



# MAA MATHFEST

July 26-29, 2017

# ABSTRACTS of PAPERS

CHICAGO



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# Invited Addresses

## Earle Raymond Hedrick Lecture Series

**Dusa McDuff** Barnard College, Columbia University

**What is Symplectic Geometry? An Introduction to Some Concepts and Open Questions**

**Lecture 1: Thursday, July 27, 10:30–11:20 AM, International Ballroom North**

**Lecture 2: Friday, July 28, 9:30–10:20 AM, International Ballroom North**

**Lecture 3: Saturday, July 29, 9:30–10:20 AM, International Ballroom North**

Symplectic geometry has many faces. It takes place in even dimensions, and can be considered a version of complex algebraic geometry that is not constrained by the requirement that functions be polynomial, or more generally complex analytic. It also gives a framework in which to describe energy-conserving flows, and so has many applications to questions in dynamics. Recently it has turned out that symplectic structures (and their odd dimensional analog contact structures) can be used to help understand purely topological questions such as the possible ways that a circle can be twisted up in three-dimensional space, i.e. knot theory. This set of lectures will first describe what a symplectic structure is, and then explain why such structures are interesting. Most of our examples will be very concrete and will concern objects in two, three and four dimensions. We will assume knowledge of multivariable calculus and basic linear algebra, but not too much else.

## AMS-MAA Joint Invited Address

**Thursday, July 27, 9:30–10:20 AM, International Ballroom North**

**Douglas Arnold** University of Minnesota

**Computational Math Meets Geometry**

One of the joys of mathematical research occurs when seemingly distant branches of math come together. A beautiful example occurred over the last decade with the development of the field of compatible, or structure-preserving, discretizations of differential equations, in which ideas from topology and geometry have come to play a key role in numerical analysis. Very roughly, instead of applying standard all-purpose algorithms, such as Runge-Kutta methods and linear multistep methods for ODEs and finite difference or finite element methods for PDEs, far better results can be obtained for various classes of problems by constructing discretization methods which exactly preserve key geometric structures underlying the equations under consideration. Such structures include, for ordinary differential equations, symplecticity, symmetry, invariants and constraints, and, for partial differential equations, de Rham and other cohomologies and associated Hodge theory. We will tour this burgeoning field, demonstrating some of the advances in numerical methods made possible by the new geometrical and topological approaches, and even present a case where the numerical point of view has enabled the resolution of a long open question in algebraic topology.

## MAA Invited Addresses

**Thursday, July 27, 8:30–9:20 AM, International Ballroom North**

**Erica Flapan** Pomona College

**An Introduction to Spatial Graph Theory**

Spatial graph theory developed in the early 1980's when topologists began using the tools of knot theory to study graphs embedded in 3-dimensional space. Later, this area came to be known as spatial graph theory to distinguish it from the study of abstract graphs. Much of the current work in spatial graph theory can trace its roots back either to the ground breaking results of John Conway and Cameron Gordon on intrinsic knotting and linking of graphs or to the topology of non-rigid molecules. This talk will present the history of spatial graph theory and survey some of the current trends in the field.



## Friday, July 28, 10:30–11:20 AM, International Ballroom North

**Steven Brams** New York University

### Is There a Better Way to Elect a President?

I describe properties of approval voting—whereby voters can approve of as many candidates as they like in a multi-candidate election, and the candidate with the most approval wins—and compare them with properties of (1) plurality voting, in which voters can vote for only one candidate; (2) ranking systems, such as the Borda count and the Hare system of single transferable vote (also called instant runoff or ranked choice voting); and (3) grading systems that have been proposed by mathematicians Warren Smith (range or score voting) and Michel Balinski and Rida Laraki (majority judgment voting). I argue that approval voting, which is used by both the MAA and AMS, among other professional societies, is a simpler and more practicable alternative and should be used in presidential and other public elections. Extending approval voting to multi-winner elections, such as to a committee or council, will also be discussed.

## Saturday, July 29, 10:30–11:20 AM, International Ballroom North

**Ronald Mickens** