminar students to use to iterate functions, create cobweb plots, create fractals and Julia sets, etc. I also envision students creating mini-documentaries on what they learned in the course for their final projects to provide them with valuable technology skills and further showcase their creativity.

Overall, my first-year seminar *The Mathematics of Chaos* gives prospective mathematics majors a taste of the richness found in upper-level mathematics, showing students a world beyond calculus.

## 26.5 Bibliography


[1] Robert Barrass, *Scientists Must Write: A Guide to Better Writing for Scientists, Engineers and Students*, 2nd ed., Routledge, New York, 2006.

[2] John Bean, *Engaging Ideas: The Professor's Guide to Integrating Writing, Critical Thinking, and Active Learning in the Classroom*, Jossey-Bass, San Francisco, 1996.

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[8] B. Hunt and E. Ott, "Defining Chaos," *Chaos*, 25 097618 (2015).

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[13] M. Vellekoop and R. Berglund, "On Intervals Transitivity = Chaos," *The American Mathematical Monthly*, 101(4) (1994), 353–355.

[14] Maryellen Weimer, *Learner-Centered Teaching: Five Key Changes to Practice*, Jossey-Bass, San Francisco, 2002.

# Appendix

## Experiment Writing Assignments

*The documents listed below on pp. 264–265 are provided to students (spaced throughout the first half of the semester) as directions for completing their computer laboratory experiments and as prompts for writing reports to summarize their conclusions. The grading guide for these Experiment Writing Assignments is found on p. 266.*

### "A First Taste of Chaos" modified from Devaney's "The Computer May Lie" [6, p. 25]

In this assignment you will use the Chaos software to investigate the dynamics of the quadratic function $f(x) = x^2+C$ for different values of the parameter $C$. Consider the following three functions:

1. $f(x) = x^2 + 1$
2. $f(x) = x^2 - 1.1$
3. $f(x) = x^2 - 2$

Find the fixed points for each function. For each of these functions, choose four different initial conditions and compute the first 50 points on their orbits. Record the results by listing the initial condition together with its long term behavior; that is, determine whether the orbit is fixed, periodic, eventually fixed, eventually periodic, some other pattern, or no visible pattern. Print out a graph of each function with a cobweb plot for one initial condition.

Write a paper summarizing what you have seen for the dynamics of each function. For a given function, do all (or almost all) of the orbits behave the same way? Which function is giving us our first taste of chaos? Why do you think so?

Your paper should be descriptive enough for a fellow student to understand it. Your writing should be clear, and concise, with proper use of grammar and punctuation.

Your essay should define orbit, fixed point, periodic cycle, and cobweb plot.

### "Windows in the Orbit Diagram" [6, p. 92]

In this assignment you will use the Chaos software to investigate the Orbit Diagrams of the quadratic function $f(x) = x^2 + C$ as well as the logistic function $f(x) = \lambda x(1-x)$.

Recall that a period k window in the orbit diagram consists of an interval of parameter values with an initial attracting period k cycle together with all of its period-doubling descendants. For example, in the picture of the quadratic function's orbit diagram, we clearly see a period 1 window and a period 3 window. Remember "period" here means the period of the attracting orbit before it begins to period double.

Procedure: The object of this assignment is to catalog a number of smaller windows in the orbit diagrams of both $f(x) = x^2 + C$ and $f(x) = \lambda x(1-x)$. The period 1 and period 3 windows are the most visible for each of the functions. Magnify the region between these two windows for the quadratic function $f(x) = x^2 + C$. You should see more windows. The two largest windows have periods 5 and 6. These windows form the "second generation" of windows. Other windows are visible, but we will record only the two widest windows at this time. We will record the windows in the chart below.

Quadratic function $f(x) = x^2 + C$

| | | | | |
|---|---|---|---|---|
| Generation 1 | 3 | | | 1 |
| Generation 2 | 3 | 5 | 6 | 1 |

Note that we put these windows in the right order. Now go to the next generation, Generation 3. Find the periods of the two "largest" windows between the period 1 and the period 6 windows. Call these periods A and B. Then find the periods of the largest windows between the period 5 and period 6 windows, C and D. Between the period 3 and period 5 windows, there should be only one largest window, so find its period E. You should now be able to fill in the chart below supplying the periods A - E, which represent the periods of the largest windows in the order given.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Generation 3 | 3 | E | 5 | D | C | 6 | B | A | 1 |

Now proceed to Generation 4. In this case we will find only the largest windows between the period 6 and period 1 windows found in the third generation, including the windows found between period 6 and period B, period B and period A, and finally period A and period 1. The largest windows have period a - e as indicated in the chart below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Generation 4 | 6 | e | B | d | c | A | b | a | 1 |

Note that you may need to magnify the windows to read off the periods.

Make four-generation charts for both the quadratic function $f(x) = x^2 + C$ as well as the logistic function $f(x) = \lambda x(1-x)$. Compare your charts for both functions. What are the similarities? What are the differences?

What is the pattern between Generation 3 and Generation 4? Use this pattern to predict the Generation 5 chart from the period A window to the period 1 window for both functions. Check your prediction on the computer.

Write a paper answering the above questions. Use your word processor to generate tables for your generation charts. Your paper should be descriptive enough for a fellow student to understand it. Your writing should be clear, and concise, with proper use of grammar and punctuation.

Your essay should define orbit diagram.

### "Fractals and Iterated Function Systems" [6, p. 197]

In this assignment you will use the Fractal Attraction software to find the Iterated Function Systems (IFS) that produce the fractals in Devaney's book [6, pp.197–198] as their attractors.

To find number of transformations in the Iterated Function System, count the number of the largest self-similar same-sized pieces in the fractal. This number tells you how many transformations (matrix functions) you will need.

Decide on a contraction factor fraction that goes on the diagonal of the transformation matrices by comparing the size of the self-similar pieces to the whole fractal. Are the pieces $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc. the size of the whole fractal?

In the "Untitled" Window, assign the fractal space to be the unit square $[0,1]\times[0,1]$. Put the contraction factor fraction on the main diagonal of the transformation matrices. Assign appropriate numbers for the horizontal and vertical shift values that go in the added vector on the right.

For each fractal, print out the Iterated Function System as well as the resulting fractal.

Write a paper explaining why you chose the Iterated Function System that produced each fractal. State the fractal dimension of each fractal. Which fractals have the same fractal dimension?

Your paper should be descriptive enough for a fellow student to understand it. Your writing should be clear, and concise, with proper use of grammar and punctuation.

Your essay should define fractal, fractal dimension, and Iterated Function System.

### "Periods of the Mandelbrot Set Bulbs" [6, p. 253]

In this assignment you will use the Chaos software to investigate the various decorations or bulbs that are attached to the Mandelbrot set M. Each bulb corresponds to a set of parameter values $C$ where the quadratic function $f(z) = z^2 + C$ has an attracting cycle of some given period. In this experiment, you will find the periods of the 30 largest bulbs on the main cardioid of the Mandelbrot set M. Record the prime periods of the cycles for each bulb on a picture of the Mandelbrot set M.

Note: The Mandelbrot set M is symmetric about the real axis, so you only need to check the periods of the upper half bulbs to know the periods of the bulbs in the lower half.

Given two large bulbs attached to the main cardioid (with no larger bulbs in between), what is the period of the largest bulb between them? What is a rule for finding the period of the largest bulb between two bulbs?

Make a list of all the fractions $\frac{p}{q}$ where $p$ and $q$ are positive integers so that $2 \leq q \leq 10$ and $1 \leq p < q$. In other words, find all the fractions where the denominator $q = 2$ and the numerator $p$ is less than $q$ (so $\frac{1}{2}$). Find all the fractions where the denominator $q = 3$ and the numerator $p$ is less than $q$ (so $\frac{1}{3}$ and $\frac{2}{3}$). Find all the fractions where the denominator $q = 4$ and the numerator $p$ is less than $q$ (so $\frac{1}{4}$ and $\frac{2}{4} = \frac{1}{2}$ and $\frac{3}{4}$). Repeat this process up to a denominator of $q = 10$. Reduce all your fractions and put them in numerical order.

What is the relationship between the periods of the bulbs and this list of fractions?

Write a paper answering the above questions. Your paper should be descriptive enough for a fellow student to understand it. Your writing should be clear, and concise, with proper use of grammar and punctuation.

Your essay should define filled Julia set, Julia set, and the Mandelbrot set.

### Grading Guide for the Experiment Writing Assignments

These four papers have a length of two to three pages and are worth 5 points each (5% of the course grade each). For mistakes in mathematics and grammar, my first-year student grading deductions are: 1 to 4 mistakes = 0.5 point, 5 to 8 mistakes = 1 point, 9 to 12 mistakes = 1.5 points, 13 to 16 mistakes = 2 points, etc.

## The Fractal Design Presentation

*The document below is provided to students (midway through the semester) as directions for creating a fractal with details for the corresponding presentation. The grading guide for this Fractal Design Presentation is found in B.1.*

In this assignment you will use the Fractal Attraction software to create a fractal of your own design. You can either use the Design window or the "Untitled" window to create the fractal.

If you use the Design window, you can create a New Transformation for each piece of your fractal. You can move the transformation by dragging it, and you can change the size of the box by clicking on the knobs. You can rotate the box by holding down the Option key on a corner knob, and you can shear the box by holding down the Option key on a middle knob.

If you use the Untitled window, assign the Fractal Space to be the unit square $[0, 1] \times [0, 1]$. Decide on the contraction factor fraction that goes on the main diagonal of the transformation matrices by comparing the size of the self-similar pieces to the whole fractal. Do you want the pieces to be $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc. the size of the whole fractal? Assign appropriate numbers for the horizontal and vertical shift values that go in the added vector on the right.

You can see your fractal by choosing the Fractal window and then Run under the Fractal menu. To clear your screen, go under the Edit menu and select Clear.

Once you have the fractal you want, save the "Untitled" window file with your name as the title. Print out the transformation matrices and a picture of your fractal.

Your fractal design presentation should be five minutes. First, explain how your fractal design is self-similar. Then, describe how you designed your fractal. How many transformations did you use? How did you adjust the sizes of your transformations? Where did you place the transformations? Did you rotate or shear any of the transformations? How did you decide on the coloring? Provide enough detail so that a fellow student could create a similar fractal.

Be prepared to answer questions on your presentation. Be sure to notify me of any equipment you will need for your presentation.

Feel free to consult the Speaking Center to take advantage of their free services.

### Grading Guide for the Fractal Design Presentation

This five-minute presentation is graded on a ✓+ = 1 point, ✓= 0.5 point, ✓- = 0 points scale for a total of 5 points (5% of the course grade) with the following criteria:

1. Explanation of how the fractal design is self-similar.
2. Description of the fractal design (number of transformations used, their size, placement, rotation, shearing, color).
3. Material is presented clearly and use of visual aids is effective.
4. Presenter is well prepared and organized and responds well to questions.
5. Mannerisms are appropriate and not distracting (dress, posture, gestures, voice, eye contact).

## The Research Project

*The document below is provided to students as a guideline for the final research paper. A listing of possible research project ideas is found on p. 267, and the grading guide for this Research Project is found on p. 267.*

Your research project will allow you to select a topic of interest from the Mathematics of Chaos to pursue in more detail. The project is due at the end of the semester with a presentation during the last week of classes. Your project topic should fit into one of these categories:

1. A formal research project on some historical topic or scientific field relevant to chaos theory, with a heavy emphasis on primary sources.
2. A biography research project of a mathematician.
3. A computer exercise with an emphasis on programming and graphics.
4. A mathematics proof of a theorem.

Once you select a topic, either from the list below or from your own investigation, you will submit a short paragraph proposal describing your topic along with an annotated bibliography. This description will help you determine that you have found a workable topic, a topic with adequate sources (preferably primary sources) and a topic you can manage and enjoy. Be sure to perform the library's CRAAP Test on each of your sources [3, p. 6].

Your project paper should have a title and a non-annotated bibliography listing your sources. You will need to use in-text citations (in the body of the paper), footnotes or endnotes (preferred) for your references to indicate where the ideas originated. You should have at least four references cited within your paper. Your project paper should be four pages long, double-spaced, Times size 12 or equivalent size font, with one-inch margins. Your paper should be descriptive enough for a fellow student to understand it. As always, your writing should be clear, and concise, with proper use of grammar and punctuation.

For a historical topic or topic related to chaos theory, give the necessary background information in addition to describing your topic. For a biography research paper, describe the life and accomplishments of the mathematician or scientist. If you wish to write a computer program, indicate the programming language you will use and whether you have access to the necessary hardware and software. If you seek to prove a mathematical theorem, meet with me so I can show you where to find explanations of the proof.

Your project presentation should be an 8-minute summary of your research project. Be prepared to answer questions on your presentation. Be sure to notify me of any equipment you will need for your presentation.

Feel free to consult the Library, Writing Center, and the Speaking Center to take advantage of their free services.

The paragraph proposal and annotated bibliography are worth 5 points; the final paper is worth 5 points; the revised paper is worth 10 points and the presentation is worth 5 points.

### Research Project Ideas

Topics related to chaos theory: horseshoe map, butterfly effect, population models, fractal basins, strange attractors, African fractals, three-body problem, double pendulum, complex functions, Cantor set, dimension, definitions of a fractal, definitions of chaos, Mandelbrot set and Julia sets for other complex functions, chaos theory in popular media, fractal art, fractal music.

Biography topics: Newton, Poincaré, Lorenz, Smale, Mandelbrot, Julia, Sierpiński, Cantor, Li, Yorke, Sarkovskii.

Computer program: write a program to generate a fractal, the Mandelbrot set, or a Julia set, or write a program to animate fractals, the Mandelbrot set, or Julia sets.

Mathematics proof: prove the fixed point theorem, the attracting fixed point theorem, or the escape criterion theorem.

### Grading Guide for the Research Project

I assess the four-page research paper for correctness in mathematics, writing, and citation. The annotated bibliography with the topic proposal is worth 5 points (5% of the course grade) and is graded similarly to the experiment writing assignments. The first draft of the project paper is worth 5 points (5% of the course grade), and the revised project paper is worth 10 points (10% of the course grade). The presentation is graded on a ✓+ = 0.5 point, ✓ = 0.25 point, ✓- = 0 points scale for a total of 5 points (5% of the course grade) with the following criteria (modified from a UMW Speaking Center rubric):

1. Introduction.
2. Background information.
3. Description of the topic.
4. Information from references and resources.
5. Conclusion.
6. Material is presented clearly.
7. Use of visual aids is effective.
8. Presenter is well prepared and organized.
9. Presenter responds well to questions.
10. Mannerisms are appropriate and not distracting (dress, posture, gestures, voice, eye contact).

# 27

# The Intersection of Mathematics, Art, Technology, and Teaching the Value of a Liberal Education

Cheryl J. McAllister and Laurie Wern Overmann

**Abstract** In a First Year Seminar (FYS) course with the theme of *The Mathematics of Art*, students are encouraged to develop liberal arts skills while researching and investigating the interplay of mathematics and art. Technology makes the connection between mathematics and the visual arts accessible to all students, even those who may lack mathematical or artistic confidence. This article outlines the structure, provides assignments, and shares the observations of the instructors of *The Mathematics of Art* themed course.

## 27.1 Background and Context

In 2000, a new type of first-year seminar was instituted at Southeast Missouri State University to introduce incoming students to the academic expectations of the university's general education program. The general education program at Southeast is called University Studies and is dedicated to providing students with a broad liberal arts background and basic academic skills to support their majors and enable them to become life-long learners. The ideal is to make the general education requirements of the university's degrees a cohesive program in and of itself, rather than a collection of unrelated courses students are expected to complete to graduate. The program includes approximately 45 hours of lower-level general education courses and an additional nine credit hours of upper-division interdisciplinary courses. Typically, in their first semester, every incoming student takes the course *UI100 First-Year Seminar*, a three credit-hour course with multiple goals. This common course provides students with a shared experience intended to foster community and transition students to the university's culture. The course explores the value of a liberal education and introduces the University Studies Objectives, a set of overarching outcomes to be achieved through a collection of general education coursework, coursework in the major, and extracurricular activity (see the Appendix, p. 274, for a list of the University Studies Objectives).

Southeast Missouri State University is a comprehensive, four-year regional university with a master's degree program. The student enrollment is approximately 11,000. The University Studies Department is the unit charged with the delivery of the university's general education program. Each college in the university is required to provide faculty to teach a certain number of UI100 sections consisting of 20–30 students each. Each three credit hour UI100 course meets for a full 16-week semester delivered as either three 50-minute or two 75-minute class meetings per week. Each course has a designated interdisciplinary theme proposed by the instructor and approved by a council of faculty representing the various colleges at the university. A proposed UI100 course theme and syllabus must comply with the objectives of the University Studies Program. As two faculty members of the Department of Mathematics, we each volunteered to teach a section of UI100 using *The Mathematics of Art* theme, which seemed an obvious fit for an interdisciplinary pairing as mathematics and the visual arts have historically enjoyed a mutually beneficial relationship. We have collaborated to develop activities and assignments and taught the theme for over a decade.

## 27.2 Mathematical Theme

Themes are incorporated into the UI100 course to provide content for students that they can explore using skills supported by the University Studies Objectives. A course theme is required to include activities and assignments to teach and practice information literacy, critical thinking, and oral and written communication. A UI100 course cannot require any prerequisites and students can choose any theme. The themes allow the instructor and students to explore a subject of mutual interest while learning how to use the library facilities, find the career services offices, visit the writing center, and become familiar with the campus and programs available to students to help them make the most of their college experience. The most successful themes are those that intertwine the required elements of the general UI100 syllabus (see the Appendix, p. 275) and the exploration of the theme. *The Mathematics of Art* themed UI100 course was developed to give students a broader understanding of mathematics and art, their purposes, the historical influences the disciplines exerted on one another, how the mathematics of diverse cultures influences the art of that culture, how certain art forms may be better appreciated when the mathematics behind them is understood, and how mathematics benefits from visual representations of abstract structures. *The Mathematics of Art* theme is one usually reserved for students who are in the university's honors program and honors instructors are encouraged to allow student input into the direction the course takes. This means that the course content for the theme is never quite the same each semester it is taught. Because there is no prerequisite for the course no specific mathematical content is part of the syllabus, but mathematical content pertinent to students' projects and interests is addressed as needed. Approximately a quarter to a third of the course focuses on mathematical content covering such topics as proportionality, scaling, symmetry, functions, tessellations, patterns, two and three dimensional geometry, fractals and sequences. As time goes on and more technology is readily available for students, projects are updated to include campus technology, smartphones, and free software which allow students with little drawing or design background to create and take pride in original artwork.

## 27.3 Course Structure

As instructors we are able to develop successful, engaging group project assignments to explore the theme. We don't always use the same projects, and in this article we are sharing a variety of different assignments we developed and used successfully. Students who enroll in UI100 with *The Mathematics of Art* theme are not necessarily proficient in either or both disciplines. With this in mind, we design activities for the class that allow students to explore ideas up to the limit and beyond their current background in mathematics and art. Thus, large class discussions on the mathematics of art, followed by small group projects and individual projects create a course structure that challenges students from a wide variety of backgrounds and disciplines to expand and then apply their understanding of the interplay of mathematics and art. The class discussions and student presentations provide the content knowledge for the theme and enhance the development of critical thinking and communication skills. Working with others in a small group encourages students to socialize and communicate with peers, develop leadership skills, and appreciate the contributions each member of a team can bring to a project. Individual project assignments are designed to encourage students to explore *The Mathematics of Art* theme through creating their own artwork, research, writing, and creating and giving presentations.

### 27.3.1 Incorporating Required Assignments with the Theme

All UI100 courses are required to contain assignments related to career readiness and self-assessment, risky student behavior, sexual and campus violence prevention, basic information literacy concepts and skills, and assessments of written and oral communication skills. Much of the required UI100 course material is covered in the first eight weeks of the course and it is sometimes challenging to tie the theme into these required activities and assignments. A few of the required activities must be accomplished without a connection to the theme, but most of them can be used to explore the theme. The required writing and oral communications assignments are easily modified to researching and presenting information about the theme to the rest of the class. The majority of the content for the course is provided by the instructor, a guest speaker, or students sharing their research with the class. Required library visits and information literacy activities are tied together by asking students to locate information related to the theme. The instructors work closely with information literacy librarians to create assignments that help students locate and properly cite books, articles, online sources, and images to use for their projects (see the Appendix, p. 276).

The required visit from career services staff leads to an activity that asks students to write a resume and cover letter for one of three job openings on teams that are created for the final class projects. The students are presented with a job ad for applicants to fill positions for a consulting firm offering custom art gallery presentations with openings for a team leader, artistic/design consultant, and mathematical consultant for each team. This assignment gives students a chance to write a resume they can update as they continue their college careers, get some feedback on the quality of the resume and cover letter, and allows the instructor to "hire" balanced working teams using students with a variety of skills and backgrounds. This project allows the students to practice marketing their skills to an "employer" and allows them to have input into the role they play in the group projects (see the Appendix, p. 277).

### 27.3.2 Mathematics and Art Theme Projects

As a way to introduce them to the physical grounds of the campus, students investigate the university archives and explore mathematics in the architecture of its buildings. With the goals of familiarizing students with campus resource offices and fostering a pride in campus aesthetics, students are assigned to groups of three (historian, interior design specialist, and architectural design specialist) to investigate and orally report on the history, current use, interior design, and architecture of carefully selected campus buildings. Immediately prior to this assignment, students work as a class to study the basic mathematics of both frieze and tessellation patterns so that students can apply their pattern matching skills as they examine the interior design and architecture of buildings. A web search yields many excellent resources for information on frieze and tessellation pattern recognition. As an unanticipated benefit of the project, students comment on their recognition of the value of primary source research and source citing through interactions with university archive staff (see the Appendix, p. 277).

Another group project for student teams is to develop a virtual art exhibit, creating a "guided tour" of the exhibit (using a PowerPoint presentation), and a visitors' brochure for background on the exhibit. The team selects a particular art form that has a significant amount of mathematical content or uses math for its creation. Topics such as origami, tessellations, Möbius strip art, computer graphics, fractal art, and pentomino art are just a few of the topics that students explore. They are also required to develop a hands-on activity for visitors to the exhibit that allows fellow students to get a first-hand experience of the mathematics and art presented (see Appendix, p. 280).

In recent years, it is reasonable to expect every student to have a smartphone, so a digital photography project is assigned that asks students to take photographs around campus that can illustrate both mathematical concepts and artistic design principles. They work in groups to select the best examples of each team member's photographs and create a small group collection of original digital art that is presented to the class with an explanation of what mathematical concepts and art design principles are illustrated by the pieces. One of us also uses a similar assignment for which students locate examples of data graphics that may or may not exemplify principles of graphical excellence. We believe the photography and data graphic assignments encourage students to think more critically and become aesthetically-minded consumers of visual information (see the Appendix, p. 279).

Our campus multimedia centers have several 3D printers and an easy-to-use video production system. When we became aware of the opportunities provided by these new technologies, we designed a course capstone assignment to address FYS course goals, provide students with a challenging and creative project to demonstrate what they have learned during the course, and encourage the students to connect with their own creative abilities. Through this assignment the students are introduced to a free, simple online computer-aided design (CAD) program and a simple video production system. The students select a topic for their group's project, create small 3D sculptures, and then record a video of each artist discussing their sculpture and how it relates to the group's topic. We arrange with our campus multimedia staff to meet with our classes in a computer lab and introduce students to the CAD program which interfaces with the 3D printers. Students create their designs using items from a menu that includes geometric objects and text. Students manipulate these objects through rotation, grouping, symmetry, and hole-making tools to create an infinitude of composite objects. Given these mathematical tools, the students engage in meaningful critical, mathematical, and


**Figure 27.1.** Students use an online CAD program to communicate to a 3D printer to produce small sculptures

artistic thinking. The multimedia staff demonstrate the video production system which allows students to create high quality videos at the touch of a button. Multimedia staff assist students with editing their video productions. Some of our students have previous experience with video production and are allowed to use other programs and equipment to create their videos. The video production process provides students with further critical thinking and problem solving opportunities (see the Appendix, p. 278).

## 27.4 Reflections

Two sections of UI100 *The Mathematics of Art* have been offered every fall for over a decade and the assignments continue to evolve with the experience of the instructors and the advent of new technologies that enable everyone to express themselves in creative ways. The instructors find UI100 an extremely challenging course to teach, but also very satisfying. Because students direct much of what happens, instructors must learn to give up some control in conducting the class and selecting what topics and activities are pursued. More activities were developed over the years this theme has been offered, but only the most developed and successful activities are presented here.

Having to leave the comfort that comes from experience in a traditional mathematics course makes teaching this course particularly challenging. For instructors not accustomed to handling course discussions, it is easy to unwittingly let class discussions get off track. One of us had a discussion turn particularly ugly when a vulgar name was applied to another student by a student who felt threatened by a class discussion. In another instance after 9/11, a class with Muslim students became part of a discussion of what occurred that day and the instructor had to set ground rules for the discussion for a balanced and meaningful exchange. In our experience teaching traditional mathematics courses, these sorts of situations rarely arise.

Scoring mathematics assessments is very different from scoring writing and oral presentation assignments. We had to learn to grade written papers after grading mathematics papers for all of our professional lives. We took advantage of holistic writing workshops offered to faculty who are interested in assisting in the monumental task of scoring student writing samples. Alternatively, faculty and staff in a campus writing center or English department can be consulted to advise and help to develop grading rubrics for written assignments. Learning the value of writing rubrics for students and holistic scoring of student writing samples has been an opportunity to expand our horizons and improve our own writing skills.

As previously full-time mathematics teachers, we had limited experiences with faculty and staff outside our department. Meeting and interacting with faculty and staff across campus was daunting. We had to develop working relationships with library staff, the career center, the writing center, the oral communication faculty, the multimedia center staff, academic advisors, and many others. While it is more comfortable to stay in a niche on campus, being required to meet and work with colleagues in other disciplines and administrative units is rewarding and makes us universal resources to not only our students, but our fellow math department faculty. In several instances at department faculty meetings one of us would share information we gleaned from experiences teaching UI100 that pertained to new course proposals or other administrative questions that were under discussion.

Working with students who grew up in a time of expanding computing power often makes the instructors feel like Luddites. However, it is addressing such challenges that makes teaching the course so professionally rewarding. Students often made suggestions or demonstrated to the instructor and their fellow students a better way to execute some of the assignments, because of their superior understanding of the proposed technology. Ceding some control in a classroom to students allows these faculty members to be impressed with the skills and knowledge that a younger generation brings to bear in any situation.

Both instructors feel that teaching UI100 has had a positive impact on teaching their traditional mathematics courses. “What doesn’t kill you makes you stronger” for us best describes the result of getting outside our traditional mathematics course comfort zone. In teaching an algebra course or a history of math course, information learned from our students and our own research into the theme enriches our content background to help students in our traditional classes make better connections between what they are learning and other coursework they are taking. In addition to strengthening our collaborative relationship, we have acquired a deeper understanding of a liberal education, have good working relationships with staff and faculty across campus, and have an ever-widening understanding of the array of campus resources available to ourselves and our students. This allows us to address student questions with confidence that previously would have elicited a shrug.

Students finish the course with an understanding of the value of a liberal education, an introduction to a vast array of campus resources and opportunities, and a better understanding of how various disciplines enhance, inform, and explain one another. Students appreciate the opportunity to express themselves creatively with an increased respect for their own artistic talents. Students often return to visit and share how what was learned in UI100 is now being used in some aspect of their more advanced coursework. End of semester comments reveal that some students better appreciate how other perspectives enhance the understanding of topics such as the nature of mathematics, the purpose of art, and why the liberal arts outcome of “learning to learn” is a skill that will serve them well in future careers and their entire life.

# Appendix

## University Studies Objectives for Southeast Missouri State University

*This document is given to students the first week of class and is the basis for all activities that follow during the course.*

One fundamental purpose of a liberal education is to ensure the acquisition of knowledge common to educated people and to equip students to integrate acquired knowledge in order to produce interconnections of thoughts and ideas. The goal of the University Studies program is to provide students with the information, ideas, and skills they need to have in order to live a happier and more intellectually rewarding life. The program is based upon nine University Studies Objectives:

1. Demonstrate the ability to locate and gather information. This objective addresses the ways to search for, find, and retrieve the ever increasing information available in a technological society.
2. Demonstrate capabilities for critical thinking, reasoning, and analyzing. Students today cannot learn all the information that is produced. Therefore, they must be able to evaluate, analyze, and synthesize information. They must be able to effectively process large amounts of information.
3. Demonstrate effective communication skills. The ability to understand and manipulate verbal and mathematical symbols is a fundamental requirement in any society, especially one that thrives upon the free exchange of ideas and information. Functional literacy is not the goal, rather, students must attain a high level of proficiency in order to be effective and happy citizens.
4. Demonstrate an understanding of human experiences and the ability to relate them to the present. The degree to which individuals and societies assimilate the accrued knowledge of previous generations is indicative of the degree to which they will be able to use their creative and intellectual abilities to enrich their lives and the culture of which they are a part.
5. Demonstrate an understanding of various cultures and their interrelationships. Understanding how other people live and think gives one a broader base of experience upon which to draw in the quest to become educated. As we become more proficient in information gathering, critical thinking, communication, and understanding our past, our need to understand other cultures becomes greater.
6. Demonstrate the ability to integrate the breadth and diversity of knowledge and experience. This objective deals not merely with the possession of isolated facts and basic concepts, but also the correlation and synthesis of disparate knowledge into a coherent, meaningful whole.
7. Demonstrate the ability to make informed, intelligent value decisions. Valuing is the ability to make informed decisions after considering ethical, moral, aesthetic, and practical implications. It involves assessing the consequences of one's actions, assuming responsibility for them, and understanding and respecting the value perspective of others.
8. Demonstrate the ability to make informed, sensitive aesthetic responses. A concern for beauty is a universal characteristic of human culture. Aesthetics, while usually associated with the fine arts, can be broadly defined to include all areas of human endeavor, for example, science, history, business, and sport.
9. Demonstrate the ability to function responsibly in one's natural, social, and political environment. Students must learn to interact responsibly with their natural, social, and political environments in order to assure continued interrelationships among persons and things. This objective presupposes an educated, enlightened citizenry that accepts its responsibility to understand and participate in the political and social process.

## Syllabus for Course

*This document is distributed and discussed the first day of class.*

First Year Seminar: The Mathematics of Art, 3 credit hours, no prerequisites.

An academic skills-centered seminar that introduces students to the University Studies Program and the value of liberal education while addressing the interdisciplinary theme of the mathematics of art.

**Purposes:**

1. To introduce students to the concept and value of a liberal education.
2. To introduce students to the University Studies program goals and structure.
3. To develop student skills in locating and gathering information.
4. To develop student skills in critical thinking.
5. To develop student skills in written and oral communication.
6. To help students connect academic and career planning.
7. To develop student skills necessary for successful participation in our university community.
8. To develop lifelong skills in managing resources, maintaining a healthy lifestyle, and meeting the responsibilities of citizenship.

**Student learning outcomes:**

1. Students will identify a variety of types and formats of sources of information.
2. Students will differentiate between facts and opinions.
3. Students will demonstrate effective written communication skills.

**Theme Rationale:** Students will explore the historical influence of mathematics on art, how the mathematics of different cultures influenced the art of that culture, how some art has been inspired by mathematical advances, and how certain art forms may be better appreciated when the mathematics behind them is understood. Modern art forms generated by computer graphics or inspired by theoretical mathematics will be explored. Students will be asked to examine their own ideas and opinions about what is "mathematics" and what is "art."

**General Course Outline:**

Part I: Liberal Education, Career Planning, University Studies, and You as a Modern College Student.

A. To introduce students to the concept and value of liberal education.
B. To introduce students to the University Studies program goals and structure.
C. To develop student skills in the first three University Studies Goals.

Part II: Locating Information, Thinking Critically, and Effectively Communicating about "The Mathematics of Art."

A. To apply the first three University Studies Goals to exploring the course theme.

**Student Evaluation:** Semester grading scale: 90% = A, 80% = B, 70% = C, 60% = D.

Your final course grade will be based on:

Class Participation: You will be evaluated on attendance, posting to the electronic forum, completion of all assignments (including informal writings and completion of FOCUS assessment), and participation in classroom discussions and activities.

Informal papers (3–6): You may actually be required to write more than six informal papers, but at least two of them will be returned to you for revisions and resubmitted for a grade. The other papers you write will count towards your participation grade.

Group Projects: We will do several group activities during the semester.

Examinations: You will have a midterm exam over liberal education and University Studies. The final exam will be over material developed as we explore our theme ‘The Mathematics of Art’.

Formal papers: You will be required to research and write 1 to 2 formal research papers. These papers will be from 3 to 6 pages in length and must have a list of cited works.

Oral presentations: You will be required to do several informal oral presentations and at least one formal presentation during the semester.

## Researching Current Artists

*This project is used in conjunction with a required introduction to library resources assignment to provide a theme-related activity to reinforce the information literacy skills presented by the instructional librarian.*

### UI100 Mini Research Project

You have been assigned the name of a current artist that uses mathematical ideas in his or her art (list below). Use the research skills you have learned today to find the following:

1. Biographical information about your artist.
2. A picture of your artist (if possible).
3. Information about how your artist uses mathematics in his/her artwork.
4. At least 3 examples of the artist’s works and some background about each one.
5. Create a short PowerPoint to present what you have discovered to the rest of the class.
6. The last slide in your PowerPoint should be a Works Cited page that is formatted according to the information on citing websites and images you learned today.
7. Your presentation should be 3 to 5 minutes long and give us insight into how many of today’s current mathematical artists (or artistic mathematicians) are using their interdisciplinary knowledge to create art.

Artist/mathematician list:

| | |
|---|---|
| Conan Chadbourne | Erik and Martin Demaine |
| Francesco De Comite | Ilana Krepchin |
| Kerry Mitchell | Kurt Wenner |
| Mehrdad Garousi | Michael Leyton |
| Janet Parke | Dick Termes |
| George Hart | Susan Happersett |
| Helaman Ferguson | Tatiana Bonch-Osmolovskaya |
| Bathsheba Grossman | Keizo Ushio |
| Mike Field | Monir Farmanfarmaian |
| Anthony Hill | Hamid Naderi Yeganeh |
| Robert Longhurst | Jacobus Verhoeff |
| Tony Robbin | Hiroshi Sugimoto |
| Daina Taimina | Emily Lynch-Victory |
| Daniel Dean | Margaret Pezalla-Granlund |
| Ben Moren | Roman Verostko |
| Tracy Krumm | |

## Preparing a Resume and Cover Letter

*This assignment is given prior to assigning students to work groups for later projects and to support a required career course component. The resumes are used to form balanced work groups.*

### UI100: Resume and Cover Letter

Assume you are applying to a consulting firm called McAllister Consulting. This firm designs various shows, exhibits, and educational programs for museums, schools, and other organizations. The following positions are available:

- Design team leader: This person would coordinate the work of the design team, overlooking all aspects of the project including developing a timeline for the work and insuring team members complete their assigned tasks. Applicants should possess good communication, team-building, and management skills. Experience in leadership roles a plus.
- Artistic/design specialist: This person would develop the artistic/design elements of the project including developing "themes" for the projects and providing artistic perspective. Applicants should have some background or experience in art or design.
- Technical consultant: This person is responsible for the mathematical element of projects and other support as needed. Applicants should possess mathematical background at least through precalculus mathematics.

Think about which of these positions you are best suited to fill.

**Assignment**

- Develop a short (approximately one page) resume and a one page cover letter applying for one of these positions.
- The resume should follow a structure similar to what was discussed in class when the representative from Career Services visited.
- The cover letter should briefly explain why you would be suited for the position you are applying for. It is your chance to "sell" yourself.
- You should address the letter to:<br>Cheryl McAllister, President of McAllister Consulting<br>123 Math Street<br>Our Town, MO 65432
- This should be turned in as a two page hard copy and will be used to assign you to a team for the group projects we will be doing in the second half of the semester.

## Exploring Mathematics and Architecture

*This assignment and rubric is used at the beginning of the semester as an introduction to campus resources and personnel. The assignment also provides an introduction to primary source research.*

UI100 *The Mathematics of Art*

### Group Presentation of Southeast Missouri State University Buildings

Students will work in groups of three on a presentation involving the mathematics, art, history, and current use of various Southeast Missouri State University buildings: Academic Hall, Kent Library, Memorial Hall, University Center, Carnahan Hall, Brandt Hall, Show Me Center, Scully Hall, and Crisp Hall. One goal of the presentation is to introduce you to campus and Kent Library's research resources; another goal is for you to practice your communication skills.

**Historian:** This individual will be responsible for relating the history and current use of the building. Information about any resource offices relevant to students must be provided. Historical information will be available at Kent Library's Special Collections room; the current use of the building can be found by visiting the building.

**Exterior architecture specialist:** This individual will be responsible for discussing the architectural features of the exterior of the building. Architectural information will be available at Kent's Library's Special Collections room and by visiting the building. The information presented captures the artistic and mathematical aspects of the exterior of the building.

**Interior design specialist:** This individual will be responsible for discussing the architectural features of the interior of the building. Information will be available at Kent's Library's Special Collections room and by visiting the building. The information presented captures the unique artistic and mathematical aspects of the interior design of the building.

We will meet with staff from the Special Collections and Archives who will help to guide you through library resources needed for this assignment. Be polite and professional when working with the Archives staff.

**Oral Presentation:** The group's 10 minute oral presentation on your assigned Southeast Campus building should include a PowerPoint presentation.

Your oral presentation will be evaluated according to the following criteria:

1. According to your role, you address the history, exterior architecture, or interior design of the building, providing information relevant to the mathematics and art theme. Any resource offices relevant to students must be provided.
2. The architectural and interior design information presented addresses the mathematics and art theme, including relevant art and mathematics background.
3. The location, history, and current use of the building are provided.
4. The PowerPoint slide information is well written; proper punctuation and spelling are employed.
5. The presentation is clear, focused, and well supported. The presentation includes a strong introduction and conclusion with good supporting points.
6. The ideas you have gotten from your sources are properly cited in the text of the presentation and in a References or Works Cited page. To cite your references properly, use the American Psychological Association (APA) style as shown on the library website. The archivists will help you correctly cite those sources.
7. The PowerPoint has effective images and is consistently formatted.
8. Individual speakers speak in a clear, audible voice and do not just read from note cards.
9. The presentation is engaging.
10. You are to be a respectful audience member to your fellow classmates.

## Exploring Art and Math with 3-D Printers

*This assignment is one of the major projects the students are given to practice the various University Studies Objectives, utilize the multimedia resources on campus and challenge the students' communication skills. A grading rubric is provided in the assignment.*

### 3D Printing Project

Each student will create a sculpture using a 3D CAD design program and a 3D Printer. However, students will work in teams and the team's sculptures must have a theme or connection linking the projects. All sculptures must have both mathematical and artistic merits.

One goal of the project is to introduce you to multimedia resources on campus; more important goals are for you to practice your oral and written communication skills and to reflect upon how working on projects very naturally encourages the application of several of Southeast's nine University Studies objectives.

Each team of students will orally present their sculptures in a short video presentation. Each student individually will write an artist's statement. As you work on your project, reflect upon and keep notes about the following aspects of this project:

Mathematics: Your sculpture must have mathematical merit. Describe and defend the mathematical merit of your sculpture by using correct mathematics terminology. Include a discussion of mathematical items such as geometric shapes used, perspective, scale, symmetry, and proportion.

Art: Your sculpture should demonstrate artistic elements of design and art. Describe and defend any elements of art (line, shape, form, space, color, texture) and any principles of design (balance, emphasis, movement, pattern, repetition, rhythm, variety, unity) employed.

University Studies objectives: Think about the nine University Studies objectives. Which University Studies objectives and in what way were these applied or demonstrated in all aspects of completing this project? Describe and defend the use of any applicable University Studies objectives. Be specific.

**Grading Criteria:**

Artist statement: Each student will write his or her own statement addressing the artistic, mathematical, and University Studies aspects of the project. The statement should be no more than two pages, double spaced.

1. The theme or connection between the sculptures of your team must be addressed.
2. The mathematical and artistic merit of the pieces must be addressed.
3. The applicable University Studies objectives must be addressed.
4. The paper is grammatically well written; proper punctuation and spelling are employed.
5. The overall statement is clear, focused, and engaging. The statement includes a strong introduction and conclusion with good supporting points.

Oral video presentation: Your team's 10 minute oral video presentation of your sculptures should show each artist describing and showing different views of his/her 3D creation, then a short segment of the team's sculptures taken as a collection and a description of how the three pieces go together as a collection. The oral video presentation will be evaluated according to the following criteria:

1. Each student should present and discuss his or her sculpture.
2. The mathematical and artistic merit of the pieces must be addressed.
3. The applicable University Studies objectives must be addressed.
4. Individual speakers should speak in a clear, audible voice and not just read from note cards, but the presentation should appear integrated and cohesive.
5. The presentation should be engaging and focused with a strong introduction and conclusion.
6. The theme or connection between the sculptures of your team must be addressed.

## Mathematics, Principles of Design, and Digital Art

*This assignment and rubric, usually given mid-semester, is designed to challenge the students to find visual images in their immediate surroundings that illustrate both mathematical and artistic design principles.*

### Mathematics and Art Creative Project

**Purpose:** The purpose of this assignment is to explore your own creative thinking about how mathematics can be expressed as photographic art.

**Medium:** Digital Photography.

**Assignment:** Using a digital camera (or your cell phone), explore your world and find visual images in your environment that can serve as artistic representations emphasizing the connection between mathematics and art.

These pictures should try to represent one or more of the elements of design: space, line, color or value, shape, texture, form, and

one or more of the principles of art: emphasis, proportion, movement or rhythm, pattern, unity, variety, balance, contrast,

and

a mathematical concept or principle: slope, symmetry, pattern, balance, geometric shape, etc.

Create a short PowerPoint using at least five of the images you have taken and be prepared to discuss which elements/principles of art/design these images illustrate.

**Rubric for grading:**

- PowerPoint created by due date.
- At least five original images.
- Identification of the art/design principles incorporated into each image.
- Identification of mathematics illustrated by each image.
- Presentation skills.

## Virtual Art Gallery Project

*This is another of the culminating major projects assigned for students to demonstrate information literacy, research, communication, and teamwork skills. A grading rubric is provided.*

### Math and Art Virtual Gallery Show

The goal of this project is to design and present to the class a presentation developed around our theme of *The Mathematics of Art*. Each design team is to research their chosen topic.

The team will produce the following: An exhibit booklet providing in-depth background information about the topic, a slide show "virtual gallery" presentation on the topic, and an activity that allows all of the class members to participate.

Each team will produce a 30 minute presentation for the class that will include:

1. A PowerPoint (or other format of your choosing) presentation that will be your "virtual gallery" of examples of art related to your topic. The PowerPoint should contain at least 15 to 20 images and citations for each one must be provided at the end of the PowerPoint.
2. A hands-on activity that you will share with the class to allow them to experience first-hand the art topic you are presenting.The hands-on activity should be something that can be explained and done in 10 to 15 minutes.
3. An opportunity for each team member to present orally to the class some aspect of the topic. Each member of the team is expected to present at least five minutes of the oral presentation.

Materials for the hands-on activity will be provided by McAllister Consulting.

**The exhibit booklet:** Each team member will produce a short chapter or section for the booklet. This should be designed to provide viewers to the gallery some background information about the "show" and the topic. One member of the team is responsible for organizing, editing, and printing one copy of the booklet to turn in the day of your presentation. The booklet must include at least three images in each section which should be properly cited at the end of the booklet. The team's booklet should cover the topic thoroughly. Each chapter should give "credit" to the team member responsible for writing that section.

Decide on your team's topic from the list below or one I approve. (This will be done on a first come, first served basis with teams chosen at random.)

**Possible topics to choose from:**

- Fractal art: How geometry helps to create a new type of art form and artist.
- Möbius strips: How mathematics and art try to capture infinity.
- Escher: How an architect became an artist and gained fame in the mathematics community.
- Origami: Ancient art form, modern applications.
- Perspective: How art inspired mathematicians to develop another branch of geometry.
- Curve stitching: How to make curves from straight lines.
- The golden ratio: A basic geometric/mathematical concept influences art.
- Optical illusions: Mathematics and art working together to fool the eye.
- Trying to capture the fourth dimension: How art helps mathematicians see where the eye cannot.
- Another topic your team comes up with that I approve.

Create a timeline for your project and assign specific tasks to team members. Everyone on the team should exchange contact information. If a team member is absent, it is the team leader's responsibility to update the missing member. The team leader will meet with me prior to the dismissal of class to report progress today (and in the future).

**Grading rubric:**

- The virtual art gallery and presentation. (Includes the PowerPoint and the hands-on activity).
- The PowerPoint should contain at least 15 to 20 images.
- Proper citations for each one must be provided.
- The hands-on activity should be something that can be explained and done in 10 to 15 minutes.
- Each member of the team is expected to present at least five minutes of the oral presentation.

**The exhibit booklet:**

- Each team member will produce a short chapter or section for the booklet.
- Some background information about the "show" and the topic.
- One copy of the booklet to turn in the day of your presentation.
- The booklet must include at least three images per section which should be properly cited.
- The team's booklet should cover the topic thoroughly.

# Part VII

# Proofs and Problem Solving

As professional problem solvers, mathematicians are uniquely positioned to teach problem solving techniques that can be implemented in both mathematical and non-mathematical settings. Mathematical problem solving can be taught using topics such as logic, games, riddles, and puzzles with very few prerequisites, making it an ideal theme for a first-year seminar. One of the first-year seminars in this section shows us that these rich topics can also be used to teach the writing process and to help our students develop their abilities to write concise and sound arguments.

# 28

# Math Without Math: Unraveling Riddles and Solving Problems

Ryan Higginbottom

**Abstract** This article describes a first-year seminar at Washington & Jefferson College that focuses on mathematical problem-solving. Though the class has no prerequisites, students encounter deep and important problem-solving strategies while exploring puzzles, brain teasers, and games. This article will highlight some of the activities, structure, and assignments from this seminar.

## 28.1 Background and Context

Washington & Jefferson College (W&J) is a liberal arts college with around 1,400 students in residence, located in Washington, Pennsylvania. The First-Year Seminar Program is a campus-wide program, recruiting faculty from every department as instructors. First-year seminars enroll 16–19 students, and the instructor serves as the academic advisor for all seminar students until they declare a major. At W&J, first-year seminar courses are four-credit classes and all classes meet for 210 minutes each week—either 70 minutes three times per week or 105 minutes twice a week. (W&J semesters are 14 weeks long.)

Teaching a First-Year Seminar (FYS) is optional at W&J, though most faculty serve as FYS instructors at least every three to five years. Faculty submit ideas for first-year seminars to a steering committee; these ideas may or may not be connected to their discipline. For example, this article describes my mathematical first-year seminar, but I have also taught a seminar on coffee that contained no mathematical content at all.

The goals of the First-Year Seminar Program are, broadly, to help acquaint students with the expectations and resources of the college as well as to provide an introduction to the ideas and aims of a liberal education. More specifically, the student learning outcomes for first-year seminars at W&J are as follows:

- Upon successful completion of the class, students will be able to apply methods of inquiry and develop skills appropriate to the seminar being offered.
- Upon successful completion of the class, students will be able to analyze a subject closely and critically, both in writing and class discussion.
- Upon successful completion of the class, students will be able to articulate the meaning of "liberal arts" and connect that meaning to experiences in and out of the classroom.
- Upon successful completion of the class, students will be able to understand the Washington & Jefferson College curriculum.
- Upon successful completion of the class, students will begin to develop short-term, intermediate-term, and long-term academic plans.

## 28.2 Mathematical Theme

The theme of this first-year seminar is mathematical problem-solving, and about 80% of the course is devoted to teaching this mathematical content. The other portion of the course is a two-week unit on the liberal arts. (In the liberal arts portion of the course we discuss the defining features of a liberal education as well as the W&J curriculum.) One of my primary goals of the seminar is to introduce students to problem-solving techniques and strategies used by mathematicians. This helps students see that these strategies are not limited to use in the ivory tower; in fact, these techniques are useful in many non-mathematical situations. Some of the tactics covered in this seminar are symmetry, the extreme principle, the pigeonhole principle, and invariants. We follow the general structure laid out in chapters 1–3 of the book *The Art and Craft of Problem Solving*, by Paul Zeitz [2]. This book serves as the primary resource for our seminar.

Because this seminar has no mathematical prerequisite, the content of the course is relatively free of mathematical content beyond high school algebra and the concept of parity. This explains the title of the course, *Math Without Math*. Students learn problem-solving strategies in the context of games, riddles, and puzzles rather than in the trenches of mathematics. About 80% of the course is devoted to teaching this mathematical content.

## 28.3 Course Structure

This course is driven by in-class problem-solving activities. Students earn participation grades from their work in small and large group discussions. Additionally, students earn assignment grades for their written reflections and writeups based on classroom activities. Each student presents a problem individually twice during the semester, and there is one paper due at the end of the term. All first-year seminars at W&J also include a one-week unit on the liberal arts.

Final grades in this FYS are assessed according to these percentages.

- Class Participation, 15%
- Problem Presentations, 40% (two at 20% each)
- Class Assignments, 25%
- Paper, 20%

Most classes begin with a short demonstration, discussion, or brainstorming session involving the whole class. But the most effective part of each class period is the small-group time that follows.

Small groups usually consist of 3–5 students who I have selected to work together. Groups are given problem sets that illustrate the problem-solving tactic of the day. (See the Appendix, p. 289, for examples.) Class time is used to discuss and solve these problems and then to devise helpful and accurate ways to explain the solutions to others.

Within this broad course structure, the following sections describe some of the more effective and engaging topics and assignments.

### 28.3.1 Reading Quizzes

Students are expected to read the relevant section in the textbook before each class period. To ensure that students are prepared for class, reading quizzes are given at the beginning of many class periods.

While not the most exciting aspect of the course—and certainly one of the least appreciated by the students—these reading quizzes are essential to make sure students come to class with the proper background. Quizzes consist of four or five short-answer questions, and I grade these responses for accuracy.

### 28.3.2 Japanese Puzzle Day

On the second day of the semester, the main activity involves four puzzles that most students have never encountered: Nurikabe, Slitherlink, Akari, and Shikaku. These puzzles primarily require logical deduction for a solution, and they resemble the popular Minesweeper game in both appearance and strategy.

Students are divided into groups and each group is given the instructions for the puzzle, a sample puzzle, and several blank puzzles.

Each group must figure out the best strategy for solving the puzzle. During the next class period, each group explains the puzzle to the class along with their puzzle-solving strategy.

At the beginning of the course, students don't yet know any formal problem-solving tactics. Yet this assignment introduces students to three important skills they will practice throughout the course: learning to solve puzzles, some of which are quite difficult; thinking about the problem-solving process; and explaining their solution to a problem to their peers.

### 28.3.3 Tactile Games and Puzzles

The most effective class sessions of the semester involve small-group work on tactile puzzles and games. The following two examples serve as illustrations.

One class on the extreme principle begins with students sitting in a circle. Each student is given an even number of pieces of wrapped candy. (Tootsie rolls work nicely.) They pass half of their candy to their right and receive candy from the student on their left. If this results in an odd number of candies, I give them an additional piece. This constitutes a "round," and more rounds follow. Before the experiment begins, and as it progresses, students are encouraged to guess what will happen to the candy. They observe an equilibrium and are encouraged to explain why it happens. Without the mathematical terminology, students are introduced to the important idea of a conjecture and the effect of small changes in the rules on the final outcome.

During a class on symmetry, each small group of students is given a bag of pennies and four large shapes on paper—a circle, a square, a star, and a large capital E. For each shape, two students play the following game. Players alternate by placing pennies within the shape in question. Each penny may not touch another penny or a side of the shape. The last player able to put a penny on the board wins. The group must determine if a winning strategy exists for either the first or second player (or neither). By the end of class, they must explain in writing why the winning strategy works. During this activity students are usually engaged and competitive. The use of different shapes forces them to confront different types of symmetry and how each can or cannot be exploited.

### 28.3.4 Problem Presentations

Each student in the class must present the solution to two problems during the semester. Students must solve these problems on their own and then present their solutions to their peers. They are also responsible for a written solution to their problems. (See instructions in the Appendix, p. 290.)

These presentations take place both at midterm and at the end of the semester, and students choose the problems they solve from a long list that I provide. (Many problems are taken from the excellent resource *Mathematical Circles (Russian Experience)* [1], by Fomin, et al.) Most of the problems available for these presentations are directly related to one of the major problem tactics covered in the class (symmetry, the extreme principle, the pigeonhole principle, and invariants). I grade the writeup and presentation and the students submit evaluations as well. Peer feedback does not directly affect a student's grade, but it helps me assess the clarity and effectiveness of a student's communication.

Students receive detailed rubrics for both the presentation and the writeup. (See the Appendix, p. 290.) Additionally, I give a sample presentation and provide two written solutions to problems as examples.

During this assignment, students are introduced to the important notions of *extensions* and *generalizations*. This requirement forces students to understand the essential ingredients of their solution. This is another aspect of mathematical thinking that most non-math major students do not normally encounter.

### 28.3.5 Sherlock Holmes

Toward the end of the semester, students each read a different Sherlock Holmes short story. They are required to type up a summary of the story as well as a description of the problem Sherlock Holmes is attempting to solve. Students must identify the key ingredients of the solution to this problem and what, if anything, they can learn about problem-solving in the "real world" as a result. The whole assignment must be at least 400 words. This is a single draft exercise. (See the Appendix, p. 290, for the instructions.)

After an in-depth look at the problem-solving strategies discussed in the course textbook, this class ends with a short unit on solving problems outside of the classroom. While not entirely realistic, the Holmes stories provide a freely available resource for students to think about problems that do not affect them personally. I want students to think about solving non-mathematical problems using the language and some of the perspective we develop over the semester.

### 28.3.6 Final Paper

The final assignment of the course is a paper in which students investigate a problem and its solution. The problems in view are not puzzles or brain teasers, but rather problems that have been documented in history. Some possible topics for the paper are the capture of an elusive criminal, the building of an innovative building or bridge, and the negotiation of a peace treaty. Students are encouraged to pick a topic of interest to them.

The paper must explain the problem and its solution in depth. Students must consult a variety of sources for multiple perspectives on the problem and are required to include in the paper something they have learned about problem-solving through their research. (This does not need to be connected explicitly to anything studied in class.) The paper should have 1500–2000 words.

This assignment requires students to submit a paper proposal, a draft, and a bibliography all before the final paper is due. (See the Appendix, p. 291, for instructions and grading rubrics.) One day of class is spent in a workshop where each student gives feedback on two other student papers.

## 28.4 Reflections

The course has been taught three times, undergoing slight modification with each successive offering. There have been many successes in this class, and there is also still room for improvement.

### 28.4.1 Strengths

As self-reported on course evaluations, students find the in-class problem solving sessions helpful and engaging. Most students have never solved problems or puzzles with as much attention to detail as is required in this class, and they find discussion with their peers essential. It is important to place weaker students in groups with stronger students and then encourage a lot of conversation. This gives the weaker students a chance to learn from their peers and ask questions in a less public setting than in front of the whole class.

The individual problem presentations are also a strong point of the class. In contrast to the small group activities, students need to work on this task by themselves, and that often forces them to grapple with strategies that they may not yet have mastered. I am available as a resource if students cannot solve the problem on their own. These assignments also help students grow in their written and oral communication, both of which are essential skills for success at W&J.

At several points in the course, students are required to give feedback to their peers. Some students have engaged in this sort of dialogue before, but many have not. I provide detailed instructions and examples, and the students seem to have good conversations. Peer feedback is included on some early, low-stakes assignments as well as the problem presentations and the final paper.

### 28.4.2 Improvements

The textbook used for this course presents the material in a direct manner, but many students find the book dry and difficult to read. I wonder if another resource could be found (or written) that is more engaging for first-year students without assuming much mathematical sophistication.

One of the aspects of the course that has changed most substantially over time is the unit on problem solving in the real world. This unit was entirely absent during the course's first iteration, and it has been added in response to student feedback. The final paper is an attempt to connect the course material to practical problems and solutions, and there are also two class days devoted to the topic. Yet, I think this portion of the course could use more development. Students want to know when they will use these strategies and techniques in practical situations, and there may be other resources available to help instructors answer these questions.

## 28.5 Bibliography

[1] Dmitri Fomin, Sergey Genkin, and Ilia Itenberg, *Mathematical Circles (Russian Experience)*, American Mathematical Society, Providence, 1996.

[2] Paul Zeitz, *The Art and Craft of Problem Solving*, 2nd ed., John Wiley & Sons, Inc., Hoboken, 2007.

# Appendix

## Small Group Work

*Here is the assignment from the second class day spent studying the pigeonhole principle.*

**Instructions**: Get into your groups and work on the problem that matches your group number. After solving your problem, generalize! At the end of class, your group should hand me a well-written solution to your assigned problem, including your generalization. I will assign each group member the same participation grade for today's class based upon this writeup.

1. Each box in a $3 \times 3$ arrangement of boxes is filled with one of the numbers $-1$, 0, 1. Take the sums along each row, column, and diagonal, and prove that at least two of these sums must be equal.

2. Let $A$ be any set of 19 distinct integers chosen from the set $\{1, 4, 7, \ldots, 100\}$. Prove that there must be two distinct integers in $A$ whose sum is 104.

3. Let $A$ be the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Prove that if any five integers are selected from $A$, then at least two of those integers will have a sum of 9. Can you generalize?

4. There are 14 people at a banquet. For the sake of this problem, assume that friendship is symmetric; that is, if person A is friends with person B, then person B is also friends with person A. Prove that among these 14 people, we can find two who have the same number of friends in the group.

*Here is the assignment from the first class day spent studying symmetry.*

**Instructions**: After we finish our discussion as a class, start work on the problem corresponding to your group number. Focus first on a solution to your problem/puzzle, but then be sure to talk in your groups about extensions and generalizations of your puzzle. For example: Does the size of the board matter? Why? In what other scenarios would this strategy definitely work or definitely not work? Why? In asking and answering these questions about your problem, you will get to the heart of the essential information in your problem.

1. Two players take turns placing knights on the squares of a chessboard so that no knight can take another. The player who cannot move loses. Which player has a winning strategy? What is the winning strategy?

2. Two players take turns placing kings on the squares of a $9 \times 9$ chessboard so that no king can capture another. The player who cannot move loses. Which player has a winning strategy? What is the winning strategy?

3. Given a $10 \times 10$ chessboard, two players take turns covering pairs of squares with dominoes. Each domino consists of a rectangle one square in width and two squares in length. The dominoes cannot overlap. The player who cannot place a domino loses. Which player has a winning strategy? What is the winning strategy?

4. A daisy has 12 petals. Two players take turns tearing off either a single petal or two petals right next to each other. The player who cannot move loses. Which player has a winning strategy? What is the winning strategy? Now analyze the game if the daisy has 11 petals instead of 12.

## Problem Presentations

*Here are the instructions for two individual problem-solving assignments. The instructions (along with the grading rubrics for the oral and written presentations of the solutions) are handed out in the third or fourth week of the semester, and the solutions are due at roughly the middle and end of the semester.*

**Instructions**: Each student must solve two problems on their own and present them to the class this semester. The first round of our problem presentations will be on October 20, October 25, and October 27. The second round of presentations will take place at the very end of the semester.

1. Sign up for both a presentation date and a problem to present. Two sheets of problems are posted.
2. On your presentation day, you will have 5–15 minutes to present your problem and the solution to your problem to the class. You may use whatever tools you wish to aid in your presentation.
3. You are also responsible for a written solution to your problem. On the day of your presentation, you will turn in a hard copy of this writeup to me, and you will also upload a digital version of this writeup.

**Presentation Grading Rubric** (50 points possible)

- Quality and accuracy of visuals (5 points possible)
- Description and setup of problem, including examples when appropriate (5 points possible)
- Extensions and generalizations (5 points possible)
- Correctness and completeness of solution (10 points possible)
- Ability to respond to questions/comments (5 points possible)
- Clarity and pace of presentation (10 points possible)
- Overall speaking style, including awareness of audience (10 points possible)

**Writeup Grading Rubric** (50 points possible)

- Description and setup of problem, including examples when appropriate (10 points possible)
- Extensions and generalizations (5 points possible)
- Correctness and completeness of solution (20 points possible)
- Style and clarity of writing (10 points possible)
- Grammar, punctuation, spelling (5 points possible)

## Sherlock Holmes Assignment

*This out-of-class assignment is handed out toward the end of the semester when the class discusses solving problems in the "real world."*

**Instructions**:

1. Go to the website `arthur-conan-doyle.com/index.php/The_62_Sherlock_Holmes_stories_written_by_Arthur_Conan_Doyle` to select a Sherlock Holmes short story to read.
2. Sign up to claim that story for yourself. (I don't want any two people to read the same story.)
3. To our next class you must bring a typed summary of the story consisting of at least 400 words. You must include a description of the problem Sherlock Holmes is attempting to solve and the key ingredients that went into his solution. Discuss what (if anything) you can learn about problem solving in the "real world" from this story.

## Final Paper

*This paper assignment is handed out near the middle of the semester and is due at the end of the semester. A paper proposal and draft are due (and graded) before the final version of the paper.*

**Instructions**: For this paper assignment you will be investigating a problem and its solution. Your problem should not be a puzzle or brain teaser, but rather some problem that has been documented in history. Here are some examples of the type of problems you should be investigating: the capture of an elusive criminal, the building of an innovative building or bridge, or the negotiation of a peace treaty. (These are just examples—the possibilities are almost limitless with this assignment, so you should pick something that interests you.)

In your paper you should explain the problem and its solution in depth. You will need to consult a variety of sources for multiple perspectives on the problem. You should also include in your paper something you have learned about problem-solving in general through your research. (This does not need to be connected to anything we've studied in class.)

Your paper should be roughly 1500–2000 words in length, and I am expecting a polished final product. Note that in order for this to be a good paper of the required length, the problem and its solution will not be trivial. You will want to gather different perspectives on your problem, possibly from some of the people who attempted its solution.

Here are the relevant dates for your paper.

- Paper proposal (22 Nov) — By this date you should have done enough research to have a good understanding of both the problem and the solution that will be the subject of your paper. You will turn in a 300–500 word typed overview of your paper on this date at the beginning of class. This proposal should include a bibliography of at least three sources that you intend to use to write your paper. I will respond to these proposals quickly to offer you feedback on your project.
- Paper draft (1 Dec) — Bring two copies of a draft of your paper to class on this date. We will be holding a peer evaluation workshop during class. This should not be a first or *rough* draft—it should be a draft that your peers can read and on which they can offer suggestions.
- Bibliography (9 Dec) — This portion of the assignment will be administered by our library staff as part of an information literacy module that all first-year students are required to complete.
- Paper due (14 Dec) — This is the date for our final exam, and you should bring one copy of your paper to me at the beginning of this period. Even though you will turn your bibliography in to the library for grading, you should also include it in your final paper.

The grading for the paper will be weighted in the following way.

- Proposal (15%)
- Draft (15%)
- Bibliography (10%)
- Final paper (60%)

**Paper Proposal Grading Rubric** (50 points possible)

- Sketch of problem (15 points possible)
- Sketch of solution (15 points possible)
- Bibliography (5 points possible)
- Style (10 points possible)
- Grammar (5 points possible)

**Paper Draft Grading Rubric** (50 points possible)

- Explanation of problem (15 points possible)

- Explanation of solution (15 points possible)
- Aspect of problem solving (10 points possible)
- How well developed is the draft? (10 points possible)

**Final Paper Grading Rubric** (50 points possible)

- Explanation of problem (10 points possible)
- Explanation of solution (10 points possible)
- Aspect of problem solving (5 points possible)
- Style and clarity of writing (15 points possible)
- Grammar, punctuation, spelling (5 points possible)
- Citations correct (5 points possible)

# 29

# Mathematics as a Creative Art: A Seven-Week Seminar

Jennifer Earles Szydlik

**Abstract** I describe a freshman Honors seminar showcasing mathematics as a creative, living discipline. In the course, students engage daily in authentic mathematical practice by solving novel problems as a community. They study mathematical philosophy and culture through readings and video, and they make sense of three classic mathematical masterpieces: Euclid's proof of the Pythagorean theorem, Euclid's proof that there exist infinitely many primes, and Cantor's diagonal argument.

## 29.1 Background and Context

The University of Wisconsin Oshkosh is a four-year comprehensive school of approximately 11,000 students. We require a first-year, interdisciplinary, team-taught seminar for all freshmen in our Honors College. The College typically runs three such seminars each fall semester focused around themes chosen by the instructors. Past themes include *Food*, *Social Justice*, and *Education*. For the last three years, I have had the privilege of team-teaching, with fiction writer Ron Rindo, in alternating weeks a version of that course titled *Creativity*. Dr. Rindo addresses the creative process in writing, music, and the fine arts. I focus on the theme of mathematics as a creative art. While Dr. Rindo's part of the course enriches and deepens the student experience, the mathematics portion stands alone. In this chapter, I describe what is essentially an innovative seven-week mathematics course that meets twice a week for 90 minutes with 35 freshmen.

## 29.2 Mathematical Theme

My part of the seminar course is designed to help students view mathematics as a human creation. To that end, we read mathematicians' views on the nature of mathematics, study the culture of the mathematical community, make sense of mathematical masterpieces, and practice math as an art each day by exploring rich problems in small groups and as a whole class. While I acknowledge the amazing and powerful applications of mathematics, the focus is on practicing mathematics for its own sake. The course is not a traditional content course, and it has no exams. Students are assessed based on participation, written reflections on readings, and detailed work on three big problems (called Problem Write-ups). Approximately half of my class time is devoted to discussing mathematical content (e.g, derivations of area formulas, proofs of the Pythagorean Theorem, results in number theory, the nature of the real numbers, the nature of infinity, and Cantor's diagonal argument). Each week addresses a mathematical theme, and I have organized the next section around those themes. Authentic problem solving is an ongoing aspect of the course. I have provided many of the course reflection questions and class problems in the Appendices, pp. 298 and 299 respectively.

## 29.3 Course Structure

### 29.3.1 Overview

*Creativity* seminar meets each Tuesday and Thursday with both Dr. Rindo and I present each day. We use the first day of class to build community by asking students to write and share 30-word creative biographies and by engaging in group tasks meant to challenge their creativity (e.g., brainstorming alternate uses for the podium or solving problems with non-standard tools). In subsequent meetings, we alternate weeks as class leader. When Dr. Rindo teaches his portion of the seminar, I participate as a student by completing readings and adding to the discussion. He joins group problem solving and class discussion for the mathematics part.

Typically, each day of the math portion of the seminar begins with small-group work on a problem or small-group discussion of reading questions, and evolves into a whole-class conversation. My role is to encourage, clarify, and record student ideas so that they are available for the class to consider and debate. I assign small groups at random (using cards) and change groups with each new theme so students remain rigorous with one another and have opportunities to work with many of their classmates. Discussions are lively and informal, and I often invite students to the board to explain their mathematical thinking to the class.

### 29.3.2 Week 1: Math is a Way of Thinking

Students come to the course convinced that mathematics is a collection of tools and procedures that they must memorize and apply. My first concern is to convince them that mathematics is a way of thinking and reasoning about certain types of objects. To this end, we do two things: first, we engage in mathematical problem solving (beginning with Penny of Death—see the Appendix, p. 299) in small groups and then discuss the problem as a class. Second, we discuss two powerful articles written by research mathematicians: Paul Halmos's gentle paper titled "Mathematics as a Creative Art" [3, p. 12] and Paul Lockhart's fiery article, "A Mathematician's Lament" [6] (see the Appendix, p. 298, for the reflection questions).

The initial problem-solving experience is aimed to convince students that a key aspect of doing mathematics is revealing hidden structure. The readings help students contrast their experience in "school mathematics" with mathematics as a way of reasoning. Most students have never conceived of mathematics as creative, and they have so much to say that this conversation fills 90 minutes. Many are frustrated that they experienced impoverished mathematics in school. While I help them to clarify their ideas and to learn to listen and debate as a class, I let them talk. Putting their experiences and ideas on the floor allows them to reflect on their past thinking about the discipline and is instrumental in changing their minds about the nature of mathematics.

### 29.3.3 Week 2: Humans Create Mathematical Meaning

Students come prepared to discuss Ernst von Glasersfeld's deep paper on learning theory and the state of school math: "A Constructivist Approach to Teaching" [8, pp. 3–16]. This paper is tough, but many students cite it as the most important reading of the term because it provides a philosophical basis for thinking of humans as creators of meaning and mathematics as a realm of ideal objects we have created. Classroom problems for this week include Handshakes and Pizzas I and Pizzas II (see the Appendix, p. 299). The first two problems allow students to collect data, look for patterns, make conjectures, create generalizations, and then justify that their solutions are based on underlying structure. Pizza II makes the vital point that justification is necessary because patterns can fall apart (in this case, the doubling pattern ceases at $n = 6$ cuts). The solution to Pizza II is sophisticated, so I leave it as an open question for the class. (When students solve it on their own, I recruit them to be majors.)

At the end of this week, I assign their first Problem Write-up (Lockers) and let them spend the last fifteen minutes of class in small groups getting a start on it. For all 25-point Problem Write-Ups, I require four sections:

1) (3 points) Explain the problem in enough detail that someone who had never seen it would understand it.

2) (6 points) Describe your problem solving process. Include initial conjectures, dead ends, and incorrect ideas as well as fruitful paths. Also include any pictures or data you generate and describe any patterns you see.

3) (8 points) Provide the solution(s) to the problem.

4) (8 points) Justify the solution(s) using some type of deductive argument.

This structure helps students distinguish how they solved a problem from an argument that the solution works and is complete. Many students have never been asked to justify mathematics beyond showing their work, and it takes time and support for them to understand what the mathematical community means by *justification*, and even more time to begin to see how to accomplish it.

### 29.3.4 Week 3: The Mathematical Community is Alive and Well

This week we focus on the influence of community and culture on the ongoing development of mathematics. I begin by creating a Venn diagram on the board with circles for analysts, topologists, algebraists, logicians, and applied mathematicians, and I overview, in layman's terms and with examples, what it is that each group studies. For example, I tell them that the algebra they learned in high school is but one algebra; there are others. I explain that algebraists study the structure of operations defined on sets of objects, and I show the example of clock arithmetic. Then I put professors they know (or have read about) on the Venn diagram.

I describe my graduate school experience, and I explain that one must produce new mathematics in order to earn a doctorate. I tell them that more math has been created in the past fifty years than in all previous history, and I show them what a few professional papers look like. I also teach them the language of mathematics, and we define and discuss the terms conjecture, counterexample, proof, and theorem, and distinguish between inductive thinking (based on examples) and deductive thinking based on logic. Then I use those terms to give them a preview of the story of Fermat's last theorem with enough explanation that they understand the statement of the theorem: equations of the form $x^n + y^n = z^n$ have no non-trivial integer solutions when $n > 2$.

The second class meeting of Week 3, we watch the PBS NOVA program titled *The Proof* (a documentary detailing Andrew Wiles's journey to prove Fermat's last theorem [7]) and students write about and discuss their observations of the culture of the mathematical community. They express surprise that people might spend years thinking about problems, that the community is collaborative, and that the mathematical practices that Wiles describes fit with their own problem solving experiences from our class.

### 29.3.5 Week 4: Math is About Proving Theorems

I use the first class of this week to prepare students to make sense of Euclid's proof of the Pythagorean theorem. We begin with two standard and accessible proofs based on finding the area of a square (composed of four right triangles and a smaller square) in two different ways. Many students have never thought about this theorem in terms of areas. In each case, I recommend that students build the squares by taping down triangle cut-outs. I push them to explore what goes wrong with the argument if the triangles are not right triangles.

The second day of class that week, I present Euclid's classic proof of the Pythagorean theorem (for a nice exposition of this argument, see Dunham's *Journey Through Genius* [2, pp. 27–59]). Their job is to understand it well enough to explain it to someone else. Then I assign the second Problem Write-Up (Dot Paper Areas: this is a Pick's formula exploration). They have 45 minutes to work together on the problem in small groups. I encourage them to collect and organize data on simple cases. When they begin to make conjectures, I help them to make the statements precise, and I encourage them to think about how to prove their conjectures.

### 29.3.6 Week 5: There Exist Mathematical Masterpieces

While my students have seen fine art and read great novels, most have never viewed a mathematical masterpiece. This week, in addition to group work on problem solving, I remedy that with Euclid's proof that there exist infinitely many primes (for this argument, see Dunham [2, pp. 61–82]); it is accessible, simple, and beautiful. I prepare them for the proof by reminding them that a prime number is a number with exactly two distinct factors, and I have them find all the primes below 100. They need to make sense of the fundamental theorem of arithmetic (FTA): all whole numbers bigger than 1 are either prime or can be written in exactly one way as a product of primes. To this end, we all find the prime factorization of 800 and talk about it in the context of the FTA. I explain that the math community decided that 1 would be considered neither prime nor composite because designating it as prime would make the FTA less elegant.

They need to hear stories like this, to know that people made these choices. I overview Greek mathematics and show Euclid's proof. Their task is to learn the argument well enough to explain it to someone else.

### 29.3.7 Week 6: Math is about Counting and Classifying Ideal Objects

This week in class, I challenge students to build all the polyhedra that use only one type of regular polygon and have homogeneous vertex arrangements (the regular polyhedra). They explore this problem in small groups using snap-together regular triangles, squares, pentagons, and hexagons made of plastic (these materials can be purchased). As they work, they discover there needs to be at least three polygons at a vertex and that the sum of the angles at each vertex must be strictly less than 360 degrees. We use these conditions to make the argument that there are exactly five regular polyhedra.

They read an excerpt from Lakatos's *Proofs and Refutations* [5, pp. 7–16] for the second day of that week, and we discuss how the mathematical community has negotiated definitions and standards for justification. I have found that it works well to perform the first ten pages of the reading aloud with me playing the teacher at the board and students reading as the historical characters. Then we begin a problem of finding all the nets for a cube. We make a list of physical restrictions (e.g., the net of a cube can have no more than four squares in a row), and then carefully organize a systematic search that allows us to argue that we had found them all. Finally, I assign Problem Write-up 3: Perimeter Magic Squares and give them some time to begin the problem in small groups. I remind them that the point here is to not only find all of them, but to argue that they have, indeed, found them all.

### 29.3.8 Week 7: Math Can Blow Your Mind

This is the final week of my part of the course, and I hope to leave students with a lasting impression that mathematical thought can be stunningly creative. To that end, I prepare them to understand one of Georg Cantor's masterpieces: there is more than one size of infinity (Cantor's diagonal argument that the cardinality of the interval (0, 1) is greater than that of the natural numbers).

We begin with a series of games. I tell them that I will think of a natural number and they can have one guess at it each day. If they guess my number, they win. I challenge them to describe a systematic plan that guarantees an eventual win. Then we discuss "good" and "bad" guessing strategies (e.g., 3, 2, 1, 6, 5, 4, 9, 8, 7, ... is a good strategy because it guarantees an eventual win, where as 2, 4, 6, 8, 10, 12, ... is a bad guessing strategy because a win is not guaranteed). Next we play the same game with the integers, and finally, the rational numbers. They work in small groups in each case, and each time we do this problem, some group creates a systematic way to guess the rational numbers and presents it to the class (e.g., $0, \frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{2}, \frac{2}{1}, \frac{-2}{1}, \frac{1}{3}, \frac{-1}{3}, \frac{3}{3}, \frac{-3}{3}, \frac{2}{3}, \frac{-2}{3}, \frac{3}{2}, \frac{-3}{2}, \dots$). The point here is to help them to understand that the key to winning is "listing" the set of numbers.

Next I provide Cantor's definition that two sets are the same size if they can be put in one-to-one correspondence, and we argue that various sets (fingers on each hand, natural numbers and integers, evens and odds, natural numbers and rational numbers) are the same "size." The second day of that week (the final day), I talk about Cantor's life and present Cantor's diagonal argument (for a nice summary of that proof, see Dunham [2, pp. 245-266]). The final week also provides an opportunity to overview the history of mathematics, highlighting ideas we have studied during the semester.

## 29.4 Reflections

I love teaching *Creativity* because the course showcases authentic mathematical practice and because it transforms my students' views of mathematics. Students agree; course evaluations are uniformly positive. In particular, they write that they appreciate the opportunity to solve interesting problems in groups and to share their mathematical thinking in a supportive environment. For most class problems, little background content is required, so everyone can participate in the problem-solving process in some way. Students write that the problems were "challenging, open-ended and allowed us to experiment" and say that the course "dramatically changed [their] thinking about math."

That said, many students (even these Honors students) are disheartened by past experiences in mathematics class and so require significant encouragement and support. Throughout the course, I tell them they are not problem solving if they are never stuck, and I admit when I do not understand an idea or when I, myself, am stuck or confused. I tell them that many mathematicians are slow, careful thinkers; that they need not be fast. Finally, I assure them that mistakes are an important part of the problem-solving process and that recognizing mistakes is useful and powerful. I encourage them to share their wrong ideas and thinking with the class (for a nice reading on how to support students' mathematical thinking, I recommend Jo Boaler's book titled *Mathematical Mindsets* [1]).

The next time I teach this course, I plan the addition of Number Talks at the start of each class (for more information on Number Talks, see Humphreys' and Parker's book, *Making Number Talks Matter* [4]). These are short (10–15 minute) warm-up discussions of a single problem solved mentally where all students' methods are embraced. For example, I might invite students to solve something as simple as $44-18$ mentally and then solicit a variety of methods (e.g., students might describe counting up, changing the problem to $46-20$ (same difference), subtracting 20 from 44 and then adding back 2, etc.). Or I might ask that they each find a way to mentally count the number of squares in a given pattern or to create a story problem that would have someone subtract $\frac{1}{3}$ from $\frac{3}{4}$. I hope that the addition of Number Talks will highlight mathematical creativity at all levels, develop number sense, and further cultivate a culture of sharing mathematical thinking as a class.

## 29.5 Bibliography


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[4] Cathy Humphreys and Ruth Parker. *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding, Grades 4–10*. Stenhouse Publishers, 2015.

[5] Imre Lakatos. *Proofs and Refutations: The Logic of Mathematical Discovery*. Cambridge University Press, 2015.

[6] Paul Lockhart. *A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form*. New York, NY: Bellevue Literary Review, 2009.

[7] John Lynch and Simon Singh. The proof. In *NOVA*. Public Broadcasting Service, Oct 28, 1997.

[8] Ernst Von Glasersfeld. "A Constructivist Approach to Teaching." In *Constructivism in Education*, pp. 21–34. Routledge, 2012.

# Appendix

## Reflection Questions for Class Readings and Video

*The following are samples of homework questions that accompanied class readings and media. Student responses were collected and graded.*

### Halmos's *Mathematics as a Creative Art*

1) What does Halmos mean when he says that most people do not know that his subject exists? Do you agree?
2) What does Halmos say mathematics is? Quote him to make your case.
3) Work on the tennis problem before reading the solution. Then read the solution. What point is Halmos making with this example?
4) Find three interesting quotes from this reading and highlight them for discussion. (No need to turn this in.)

### Lockhart's *A Mathematician's Lament*

1) In what ways does Lockhart seem to agree with Halmos on the nature of mathematics and how might they disagree? Explain. Use quotes from the authors to help you make your case.
2) Lockhart pretty much says that our enterprise of school mathematics is a senseless, soul-crushing, alienating, masturbatory following-of-directions that utterly misrepresents and distorts the true creative nature of mathematics. In what ways do you agree? Disagree? Write at least two good paragraphs—based on your own experiences—responding to his lament.
3) Highlight three particularly shocking quotes from this reading for class discussion. (No need to turn this part in.)

### von Glasersfeld's *A Constructivist Approach to Teaching*

1) How is the constructivist theory of knowledge different from the traditional view? What are the primary tenets of constructivism? Pick out three quotes from the article that you think best describe constructivism.
2) Why does von Glasersfeld say this new theory (constructivism) is needed? Do you buy his arguments? Explain.
3) True or False? Von Glasersfeld believes that all "truths" are equally valid. Explain.
4) What point is von Glasersfeld trying to make using the examples of a constellation, a coastline, and an equilateral triangle?
5) What does this reading have to do with human creativity?

### Lynch and Singh's documentary *The Proof*

1) How do the mathematicians in that movie describe doing mathematics?
2) In what ways was your mathematical experience of working on the dot-paper area problem like the mathematical experience described by Wiles?
3) Would Wiles agree with Halmos about the nature of mathematics? Explain your thinking.
4) What was notable or surprising to you about the mathematical culture?
5) How does Wiles's creative process fit with what we have learned so far in class? Use quotes to make your case.

### Lakatos's *Proofs and Refutations*

1) Search for relationships among the numbers of vertices (V), edges (E), and faces (F) of the five regular polyhedra we built in class. Do any of the relationships you found work for other types of polyhedra?
2) What is the difference between a "local" counterexample and a "global" one? Explain, and then give examples.
3) What is the method of monster-barring? Give an example of it from the reading.
4) What do you think von Glasersfeld would have to say about this reading? Quote him to make your case.
5) Has this reading altered your idea of mathematical proof? Explain.

## Class Problems

*Students worked on the following problems in small groups in class. Sometimes we discussed ideas and solutions and arguments as a whole class. Other times, students were asked to provide written solutions and arguments as part of their homework.*

### Penny of Death

The game is this: place $n$ pennies on the table between you and one opponent. Now you and your opponent must take turns removing pennies from the table. Either one or two pennies may be removed in each person's turn. The object is to force your opponent to take the last penny. Your group's job is to write careful instructions (that some innocent person walking by in the hall could understand) for how to win at this game for every number of pennies.

### Handshakes

Our club always starts off a meeting with a ceremonial "shaking of the hands" in which everyone shakes hands with everyone else present. If there are ten club members, how many handshakes will take place? Justify that you are correct. If our club grows to $n$ members, then how many handshakes would there be? Justify that you are correct.

### Pizza I

Suppose that you take a round pizza and make $n$ straight cuts across it. What is the maximum number of regions into which you could divide the pizza? Argue that you are correct.

### Pizza II

Place $n$ points on the edge of the pizza (on the rim of crust). Now connect those points with all possible chords. What is the maximum number of regions you can divide the pizza into using $n$ points? Argue that you are correct.

### Locker Problem

The teachers at Read Elementary School play the following game after all the students have gone home. The first teacher opens an entire row of 100 lockers. The next teacher closes every other locker starting with the second locker. The third teacher changes every third locker beginning with the third. (If a locker was closed, she opens it, and if it was open, she closes it.) The fourth teacher changes every fourth locker starting with the fourth, and so on. After the 100th teacher has completed her work, which lockers end up open and why?

### Pythagoras Proofs 1 and 2

On the attached sheet you will find eight copies of a right triangle. For each one, the shortest side has length $a$, the other leg has length $b$, and the hypotenuse has length $c$.

Proof 1: Cut out four identical copies of your right triangle and arrange them to form a square with area $(a + b)^2$. Tape them down. If you find that square's area another way, it will help you to prove the Pythagorean theorem. Do that. Where in the above proof do you need the fact that the triangle is right?

Proof 2: Cut out and arrange the remaining four identical copies of the right triangle to form a square with area $c^2$. Tape them down. If you find that square's area in terms of $a$ and $b$, it will help you to prove the Pythagorean theorem. Do that. Where in the above proof do you need the fact that the triangle is right?

### Dot Paper Areas

There is a shortcut method for finding the area of any polygon made with its vertices on the dots of square dot paper. The formula involves both the number of boundary dots and the number of interior dots for the polygon. Your job is to find that shortcut and create an argument that your shortcut works for the case of rectangles.

### Nothing but Net

A net for a three-dimensional figure is a two-dimensional pattern that can be folded to create the figure. How many different nets (of six squares aligned along edges) exist for the cube? Find them all and argue you have done so.

### Perimeter Magic Squares

Find all the different ways you could place the digits 1, 2, 3, 4, 5, 6, 7, and 8 into the empty boxes of a 3 by 3 square grid so that every perimeter row and every perimeter column has the same sum. Then justify that you have indeed found all the possible ways to do this. (You may need to think about what configurations should be counted as different here.)

# 30

# A Writing-Intensive FYS Course on Recreational Mathematics

Enrique Treviño

**Abstract** In this article I discuss my writing-intensive first-year studies course, *Recreational Mathematics*. I describe in-class activities on Benford's law and Buffon's needle, the class's main writing assignments (in which students maintained a course blog about Martin Gardner's columns and other mathematical topics they found interesting), and the final poster presentation assignment.

## 30.1 Background and Context

Lake Forest College is a liberal arts institution in the greater Chicago area with around 1,500 students. The college has a campus-wide first-year studies program (FIYS) with the aim of introducing students to the college and preparing them to write at a college level. The program invites faculty from all departments to submit a proposal for an FIYS course. Every first-year student must take an FIYS course. The typical FIYS course has around 15 students. The students select five FIYS courses they are interested in (out of around 25), and then the administration chooses which class to assign them to from their five choices. This means that many of the students in the FIYS class have an interest in the subject area being taught. A professor teaching an FIYS also acts as the advisor for the students in their class. Because of this extra responsibility, there is a stipend to teach the course. The FIYS course meets three times per week for 50 minutes in a 15-week semester. The course counts as one Lake Forest College credit, which is the equivalent of four credit hours at most US institutions.

## 30.2 Mathematical Theme

The theme of the course is *Recreational Mathematics*. I chose this topic because I think it engages students regardless of their background, and it is a topic open to writing assignments. One way the course reaches students is that we study games and we look at surprising phenomena. For examples about games, we analyze strategies in Sudoku, we explore variations of Sudoku, and we look at the frequency distribution of the properties in the game of Monopoly. Examples of surprising phenomena we cover in class are Benford's law and Buffon's needle. In Section 30.3, I go into detail about how I do this in the course. My goal is to showcase that mathematics is not just about calculating and doing rote computations. At the same time, I want to have students not fear mathematics and to appreciate its beauty. By showing them that many games are mathematical, they can understand that they like mathematics without even knowing it.

The theme is also good for writing assignments, which is one of the requirements for FIYS courses at Lake Forest College. The goal is for students to be able to write at a college level. I will describe the assignments I give students in Section 30.3. About 85% of the classes were dedicated to mathematics. The rest were dedicated to college life such as explaining how to use the library or the resources they have at the career advancement center. There was no class time dedicated to teach writing techniques, yet there was class time used to explain the assignments and my expectations. There was also one class period dedicated to discussing the summer reading assignment (discussed in Section 30.3.1).

## 30.3 Course Structure

The course meets three times a week for 50 minute sessions. Some sessions consist of lectures, most include in-class activities, and there are three important assignments.

As mentioned in Section 30.2, we cover a lot of surprising phenomena in the classroom. For the classroom activity on Buffon's needle,[1] I bring sheets of paper with equidistant parallel lines, and I bring toothpicks to throw on the sheet of paper.[2] The students then record the number of toothpicks that "hit" a line. At the end of the class, we look at how the number of hits per student are remarkably close to each other, and that the average proportion of hits is close to $2/\pi$. For the classroom activity on Benford's law,[3] I bring newspapers to the classroom, and I ask students to record how many times each digit appears as a leading digit.[4] I ask students to ignore numbers that are used for dates or ages, as those skew the data. After all of this, we look at the distribution and find out that it's not uniformly distributed. The digit 1 is more likely than the digit 2, which is more likely than the digit 3, and so on. We then dive into the mathematical reasons for this phenomena.

My main references for topics to cover in class are articles by Martin Gardner [3], puzzles from Pete Winkler [7], [8], articles by Ian Stewart [4], and articles from the Art of Mathematics website [2] (especially for good Sudoku activities).

The course grade evaluates four main areas. Class participation is worth 15%. There is a summer assignment that is worth 5% of the grade. There are three blog posts the students have to write that are worth 10%, 15%, and 15%, respectively. Blog participation is worth 15%. Finally, there is a poster presentation at the end of the semester worth 25%.

### 30.3.1 Summer Assignment

FIYS courses at Lake Forest College include a summer assignment. The goal of the summer assignment is to have instructors engage with students before arriving on campus, since the instructor is their advisor, and to have a topic of discussion on orientation week. In my course, I assign students to read *Logicomix: An Epic Search for Truth* written by Apostolos Doxiadis and Christos H. Papadimitriou with art by Alekos Papadatos and Annie Di Donna [1]. *Logicomix* is a graphic novel that gives a biography of Bertrand Russell and explains important concepts from set theory. I chose *Logicomix* because I think it conveys hard mathematical ideas through an interesting story. The book also gives motivation for abstract thinking. Furthermore, I think the book illustrates why many of us love mathematics so much. Finally, I think the book makes students think differently about mathematics. With this in mind, the summer assignment is to write a 750-word essay that touches on the following two questions:

- What differences do you find between mathematics as described by the characters in *Logicomix* and mathematics as taught in high school?
- Did the reading make you view mathematics differently? If so, what changed?

The students have to turn in the assignment a week before classes start. Then, on the first day of class, students get feedback on their assignment, and then they revise it. We discuss *Logicomix* during a class period a few days later.

### 30.3.2 Course Blog

The course has a blog.[5] I ask students to submit three blog posts during the semester. I schedule it so that each day has two new posts. For example, one year, the blog had daily student posts from early September to late November.

[1]Consider parallel lines in the plane equally spaced at distance $d$ from each other. Consider throwing a needle of length $\ell \leq d$ onto the plane. Buffon proved that the probability of the needle intersecting one of the lines is $\frac{2}{\pi} \cdot \frac{\ell}{d}$ [6].

[2]The toothpicks have the same length as the distance between two parallel lines.

[3]In numerical data that includes numbers from different orders of magnitude, the frequency distribution of the leading digits is not uniform. Benford's law predicts that 1 is the leading digit with proportion $\log(2) \approx 0.301$, 2 is the leading digit with proportion $\log(3/2) \approx 0.176$, and in general $i$ is the leading digit with proportion $\log((i+1)/i)$ for $i = 1, 2, \ldots, 9$ [5].

[4]We say that the leading digit is the leftmost digit in a number. For example, the leading digit of 75435 is 7.

[5]The URL for the course is: `http://fiys169.blogspot.com/`.

Each of the three blog posts is different. For the first blog post, each student has to write a summary of one of the chapters in *The Colossal Book of Mathematics* [3] emphasizing what the student thinks is most interesting in the chapter. My goal for this post is for students to read recreational mathematics and to write about it.

For the second blog post, each student has to write a more detailed essay on another chapter of *The Colossal Book of Mathematics*. For this essay, they have to read at least three of the references cited by Martin Gardner and incorporate them into their blog post. For example, I ask the student to think of what parts of the references they think should have been part of Gardner's essay. My goal for this post is for students to read sources and to think of the choices an author makes when deciding what to include in an article.

For the third blog post, the student has to write an essay on some mathematics content they discovered for themselves. I provide suggestions for students that request them, but the idea is for students to find some mathematical puzzle or mathematical idea that they find fascinating and want to write about. My goal for this post is for students to be active in choosing what they want to write about and for me to see what mathematical aspects intrigue them. The prompts for the three blog posts can be found in the Appendix, pp. 304–305.

Finally, to ensure the blog has a life of its own, students are required to post one comment (or more) on at least two posts each week. The comment has to be a well-written post of at least 200 words. The blog participation grade also considers the quality of these comments.

The reason I like to use a blog in the course is that it is a way to get students to read their peers' work. Given that I frequently read blogs online, it also makes it easier for me to keep track of my students' progress during the semester.

### 30.3.3 Poster Presentation

The students are asked to give a poster presentation in groups of two or three. They can choose the same topic from the third blog post or find a new topic. They have to submit a draft first, then print the poster and present in front of an audience. I invite other faculty to attend the poster presentation. Students place their posters at different locations in a room and people can roam around as the students present their posters multiple times over the span of 45 minutes. Examples of topics of posters my students have done include "What's the best Monopoly strategy?", "The Tower of Hanoi", and "The mathematics of snowflakes."

## 30.4 Reflections

I have taught *Recreational Mathematics* twice. The course structure described in Section 30.3 comes from the second time I taught the course. The first time I taught it, I had a final paper assignment instead of a poster presentation. The final paper assignment was to write a brief biography of a mathematician (chosen from a list I gave them) and talk about one of their main contributions to mathematics.[6] The main reason I changed this assignment was that I wanted my students to develop presentation skills. Another motivating factor was that my favorite assignment was the third blog post, where they chose a topic and explored the mathematics surrounding it. The students' enthusiasm shined through, and I think students learn more by researching topics they are interested in. By assigning a poster presentation, I essentially made the students focus on a project more to their liking.

Changing the final paper to a poster presentation was very successful. The students did a better job with the assignment. They practiced a different skill (presenting), and they interacted with their classmates outside the class. The students enjoyed the poster session and seemed eager to share the mathematical topic they chose with their peers. Furthermore, some posters were so well received that my colleagues encouraged me to nominate them for the College Symposium[7] the following semester. Two of the posters from the FIYS class were presented the following semester at the Symposium.

One of the challenges I felt teaching the class was that I have no training in teaching writing. Because of this issue, I worried that my assignments did not have good prompts, and I worried about whether I graded fairly. Another obstacle is that English is not my first language.[8] If I had to teach the class again, I would ask my Humanities colleagues for advice on grading rubrics and writing prompts.

[6] The prompt for the final paper is in the Appendix, p. 306.
[7] Lake Forest College has a symposium every year, where students present to each other.
[8] My first language is Spanish.

I think the main strength of the course was showing students how mathematics is more deeply connected to their everyday life than they might have realized. A professor in the education department surveyed my students (informally) about what they got from my class, and their reply was that mathematics was everywhere. I thought the lectures were usually fun for the students (or at least had them engaged). There are certainly things to improve in my lectures. For example to give a justification for Benford's law, I introduced the concept of logarithmic density.[9] This was too advanced for students with little calculus background.

Another strength of the class was the activity in the course blog. While most of the comments were written mainly because students had to do it for their grade, the creation of daily mathematical content was something I was proud of. The blog assignment that generated more excitement among the students was the third assignment, where they were able to choose their own topic. Because students chose their own topics, the other students would get more involved with the comments on these posts. That assignment added a community-feel to the class. One way to improve future classes might be to allow students to have more choices in topics instead of assigning chapters from Martin Gardner's book [3].

## 30.5 Bibliography


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# Appendix

## Course Blog Prompts

*The following are the three blog post prompts assigned throughout the semester. Students' essays were posted online at a course blog.*

### First Blog Post

You have to read the assigned chapter and write a post containing at least 500 words and at most 1000 words. The main purpose of this assignment is to read carefully and critically. Your blog post will consist of:

- A summary of the chapter. For this summary it would help to answer questions such as "what is the chapter about?" and "what is the main concept or idea the author is communicating in the chapter?".
- Your thoughts on the chapter. Some motivating questions are "was the chapter interesting?" and "was the chapter surprising?". You should also include questions the chapter raised for you.

[9] Let $A$ be a set and $A(x) = \{a \leq x \mid a \in A\}$ be the counting set of $A$. The natural density of $A$ is the limit of $A(x)/x$ as $x \to \infty$. The logarithmic density of $A$ is $\lim_{x\to\infty} \frac{1}{\ln(x)} \sum_{a\in A(x)} \frac{1}{a}$. If we let $A$ be the set of numbers with leading digit 1, the natural density of $A$ does not exist. However, the logarithmic density does and it matches what Benford's law predicts.

Your assignment also includes reading the chapters the other students write about and comment on at least four blog posts. To insure that you do not leave all comments to the last minute, you should post each comment on different days. This means the day you post a comment is different for each comment you make. The comments you write should be written seriously and contribute to the discussion. The comment has to be a well-written post of at least 200 words. Your blog participation grade will depend on the quality of these comments. When commenting on a post about a chapter in the book, you need to read the chapter yourself as well. An example of a good comment would be a question the reading raised for you. Another example would be a comment describing what you think about the reading or a comment regarding whether you agree with the student's blog post.

### Second Blog Post

You have to read the assigned chapter and three references cited at the end of the chapter. Your blog post will consist of:

- A brief summary of the chapter. For this summary it would help to answer questions such as "what is the chapter about?" and "what is the main concept or idea the author is communicating in the chapter?".
- For each of the three references you picked, find something you wished the author had included in his chapter.
- For each of the three references you picked, find something you are happy the author omitted in his chapter.

Your assignment also includes reading the chapters the other students write about and commenting on at least four blog posts.

### Third Blog Post

You have to find a subject on your own and write about it. The assignment has the following rules:

- First, pick a mathematical subject. It can be a cool puzzle, an interesting game (that involves thinking), an application of mathematics to some field (maybe an application of math to sports) or an exciting theorem.
- Once you pick your subject, find three references to give you background on that subject.
- Send the subject and the three references to my email at least one week before your blog is due (for example, if your blog is due Nov. 12, you have to send me the information by Nov. 5). I will then approve the topic, if it is suitable. If not, you have to find one right away. I suggest that you have three subjects in case I don't approve one of them.
- Once you have my approval, write a blog post talking about the subject. You can think of how Martin Gardner wrote his essays (each chapter in the textbook) as inspiration. Your blog post should be between 800 and 1200 words. No longer and no shorter. You are welcome to include pictures and you should include a bibliography.

Your assignment also includes reading the posts the other students write and commenting on all of them by the last day of class. The comments you write should be written seriously and contribute to the discussion.

## Poster Presentation Prompt

*The following is the prompt given to students for their poster presentation. The students receive this prompt on the first week, references are due on week 12, and they present their poster on week 14.*

You will have to present a poster on the last day of class. You will work on the poster in a group of two people (I will accept groups of three in some cases, but you have to request it via email). You must find a mathematical topic of your interest (it can be the same as one of the third blog posts) and create a poster for it. You will have to be ready to present a draft (a pdf version of your poster) five days before the poster presentation is due. The printed version of the poster will be due two days before the presentation is due. On the day of the presentation, we will keep the posters up all day, but you will have to be next to your poster from 9 AM to 9:50 AM.

Furthermore, you need to do the following two weeks before the presentation is due:

- MLA style citation to a journal article you intend to use as a reference in your poster presentation. The article must come from using one of the college databases; include a permanent link to the article in the database.
- MLA style citation to a book you intend to use as a reference for your poster that is in our library or that you requested from I-share.[10]
- MLA style citation for at least three other references.

## Final Paper Prompt

*The following is the prompt given to students for their final paper. The students receive this prompt on the first week, the reference list is due on week 11, the first draft on week 13, the final draft on week 15.*

You must write about one of the mathematicians listed in the course website.[11] The paper should give a brief biography of the mathematician and you should talk about one of their main contributions to mathematics. For example, if you could choose Bertrand Russell (you can't because he's not on the list), then you would give a brief biography of Russell and talk about his contributions to set theory with his book *Principia Mathematica*. The book you read in the summer (*Logicomix*) gave a very nice example of a way to give a biography and talk about the math contributions from a mathematician (in this case Bertrand Russell). This paper should be a little longer than the blog posts. It should be at least 1200 words and at most 2000 words (if you need more space to write about the mathematical concept, you can request an extension).

There are three deadlines associated to the final paper. The first deadline is for the annotated bibliography. For the Friday of week 11 you must have the following (and turn it in):

- MLA style citation to an article you intend to use in your final paper that you found using one of the college databases; include a permanent link to the article in the database.
- MLA style citation to a book you intend to use in your paper that is in our library or that you requested from I-Share.
- MLA style citation for at least three other references.

The second deadline is for a first draft for the paper. This draft is due one week before the final day of classes (week 13). This should have at least 1200 words and a bibliography.

The third deadline is for the final paper. After I hand you back comments on your first draft, you work on the final paper and turn it in on the day of the final exam.[12] You can turn it in before, but failure to turn it in by the final exam date will mean an automatic fail for the class.

[10] I-Share is a catalog of over 80 Illinois academic libraries that are connected to Lake Forest College.
[11] The URL was: http://campus.lakeforest.edu/trevino/Fall2014/FIYS169/ListOfMathematicians.html/.
[12] Every class in Lake Forest is assigned a day for its final exam. If a class has a final paper instead of a final exam, the paper is due on that day.

# 31

# Technical Writing: A Mathematical Approach

Nathan Shank

**Abstract** The first-year writing seminar, *Technical Writing: A Mathematical Approach* is designed for a general audience with no mathematical prerequisites. The course focuses on understanding and constructing concise, evidence-based arguments grounded in truth. We develop writing skills by applying the general writing process (brainstorm, outline, draft, edit, final) to mathematics problems. In this paper I present unique assignments that provide students with an opportunity to learn and reflect on the importance of precision of language, the writing process and its application to technical writing, and the interconnectedness of writing, mathematics, and language.

## 31.1 Background and Context

Moravian College is an independent liberal arts college with approximately 1,800 undergraduate students. Founded in 1742, Moravian College is affiliated with the Moravian Church of America and is recognized as the sixth oldest college in the United States and the first to educate women. The College mission, to provide a liberal arts education that prepares each individual for a reflective life, fulfilling careers, and transformative leadership in a world of change, is grounded in Moravian's revolutionary educational spirit: believing everyone deserves to be educated, independent of social status, gender, race, ethnicity, and religious background. The mission of the mathematics program at Moravian College is to foster a community of faculty and students who promote the aesthetic, theoretic, and pragmatic qualities of mathematics in order to develop in its students communication and problem-solving skills applicable to many disciplines that prepare them for fulfilling careers.

The First-Year Writing Seminar (FYWS) is a campus wide program taught by full-time faculty across all disciplines and is overseen by the English Department. The current structure of the FYWS was developed by combining two transition courses: an introduction to college life and an introduction to college-level writing into a one semester FYWS experience. The unit course (four credits) meets for 70 minutes three times per week for a traditional 15-week semester. Each FYWS has a common set of goals that include developing students' writing skills, building community, and providing an introduction to college life. Each instructor can add additional course objectives and tailor the theme and content of the course. In addition, each instructor has full academic freedom in developing assignments, class structure, and means of assessment. First-time freshman (approximately 450 students) are placed into one of their top three FYWS selections. Students do not enroll at the college with a major declared, but most students select an FYWS based on academic interest. Enrollment in FYWS is capped at 18 to help facilitate student-student and student-professor interaction.

## 31.2 Mathematical Theme

*Technical Writing: A Mathematical Approach* is an FYWS designed for a general audience with no mathematical prerequisites and has a focus on developing precise and concise evidence-based arguments grounded in truth. Students' mathematical abilities range from precalculus to calculus III and about half of the students are interested in studying mathematics or computer science. There are two required texts for the course: *An Introduction to Mathematical Thinking* by Keith Devlin [3] and an academic writing text chosen by the instructor ([1], [2], or [4]). Students in the course are introduced to different logical constructs including Boolean algebra, contrapositive, contradiction, and induction. Students learn how and when these constructs are applied in mathematical and nonmathematical settings. Mathematical content is not the focus of the course and only constitutes about ten percent of the course material. The main focus of the course involves developing the general writing process (brainstorm, outline, draft, edit, finalize) and applying it to mathematics, for example, in a calculus course. Readings for the course include mathematical and nonmathematical pieces related to college life, academic maturity, the writing process, proper and improper ways to create arguments, and understanding the importance of mathematical and technical thinking (see [6], [7], and [8]).

## 31.3 Course Structure

The FYWS course meets three times per week for 15 weeks. However, the first six Fridays of the year are devoted to presentations by the Division of Student Affairs that introduce students to many of the resources and opportunities on campus. The remainder of the class time can be used at the discretion of the instructor.

This course is devoted to three themes: college life (including academic maturity), the writing process and college-level writing, and mathematical thinking. At the beginning of the semester, we focus primarily on college life. About three weeks into the semester, we begin to focus on technical writing and the writing process starting with an introduction to Boolean algebra, implications, and quantifiers. This serves as a background for discussions on precision of language, the importance of making unambiguous definitions, and logical constructs. By the end of the semester, the course is completely focused on technical writing.

Students generally work on three or four assignments concurrently which mimics the workplace/research environment. Each assignment focuses on being precise and concise and has a specific purpose, audience, genre, and final draft instructions that may include a page maximum (not a minimum), font size, minimum margin size, and detailed submission format.

For assessment purposes, most written assignments are grouped into one of two categories: reflective/free-writing assignments and writing-process assignments.

### 31.3.1 Reflective/Free-Writing Assignments

Reflective/free-writing assignments are designed to introduce students to a new topic, encourage students to think more deeply about a topic, or see the interconnectedness of writing, mathematics, and language. A reflective/free-writing assignment is given approximately every three weeks for a total of five or six throughout the semester. Two examples of reflective/free-writing assignments are:

1. Bean Game (Nim):
   (a) Setup: During the first day of class, as an icebreaker to encourage student-student interaction, we play variants of the game of Nim. Nim is a well studied two player impartial game consisting of piles of beans. A turn consists of removing any nonzero number of beans from a single pile. The player to remove the last bean loses the game. Students are randomly assigned a partner and play the game several times. We start with one pile of beans and students are allowed to remove one or two beans during a turn. The students rotate around the room playing with several classmates. As they play, they are asked to think about a strategy for winning the game. At this point the idea of "Mathematical or Scientific Research" is introduced by explaining that we often want to solve a very complicated, possibly impossible, problem that we simplify until we can solve the problem. We then gradually make the problem more general. As a class, we simplify to a one bean pile then gradually add beans to the pile to develop a conjecture for a winning strategy. Although the students do not understand the nuances of a formal proof, they can understand some of the logic as to why the strategy

works. Next, we play the two pile game where a turn consists of removing any nonzero number of beans from one pile. We mimic the process we followed in the one pile setup: play a few games, simplify to small piles, gradually increase the number of beans, make a conjecture for a winning strategy, and rationalize why the strategy works.

(b) Assignment: For the reflective/free-writing assignment, students write precise descriptions of the two pile problem, the winning strategy, and a rationale for why the strategy works.

(c) Specific Requirements: The audience for the paper is a peer who is not in the class and the written reflection must be single-spaced with one inch margins in ten-point font and be one-page maximum.

(d) Assessment and Follow Up: Students are graded on completeness rather than correctness. The assignment lends itself to a fruitful discussion on the importance of language and definitions. Ambiguous statements are discussed (e.g., both piles even) so that students can start to understand the importance of precision in writing. Examples from the assignment are used as discussion points during the class.

2. Understanding Quantifiers and Implications:

   (a) Setup: Prior to class, students read an introduction to implications and quantifiers ([3], pp. 10-50). During class if-then and if-and-only-if statements and universal, existential, and uniqueness quantifiers are presented. We discuss both mathematical and nonmathematical examples of these implications and quantifiers. Students work on translating between English statements and mathematical expressions. We begin with expressions that are logically true (e.g. if $x \neq 0$ then $x^2 > 0$) and end with expressions that are not true. False statements force students to use what they have learned rather than rely on intuition (e.g., for all $x > 0$, there is a $y > 0$ so that $xy < 0$).

   (b) Assignment: For this assignment, students need to find ten nonmathematical uses of quantifiers and implications. For each example they are to write three short paragraphs. The first paragraph explains the quantifier or implication in its context. The second paragraph contains an exact copy of the text which they are using and includes any additional context. The third paragraph explains how quantifiers or implications are being used and why they are important. They must explain whether the mathematical interpretation is the socially accepted interpretation. Misconceptions and ambiguities should be addressed. See the Appendix (p. 313) for the prompt and grading rubric.

   (c) Specific Requirements: Students cannot use a source more than once and must find examples of all three quantifiers and both implications. At least four examples must use combinations of quantifiers and implications and at least one example must reflect how the mathematical interpretation is different than the socially accepted interpretation. At least one example must come from each of the following: newspaper, song, movie, research journal, commercial, magazine, government document, financial document, and something college related.

   (d) Assessment and Follow Up: Often students have trouble with multiple quantifiers and more technical material. Therefore students are graded mostly on completeness since complicated combinations are encouraged. Interesting examples that come from this assignment are then discussed as a class. These have included:
      - We all need somebody to lean on.
      - Everybody is way too nice to everybody.
      - Not everybody agrees with me.
      - If I try to make everyone happy, I'm going to be miserable.
      - There's no place like home.
      - My mom doesn't like you and she likes everyone.

### 31.3.2 Writing-Process Assignments

Writing-process assignments are designed to have students engage in all parts of the writing process. Throughout the course there are typically five writing-process assignments with varying degrees of depth. The assignments are scaffolded so that the students receive formative feedback during each step of the process. Scaffolding assignments also serves as a mechanism to prevent procrastination which ties in well with college-life objectives. Students engage in a peer-review exercise that provides anonymous feedback through comments on their paper as well as a one-page, unbiased, factual (not opinionated) critique of the content of the paper. An early class session on peer review and editing, beginning with the understanding that peer editing is as valuable for the editor as it is for the author, sets the stage for students to give constructive feedback.

In the beginning of the course, students are introduced to LaTeX through Overleaf. Many early reflective/free-writing assignments with little or no mathematical equations or symbols require students to use LaTeX to familiarize them with the program. For each assignment a template is shared with the class that contains header material, fill in the blank responses for personalized information (name, date, title, section headings, etc.) and some helpful hints. For writing-process assignments students must share their Overleaf link with the instructor to help facilitate feedback and expedite error correcting. The use of LaTeX allows students to be able to see mathematical thoughts and expressions in sentence and paragraph form. Additionally, students are more amenable to editing and revisions with typed work rather than hand-written work.

Two examples of reflective/free-writing assignments include:

1. Mathematical Problem
   (a) Setup: This assignment engages students in the writing process centered on a mathematical problem. Most students who have not had a proofs course only work through the beginning stages of the writing process for mathematics work: brainstorm and outline. They rarely use complete sentences to explain a solution to a mathematics problem. This assignment encourages students to see that mathematics work, like work in other disciplines, requires skills in writing.
   (b) Assignment: This assignment requires students to write a solution to a mathematical problem by going through the entire writing process. Each student is allowed to pick his or her own problem which may come from a current course (like calculus or statistics) or a previous mathematics course. The idea is not to introduce new mathematics, but to have the students write a solution to a problem they know how to solve. For example, the problem may be to find the derivative of a function $f(x)$. Each stage of the writing process is closely monitored. For brainstorming, students need to understand the problem. Students must write the problem on the board and define terms, draw pictures, etc. to understand what the problem is asking and providing. For outlining, students work on scrap paper or the board to solve the problem and find an appropriate answer if there is one. They must be sure to write justifications for each step. In constructing a rough draft, students convert their scrap work into written form using sentences and paragraphs. They must incorporate the equations and expressions into sentences correctly. This is the first stage in LaTeX. Editing includes self edits and mathematical peer edits. It is useful to group students based on mathematical ability when doing peer edits. Students produce a final version in LaTeX with appropriate white space, title, problem, name, and date that conforms to the details in the assignment. Generally a LaTeX template is provided that includes the header and setup for the file. See the Appendix (p. 314) for the prompt and grading rubric.
   (c) Specific Requirements: The final product should be in problem-solution form that is easily understood by a mathematical peer. The final paper should be similar to what the students would read as an example in a textbook. Others should be able to follow along and understand the flow and validity of the solution without the use of scratch paper.
   (d) Assessment and Follow-Up: Students' work is assessed throughout the process. However, many students choose to do the majority of the work on the board. Students are informed that above all else, truth is the most important aspect. A follow-up discussion on level of detail, audience, and the challenge of writing mathematics in appropriate sentence and paragraph form usually follows.
2. Grant Proposal—Getting Groups Together Grant
   (a) Setup: One of the objectives of the FYWS is to build community not only within the FYWS class but also on campus. This assignment, the final assignment of the course but not the most substantial portion of the final grade, accomplishes both. An auxiliary purpose of this assignment is for students to engage in all parts of the grant writing and review process. Students need to provide a preliminary letter of intent, develop a proposal that conforms to specific guidelines, and work in groups to read, critique, and provide feedback. Although the proposals do not require a mathematical theme, the idea of making arguments based on truth while being precise and concise still applies. In addition, developing grant writing skills is important for many disciplines, including mathematics where grant writing is often expected.

(b) Assignment: The assignment, *Getting Groups Together Grant*, requires students to write a grant proposal that would create a significant common experience for at least two distinctly different groups of college students on campus. Students work in pairs to develop a competitive proposal. The proposals must contain a cover form, proposal (two pages maximum including plan and budget), personal statement, and at least two letters of support. No other supplementary documents are accepted. See the Appendix (p. 316) for the prompt and grading rubric. The winning proposal is funded and the proposed activity serves as a class project the following semester. Although this is not a college-wide grant program, students compete against other students in the class. Participation in the following semester is not required; however, about two-thirds of the students in the class attend and help organize the event, which is open to the entire campus. Approximately 30-60 participants attend the event. Funding for the winning project is provided by a $200 stipend given to the FYWS class as well as a small contribution from the mathematics program.

(c) Specific Requirements: The specifications follow general grant submission guidelines. Students develop a one-page letter of intent that is due three weeks before the grant proposals are due. This ensures that student groups have different proposals and ensures proposed projects would be acceptable for the campus community. In order to emphasize precision and details, the proposal specifications include: 10-point font, one-inch margins, signed signature page in red or blue ink, double spaced, five stapled copies with specific order, specific header, page numbering, etc.

(d) Assessment: Groups of three to four students evaluate five other proposals. The intention is to have each proposal evaluated by at least three groups. As a class we develop a rubric which generally contains content, compliance, support letters, personal statement, and other factors. Students must evaluate the proposal based on the rubric and then develop a two-paragraph response to the authors outlining strengths and weaknesses of the proposal. In addition, student groups must make a recommendation for priority funding, fundable, or not fundable. The written responses are then returned to the original authors and the overall class rankings are made public to the class. The highest ranked project becomes our class project in the spring semester. Two examples of winning proposals are:

- Mathematics and Music: A laptop orchestra club was created and they performed a show during a math club meeting and a music club meeting. Members of the club include both mathematics students and music students.
- Mathematics of Art: Students competed to see who could accurately draw geometric objects by having another person describe them orally. Teams consisted of art only students, mathematics only students, and mixed students from many disciplines.

## 31.4 Reflection

This course was originally developed because of the lack of a technical writing FYWS section that would prepare students for college-level writing in mathematics and the sciences. Since its development, this course has been offered for three consecutive years and enrollment has reached the maximum each offering.

### 31.4.1 What Went Well

Because the mathematical ability of students varies greatly, many short assignments, including textbook homework problems [3], are graded on completeness rather than correctness. This allowed the students to concentrate on understanding concepts based on their abilities. For example, students who were more prepared in mathematics or computer science were asked to do more complex boolean algebra problems. This allows each student to expand their depth of understanding at their appropriate level.

Students with a limited mathematical background often had difficulty explaining abstract concepts. Therefore, many short writing and reading assignments related to mathematics tend to have a mathematical and nonmathematical component. This allowed students to reflect on concepts in both setting. For example when trying to make arguments based on universal quantifiers students would start with a small finite set (e.g., students in the room), then expand to a larger set (e.g., students in the United States), then to a larger finite set (e.g., grains of sand in the world), then to an infinite set (e.g., the set of all rational numbers). This progression allows the students to understand limitations in their rationale and appreciate other persuasive (proof) techniques.

Students have reported at the end of the course that they have a much better understanding of the writing process. Many commented that engaging in the writing process for a computational mathematics problem (or a proof) helped them understand the importance of the writing process in other courses. This could be because at the beginning of the course students report having a disdain for many parts of the writing process, particularly outlining and editing. However, these two stages are crucial, almost innate parts of mathematics and problem solving. Additionally, students who have completed this particular FYWS course are more prepared for a sophomore level introduction to proofs course. Having learned LaTeX and engaged in using the writing process to write mathematics, students can focus on learning proof skills and techniques rather than on learning to write formal mathematics.

### 31.4.2 Modification and Challenges

The entire course is divided into three topics: introduction to college life, the writing process and college-level writing, and mathematical thinking. We devote about 25% of the course to college life. The course would have more flow and depth if we were able to focus more on the writing process, college-level writing, and mathematical thinking and less on college-life objectives. Although students do a proof as a writing assignment in the course, having more time would allow us to see the aesthetic quality of mathematical writing. Examining advantages and disadvantages of certain mathematics proof techniques, wrestling with mathematical paradoxes, and an excursion into descriptive mathematics related to geometry would add a deeper understanding of the importance of reasoning and language in the course.

Generally, students understand that the transition from high school to college includes a shift in responsibility. For example, it is no longer the teacher's responsibility to make sure that a student submits assignments; it is the students responsibility. Although students are made aware of this shift many times, they are too often not prepared for it. Students struggle with the lack of structure and the increased freedom that hinders their ability to keep track of assignments and due dates. Larger assignments that are scaffolded help students make the transition easier and allow them to manage new responsibilities in smaller, less formidable chunks. Many smaller graded assignments peppered throughout the semester serve as a tool to assess the academic maturity of a class. These assignments, generally graded on completeness rather than correctness, include class discussion boards, short email responses, written mathematics homework, and a small research focused initiative on evaluating the credibility of a source. Additionally, these assignments are used to motivate class discussions and encourage student-teacher interaction.

Although developing and implementing this type of course demands a lot of time and effort on the part of the instructor, the overwhelming response of the students indicates that the course prepares them for college-level work and improves their mathematical writing in upper-level courses.

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# Appendix

## Understanding Quantifiers and Implications

*The "Understanding Quantifiers and Implications" assignment and rubric below is assigned in week three of a typical 14-week semester after an in-class discussion of quantifiers. The goal is for the students to see how pervasive and important quantifiers are in everyday language.*

### Assignment

For this assignment you are to find ten different non-mathematical uses of quantifiers and implications.

For each one that you find you should have three short paragraphs:

P1: Explain the context in which you found the quantifier or implication. This should be written in your own words, in full sentences, and not using a citation or a quote.

P2: Copy, word for word, the text you are quoting. Be sure to include any important information before and after the quantifier statement to add context if needed.

P3: Explain how quantifiers and implications are being used and why they are important. Explain their importance and interpretation if they are used correctly and the potential misinterpretations if they are used incorrectly.

For each one you will have about a half-page write-up when completed.

Other requirements:

1. You may only use each source once. So if you use the DMV website for an example, you can only use it once.
2. You must find examples of all three quantifiers (existential, universal, and unique) and both implications (if-then and if-and-only-if).
3. You should find at least four examples using combinations of quantifiers and implications.
4. At least one of your examples should reflect how the mathematical interpretation is different than the socially accepted interpretation.
5. You must have at least two examples of negations of quantifiers or implications. For example: "If there does not exist someone in the home under the age of 18, then ..."
6. You must have at least one example from each of the following:
   - Newspaper
   - Lyrics in a song
   - Movie quote
   - Something Moravian related
   - Research journal (not math or CS)
   - TV commercial or magazine advertisement
   - Government document (local, state, or federal)
   - Financial document

### Rubric

| Example | Quantifiers and Implications | Correct Interpretation 4 | Expanded Explanation 4 | Total Points 8 |
|---|---|---|---|---|
| 1. Newspaper | | | | |
| 2. Lyrics | | | | |
| 3. Movie | | | | |
| 4. Moravian | | | | |
| 5. Journal | | | | |
| 6. Commercial | | | | |
| 7. Govt. document | | | | |
| 8. Financial document | | | | |
| 9. Other | | | | |
| 10. Other | | | | |
| TOTAL | | | | / 80 pts |

1. ________ / 2 pts: You may only use each source once.
2. ________ / 2 pts: You must find examples of all three quantifiers (existential, universal, and unique) and both implications.
3. ________ / 2 pts: You should find at least four examples using combination of quantifiers and implications.
4. ________ / 2 pts: At least one of your examples should reflect how the mathematical interpretation is different than the socially accepted interpretation.
5. ________ / 2 pts: You must have at least two examples fo negations of quantifiers or implications

Overall Quality of Writing ____________ / 10 pts

TOTAL: ____________ / 100 pts

## Writing a Mathematical Solution

*The assignment below comes during week 7 of the semester. A blank LaTeX template is provided to the students and basic LaTeX commands are covered in class through Overleaf. The goal of this exercise is for the students to follow the writing process for a mathematical problem.*

### Assignment

For this assignment we will be working through the writing process on a mathematics problem. Our goal is to produce a technically sound argument that solves a mathematical problem and our solution will be in paragraph form.

**Homework:** Over the weekend, choose a mathematics problem that you have already done for a class or already know how to do. The problem should come from your most recent mathematics class (or your current one). Bring a copy of the problem with you to class on Monday when we will begin working through the problem to prepare to write our solution. If the problem you bring in seems to be inappropriate (too hard or too easy) we will find a new one during class. If you are unsure if your problem is appropriate, feel free to email it to me over the weekend and I will try to look at it.

**Specifications:**

1. Purpose: Write the solution to a mathematical problem.
2. Audience: A mathematical peer (someone who has the same mathematical ability as you do).
3. Genre: You should write in problem-solution form, similar to what you would see in a textbook. Everything you write needs to be incorporated into a sentence. Every sentence needs to be incorporated into a paragraph. Every paragraph needs to be incorporated into the structure of the piece.
4. Extras: You need to write your rough draft and final in LaTeX. Be sure to compile as you go along. Finals will be due at 9 a.m. on Friday. Bring four printed copies of a rough draft with you to class on Wednesday.

**Writing Process:** Brainstorm: rewrite the problem, either on the board or on scrap paper, and be sure you understand every word in the problem. Annotate the problem. What are you being asked to find? What is given? What mathematical terms do we have to know?

**Outline:** Work on the board to try to solve the problem. This may involve only variables, expressions, or pictures and no words. See if you can get an answer. This is what a mathematician would call their scrap work. When you have solved the problem, go back and try to justify each step. Why are you allowed to do what you did? You may need to remember some of our common algebraic properties such as associative, commutative, distributive, and closure properties. If you make an algebraic manipulation in your work, you need to be able to justify why you are allowed to do it. Be sure to organize your thoughts from top down. What are you allowed to assume? Who are the characters in your story and are they well defined?

**Rough Draft:** Here is where we convert our scrap work into sentence form. Hint: think about how you would say the problem out loud. We will work in pairs to read our rough drafts. This will take the majority of an entire class to convert an outline to a rough draft.

**Edits:** We will work in groups to edit our problems/solutions. Think about paragraphs and the effective use of white space. Be sure to vary your sentence structure.

**Final:** The final version should be something you might see in a textbook.

### Rubric

| | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| Organization | Paper is well organized. Sentences and paragraphs flow nicely and in logical order. | Paper is organized nicely. Most sentences and paragraphs flow nicely and are in logical order. | Paper lacks organization. Most sentences flow nicely, but paragraphs are not in logical order. | Sentences and paragraphs lack organization and are not in logical order. |
| Audience | The paper is written to an appropriate audience. | The paper is mostly written to an appropriate audience but lacks background. | The paper is not written to the appropriate audience, but is consistent. | The paper is not written to the appropriate audience and has no clear audience. |
| Clarity and Truth | The content is clear and logical with appropriate level of details. | The content is clear and logical but has too much or too little detail. | The content is not clear and has small logic problems. | The content is not clear and has major logic problems. |

| | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| Writing Style | The paper fits the appropriate style for a mathematical problem/solution found in a textbook and has appropriate level of details. | The paper fits the appropriate style for a mathematical problem/solution found in a textbook but is lacking in one or two details. | The paper fits the appropriate style for a mathematical problem/solution found in a textbook but is severely lacking in details. | The paper does not fit the appropriate style for a mathematical problem/solution found in a textbook. |
| Grammar, Punctuation | No errors in punctuation or grammar. | Very few errors in punctuation or grammar, and they do not take away from the content. | Several errors in punctuation or grammar that slightly distract the reader. | Several errors in punctuation or grammar that distract the reader. |
| Mathematical Notation and Conventions | No errors in mathematical notation or conventions | A few minor errors in mathematical notation that did not distract the reader. | Several errors that slightly distract the reader. | Several errors that distract the reader. |
| Answers the Problem | Correct solution and states the solution in the context of the problem. | Correct solution but missing details (units or context). | Computational error leads to wrong, but plausible solution. | Wrong answer or no answer. |

## Getting Groups Together Grant

*This assignment comes during the last three weeks of the semester. The grant solicitation* (see Assignment below) *is discussed in-depth as a class to practice technical reading. The goal of this culminating assignment is for the students to engage in technical writing in a non-mathematical setting.*

### Assignment

Adapted from [5]

The purpose of this grant is to create a significant common experience for at least two distinctly different groups of college students on campus. The grant can have a maximum budget of $200. The winning award will be made to a student currently enrolled in our course. Materials may be in the form of supplies, prizes, food, books, or other appropriate expenses. No funds can be used for stipends or wages.

Grant proposals should address the following:

- Project participants (at least two distinctly different campus groups)
- Project goals
- Plan for development of the project including how the project goals will be met
- How the entire class can be involved in the project
- Importance of the project on the groups and the general community
- Detailed budget

Projects that bring distinctly different groups together, have a learning component, have an impact beyond the groups directly involved, and have the potential to have a lasting impression on those involved will be given priority. Activities are to be completed between January 15 and May 1. All students must submit a proposal. The GGTG Proposal Cover Form must be completed and serve as the top page of each copy of the proposal. The proposal must be typewritten, double-spaced and single-sided (please organize as outlined below), with margins of at least one inch on standard-sized (8.5 in. x 11 in.) paper. Font size must be no smaller than 10-point (Times Roman suggested), and the width between characters should be normal (100%). Five copies (one original and four copies) of the proposal should be included in a single packet addressed to the instructor. The application packet must be postmarked by December X, 20XX. Faxed or emailed copies will not be accepted. Duplicate applications will not be considered. Lack of an applicant's signature will automatically disqualify the proposal.

### Proposal Requirements

1. Proposal Cover Form which contains:
   - Title of the proposal
   - Author information (names and current intended majors)
   - Anticipated date of event
   - Total budget request
   - Author signatures
2. Proposal (two page maximum)
   - Plan. Describe the who, what, how, and why of the plan. Include references and supporting documents. The bibliography/reference page does not count towards the two page maximum.
   - Budget. Include an itemized budget (presented by line item in a table format).
3. Background and Experience (one page maximum each; outline format preferred)
   - Personal Statement: Indicate your personal involvement in the project and your personal thoughts on why the project should be funded. Indicate your experience or association with the groups and any other personal information that might enhance the outcome of the project.
4. Letters of Support (one page maximum each)
   - Two letters asking for support: the letters must be written to the faculty member and student "advisor" of the groups you want to participate. The letters should be signed by the student and the faculty member. The signature does not imply they agree to the proposal, but they will consider engaging in the activity.

An initial letter of intent (one page maximum) is due by 11/XX/YY and should include the project coordinators and a brief description of the project.

### Rubric

GOAL: Create a significant common experience for at least two distinctly different groups of college students on campus.

**Overall Proposal Content**

| | Excellent | Very Good | Good | Fair | Poor |
|---|---|---|---|---|---|
| Project Participants | | | | | |
| Project Goals | | | | | |
| Plan: Who, What, How, Why | | | | | |
| Entire Class Involved | | | | | |
| Importance on Groups and the General Community | | | | | |
| References | | | | | |

**Basic Details**

| | Excellent | Very Good | Good | Fair | Poor |
|---|---|---|---|---|---|
| Budget | | | | | |
| Cover Page (with Signatures) | | | | | |
| Date | | | | | |
| Typed | | | | | |
| Double-spaced, Single-sided | | | | | |
| Organized Correctly | | | | | |
| Font Size 10 or Larger | | | | | |

**Personal Statement**

| | Excellent | Very Good | Good | Fair | Poor |
|---|---|---|---|---|---|
| A: Peronsal Involvement | | | | | |
| A: Rationale | | | | | |
| B: Personal Involvement | | | | | |
| B: Rationale | | | | | |
| C. Personal Involvement | | | | | |
| C: Rationale | | | | | |

**Letters of Support**

| | Excellent | Very Good | Good | Fair | Poor |
|---|---|---|---|---|---|
| At Least Two Letters | | | | | |
| Appropriate Form | | | | | |
| Appropriate Content | | | | | |
| Signatures | | | | | |

**Priority Items**

| | Excellent | Very Good | Good | Fair | Poor |
|---|---|---|---|---|---|
| Distinctly Different Groups | | | | | |
| Learning Component | | | | | |
| Impact Beyond The Groups Directly Involved | | | | | |
| Potential To Have Lasting Impression On Those Involved | | | | | |

**Overall Rating**

__________ Priority Funding     __________ Fundable     __________ Not Fundable

# Part VIII

# Quantitative Literacy

We live in a time when information abounds. Data touches all aspects of our lives and continues to grow in quantity and complexity. The first-year seminars in this section highlight different ways students can learn how to critically evaluate and use quantitative data to make well-informed decisions and arguments in their personal and professional lives. This section also provides an example of a first-year seminar that is linked with an introductory quantitative reasoning mathematics course, a pairing that provides numerous benefits when teaching quantitative literacy.

# 32

# Nine Out of Ten Seniors Recommend this First-Year Seminar: Statistics in the Real World

Johanna Hardin

**Abstract** The first-year seminar described in this article is a celebration of statistics in the real world. As a statistics professor in a department of mathematics and statistics, my primary teaching assignments focus on the technical and computational aspects of statistical modeling. However, with first-year students and in a seminar setting, I am able to delve deeper (in discussion and in expository writing) into how statistics is used and misused in the world around us. As a science, statistics is not an unbiased arbiter of truth. Instead, it is one tool for interpreting our individual and varied experiences. Throughout the semester, the students do weekly readings and reflect on the readings in class discussions. There are three formal essays that include peer review and draft revisions. I hope that after leaving the course, students understand the statistical ideas and historical contexts well enough that they are able to critically evaluate the news, the media, medicine, and any other reports of data or variability with which they are presented.

## 32.1 Background and Context

Pomona College is a private residential liberal arts college with approximately 1,700 undergraduate students (there are no graduate students at Pomona College). Every incoming first-year student is required to take a first-year seminar. The summer before their matriculation, the incoming students are provided a list of the upcoming seminars, and they send in a ranked list of their top five choices. In a large majority of cases, the students are placed into one of their top two or three choices. Each first-year seminar has approximately 15 students, and once a student is enrolled in the class, that student is unable to drop the class or transfer sections. It is the one class in their time at Pomona that they have no flexibility to switch or drop. The class runs as a full semester-long course that meets twice a week for 75 minutes each time. At Pomona we do not distinguish between, for example, a 3-unit class and a 4-unit class. The seminar represents one course in a typical four course schedule.

The seminars are offered across the disciplines. Typically, bigger departments offer more seminars, but due to enrollment pressures, some science departments are unable to staff first-year seminars, and some humanities departments have many people who want to teach the first-year seminars. In recent years in mathematics, we have offered two first-year seminars annually. The faculty who teach first-year seminars are those who volunteer to teach them; some of us enjoy the writing aspect, some have particular topics that we want to explore, and others enjoy the mentoring aspect of working with first-year students.

At Pomona, the topic of the first-year seminar is a means to exploring writing and critical thinking. That is, the discipline specific content is secondary to learning to engage effectively in writing. Pomona College does have learning goals for the first-year seminar, but recently the Director of College Writing and a subcommittee have been working to update the goals. The goals are currently lengthy, and they include ideas of making a strong argument, using many sources (and a variety of types of sources), engaging multiple sources with each other, knowing how to revise and edit, and critical reading.

## 32.2 Mathematical Theme: Statistics in the Real World

Many people consider the field of statistics to be synonymous (or at least interchangeable) with probability and probability models. While using probability models is integral to statistical methods, statistics is more accurately described as the set of tools applied to datasets in order to make claims about larger populations. (N.b., one way to think about probability is in the opposite direction: a set of tools used to describe the datasets that are chosen from populations.)

The focus of the mathematical content is on teaching statistical intuition. In more mathematical courses, this might be akin to teaching the logic of proof writing. While not directly a mathematical "topic" in the traditional sense, statistical intuition (and proof writing logic!) are integral pieces of the content. Other more standard statistical topics covered in the class (although mostly not covered in technical detail) include: sampling, experiments and observational studies, reproducibility, causation, visualization, and p-values. Except for a few days spent workshopping papers, every class period is dedicated to understanding some aspect of the statistical pipeline. In total, approximately 50% of in-class time is devoted to topics in statistics. The students are not learning the material (or, for example, practicing problem solving using software) in the same way as would be covered in an introductory statistics course.

Throughout the first-year seminar course, I work to demonstrate the myriad ways statistics are used and misused in daily life. For example,

1. Using the book *Fat Politics* [4], we discuss the arguments that are made for and against the idea that "being fat" is unhealthy. Given a statistical framework, we are able to understand how causal arguments are made and when they cannot be justified.

2. In the movie *12 Angry Men* [2], jurors spend the entire film debating the evidence of the case. The movie is used in our class as a way to understand the structure of hypothesis testing. In neither law nor statistics do we ever know *truth*, so we need to weigh different pieces of evidence and think carefully about what they say about the legal/research question of interest.

3. ProPublica's article on "Machine Bias" [1] describes how algorithms can be systematically biased despite being tuned by a computer. In class we discuss the ethics of using any kind of large algorithms which will naturally pick up on the dominant trends (read: existing biases) that already exist in a culture, criminal profiling, social media, etc.

4. With the brilliant work of Giorgia Lupi and Stefanie Posavec in *Dear Data* [3], we discuss the difficulties associated with knowing what data to collect, actually collecting data, and visualizing data effectively. We also discuss poorly visualized data, and how different conclusions can be drawn from images that misrepresent the story of the data.

## 32.3 Course Structure

By institutional design, the course is set up as a discussion course with evaluation based primarily on written assignments. The readings cover a large variety of materials including statistical texts, media articles, scientific literature, fiction, and non-fiction.

### 32.3.1 Classroom Activities

The majority of class meetings consist of a group discussion based on a set of readings. The students are responsible for the discussions in pre-defined roles. Throughout the semester, each student will take complete responsibility for leading the class discussion. The other students submit discussion questions the night before, but it is the leader's responsibility to organize and structure the discussion. A second student actually starts the discussion by adding context to the readings. The context might include a biography of the author or an extension to the scientific work. As a way to close the circle on the discussion, a third student is responsible for taking notes and writing up a summary of the class discussion.

### 32.3.2 Writing Assignments

With the exception of one data collection assignment (see below), all of the student work is essay writing. One of the institutional learning goals for the course is for student to be able to "write a *good* (clear, concise, logical) college-level essay." There is general agreement that one way to achieve that goal is through repeated practice with writing and feedback throughout the first-year seminar. Every week the students are required to write. Sometimes they write short micro-essays (500 words), and sometimes they write longer essays (1000-2000 words) or revisions of their longer essays.

#### Essay prompts

The main, stated goal of the first-year seminar is for students to develop their writing skills. In my course, however, I am also hoping that they engage with the discipline of statistics. As such, I've given writing prompts that aim to teach them about technical details of statistics (e.g., evaluating political polls) as well as develop an appreciation for the historical context around the role of statistics in systemic inequities. (see Essay 3).

**Essay 1** Find a fictional character for whom something random happens. Interpret their resulting behavior through the lens of a nonfiction text of your choice. Your essay should be accessible to a general reader who may be unfamiliar with the work of fiction and the lens text. 1500 words maximum.

1. Pick a fictional text (novel, film, etc.) and a character of interest to you in that text. At some point in the text, the character should be involved in a *random*, *chance*, or *fateful* occurrence. The occurrence could be a single event or a sustained pattern of behavior. The behavior should have consequences.
2. Select a *lens text* through which you will describe the character's choices and behavior. Your lens text may be one of the class readings or something you find on your own. Note that *Seeing Through Statistics* [5] (the main text used in the course) lists many articles you could choose as your lens.
3. Write an essay in which you explain the behavior of your primary character in terms of your primary lens. The idea is for you to use your lens text to illuminate the character and their actions. When it goes well, you and your reader will learn something new and interesting.

**Essay 2** Find a political race that you find engaging and that will be decided in November's election. Use the readings and other sources you find to frame the information being presented in the media. Your essay should be accessible to a general reader who may be unfamiliar with the political race and political polling in general. 1000 words maximum.

1. Pick a political race about which there is enough media attention that you can find ideas and sources.
2. Find a poll describing the race and get as much information as you can about the poll (Who they asked, What they asked, Who funded it, etc.). Then ask as many follow-up questions as you can about the poll (How do they know that? How were the people selected? Who did they miss? ... ).
3. Write an essay in which you explain the way the media has portrayed the chances of your candidate winning their race. The idea is for you to use an outside text(s) (readings from class are fine, lens text does not need to be academic or peer reviewed) to get deeper insight into how the candidate will fare on election night.
4. Your essay should include a claim (thesis statement) that you argue throughout the paper. You may be arguing that some aspect of the campaign, polling, or media is similar to the candidate's previous race, or it may be different. Or you may argue that there is something particular to your race that makes polling important or unimportant. The idea is to make a claim and use the the readings to back up or contextualize your claim.

**Essay 3** Your final paper will center around a research question which will be argued using sources that you find. The paper should address how statistics (as a discipline, as a set of data analysis tools, as individual people, etc.) are/were involved in creating or reinforcing systemic inequality or social injustice. The research question will likely evolve over the life of the project, but you should keep coming back to the (evolving) idea you want to argue. In addition to a succinct and arguable thesis statement, your paper should include:

1. Details of the event of interest.
2. Historical context (*why* the thing you describe happened or was able to happen).
3. You might also give the reader an idea of possible solutions (how to make it better) or maybe a counterargument.
4. A potential audience for your paper is a STEM undergraduate who feels disempowered by the traditional canon.

Along with practice putting ideas into words, another goal is for a student to be able to "read for the author's argument, perspective, and *stance*—that is, with an awareness of the perspectival nature of knowledge." In class, we discuss what it takes to make an argument, and for each of their three main essays, the students are required to peer-review two of their classmates' essays. Materials and instructions for essays and peer-review are provided on the course website, `http://st47s.com/ID1/`.

Because writing is an integral part of Pomona's FYS, the students are encouraged or required to make use of our writing resources. The Writing Center at Pomona is staffed with a director and co-director as well as three dozen students who are trained to help students improve their writing. My course has a dedicated Writing Partner who meets with the students regularly, but the students are also able to drop in at the Writing Center for additional help. One class session is spent working with a librarian. During that session we focus both on *what* makes a good source as well as *how* to find a good source.

**Personal Data** One additional assignment based on *Dear Data* is worth mentioning. After deciding, as a group, what to measure, each student spends a weekend collecting data from the world around them and presenting it to the class as an image with an associated legend. Examples of student work are given in Figures 32.1 and 32.2. Note that the two students took remarkably different approaches to both the data collection and data visualization tasks. However, both figures are quite effective at displaying a weekend of greetings. The assignment has a few different learning goals. First, it is important for the students to understand how hard it is to collect data accurately. The difficulty can be in the measuring, the remembering, or the obtaining. Ideally, after completing the assignment, students should find themselves asking "how in the world did they collect *that* data?" every time they encounter the analysis of a new data source. Additionally, the assignment should convey the difficulty, power, and importance of being able to effectively present visual information.

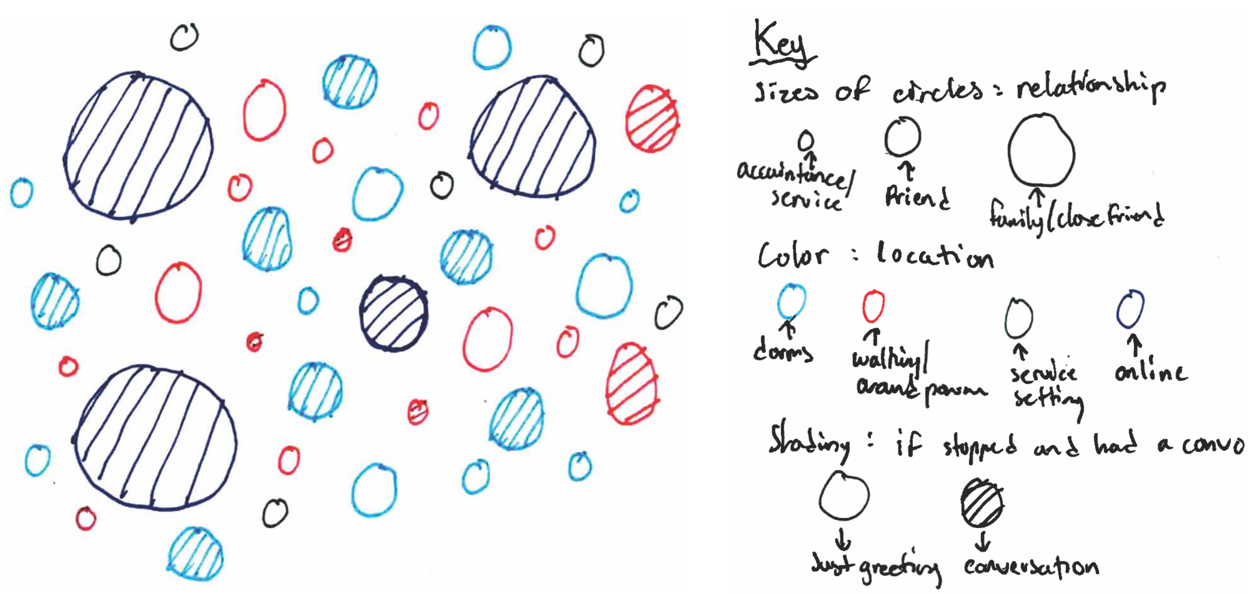


**Figure 32.1.** Franco's greetings over the weekend. Franco decided to keep track of whether the greeting turned into a conversation or not and also the variables including who it was and where the greeting happened.

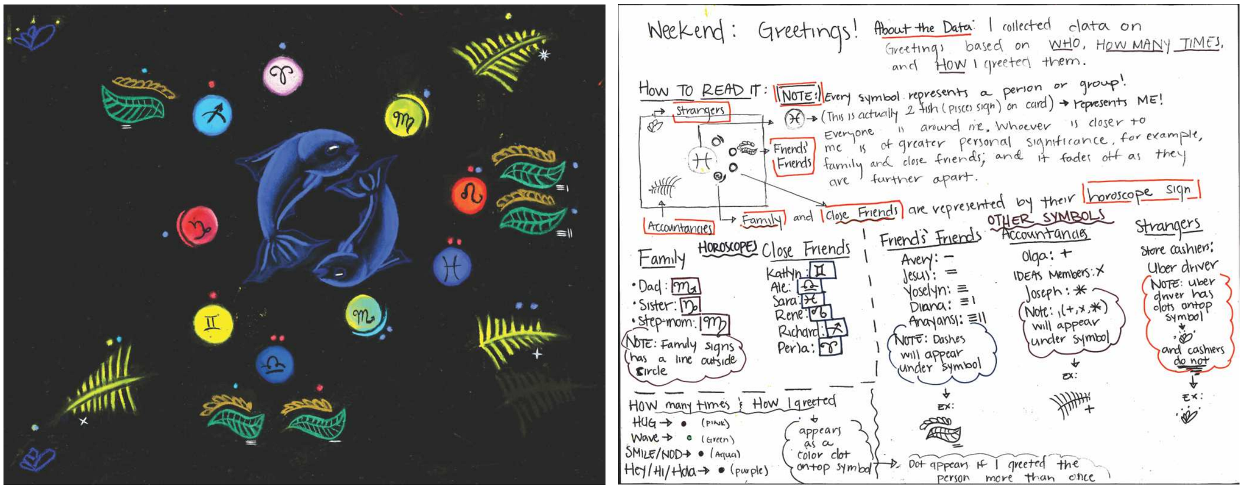


**Figure 32.2.** Maelvi's greetings over the weekend. Maelvi collected a lot of information about the individual she greeted and their relationship to her. Additionally, she measured the type of greeting (hug, wave, smile, hi).

## 32.4 Reflections

I have taught the course four times. The readings and assignments have been updated each year, but the general course framework has remained the same. My training as a statistician has not given me much experience in creating writing assignments, teaching how to write well, or assessing writing. I have relied heavily on talking to colleagues and working with the Director of College Writing.

The students in my first-year seminar are all first semester students who are used to writing a "five paragraph" essay. We spend a lot of time talking about the parts of an essay including the thesis statement and the supporting evidence (i.e., the paragraphs, but not necessarily three of them). Each time I teach the course, I see tremendous growth over the semester. Not only do the students learn to communicate more effectively, but they also become more confident in their abilities. With feedback from their peers, the course Writing Partner, and me, they learn to recognize how others perceive their work which helps them to defend their ideas more successfully.

### 32.4.1 Strengths

My students generally give very positive reviews of how they grew as writers throughout the semester. And while my feedback to them is undoubtably valuable, I also stress throughout the semester that there are many different ways to communicate effectively in writing, and surely throughout their time at Pomona College they will get lots of different feedback, sometimes in perfect alignment and sometimes in direct conflict with previous advice they have received. Generally, the students appreciated the feedback on their writing and the opportunities to lead class discussions. They also love the topic and the readings. Some students thought that the readings were too long and that the full class discussions could have been more productive if the students had talked in smaller groups first.

### 32.4.2 Challenges and changes

As might be surmised from the comments above, reading, grading, and discussing multiple versions of essays takes an extraordinary amount of time. Of course, practice in grading does help speed up the process, but nothing can substitute for one-on-one conferences with individual students. Asking them to talk through their paper and tell you what they will accomplish in writing is a productive way for a student to learn to become a better writer.

One recent change to the course is to include more peer assessment (see the course website at: `http://st47s.com/ID1` for the assignment schedule as well as guidelines on peer assessment). For each of the first two essays, we work through peer-review in groups of three. Each triple reads, comments on, and reviews the papers of two peers before coming to class. In class, the triples get together and discuss each individual's paper, and the written reviews of the paper are provided to each author who uses them to revise their work.

With peer assessment, each student is able to talk through their argument multiple times. Additionally, they will be able to "read" their essay through someone else's eyes. Not only do the peer assessments allow for additional and valuable feedback, but they take some of the onus of review from the professor. They also give the students practice editing other people's work.

Overall the class has been quite successful, and the student evaluations report an acknowledgment of learning a lot about writing and statistics. I look forward to every discussion, and I find the readings to be extremely engaging. I never have trouble getting students to show up to class, and I often get emails from past students who contact me after the class is over to report on something related to the course material. Some small adjustments in readings, writing assignments, structure, and increased peer review will hopefully serve to improve the course even more. One piece of advice that I continue to revisit is that less is more. Giving up multiple days to peer review and writing instruction will benefit the students much more than an additional set of readings on political polls.

One other piece of advice I give myself is: have fun. I am incredibly lucky to be able to share some of my favorite aspects of statistics with a group of engaged students. I hope that you, too, will find an opportunity to spend three hours a week discussing your favorite aspect of mathematics.

## 32.5 Bibliography

[1] Julia Angwin, Jeff Larson, Surya Mattu, and Lauren Kirchner. "Machine Bias," *ProPublica*, 2016. `propublica.org/article/machine-bias-risk-assessments-in-criminal-sentencing`

[2] Henry Fonda, Reginald Rose (Producers) and Sidney Lumet (Director). *12 Angry Men*, Los Angeles, Orion-Nova, 1957.

[3] Giorgia Lupi, Stephanie Posavec. *Dear Data*, Princeton Architectural Press, New York, 2016.

[4] J. Eric Oliver. *Fat Politics*, Oxford University Press, New York, 2006.

[5] Jessica Utts *Seeing Through Statistics*, Cengage Learning, Kentucky, 2015.

## Appendix

### Note

Further details about the course, including the daily schedule, the syllabus, specifics of writing assignments, and some general thoughts on writing, can be found at `st47s.com/ID1`.

# 33

# A First-Year Seminar on *Lies, Damned Lies, and Statistics*

Aaron M. Montgomery

**Abstract** The *Lies, Damned Lies, and Statistics* first-year seminar is designed to explore sources of quantitative misinformation. This course makes use of current political topics as a means of engaging student interest and demonstrating the relevance of the course content. Among other assignments, each student writes a pair of essays citing statistics; one essay is in support of some thesis, and the other is in opposition to it. The primary goal of the course is to dispel the notion that numbers and data constitute an objective, unambiguous view of reality. Assessment data suggested some success in achieving this outcome.

## 33.1 Background and Context

Baldwin Wallace University (BW) is a private institution in northeast Ohio with an undergraduate enrollment of just over 3,000 students. All first-year students entering BW enroll in a First-Year Experience (FYE) course as a compulsory seminar. The explicit learning outcomes for this course are that students will demonstrate effective academic communication, engage in critical thinking, recognize the BW mission, and demonstrate intellectual curiosity. At an institutional level, the FYE is also understood to serve the auxiliary purposes of retention and college acclimation. The FYE is a three credit course; two credits' worth of the FYE are taught by the instructor responsible for the primary seminar content, and one credit is taught by a transition instructor whose specific focus is on the mission statement, adjustment to college, and similar goals that may not obviously fit the primary content. The two instructors of the FYE are expected to collaborate and coordinate their efforts whenever possible. Each credit hour corresponds to one 50-minute meeting per week for a total of three of such meetings each week (sans holidays) across a 15-week semester.

In keeping with Baldwin Wallace's tradition in the liberal arts, instructors for these seminars are drawn from all corners of the BW faculty and are granted substantial freedom to select seminar topics that will generate student interest and foster intellectual exploration. Students have an opportunity to indicate their preferred topics, but they are ultimately sorted into classes of roughly equal size (about 16 students) in which their classmates often hold different majors and interests than their own. Each student in an FYE course completes at least ten pages of process writing in which they submit multiple versions of the same paper and respond to feedback from their instructor.

## 33.2 Mathematical Theme

The FYE seminar *Lies, Damned Lies, and Statistics* explores some of the nuances that reside in the margins of many statistics classes. The main purpose of the course is to probe how statistics can be used to mislead or deceive. Some topics, such as the distinction between correlation and causation, are commonly discussed in many introductory statistics curricula. Other topics in the seminar, such as confirmation bias and the necessity of replication studies, are often de-emphasized or absent entirely from traditional statistics courses. The *Lies, Damned Lies, and Statistics* FYE seeks to fill in these gaps of introductory statistics by exploring sources of numerical misinformation and addressing the pervasive problem of an uncritical acceptance of statistics in forming or evaluating opinions. Notably, introductory statistics is not a prerequisite for this FYE; the ability to think critically is much more necessary in a course of this nature than is the knowledge of how to execute a hypothesis test or construct a confidence interval. This course emphasizes mathematical context over mathematical content and eschews computations in favor of critical analyses of stated figures. Around one-fourth of the course contains material that may or may not overlap an ordinary statistics course (depending on whether the statistics instructor would see fit to include it), but the FYE aims to provide further development for those common topics.

To fully explore the theme of misinformation, the *Lies, Damned Lies, and Statistics* FYE seminar also draws heavily from political news. It is very useful to be able to run this seminar during the same term as a major election, such as a U.S. presidential election (or at least a midterm election) so that students can see the obvious relevance of the content to modern society. In addition, such elections tend to provide a veritable fount of discussion topics for the class. Students who may otherwise be disinclined to take a seminar that they perceive to be about mathematics can be attracted to this seminar if it is billed as a course examining politics through a numerical or statistical lens.

## 33.3 Course Structure

The official course textbook is *Damned Lies and Statistics: Untangling Numbers from the Media, Politicians, and Activists* by Joel Best [1]. The author of the text is a social scientist, and the book has a nontechnical focus with the intent of accessibility to the general public. Students receive periodic reading assignments as homework, and several in-class activities and discussions center around the content of the chapters. Many topics from the assigned reading reverberate in other unrelated discussions and activities during the class as well.

The central assignments in the course, and the method by which students fulfill the requirement of ten pages of process writing, are a pair of writing assignments called the Point and Counterpoint essays. In the Point essay, students write a five-page paper arguing any point of their choosing and support their argument with at least five quantitative citations from at least three different reputable sources. In the Counterpoint essay, students make the opposite argument to their Point essay, again with at least five quantitative citations from at least three different reputable sources. In keeping with the goal of fostering intellectual curiosity, students have complete reign over the choice of topic and are subject only to the restriction of being able to find five credible statistics to support and oppose a thesis. Students are encouraged, but not required, to write about a topic on which they personally hold mixed feelings or on which they want to research multiple sides of an issue to clarify their own opinions.

A student's first task in the Point/Counterpoint assignment is to submit a proposal for their topic. This proposal must include one statistic in favor of some argument as well as one statistic in opposition to that same argument. This is intended as a proof of concept to verify that students choose topics which plausibly contain sufficient depth to meaningfully argue two sides of a discussion. Upon receiving approval of a proposal, each student begins writing their Point essay. After some time to research and write, each student brings a first draft of their essay to class and exchanges them with peers for review. They then submit a second draft to the instructor who provides extensive feedback on the work. Students then turn in a third and final draft of their Point essay to the instructor, and the entire process (except the proposal) begins again for the Counterpoint essay. Students are asked to withhold their true opinions (if they have any) during the writing process so that the instructor can evaluate the quality of students' arguments in an unbiased fashion.

This course also features two required short presentations from each student. The first of these assignments, called Bad Graph Monday, functions by asking one or two students each Monday to present on a graph or chart conveying some numerical information in a misleading or unclear way. The instructor forms a random presentation schedule at the beginning of the semester and distributes it to students so that they know the particular Monday on which they are responsible for presenting. Students select a bad graph from a central repository during the week leading up to their presentations; safeguards should be established to ensure that students do not select graphs that have already been used by a peer. The presentations are short (three to seven minutes) and include a discussion of the context of the graph, an explanation of why the graph was bad, and some suggestions for how the graph might be improved.

The second recurring presentation assignment, Fact Check Wednesday, is the feature of the course that most directly connects it to a concurrent election. Each Wednesday, one or two students make a presentation based on a fact checking article published in the previous week. As with the Bad Graph Monday assignment, the instructor determines a random presentation schedule in advance, although this schedule may need to be adjusted to ensure that a student does not have to deliver both presentations in close proximity to one another. When presenting, students analyze some quantitative claim made by a politician or other public figure that has been judged to be false or misleading by a reputable fact checking organization. Required elements of the presentation include an analysis of the context of the speaker, the nature of the incorrect claim, possible motivations or explanations for the incorrect claim, and suggestions for how to repair the claim.

A typical day in the class begins with one of the aforementioned presentations and continues with a discussion or activity involving some numerical nuance that fits into the general theme of the course. The format of discussions may vary; sometimes, the instructor may serve as the moderator for a class-wide discussion, and at other times students may be divided into pairs or into groups of four to hold small discussions before reconvening as a larger group for a summary discussion. These lessons are somewhat flexible in nature in order to give the class opportunities to discuss quantitative issues related to current events when appropriate.

One example of a politically-themed class discussion is an analysis of newsworthy statements by political figures. For instance, the class could draw a contrast between Hillary Clinton's statement that, " ... [Y]ou could put half of Trump's supporters into what I call the basket of deplorables ... the racist, sexist, homophobic, xenophobic, Islamophobic—you name it," [2] and a comment from Mitt Romney from four years prior to that, "There are 47 percent of the people who will vote for the president no matter what. ... These are people who pay no income tax." [3] In this case, students could be asked to discuss in groups whether either comment was intended to contain statistical information, and assuming that they were, the extent to which the claims are true or verifiable.

In a different activity, students could form small groups and work to classify several incorrect numerical claims taken from fact checking articles as either generalization errors, transformation errors, confusion errors, or compound errors (terms found in the *Damned Lies and Statistics* text). Another discussion related to modern political topics could center around the difference between genuine scientific polls and online voluntary response pseudo-polls (also known as Voodoo polls) that are often found on partisan corners of the internet. Such a conversation could highlight an apparent disparity between the two methods regarding which presidential candidate is regarded by the public as having won a presidential debate.

Another discussion could explore the concept and importance of the term "likely voter". As a related activity, students might answer a series of questions to determine if they would be regarded as likely voters by the Gallup screening process. Similarly, on the day before a presidential election, the class could discuss both the state of the race according to polling data and the historical magnitude of polling errors. Such a conversation might include a parallel to the premature "Dewey Defeats Truman" headline from the 1948 *Chicago Daily Tribune*. On the day after the election, the class could conduct a postmortem of any polling errors.

The portion of class time following the presentations can also be used for topics not directly related to politics. One such discussion might involve the necessity of replication studies and include an activity in which students are given opportunities to demonstrate apparent extra-sensory perception (ESP) abilities with a partner by:

- guessing the result of a coin flipped in secret by the partner five times consecutively
- guessing the suit and value of a card drawn from a half-deck of cards and held in secret by the partner
- guessing the faces of two differently-colored dice rolled in secret by the partner and
- guessing the two-digit number at the end of a partner's student ID number within two.

In this activity, each student performs each action as the guesser exactly one time. These particular ESP activities work well because of their low (but not minuscule) probabilities of about two to five percent. Any particular student is fairly unlikely to successfully demonstrate ESP on any of the activities; however, for a class of at least 15, it is likely that a small number of students will successfully demonstrate ESP at least once. Students who successfully demonstrated signs of extra-sensory perception are then asked to repeat their performances; all or most will generally fail to do so. This phenomenon can ignite discussions on the definition of a p-value (from a non-computational perspective), on why it is plausible that a substantial subset of peer-reviewed scientific literature is false, and on the role replication studies can and should play in the scientific process.

In addition to the Point/Counterpoint essays and the assigned presentations, sporadic homework assignments are given throughout the course. Students are occasionally assigned chapters from *Damned Lies and Statistics* as required reading; other times, online articles or videos are assigned on topics such as confirmation bias, statistical misinformation in marketing, and scientific publication and the replication process. In all cases, these readings inform the following class session. An assignment could consist, for instance, of students finding a marketing statistic in the real world, considering the basis for its claim, and bringing it to the class for a discussion about its plausibility.

The students' final assignment is to write a two to three page reflective essay addressing what they learned when writing the Point/Counterpoint essays, whether their perception of statistics had changed, and the role of statistical literacy in the goal from the BW mission of creating "contributing, compassionate citizens of an increasingly global society." Students are also required to give a presentation about their Point/Counterpoint essays in which they read one representative paragraph from each essay and optionally comment on their actual opinions on the subject.

## 33.4 Reflections

As a teacher who is accustomed to typical math subjects such as calculus or statistics, the FYE course was a unique experience. This is the only class I have ever taught that felt genuinely relevant to a culture larger than the usual STEM domain. When I submitted the course proposal, I could not have predicted the extent to which the concepts of fake news and untruthfulness would become a cultural touchstone. For instance, one early discussion in the *Damned Lies and Statistics* book centered around the inherent difficulty in counting people in crowds; this topic became a multi-day news story after the presidential inauguration in January following my FYE class. Before the semester started, I was modestly concerned about the feasibility of having students find a unique article each week for Fact Check Wednesday about an incorrect numerical claim made by a public figure. By the time the class began, it was obvious that my fear was unfounded.

In hindsight, my decision to hitch the seminar to the U.S. presidential election was the best thing I could have done for the course. Topics that did not relate to the election went fairly well and generated a sufficient amount of student engagement, but political topics seemed to resonate more and spark more thoughtful conversation. (I will acknowledge that this comment is based only on my anecdotal observations.) I am convinced that the backdrop of an important election for a course like this is as crucial as the wind to sails, particularly in a course for first-year students who have most likely not yet meaningfully engaged in science.

Aside from the link to the election, I believe the Point/Counterpoint essays were the single best component of the class; this idea is supported by evaluations from the end of the semester. I was deeply impressed by the students' creativity in selecting topics (Appendix, p. 332) and felt that every student meaningfully engaged with the process of trying to argue two sides of the same issue. The peer review component of the essays was very helpful, and the summary presentations were illuminating. I was surprised during the final presentations when most students said they felt less sure about their stances on their chosen topics than before they wrote the essays.

Other successful elements of the course included a first-day anonymous survey of the class to ask about students' general political leanings and preferences in the upcoming election. The students in my class were not all of the same political persuasion, and it was useful to make that known on the second day of class in order to set a tone of understanding and patience for those with differing views. The Bad Graph Monday and Fact Check Wednesday assignments served their purposes well and gave many opportunities to discuss numerical falsehoods. I was anxious before the semester started about the breadth of topics that could be explored without meaningful statistical tools, but this never presented a problem during the semester. I was ultimately pleased with the amount of math-adjacent content I was able to find, and I was glad to be able to shoehorn it into an FYE curriculum that is traditionally devoid of mathematics. I also prioritized finding current news topics with meaningful quantitative elements for sources of discussion, and I believe this improved the quality and impact of the corresponding lessons.

Just like any other course, there are several things that I would do differently or improve in a second attempt. First: as a math teacher, my ability to assign and grade essays was, at best, underdeveloped. I took an inordinately long time to return essays with feedback, most likely because my comments had too much detail (much of which may have been ignored by the students anyway). I would have been well-served to find a colleague on campus who frequently grades essays and ask for novice-level tips. I also spent a significant amount of time developing a rubric for the Point/Counterpoint essays from existing ones I had found, but I suspect that I could have saved a great deal of effort and borrowed something from a colleague in the humanities. In addition, I felt as though I missed an opportunity in the class to incorporate debates between students. The Point/Counterpoint essays invited this type of thinking, but debates could have been a useful venue to have students practice their public speaking in a low-stakes environment.

Overall, I was quite pleased with the seminar and the extent to which it met the stated goals. Every activity in the course demanded at least a basic level of critical thinking. Students demonstrated intellectual curiosity in their selection of topic and their research for their Point/Counterpoint essays, and the essays demonstrated effective academic communication. Students were also given an explicit chance to recognize the Baldwin Wallace mission in their final reflection, and most did so. But perhaps more importantly, my goal of helping students to think more carefully before blindly trusting information presented numerically was met, as evidenced by a reported shift in attitude to the prompt, "I believe that for most political and social issues, numerical data presents an unambiguous story" (Appendix, p. 333).

Finally, I would like to acknowledge a weakness in the aforementioned study. Due in part to a miscommunication between myself and the BW Institutional Review Board, this study had a suboptimal design. The ideal instrument would have involved a survey that asked whether students agreed with various prompts, and such a survey would be administered once before the semester started and again after the semester so that responses could be compared after the experience of the class. Instead, I only administered a single study at the end of the semester that asked students to consider the seminar and reflect upon whether their opinion on each prompt had changed. It is well-known that this kind of self-reported result is a poor substitute for a direct measurement and that self-evaluation even of opinions may not be reliable. This error in the study design could make a suitable topic for the *Lies, Damned Lies, and Statistics* seminar in a future iteration.

## 33.5 Bibliography

[1] Joel Best, *Damned Lies and Statistics: Untangling Numbers from the Media, Politicians, and Activists*, University of California Press, Berkeley, California, 2012.

[2] Katy Reilly, "Read Hillary Clinton's 'Basket of Deplorables' Remarks About Donald Trump Supporters," *Time,* `time.com/4486502/hillary-clinton-basket-of-deplorables-transcript/` (accessed January 21, 2019).

[3] Lucy Madison, "Fact-checking Romney's '47 percent' comment," cbsnews.com, `cbsnews.com/news/fact-checking-romneys-47-percent-comment/` (accessed January 21, 2019).

## Appendix

### Point/Counterpoint Essay Evaluation Rubric

*This rubric was given to students prior to their construction of the Point and Counterpoint essays, which were argumentative essays written about the same topic but with opposing viewpoints. The rubric was used both as a guideline for students and for evaluative purposes.*

| | Exemplary (4) | Sufficient (3) | Developing (2) | Insufficient (1) |
|---|---|---|---|---|
| **Thesis / controlling idea (x2)** | Clearly defined; highly engaging and immediately captures readers attention | Clearly defined; somewhat engaging, but predictable and familiar | Controlling idea not clearly defined but can be successfully inferred; not engaging | Not defined; no apparent effort to engage |
| **Organization / Structure** | Consistently logical structure in paragraphs and essay as a whole | Logical structure in paragraphs and essay as a whole with some minor inconsistencies | Inadequate structure in paragraphs and essay as a whole | No apparent attempt to provide structure in paragraphs and/or essay as a whole |

| | | | | |
|---|---|---|---|---|
| **Evidence of deep processing / reflection (x2)** | Strong evidence of thought regarding central idea and its significance; strongly imaginative or creative in approach | Some evidence of thought regarding central idea and its significance; some creativity or imagination used in approach | Little evidence of thought regarding central idea and its significance; little evidence of creativity or imagination in approach | No evidence of thought regarding central idea and its significance; no evidence of creativity or imagination in approach |
| **Logic and argumentation (x2)** | Ideas have a distinct logical flow; argument is well-supported and is convincing; potential counterarguments are anticipated and defused | Ideas mostly flow logically; argument is mostly supported and fairly convincing; potential counterarguments may be acknowledged without being answered | Occasional logical failures or inconsistencies; argument may be unclear, be unconvincing, or lack flow; counterarguments not addressed | Ideas possess no flow; central argument is not clear or nonexistent; grasp on topic is overly simplistic and shows no depth of understanding |
| **Support from sources (x2)** | At least five different sourced, quantitative statistics given to support argument; each statistic contributes a unique and meaningful facet of the argument | At least five different sourced, quantitative statistics given to support argument; a few of the statistics overlap or do not provide meaningful support to argument | Fewer than five sourced, quantitative statistics, or many statistics are redundant, or many statistics do not meaningfully support argument | Statistics not present, not sourced, or not quantitative in nature, or statistics provide no meaningful support in argument |
| **Quality of sources** | At least three distinct sources cited; all sources are clearly reputable and trustworthy | At least three distinct sources cited; one source has questionable validity | Fewer than three distinct sources, or multiple sources have questionable validity, or one source is obviously biased or untrustworthy | No sources cited, or multiple sources are clearly biased or untrustworthy |
| **Word choice** | Word choice is clearly creative and enhances the argument | Word choice is somewhat creative and somewhat enhances the argument | Creativity in word choice is evident but limited; word choice does not detract from argument | Word choice lacks creativity or detracts from argument |
| **Mechanics / correctness** | No or very few errors in grammar, punctuation, or word usage | Few errors in grammar, punctuation, or word usage; errors do not interfere with understanding | Frequent errors in grammar, punctuation, or word usage; errors mildly interfere with understanding | No evidence of attention to grammar, punctuation, or word usage; errors greatly interfere with understanding |

Two additional points will be awarded based on how plausible it is that you actually believe your argument. (Quite plausible = 2, somewhat plausible = 1, implausible = 0.)

Score: ________________ / 50

## Point/Counterpoint Essay Topics

*The following is a list of pairs of topics chosen by students for the Point/Counterpoint essays.*

- a particular treatment for bipolar disorder is/isn't effective
- nuclear energy is good/bad
- late-term abortions are/aren't ethical
- enhanced interrogation is/isn't ethical
- prostitution should/shouldn't be legal

- print textbooks are better/worse than e-texts
- labor unions are good/bad
- Black Lives Matter is good/bad
- affirmative action is good/bad
- golf should/shouldn't be considered a sport
- college tuition should/shouldn't be free
- animal testing is good/bad
- the drinking age should/shouldn't be lowered
- recreational marijuana should/shouldn't be legal

## Attitudinal Assessment

*A survey was given at the end of the semester to determine students' attitudes toward various topics related to the theme of the course. Students were not asked whether they agreed or did not agree with the prompt; instead, they were asked whether their opinion on the prompt had changed during the course of the semester. Numbers in the tables below report the counts of responses. This study was completed with the approval of the Baldwin Wallace Institutional Review Board* (*BW IRB Archival code FA16-9605*).

- A: Opinion did not change
- B: Opinion changed only a little
- C: Opinion changed somewhat
- D: Opinion changed significantly

| | A | B | C | D |
|---|---|---|---|---|
| Q1 | 1 | 4 | 9 | 2 |
| Q2 | 3 | 2 | 4 | 7 |
| Q3 | 4 | 8 | 3 | 1 |
| Q4 | 3 | 8 | 3 | 1 |

| | A | B | C | D |
|---|---|---|---|---|
| Q5 | 8 | 2 | 2 | 4 |
| Q6 | 2 | 7 | 5 | 2 |
| Q7 | 2 | 8 | 3 | 3 |
| Q8 | 2 | 6 | 3 | 5 |

- Q1: I believe that when numerical data is presented honestly and with good intentions, it generally points to a single correct conclusion.
- Q2: I believe that for most political and social issues, numerical data presents an unambiguous story.
- Q3: I feel that my own political beliefs are generally well-supprted by numerical data.
- Q4: I feel that the political beliefs of those who disagree with me are generally well-supported by data.
- Q5: I believe that people are more likely to be convinced by data when it supports an opinion they already have.
- Q6: I believe that politicians and public figures enhance their credibility by using data to support their arguments.
- Q7: I think that misuse and abuse of numerical data in politics is committed by one party or ideology more than others.
- Q8: I could be convinced to change my mind about certain political opinions I hold by seeing relevant data.

# 34

# Why Do We Choose to Do What We Do? Using Statistics to Investigate Our Actions

Tricia Muldoon Brown

**Abstract** This article describes the activities of a one-semester first-year experience (FYE) course linked to an introductory quantitative reasoning mathematics course. The required text is *Freakonomics* by Levitt and Dubner, which inspires the theme of the course as well as the main student paper in which the students research a topic that explores the economic, social, and moral incentives and disincentives for why people choose to act in a certain way. Students must critically evaluate sources and provide relevant statistics to support their conclusions. Here I discuss these class activities and assessments, including teacher observations, evaluation strategies, and rubrics. I relate successes and pitfalls and suggest modifications for future seminars.

## 34.1 Background and Context

Georgia Southern University is a large, comprehensive, public university composed of three campuses: Armstrong, Liberty, and Statesboro. The course under discussion is taught at the Armstrong campus located in Savannah, Georgia which has a diverse student body encompassing many non-traditional students, first-time college students, and military-affiliated transient students. The first-year experience (FYE) program is campus-wide and required for all freshman. Multiple instructors teach sections of the course with each having the freedom to choose their own theme and course assignments as long as the two overarching goals of campus engagement and information literacy are satisfied. In fact, the themes are advertised in the course catalog, so students have the ability to self-select into courses meeting their interests. The FYE classes are smaller, capped at 24 students, and the themed course can be run as a stand-alone course or linked with a core course providing a learning community experience. FYE courses have historically met for 50 minutes a week over the entire 15-week semester, though the university has recently moved to a two-credit course which meets for 50 minutes twice weekly throughout the semester.

## 34.2 Mathematical Theme

My FYE course is taught within a learning community model by being linked with the introductory mathematics course, Quantitative Skills and Reasoning, meaning students must be enrolled in both classes. The benefits I have observed from such a pairing are increased flexibility for content to be utilized in various ways between the two courses and greater community among students, who are working together for more class time on the course goals. Further, the FYE course teaches skills that support learning in the math class, while the mathematics content provides a real-world framework for the research done in the FYE course. The mathematical content encompasses about 20% of the FYE coursework, and the mathematical themes in the FYE course mirror those of the quantitative reasoning course. Broadly, the course objectives of the math class are for students to learn practical mathematics and critical thinking skills applicable to authentic life situations and to prepare them for their next course in introductory statistics. Therefore, in a unification of general FYE and quantitative reasoning goals, the FYE activities are designed to teach data literacy in conjunction with information literacy. Specifically, the primary mathematical goals associated with this FYE course are to analyze and interpret data and the descriptive statistics describing the data before critically assessing this information for bias, misconceptions, and assumptions. The theme of critically assessing our underlying assumptions runs through the entire semester in the first-year course.

I chose this theme for two reasons, each of which supports one of the general course objectives of data and information literacy. First, our students are living in a world with ever increasing access to data, more so than any other generation before, and I believe data literacy is crucial. If they do not know how to properly process data and the conclusions that others draw from it, the students leave themselves open to weak arguments or manipulation. I would like the students to be able to understand and interpret graphics and statistical studies in order to discern reputable information from disreputable. Thus they can make well-informed decisions in their professional and personal lives. Second, I think it is the responsibility of a college education to challenge student viewpoints. The students should use information literacy skills to consider both sides of an issue through scholarly research, providing credible arguments to support their conclusions. The next section presents specific assignments and class activities in support of these course goals.

## 34.3 Course Structure

My FYE course activities and assignments fall under four roughly-equal categories: traditional freshman experience topics such as time management, study skills, advisement, and campus resources; student presentations and reflections; book club readings and discussions; and preparation and writing of a research paper. As the former two categories proceed in a conventional fashion, this article will mainly focus on tasks associated with the book club and the research paper. First, I describe a typical daily class meeting and then offer specific activities and assignments related to the FYE book club and research paper.

### 34.3.1 Daily Structure

The FYE course meets for one hour a week immediately following the quantitative reasoning class. Because the FYE class is part of a learning community, it is run in a manner that is much more informal than traditional core courses. I begin each class with 15 minutes of student-led discussion. In the first few weeks, I usually have to get the discussion going. I will ask the students to share something good, bad, or funny that happened to them over the last week. I often find that other students share the same experience, so they can give guidance for the difficult incidents, reinforce in the positive experiences, and laugh along as a student relates their comical encounter. These shared experiences allow students to get to know and help each other. Fifteen minutes may seem like a lot of class time to devote to "non-academic" material, but students regularly tell me in evaluations that these discussions were very helpful to them. After the student dialogue, I move to the instructor-led content. For most classes the students have an assignment to be done before class such as a book club reading or an intermediate assignment for the research paper. The homework tasks are either assessed with a short quiz or submitted, usually online, and the remainder of the class is a combination of lecture and discussion of the topic for that day.

### 34.3.2 *Freakonomics* Book Club

The required text is *Freakonomics: A Rogue Economist Explores the Hidden Side of Everything* [2] by Steven Levitt and Stephen Dubner. This book is a collaboration between an economist and a journalist who explore commonly held assumptions relating to subjects of interest such as crime, education, or parenting. The authors use well-documented statistical studies to provide evidence, often against conventional wisdom, to challenge these assumptions. As we are part of a learning community, I lead these classes as a book club that has chosen to read this book for education and pleasure. *Freakonomics* has six chapters as well as an introduction and epilogue. The book club meets four times, each time covering two sections of the book. Throughout the 15-week semester I typically have book club classes in the fourth, sixth, eighth, and tenth weeks, allowing us to finish the reading before the final paper is due near the end of the semester. For each book club meeting, the students are expected to read the chapters before class and come prepared for a short quiz and a class discussion. The quiz is given at the start of class in order to incentivize the students to read and be prepared to contribute to the discussion. These quizzes are easy; the aim is not to punish students, just encourage them to come to class prepared. The quizzes contain seven multiple choice questions on themes from the chapters. Usually one of the questions is dropped when grading. That way if a student reads the section but fails to remember every detail or disagrees with my interpretation, they can still achieve a good grade on the quiz. In my experience, only the students who refuse to read end up with a poor quiz grade. Two examples of such quizzes are provided in the Appendix, p. 340.

After spending about five minutes to administer the quiz, we have a class discussion. The desks are moved around so the students see each other, and the quiz questions are used as a outline when leading the discussion. The goals of the discussion are to promote discourse, clarify confusions, and develop critical thinking skills. While the quiz questions are direct, the motivation and reasons behind the answers to the questions are often not as obvious. As a group, the students are expected to discuss the supporting evidence provided in *Freakonomics* and evaluate the authors' conclusions. These formative discussions satisfy the mathematical goals to analyze statistics and their sources as well as the critical thinking and data literacy objectives, but also address topics that may be considered contentious or controversial. Students learn to address issues that may make them uncomfortable, such as abortion and racism, and see that it is possible to have an academic dialogue even among people with differing emotional feelings about an issue.

### 34.3.3 Research Paper

The research paper is in essay format and requires the students to choose a topic, select sources, analyze appropriate statistics, and write a paper. For each of these four requirements, one week of the semester is allocated to help the students attain the assignment's objectives.

The first of these weeks is devoted to choosing a topic. I ask the students to come to class with a topic satisfying the following instructions:

> Write a *Freakonomics* style essay to explore economic, social, and moral incentives for some action a person could choose to take. The paper should explore both positive and negative incentives for and against the action. It is important that the topic focuses on choice and is not about biological imperatives or truly unconscious decisions and should fit a theme of "Why do people ________?" Conventional wisdom and anecdotal evidence are not sufficient to document an incentive, so claims made in your paper must be supported by scholarly sources as well as reliable statistics or data.

Some commonly chosen topics are "Why do people eat fast food?", "Why do people get married?" and "Why do people go to college?" These topics can result in good papers because there are motivations both for and against these actions, the motivations tend to vary across the categories of economic, moral, and social, and source material is easily accessible. In class everyone shares their topic, and we discuss each one individually to see if the following questions are answered affirmatively.

*Q1: Does the topic satisfy the requirements of the instructions, i.e., is it about motivation and the choices that people consciously make?* An example of a topic that fails to satisfy this criterion in this question is "Why do people dream?" because dreaming is an unconscious choice. Topics that fail this requirement can be more subtle. One semester I had a student choose the topic "Why do people become terrorists?" While it is possible to write a paper on this topic using only conscious incentives, when researching the topic he found much more information about the psychological impact of upbringing—abuse, religion, freedom, even geography that were outside the realm of choice. If the student cannot list at least two or three potential conscious incentives at the outset, I usually advise them to consider another topic.

*Q2: Is the topic appropriate is scope? Alternatively, is it too broad or two narrow?* This question is asked for essay assignments in any class. Here, I try to help students broaden or narrow their topic as necessary. If a student suggests the topic "Why do people work?" I point out that the reasons people work are often very dependent on the type of work itself, and try to get them to focus the topic to something like "Why do people become teachers?" On the more narrow end, I had a student select the topic "Why do people keep golden retrievers as pets?" Again, she might be able to find enough information on golden retrievers, but changing to the broader topic of keeping dogs or pets in general could provide more opportunities to find statistics and source material. Topics that are too narrow are also associated with the third question.

*Q3: Can you find sources? Where or what kinds?* In the digital age, practiced researchers can find good scholarly information to support many topics, but first-year students are not practiced. For example, a student chose to ask "Why do people go on vacation?" It seemed like a good topic, because there are several plausible reasons. However, when she began her research, most of the sources she found were colloquial or anecdotal. That is not to say that the scientific sources did not exist, because some of the popular articles were probably reporting on findings from studies, but she was unable to locate the primary sources. With only magazine articles and online polls with little credibility, her paper suffered. I try to remind students that the topic must be interesting enough and main-stream enough that scholarly sources for this topic exist.

Choosing an appropriate topic is essential to a good paper, so this class provides an important starting point for their research. In the next class, the students take a trip to the library. The librarian leads a class to introduce the students to the on-campus and online options for finding appropriate and diverse sources. Because they have already chosen a topic, they are able to research sources during this class. As a participation grade, I ask them to email me at least one scholarly source that they plan to use in their research paper. The paper requires at least five different sources with some popular sources allowed, but with an emphasis on diversity among publications and types of publication.

To further prepare the students to write their research paper, the subject of the third class is misleading statistics. In their quantitative reasoning class, the students have been studying bias in statistical studies, so these themes are continued in the FYE class. I hand out a worksheet written by Lori Alden [1] containing statistics reported in the media, and in small groups the students evaluate each example with regard to source, relevancy, and misleading conclusions. We go over the answers together in the last few minutes of class. For homework they are to submit at least one statistic that they will be using in their research paper. Students are usually successful with this assignment; some examples they have offered are: "Americans tend to spend $552 more per year when dieting and eating healthy foods," or "Study finds that up to 95% of college students procrastinate," as well as "In 2004, 74% of women who had an abortion said that a baby would interfere with their life."

Finally, in the fourth class, the students must bring a rough draft for peer review. This is usually the twelfth week of the semester. The students exchange papers and make comments and suggestions according to the guidelines provided. The guidelines are found in the Appendix (p. 342) and are adapted from feedback forms by the Mānoa Writing Center at the University of Hawai'i [4] . These guidelines ask the students to evaluate the structure of the paper as well as the validity and diversity of the sources and the strength of the supporting arguments. I also collect the papers to provide instructor feedback. This a time consuming part of the semester, because the papers need to be read quickly in order to allow enough time for the students to make corrections. However, this feedback is often necessary in order to improve the quality of the final paper and prevent some students from turning in a failing assignment.

The final paper is due in the fourteenth week of the semester. It should have three to five pages of content with an additional works cited page. The fourteenth week is also one of the two presentation weeks. Their research paper presentations are very basic. The instructions are to provide a two-minute presentation with some kind of visual aid in order to disseminate their main results to the rest of the class. Most students prepare a slide show of three or four slides using PowerPoint or Prezi containing their title, the motivations for an action, the motivations against an action, and the references, although occasionally a student will produce a poster or a movie. The presentation is required for two reasons. First, the students are more likely to actually complete the research paper when they realize the other students are expecting an output, and second, and more importantly, it is good practice to prepare the students for future classes and careers where they will be expected to present and explain their results to other researchers, co-workers, or employers.

A rubric, found in the Appendix (p. 343), is used to assess their research paper and presentation. It is graded on the scale of proficient, developing, and novice in information and data literacy skills. As long as a meaningful attempt is made on the research paper and the presentation, students typically score in the developing range on the criteria for determining the extent of information needed, critically evaluating source material, and integrating source material. Many achieve proficiency for finding varied sources and responsibly documenting these sources in the paper. Most do a good job utilizing accurate statistics and preparing an effective presentation. After the presentations, the students have completed their program requirement involving information and data literacy.

## 34.4 Reflections

In this final section, I discuss a few of the changes that have been made over the five years I have been teaching my FYE course as well as changes planned for the future

### 34.4.1 Implemented Changes

First, beginning with changes to tasks associated with the *Freakonomics* book club, I learned very quickly that I needed to have a book club quiz and not a participation grade. The first sections did not take the reading seriously and the class discussions suffered as a result. As an instructor, it is frustrating when half the class is trying to make comments based solely on the titles of the chapters or the first paragraphs. The quizzes were just enough of an enticement to push the majority of the students to read.

Second, I realized that if I want to treat the classes as a book club, the choice of the book makes a big difference. The discussions lag when the students become disinterested and bored, as I learned when I assigned an unpopular common-read book in a prior section of FYE. Thus the book needs to be fairly easy to read and on a subject that students will find interesting. I found *Freakonomics* does achieve these goals. Very little technical knowledge or jargon is used throughout the book, and the constant surprising or curious conclusions keep the students' attention.

Third, it is a good strategy when leading book club discussions to model what you expect from the research paper. When asking why the teachers cheat, for example, or why the sumo wrestlers throw matches, have the students identify the incentives and disincentives in the categories of economic, moral, and social.

I also made several changes to assignments associated with the research paper. In particular, the students need to have several intermediate deadlines for the research paper. In my first classes, the only intermediate assignment they had to turn in was the topic. Now, after reading multiple papers on why students procrastinate, I realize many students are partial to waiting until just before the deadline to complete a paper. However, first-year students in particular are the least able to actually complete a quality assignment in a short time frame. Forcing them to chose a topic, find sources in the library, and find statistics early helps them meet most of the essential requirements for the paper. The peer-review exercise forces them to have some product written down (even if it is terrible) before the deadline. At least in this way almost all the students will turn in a paper.

Finally, the presentations to disseminate the results of their research papers are also new additions. While not essential, watching the students present their research and receive recognition from their peers is enjoyable, and the fact that the students are learning interesting facts researched by their peers is also a good consequence.

I conclude with a few more specific suggestions for implementing the tasks associated with the research paper. For one, when choosing topics, do not let students choose the same topic. On top of concerns of academic dishonesty, repeated topics can cause the presentations to become tedious, not to mention the grading. Next, student peer review is usually not enough. As a mathematician by training, it can be daunting read through all the papers not once but twice, still the students are usually not up to the task of critically evaluating each other's papers.

### 34.4.2 Future Changes

For the future, there are two modifications I am planning to implement. As previously mentioned, this course is transitioning from one credit-hour to two. Therefore as an instructor I will have a lot more class time to work with the students. For future papers, I want the students to be involved in the collection and very basic analysis of data, in addition to the usual research to find and use appropriate scholarly sources and statistics. With regard to the paper on incentives, this would involve designing a social media poll asking others why they choose or do not choose to take the action in question. The students can collect a substantial amount of data quickly that can then be incorporated into their paper to support their claims. I would devote a class to designing polls, discussing how to write questions without bias, the merits of multiple choice versus open response, and how to recruit participants. Another class can be used to analyze the results, create a distribution, consider features of the data, and identify possible biases in the data collection.

Further, I do have the freedom to change the course description in the catalog. For some sections I plan to narrow and change the focus, still with the *Freakonomics* idea that conventionally held wisdom and assumption are not always correct, but now in the realm of sport. I will assign the text *Moneyball: The Art of Winning an Unfair Game* [3] by

Michael Lewis. This book relates the story of Billy Beane, manager for the Oakland As, who, contrary to popular belief at the time, used analytics instead of tradition to build a successful baseball team. Instead of assumptions about why people choose to act the way they do, the class and the final paper will target assumptions in sports. Believing his teams are "cursed," a student could investigate if Atlanta-based sport teams perform worse in the playoffs than their regular season record would predict by gathering data on upsets in the playoffs over time. Or noticing that her team wins more often at night, a student could collect data to determine if the University of Georgia football team has a higher win percentage in night games versus day games. I have wanted to explore these kinds of themes in the past, but felt it was not fair to restrict the topics to sports, of which some students have little interest or experience, without ensuring the students self-select into the course.

There are certainly many other ways to make the theme of challenging assumptions more specific to your student population. For example, if you have a group of future health care professionals the final paper could involve investigating assumptions in medicine and health care such as evaluating the veracity of "old wives' tales." For a more general population, another paper could ask the students to choose a topic in the news over the last year and gather data from as many sources as possible to investigate how the coverage changes from one outlet to another. Whichever of these themes is implemented, the students will benefit by learning to challenge commonly held assumptions, question statistics, and think critically before drawing conclusions.

## 34.5 Bibliography

[1] Lori Alden, *Statistics can be misleading*, Econoclass.com, (2005–2008). Retrieved August 31, 2018, from `econoclass.com/misleadingstats.html`.

[2] Steven D. Levitt and Stephen J. Dubner, *Freakonomics: A Rogue Economist Explores the Hidden Side of Everything*, HarperCollins Publishers, New York, 2005.

[3] Michael Lewis, *Moneyball: The Art of Winning an Unfair Game*, W. W. Norton & Company, Inc, New York, 2003.

[4] Mānoa Writing Program, *Peer Review*, University of Hawai'i at Mānoa, (1997–2014). Retrieved August 31, 2018, from `manoa.hawaii.edu/mwp/faculty/teaching-tips/syllabus-design/writing-activities/peer-review#sample3`.

# Appendix

## *Freakonomics* Quizzes

*The following are the first two book club quizzes of the four assigned throughout the semester.*

### *Freakonomics* Quiz 1 - Introduction and Chapter 1

1. What is the explanation given for the drop in crime in the 1990s?
   (a) Gun control
   (b) Stricter prison sentences
   (c) Economic upturn
   (d) Roe versus Wade decision

2. Does election spending have a significant impact on the outcome of the election?
   (a) Yes
   (b) No

3. What happened to the number of late pick-ups at a daycare when a monetary penalty was introduced?
   (a) The number of late pick-ups decreased.
   (b) The number of late pick-ups stayed the same.
   (c) The number of late pick-ups increased.
4. What happened to the number of blood donations when a monetary reward was introduced?
   (a) The number of donations decreased.
   (b) The number of donations stayed the same.
   (c) The number of donations increased.
5. How did the Chicago Public School System detect cheating teachers?
   (a) A computer algorithm
   (b) Student complaints
   (c) An undercover teaching aide
   (d) Teacher confessions
6. How were the cheating sumo wrestlers caught?
   (a) Opponent complaints
   (b) An undercover sumo wrestler
   (c) Confession by retired wrestlers
   (d) They were not identified.
7. Which type of business was more likely to steal bagels?
   (a) Large businesses
   (b) Small businesses

### *Freakonomics* Quiz 2 - Chapters 2 and 3

1. According to *Freakonomics*, what is the primary reason for the decline in the KKK's numbers in the late 1940s and 1950s?
   (a) World War II
   (b) Jim Crow laws
   (c) Publicizing inside information
   (d) Motorcycles
2. How was J.T. different from most of the members of his gang?
   (a) He was college educated.
   (b) He had 8 children.
   (c) He was born in Mexico.
3. According to *Freakonomics*, what contributed to the drop in term life insurance costs in the 1990s?
   (a) Health care improvements
   (b) Internet
   (c) Grunge rock
   (d) Cryogenics
4. Black Americans were hurt more by ______________ than by any other single cause since Jim Crow.
   (a) The Ku Klux Klan
   (b) Crack cocaine

5. The TV show *Weakest Link* showed discrimination against which group(s)?
   (a) Elderly
   (b) Women
   (c) African Americans
   (d) Hispanics
6. What term used in real estate ads correlates to higher sale prices?
   (a) Spacious
   (b) Fantastic
   (c) Gourmet
   (d) Great neighborhood
7. How much did the typical foot soldier for the Black Gangster Disciples Nation make?
   (a) $214.05/hr
   (b) $25.43/hr
   (c) $15.75/hr
   (d) $3.30/hr

## Guideline for Peer Review

*This guideline is adopted from a sample peer review feedback form from the Mānoa Writing Program at the University of Hawai'i at Mānoa [4] and is used to evaluate the first draft of the students' culminating research papers.*

### Peer Review Feedback Form

Author________________ Reviewer________________

The goals of peer review are to help improve your classmate's paper by pointing out strengths and weaknesses that may not be apparent to the author, and to help improve editing skills.

**Instructions**

Read the paper(s) assigned to you twice, once to get an overview of the paper, and a second time to provide constructive criticism for the author to use when revising his/her paper. Answer the questions below.

**Organization**

1. Were the basic sections (Introduction, Body, Conclusion, and Works Cited) present? If not, what is missing?
2. Was the material ordered in a way that was logical, clear, easy to follow? Explain.

**Citations**

3. Did the writer cite sources appropriately? Note any incorrect formatting.
4. Were all the citations in the text listed in the Works Cited section? Note any discrepancies.
5. Is the Works Cited section made of up a variety of high-quality sources? Explain.
6. Did you notice any text where a citation is needed? For example, were there passages of text that do not seem to be in the author's own words or facts that are not common knowledge without justification?

**Grammar and Style**

7. Were there any grammatical or spelling problems?
8. Was the writer's writing style clear? Were the paragraphs and sentences cohesive?

**Content**

9. Could you identify the topic? Write it here.
10. Identify the main points made in this paper. Write them here.
11. Are these main points cohesive and relevant to the topic?
12. Is there reference material supporting each main point? Is the reference material adequate? Which points need more support?
13. Did the writer make some contribution of thought to the paper, or merely summarize data or publications? Explain.

## Rubric for the Research Paper and Presentation

*This is the document that was provided to students as a guideline and grading rubric for the final research paper and presentation. Note, the first five learning outcomes were defined at the university level.*

| **Student Learning Outcomes** | **Proficient - 3 points** | **Developing - 2 points** | **Novice - 1 point** |
|---|---|---|---|
| Determine the nature and extent of information needed to complete a project or solve a problem. | Directly defines the scope of the research question or thesis. Can determine key concepts. Types of information (sources) selected relate to concepts or answer research question. | Ineffectively defines the scope of the research question or thesis (parts are missing, remains too broad or too narrow, etc.). Can determine key concepts. Types of information (sources) selected partially relate to concepts or answer research question. | Has difficulty defining the scope of the research question or thesis. Has difficulty determining key concepts. Types of information (sources) selected do not relate to concepts or answer research question. |
| Select information resources including but not limited to print and electronic media. | Accesses relevant information sources. Demonstrates ability to refine search. | Accesses information from limited and similar sources. | Accesses information that lacks relevance and quality. |
| Critically evaluate information for currency, relevancy, bias, authority, and accuracy. | Thoroughly (systematically and methodically) analyzes own and others' perspectives and evaluates the relevance of contexts when presenting a position. | Questions some positions. Identifies relevant contexts when presenting a position. | Has difficulty identifying some contexts when presenting a position. Has difficulty analyzing own and/or others' perspectives. |

*Continued on the next page.*

| **Student Learning Outcomes** | **Proficient - 3 points** | **Developing - 2 points** | **Novice - 1 point** |
|---|---|---|---|
| Communicates information responsibly with respect to integrating information from sources. | Organizes, synthesizes, and communicates information from sources. Intended purpose is achieved. | Organizes and communicates information from sources. The information is not synthesized, so the intended purpose is not fully achieved. | Communicates information from sources. The information is fragmented and/or used inappropriately (misquoted, taken out of context, or incorrectly paraphrased, etc.) so the intended purpose is not achieved. |
| Communicates information responsibly with respect to documenting sources. | Students use correctly three of the following information use strategies: use of citations and references, choice of paraphrasing, summary, or quoting; using information in ways that are true to original context; distinguishing between common knowledge and ideas requiring attribution. | Students use correctly two of the following information use strategies: use of citations and references, choice of paraphrasing, summary, or quoting; using information in ways that are true to original context; distinguishing between common knowledge and ideas requiring attribution. | Students use correctly one or none of the following information use strategies: use of citations and references, choice of paraphrasing, summary, or quoting; using information in ways that are true to original context; distinguishing between common knowledge and ideas requiring attribution. |
| Communicates statistics responsibly and accurately | Uses accurate and appropriate statistics to support the claims. | Statistics are accurate but are not completely appropriate for supporting the claims. | Statistics are inaccurate, misused, or inappropriate. |
| Presentation | Presentation effectively and accurately disseminates the results of the paper. | Presentation is inaccurate or fails to use an effective mode of presentation. | The presentation is inaccurate and ineffective. |
| Spelling, grammar, and writing style | The paper is free from grammatical and spelling errors. Sentence structure is varied and appropriate. | The paper may contain a few small grammatical or spelling errors, or the quality of the writing is not completely clear or effective. | The paper has many grammatical and spellings errors, and/or the writing is unclear and hard to understand. |

# 35

# *The Signal and the Noise:* Why Numeracy Really Matters

Jennifer R. Bowen

**Abstract** This paper reflects on my experience teaching the First-Year Seminar, *The Signal and the Noise: Why Numeracy Really Matters*, with emphasis placed on the course structure and assignments. The course explores quantitative literacy and numeracy/innumeracy in the United States. Students examine quantitative experience using Nate Silver's writing, informed by big data, social and natural sciences, popular culture, and the emergence of "Massively Open Online Courses" (MOOCs) and Khan Academy as educational technologies in mathematics. In this paper, I examine some select writing assignments: personal autobiography, comparison/contrast essay, briefings, and a final research paper.

## 35.1 Background and context

The College of Wooster is a small, residential, undergraduate-only, liberal arts college in rural Ohio. It has approximately 2,000 undergraduate students, enrolling a diverse student body: 20% domestic students of color, 13% international students, and 55% female. Students hail from 47 states and 45 different countries. Wooster's First-Year Seminar in Critical Inquiry (FYS) is required of all students in their first semester. Each writing-intensive seminar enrolls approximately 15 first-year students; Wooster offers, on average, 37–39 FYS sections each fall. The course meets for the entire 14-week semester, three contact hours per week (50 minutes three times a week or 80 minutes twice a week), for a full course credit for the student. The FYS Program is led by the Dean for Curriculum and Academic Engagement's administrative offices; FYS instructors are selected from all disciplines and faculty ranks, and faculty are encouraged to select their own course topics. FYS instruction is an expectation of all tenure-track faculty before their tenure review in their sixth year of service to the College. In addition, each FYS instructor also fulfills the role of academic advisor until a student declares a major (which is not required until the spring semester of a student's sophomore year). Learning goals for the College's FYS Program include "engag[ing] in a set of issues, questions, or ideas that can be illuminated by the interdisciplinary perspectives of the liberal arts."[1] The FYS learning objectives include writing essential to critical thinking, deep reading, construction of arguments, evidentiary support, oral presentation, and critique.

## 35.2 Mathematical Theme

Is algebra really necessary? When will I use this? What is math for? My FYS course[2], *The Signal and the Noise: Why Numeracy Really Matters* explores quantitative literacy, quantitative reasoning, and numeracy and innumeracy

[1] `wooster.edu/academics/fys/`

[2] I presented an MAA Contributed Paper talk on my FYS at the 2018 Joint Mathematics Meetings.

in the United States. In our context, students examine their quantitative experience, preliminarily defined by John Allen Paulos in *Innumeracy* [7]. We use Nate Silver's writing [9], informed by big data and quantitative themes in social and natural sciences, and popular culture to apply quantitative literacy to everyday life. At the end of the course, we investigate the emergence of "Massively Open Online Courses" (MOOCs), that is, open and free online courses available for anyone to enroll, and Khan Academy [5] as educational technologies in mathematics all to inform our discussion about the College's curricular initiative to update the quantitative reasoning requirement. No instruction of mathematical content was included in this course.

## 35.3 Course Structure

In practice, Wooster's FYS classes include a minimum of five graded writing assignments that consider, synthesize, judge, and compare different approaches or points of view and draw on several sources in constructing an argument. This paper highlights a few of the meaningful assignments from my FYS. The main goals of FYS are to provide students with the opportunities to improve their writing, research and presentation skills, and critical thinking abilities. To that end, over the course of the semester, students write and revise a variety of papers, do research, and develop their presentation skills. Each writing assignment examined here focuses on different skills, including informal persuasion, formal presentation, writing style and abilities, organization, argumentation, logic, form, research, and citation. This course, heeding sage advice from the College's Writing Center and fellow senior colleagues, focuses on scaffolding writing assignments throughout the semester (and within larger assignments). In this paper, I examine some select writing assignments: personal autobiography, comparison/contrast essay, briefings, and a final research paper.

### 35.3.1 Learning Objectives

I recommend spending time contextualizing the learning objectives that you have for your FYS in advance of creating assignments. With this in mind, I focus on the following learning objectives in my FYS:

- *knowledge*: recalling basic information about the subject presented,
- *comprehension*: understanding the meaning of the information provided,
- *application*: using information in a different context,
- *analysis*: examining information, close reading, and text interpretation,
- *synthesis*: presenting the information in a new way or suggesting a different solution,
- *evaluation*: presenting opinions on the content, validity, or quality of a text or texts,
- *citation*,
- *revision*, and
- *research*.

With these learning objectives in mind, when I create writing assignments or longer term projects in FYS, my prompts to students are more clear, goals in these activities are more observable and measurable. In addition, rarely, do I ask for "too much" in an assignment.

### 35.3.2 Personal Quantitative Autobiography

The first day of class, students participate in an in-class discussion about their quantitative or mathematical backgrounds. Just prior to the second class, students view Jim Fowler's (Ohio State University) TEDxColumbus talk "The Humanity of Calculus" [4]. As a group, the class discusses Fowler's argument for the "human story of calculus" and his open classroom initiatives. In addition, students complete an Attitudes Toward Mathematics Inventory [12]. After having read about our operative definition of quantitative literacy [11] for the course, students compose a two to three page (500-750 words) essay outlining the definition of quantitative literacy from their reading, their personal quantitative autobiography, and how these two topics properly fit together. In addition, students explain the definition of quantitative literacy using points and examples from their own personal narrative including the factors that have contributed to their success or non-success in quantitative disciplines. I find it important to offer a pertinent and short assignment early on in the semester to set class expectations, gauge writing levels, and get to know these first-year students. This assignment is a fitting way to start the term as a class—students provide key details about their learning styles and also their writing styles within this essay. Learning objectives for this assignment include *knowledge* and *comprehension*. Students are assessed on their explanation of issues, personal autobiography, evidence, and reflection.

### 35.3.3 Compare and Contrast Essay: Paulos and Texts in Conversation

After defining quantitative literacy early in the semester, we read Carnevale and Desrochers' "The Democratization of Mathematics" [2] to unravel and analyze the presented arguments to conclude about best practices and policy on the course topic. As a class, students' first major book reading assignment is John Allen Paulos's *Innumeracy: Mathematical Illiteracy and Its Consequences* [7]. This book is written by a mathematics professor arguing that the public has an inability to understand numbers and quantitative concepts. This results in destruction of democracy, poor decision making, and pseudoscience. After finishing Paulos's text, students read a piece Paulos wrote for the *New York Times* Book Review [8] and the book review published in the *American Mathematical Monthly* [3] as the basis to write a two to three page comparison and contrast paper. I do not let them know that the *New York Times* piece is authored by Paulos, but let them work that out for themselves. Not all students realize this during the assignment. The assignment asks students to address all of the following about the two "review" pieces they read:

- In what ways are the two works similar? Why do you think this is the case?
- In what ways are the two works different? Why do you think this is the case?
- How would Paulos react to each of these sources?
- As a short and concise conclusion, which of these works most matches your own views of Paulos's text?

Learning objectives for this assignment include *knowledge*, *comprehension*, *application*, *analysis*, and *citation* (also see the rubric included in the Appendix, p. 351). Students are instructed to keep their comments and analysis applicable to Paulos's text and the two book reviews. They are assessed on clear explanations of similarities and differences of the two book reviews, including evidence in their arguments. In addition, students need to predict Paulos's reaction if he were to have read both pieces and include a personal but formal conclusion incorporating the two reviews. This assignment was a perfect way to add on brief additional and outside of class reading and sources and to incorporate them into a relatively short student writing assignment.

### 35.3.4 Briefings

At the recommendation of my colleagues in political science, I have students in this FYS write a series of one to two page (250-500 words) *briefings*. A briefing, a document usually meant for a political official, outlines the key points of an issue. Policy makers do not usually have enough time to investigate all the sides of a topic. Therefore, a briefing, prepared by their researchers, is a concise, well-organized, informative and persuasive piece of writing. I use these briefing writing assignments throughout the semester, students write about on-campus events and seminars, but also about information related to our reading and discussion assignments. I outline just two of these here, relevant to the quantitative literacy FYS theme, but I assign others throughout the semester as report-outs to the rest of the class (distributed as handouts) or short "proof-of-attendance" at on-campus academic presentations.

The class also reads Nate Silver's *The Signal and the Noise: Why So Many Predictions Fail - but Some Don't* [9]. Silver is a popular blogger and political forecaster who has built predictive models for baseball and is well known for correctly predicting the 2008 U.S. election. His book is a set of case studies from a range of areas—he writes about their successes, challenges, and patterns they find. Silver believes a dynamic knowledge of probability is the key to solid prediction models. As an introduction to reading this text, students examine one of Silver's blog posts on FiveThirtyEight.com [10]. They need to find a post pertaining to their academic or personal interests, and submit a one to two page briefing that answers the following questions:

- What is the blog post title? When was it written?
- In your own words, what event or phenomenon is Silver commenting on in the post you selected?
- What evidence of quantitative literacy and quantitative reasoning is contained in the blog post? In what ways does Silver display quantitative information?
- Include a graphic from his writing in your briefing. Did you understand this graphic immediately? How does Silver explain it? Is there more information necessary to understand this graphic representation? Would the average blog reader understand Silver's meaning quickly?

- What are Silver's conclusions? Do you agree with his conclusions?

Students are also asked to do a similar introductory activity in advance of reading Salman Khan's *The One World Schoolhouse: Education Reimagined* [5]. The author of this book is the mind behind Khan Academy, an online high-quality educational resource in mathematics and other academic disciplines. Khan simultaneously writes his own story, but vision and philosophies for free global education and associated educational technologies. Students explore Khan Academy's website [6] and videos for a specific mathematical topic that is unfamiliar to them. Students need to answer the following questions in the one to two page assignment:

- How well does Khan explain this topic?
- What methods are particularly effective? Ineffective?
- Can you imagine learning course content using online videos with active learning instruction in our college's classrooms?

Learning objectives for all of their briefings assignments during the term are *knowledge*, *comprehension*, and *citation*. In all the assigned briefings throughout the course, students are assessed on the following traits of their writing (see the rubric included in the Appendix, p. 351):

- Concise: The briefing quickly informs the reader about an issue or topic.
- Reliable: The briefing contains accurate, sound, and dependable information. Any missing information or questions are pointed out to the reader.
- Readable: The briefing uses plain language, including definitions or explanations when necessary.

Using the briefing assignment structure, students are reminded to keep arguments effective but efficient. Requiring students to keep their writing clear, concise, and accurate, has big payoffs later in the semester in our culminating final course project planning and writing.

### 35.3.5 Culminating Assignment: Research Paper with Group Presentations

At the time that this FYS course was offered, the College was embarking on a multi-year long project to reevaluate the governing curricular documents for students. Given our examination of quantitative literacy and quantitative reasoning, and discussion in this course, students formulate a clear proposal, in the form of a final research paper, for Wooster's future Quantitative Literacy requirement toward graduation. I will continue to keep this assignment for future offerings of this course, but I will change it to a "curricular recommendation." This assignment is split into six sub-projects and deadlines over the latter seven weeks of the semester.

#### Pick a Topic: All Killer No Filler

As an initial assignment toward the research paper, students write a one to two page "All Killer No Filler" argument. This is a short, to-the-point persuasive essay, with no citations required, to articulate and convince the college's Dean of Curriculum and Academic Engagement of the virtues of their new proposal and plan for the college's future Quantitative Literacy requirement. The learning objective for this assignment is *application*. Students are encouraged not to use any sources here, but rather get their ideas, potential arguments, and preliminary, but anecdotal, evidence on paper. Students are assessed on their concise and reliable argument that clearly articulates the new graduation requirement, as well as the preliminary evidence they would use to persuade the Dean to approve their version of the new requirement. The experience writing the briefings assignments earlier on in the semester also proves beneficial here.

#### Annotated Bibliography

As the next step in the research paper process, students need to write an annotated bibliography. In this paper they should find and present four to six sources (Wikipedia and associated websites are forbidden) that support their arguments to the Dean for the College's future Quantitative Literacy requirement. Each source needs to be cited correctly using proper citation standards and to include a four to six sentence annotation. Learning objectives include *knowledge*, *comprehension*, and *citation*. Students are assessed on including an introductory paragraph to the annotated bibliography, using correct citation style format, and writing informative annotations for each source.

### Research Paper Proposal

Given the argument presented in the earlier "All Killer No Filler" assignment, and incorporating evidence from the four to six sources that they present in their annotated bibliographies, students now develop a two to three page formal research paper proposal answering the question: What is your proposal and plan for Wooster's future Quantitative Literacy requirement? The proposal includes:

- the plan and outline for Wooster's future Quantitative Literacy requirement,
- at least one paragraph of basic preliminary support for the viewpoint, what evidence and key points support the plan?
- at least one paragraph acknowledging alternate point of views – who might imagine that this is a "bad idea"?

In this assignment, students are not required to include quotations or paraphrases from sources, rather, they can reference the author or speaker by last name in their writing.

Learning objectives for this assignment include *knowledge*, *comprehension*, and *application*. The assessment evaluates the proposal's explanation of issues, a clear outline and plan for the proposed graduation requirement; evidence with key points of support for the plan; and *reflection*, acknowledging objections or difficulties within the plan and providing appropriate replies.

### Peer Review

As the penultimate step in the research paper writing process, students complete a full draft of their research paper and complete two peer review responses. Students workshop their full draft of their research paper within a peer group of three students. Copies of their paper are made available to each member of the peer review group (and the instructor), in advance of our in-class workshop. This allows peer group members to read and critically analyze their classmates' paper, and to respond in writing. Each student is responsible for a one to two page written peer review of each paper they read and respond to. Each response to their classmates is an additional brief writing assignment for the course. In their peer review, students answer the questions:

- What is the stated thesis (or plan) for this essay?
- Does the stated thesis effectively reflect the full argument that the writer is making?
- What reasoning, evidence, or support does the writer provide for this argument?
- Is this reasoning, evidence, or support sufficient? What else might the writer include or develop?
- Are all the ideas and examples in this essay fully described and developed?
- How is the essay organized? Is the ordering of the paragraphs logical? What about the transitioning?
- Does the writer explore implications and consequences for this argument?
- Does this essay consider alternate perspectives?

Learning objectives are *knowledge*, *comprehension*, *application*, *analysis*, *synthesis*, *citation*, and *revision* and *research*. It is essential that students complete a full draft of their research papers in time for the peer review in-class. As a class, we discuss how necessary this "buy-in" well in advance of the deadline. In addition, these one to two page peer responses prove helpful in having students take the peer review seriously, thoughtfully, and civilly for their fellow classmates.

### Research Paper

In their research papers, students should combine our theoretical work with consideration of relevant secondary sources in order to develop their original argument and express it in precise and effective writing. This research paper is expected to be six to eight pages (1500–2000 words) in length. Students are encouraged to refine their arguments from the responses received from the paper proposal and peer review assignments.

Learning objectives are *knowledge*, recalling basic information about the subject presented; *comprehension*, understanding the meaning of the information provided; *application*, using information in a different context; *analysis*, examining information, close reading, and text interpretation; *synthesis*, presenting the information in a new way or suggesting a different solution; *evaluation*, presenting opinions on the content, validity, or quality of a text or texts; and *citation*. Students are assessed on explanation of the issues, evidence, influence of context and assumptions, the position, and conclusions – all these criteria were adapted from the Association of American Colleges and Universities (AAC&U) Critical Thinking VALUE rubric [1]. This VALUE (Valid Assessment of Learning in Undergraduate Education) rubric initiative started in 2007–2009. It involved over 100 campuses that examined 16 different learning outcomes as a part of AAC&U's Liberal Education and America's Promise (LEAP) Initiative. "VALUE rubrics provide needed tools to assess students' own authentic work, produced across students' diverse learning pathways, fields of study and institutions, to determine whether and how well students are meeting graduation level achievement in learning outcomes that both employers and faculty consider essential" [1].

### Group Presentation to the Dean

As a culminating activity, I group students into teams of two to three to present a final persuasive 10-minute explanation of their proposed curricular changes. Throughout this research paper process, I have a good idea of the major ideas of the final research projects. Shortly after the final paper is due (with two weeks remaining in the semester), I create groups of students with similar proposals (e.g., additional course requirements, loosening the quantitative reasoning definition in the curriculum, etc.). Our class invited the Dean for Curriculum and Academic Engagement to their final exam presentation period so that the Dean could consider their results.

Learning objectives are *knowledge*, *comprehension*, *application*, *analysis*, *synthesis*, and *evaluation*. Students are assessed on the following characteristics of their presentations:

- clarity of introduction to the topic,
- evidence of good preparation and practice,
- clear, concise language,
- quality of slides and other visuals,
- professionalism and audience awareness,
- presentation flow,
- completing the group member evaluation.

## 35.4 Reflections

These four assignments presented here and their corresponding sub-assignments were the most effective scaffolding of tasks for first-year students, among others given this semester. I was able to make good judgement of students' writing abilities early on in the semester, while also breaking down a major project into small, more palatable, parts throughout the latter half of the term. From my planning perspective, preliminary planning visits to the College's Writing Center proved invaluable in creating assignments, learning objectives, and rubrics, all before the semester began. In the end, after this seminar was offered, the Quantitative Literacy requirement for the College was officially approved, to take effect the following year for future College students. The official proposal came from me and another faculty member who had been a part of a Quantitative Literacy Faculty Learning Community on campus. Merging faculty discussions with student views from this course was invaluable to our proposal.

What went well? Students viewed any one to two page writing assignment as manageable; in reality, they were writing at least one to two pages each week of the semester. From this perspective, students came to expect short, frequent writing assignments. By the time the final research project's full draft came due, fewer students found themselves up late the night before. The scaffolding of assignments helped to "save them from themselves" facing the longer paper at the end of the semester. In addition, a six to eight page research paper is a perfect assignment length for first-year students. Many of my colleagues assign 20–25 page "term papers" and as a mathematician teaching a first-year seminar, this type of assignment seemed unwieldy and cumbersome.

The most challenging task in teaching this FYS was course and assignment creation. Now that the Quantitative Literacy requirement has been changed in the curriculum at Wooster, the course's final research project would be somewhat silly in future iterations. This is not problematic; these assignments are effective and transferable in developing a tight knit course for first-year students. Another course topic could easily be infused. I have thought about new topics for discussion and implementation in the fields of data science and ethics, women in mathematics, or mathematics and identity in popular culture.

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## Appendix

### Sample Rubrics

*The following two rubrics were helpful in scoring students' essays and briefings in the course. Students receive copies of each rubric when they start each assignment.*

#### Compare and Contrast Essay: Texts in Conversation Writing Rubric

**First-Year Seminar: The Signal and the Noise, Why Numeracy Really Matters**

5 = No problems and overall quality is excellent for this trait
4 = A few minor problems, but overall quality is very good for this trait
3 = A few salient problems, but overall quality is good for this trait
2 = Enough problems so that overall quality is fair for this trait
1 = Numerous salient problems so that overall quality is weak for this trait
0 = Numerous major problems so that overall quality is very week for this trait

| | | | | | | |
|---|---|---|---|---|---|---|
| **Similarities**: This paper contains a clear explanation of similarities of the two book reviews and includes evidence. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Differences**: The paper contains a clear explanation of the two book reviews and includes evidence. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Authors' Reaction**: The paper illustrates the evidence that indicates what Paulos's reaction would be to each review. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Reflection/Conclusion**: The paper includes a personal but formal conclusion incorporating the two reviews. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Organization**: The paragraphs are logically ordered within the paper. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Clarity**: The paper presents ideas and arguments with clarity and precision. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Grammar and Mechanics**: The paper uses correct grammar, punctuation, and spelling. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Style:** The paper is written in an engaging and interesting manner that appropriately integrates elements such as vocabulary, phrasing, and voice. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Citation**: The paper includes evidence from each review, cited correctly. | 0 | 1 | 2 | 3 | 4 | 5 |

### Briefing Rubric

**First-Year Seminar: The Signal and the Noise, Why Numeracy Really Matters**

5 = No problems and overall quality is excellent for this trait
4 = A few minor problems, but overall quality is very good for this trait
3 = A few salient problems, but overall quality is good for this trait
2 = Enough problems so that overall quality is fair for this trait
1 = Numerous salient problems so that overall quality is weak for this trait
0 = Numerous major problems so that overall quality is very weak for this trait

| | | | | | | |
|---|---|---|---|---|---|---|
| **Short and Concise**: The briefing quickly informs the reader about an issue or topic. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Reliable**: The briefing contains accurate, sound, and dependable information. Any missing information or questions are pointed out to the reader. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Readable**: The briefing uses plain language, including definitions or explanations when necessary. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Organization**: The paragraphs are logically ordered within the briefing. The briefing contains a purpose, a summary of the facts, and a conclusion/recommendation or advice. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Clarity**: The briefing presents ideas and arguments with clarity and precision. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Grammar and Mechanics**: The briefing uses correct grammar, punctuation, and spelling. | 0 | 1 | 2 | 3 | 4 | 5 |
| **Style:** The paper is written in an engaging and interesting manner that appropriately integrates elements such as vocabulary, phrasing, and voice. | 0 | 1 | 2 | 3 | 4 | 5 |

# 36

# My Hometown Exploration Unit in a First-Year Seminar

Maria G. Fung

**Abstract** This article describes a series of activities from a first-year seminar on quantitative literacy. The featured activities center on the topics of population, climate change, and human trafficking. They allow explorations into the mathematical themes of measurement, percentages, measures of center and spread, and elementary mathematical modeling.

## 36.1 Background and Context

Worcester State University is a public urban institution of about 5,500 undergraduate students located in the second largest city in New England. The First-Year Seminar (FYS) program is a required component of the most recently adopted general education curriculum. This program mandates that all incoming first-year students complete a three-credit seminar of their choice, typically in their first semester of study. First-year seminars meet for 150 minutes per week in a 15-week semester. Roughly half of these seminars meet for three 50-minute sessions, the rest for two 75-minute sessions. Generally, over 45 seminars are offered each fall semester and only a handful are offered in the spring semester. Even though these seminars are listed through nearly every department on campus, they are meant to be cross-disciplinary and focus on content themes inasmuch as these themes serve as a vehicle for developing the students' skills that ensure college success. Seminars are limited to 25 students and emphasize community building, active collaboration, and discussion.

The learning goals of the FYS are three-fold: focus on the development of critical thinking, written and oral communication, and information literacy skills. These learning goals accompany adjustment-to-college-life topics such as time management, selection of courses, use of campus resources, and a library resource session.

Instructors from every department at Worcester State University are offered the opportunity to design their own seminar based on personal interest and expertise. Extensive training on the pedagogy of these courses has been provided through workshops and special sessions organized by Worcester State University's Center for Teaching and Learning.

## 36.2 Mathematical Theme

I have designed a first-year seminar called *What the Numbers Say*. The name of the seminar is inspired by the book *What the Numbers Say: A Field Guide to Mastering Our Numerical World* [5]. The focus of the seminar is on quantitative literacy—the ability to navigate numerical information; the collection, organization and introductory analysis of data; the limitations and advantages of thinking quantitatively, in various contexts and situations [2]. The mathematical topics of the course are measurement, percentages, measures of center and spread, and modeling using functions of best fit. These themes appear in the course in conjunction with readings, discussions, in-class activities, and longer outside of class projects. Roughly 60 percent of the class time is dedicated to learning mathematics.

## 36.3 Course Structure

FYSs at Worcester State University are student-centered and discussion-based. Throughout the semester, my students learn how to prepare for in-class discussions and activities by completing scaffolded daily homework assignments (which include more details than a typical college assignment). They come to class ready to work with their peers on reviewing homework, asking and answering questions, and participating fully in activities and problem solving sessions. These expectations are set on the very first day of class and I have my students create a set of classroom participation norms that I hang in the classroom.

A typical day in my first-year seminar consists of checking in, discussion on the assigned readings or doing a short reading, and then working on an activity together, culminating in a short debrief. The readings come from two texts *What the Numbers Say: A Field Guide to Mastering Our Numerical World* [5] and *The Numbers Game: The Commonsense Guide to Understanding Numbers in the News, in Politics, and in Life* [1]. My students typically read all of the latter and most chapters of the former. Students alternate reading the two texts, so that similar topics such averages, measurement, and probability are considered from two similar but distinct perspectives.

I assign a variety of writing assignments based on the daily readings, a bulleted summary, a letter to a relative explaining the main points of the text, a detailed list of possible discussion questions, a memo to the Provost, an op-ed piece, and even a creative piece like a story or a short one-act play. The goal of these writing assignments is not only to help students extract the main ideas of the text, but to allow them to practice written communication across a variety of styles and audiences. Students bring their writing assignments to class in order to take part in the discussion and follow-up activity.

For example, early in the semester my students read through the chapter on size in *The Numbers Game*. Their homework is to bring a detailed bulleted summary of the ideas in the text. We discuss the types of summaries one can write during the previous class session. After a think-pair-share debrief at the beginning of class, where they discuss their summaries in groups of two to three students and work on finding easy-to-understand ways of visualizing and thinking about millions, billions, and trillions. Some examples include figuring out how long are 1 trillion hours in years or one billion hours in years, or comparing one million steps to the distance from Worcester to Boston. A follow-up to this activity involves two different homework assignments—writing a short children's book similar to *How Much Is a Million* [6], and working through a problem that asks students to think about the buying power of a trillion dollars in terms of buying a home worth 150,000 dollars and in terms of the US military spending.

Later in the semester, my students read about the difference between causation and correlation. Their homework assignment is to get familiar with the genre of op-ed by reading several such pieces from the *New York Times* first, then to write one about their reactions to the chapters in both required books that deal with this important topic. An alternative to this assignment is to write a letter to the editor of their town's newspaper expressing their opinion on the lack of knowledge in the general public on the difference between correlation and causation. In class students peer-review their writing, then work on creating examples different from those in the text, that make it obvious that correlation is not the same as causation. Then my students find and read a couple of research articles about consuming a controversial food such as soy. My students are often shocked to discover how difficult it is to set up a controlled study and to establish solid correlation.

In this paper, I focus on a unit of my seminar called "Exploring My Hometown." This unit focuses on all of the mathematical content from the course and can easily be modified and adapted by instructors in other institutions to other geographic locations. The unit comprises the following subunits: the Census and Worcester; the population change of Worcester in comparison to Boston, Massachusetts, USA, and the world; human trafficking and Worcester; the climate of Worcester. Each of these subunits is centered on working with one of the mathematical themes in a real life context.

### 36.3.1 Worcester and the Census

This project begins with discussing the collection of reliable data. At this point of the course, my students are familiar with sampling, population, and measures of center and spread for data. They have read the sections on population, sampling, and averages. Students are also very familiar with percentages. One primary example as it pertains to population statistics is the Census [3]. Students are asked to look at the data for Worcester and compare at least five different categories from 2000 and 2010. What are the changes? How do we measure these changes and what are their implications? Here we focus on percentage rise and fall and how we can interpret these measures in context. Then students focus on the percentage of the Hispanic population in Worcester. A lot of them are shocked to see the numbers because their anecdotal experiences of living in Worcester do not correspond to what the numbers in the Census reveal. It seems that there should be a lot more Hispanics than those accounted for.

Next students are instructed to look at data about the percentage of Hispanic students in the Worcester public schools [4]. Is there a sizable difference? Why is this?

The idea is that students recognize a major issue with the Census—self-reporting plays a critical role in the collection of data, which allows many Hispanics to count themselves as white, thus skewing the numbers.

This project is completed in class, and presented by small groups.

### 36.3.2 Population Models

After reading the section "Throwing a Curve" in [5], my students collect data from the past 100 years through 2010, in increments of 10 years, from the United States, Massachusetts, Worcester and Boston, and the world. They use *Google Sheets* to make scatter plots with the years being the predictive variable. Then they use a function of best fit for each of the plots.

The interesting point of this in-class project is that the populations of the United States and Massachusetts have both been increasing (the former faster than the latter), while Boston and Worcester have not recovered from a population drop in the 1980s. It is interesting to compare Boston to Worcester relative to this drop and subsequent increases. This allows for a lively discussion about immigration and the local economy. While the first two graphs are better suited for an exponential or logistic model, the latter are best suited for a polynomial model due to their falling and then rising behavior.

This project is again completed in class, with presentations from different groups on each of these cases. There are follow-up questions of what each model predicts for 2020 and 2050. In particular, how do these figures compare with this year's data? Which ones are off? Why? Students are given the opportunity to reflect on the use and limitations of mathematical models in making predictions.

### 36.3.3 Human Trafficking and Worcester

This part of the project follows immediately after the first two parts are completed. Students are largely unaware of the problem of human trafficking. An in-class activity introduces them to this disturbing issue. They explore recent data from Massachusetts and study how it has changed over time. Why are there fluctuations? What could be done to combat this problem? Is the mean or the median a better measure for the cases of trafficking? Why?

This is an example of an in-class project that takes approximately two class sessions to complete. During the first one, students research human trafficking and watch a video (see the Appendix, p. 357, for details). During the second period, students organize the data in Google sheets, and discuss which measure of center is appropriate in this situation.

### 36.3.4 Climate and Worcester

This is one of the longer projects in the course and a capstone for this entire unit. It is completed outside of class. Students turn in their data analysis, a one page report on their findings, and a letter to a climate change doubter.

Using a weather archive site [7], students collect mean monthly temperatures from the past 60 years. They input these in a Google sheet. Then they compute the average of the mean temperatures for the first 30, the last 30, and all 60 years. They also find the standard deviations in all three cases. Students construct a scatter plot with years as the predictive variable and then they use a line of best fit to calculate the correlation coefficient. What is the trend in the data in all three cases? How are they the same and different?

Students observe an overall increasing trend that is more pronounced in the past 30 years. They also notice that the standard deviation is larger, which indicates more extremes. The Appendix (p. 357) provides more details on this project.

### 36.3.5 Assessment

The first three parts of this unit are assessed on the basis of completing all the tasks and taking an active part in the presentations. The Climate assignment is evaluated according to a scoring rubric (found in the Appendix, p. 357) and accounts for a larger part of the students' grade, since it obviously requires a more significant amount of time and effort.

Instructors have many options when it comes to assessing projects such as the ones presented here. They can use a generic rubric or they can create one themselves, tailored to the assignment. The rubric used in my course is a collaborative effort between the students in my honors first-year seminar and me. After telling them about the Climate and Worcester project, I ask them to brainstorm in small groups the facets of the projects that should be assessed. Then we combine the ideas from the entire course and created the rubric. The advantage of this method of rubric creation is that students are invested in following it.

## 36.4 Reflections

As of the time of the preparation of this manuscript, I have taught the *What the Numbers Say* first-year seminar three times to three distinct groups of students—honors, general, and elementary education majors who were a part of a remedial mathematics cohort. Each time the course has turned out to be slightly different, depending on the student population. Overall, students seemed to enjoy it and informally reported to me that they found the content very useful. Three mathematics majors were recruited from the three cohorts, two of whom are Mathematics for Elementary Education majors.

The group of the elementary education majors was by far the most challenging group to work with, due to their gaps in mathematical knowledge. I spent more time reviewing basic arithmetic and statistical literacy skills, which undoubtedly proved highly beneficial for this group. At this time, I am offering this seminar again to another elementary education remedial cohort and I am using only *The Numbers Game* [1]; the other text proved too challenging to be followed independently.

Every time I teach this first-year seminar I try to adjust some of the data collection to the interests of the particular students in the course. For example, in one of the iterations of the seminar, students collected data on school shootings, looking to see if there are any patterns. Each seminar has a final paper assignment, called a *Signature Assignment*, which is a general requirement for all seminars at Worcester State University. This assignment is required to show that students have met all three of the goals of the seminar (critical thinking, communication, and information literacy) successfully. The honors seminar's students focused on the presidential elections and wrote a comparative essay about the election system of the United States and that of either India or Brazil. My general seminar focused on exploring arguments for and against a specific diet such as high-fat, low-fat, or low-carbohydrate. My elementary education majors course researched the educational system of a country of their choice and compared it to that of the United States. In all cases, students had to use quantitative arguments to support their claim, as well as a number of scholarly sources.

Other projects generated by student interest include an "Are Meatless Mondays Worth It?" exploration and a population reduction plan for the most populous nation in Africa, Nigeria.

Overall, my course helps students to see mathematics as a worthwhile experience that allows them to function in the quantitative realm with understanding of where numbers come from, where they are useful, and where we can make successful predictions.

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# Appendix

## Climate and Worcester

*This was the document that was provided for students for the capstone project in this course. It includes detailed instructions and a scoring rubric.*

Go to the website: wunderground.com/history/, Then enter a month in 1957. Record monthly average temperatures for every month from January 1957 to December 2016. Then find yearly mean temperatures for these 60 years.

Enter your data in Excel or Google Sheets, putting the year in Column 1 and Average Temperature in Column 2. You should have 60 entries, organized in two columns.

Construct a time series graph to look for trends. What do you observe?

Then construct a scatter plot and fit the data to a trendline. What do you notice?

Now make a second and third sheet from your data. In the second one, record all the average temperatures from the first 30 years; in the third one, record all the average temperatures from the last 30 years. Repeat the steps above, making all the graphs. Do you notice a difference among the three different graphs/trends you obtain? Explain your observations in detail.

Finally, compute the standard deviation of the average temperatures in all three cases. What do you observe? What do the differences mean here?

Write a complete report on your findings, including all numeric information you computed.

Summarize your findings in a short letter to climate change doubter.

Your score will be determined by the following rubric:

- Assignment follows directions in terms of content.
- Assignment follows directions in terms of graphs and computations.
- Assignment is clear and well-organized.
- Letter is well-researched and supported.

Each of these are worth 1-5 points, depending on the extent of adhering to the criteria.

## Human Trafficking and Worcester

*These are instructions for the third part of the hometown exploration unit.*

Define what human trafficking means, using the following website: unodc.org/unodc/en/human-trafficking/what-is-human-trafficking.html.

Focus on the acts, the means, and the purpose of trafficking.

Then read through this website: unodc.org/unodc/en/human-trafficking/global-report-on-trafficking-i html

Did any of the facts presented there shock you? Compare the world statistics to what is happening in Worcester: humantraffickinghotline.org/state/massachusetts

What are the similarities and differences? Collect as much data as you are able from the website. Organize and analyze its measures of center and spread in a Google sheet.

Listen to the following podcast: `unodc.org/documents/audio///TIP_English.mp3`

List the main statistics presented in this interview. What do you see as the main difficulties with collecting this type of data? Be specific here.

Do you think people are aware of human trafficking? Why or why not? What are some things we could do to prevent trafficking, both locally and globally?

## Example of a Population Graph

*The following is an example of a polynomial model for the change in the population of Boston.*

The figure below represents an example of student work that reflects the change of population of Boston, which is modeled using a polynomial function.

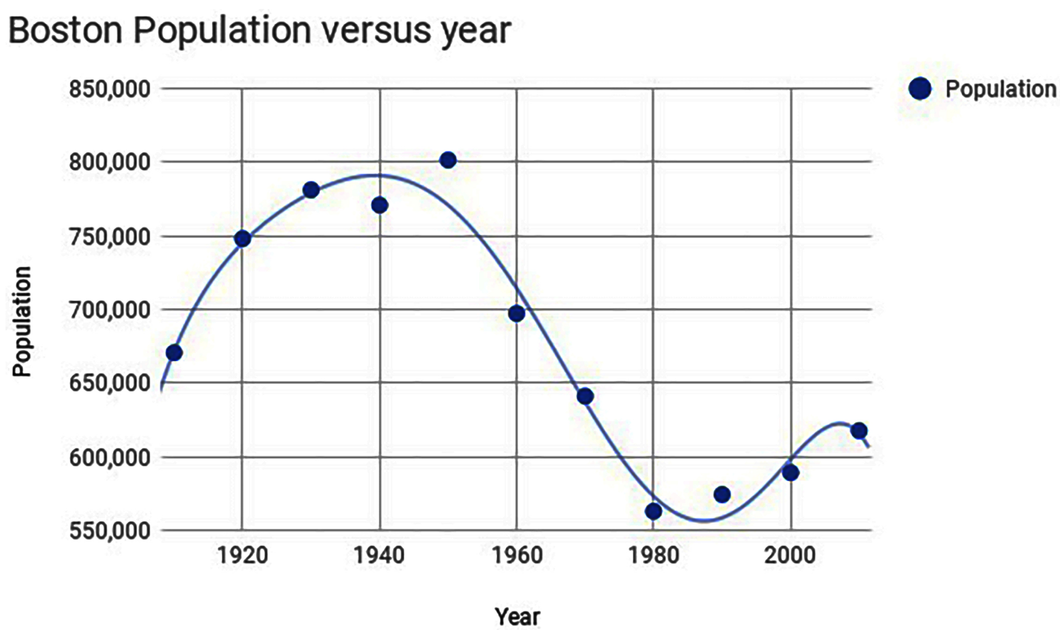


**Figure 36.1.** This is a model of the population of Boston from 1910 to 2010.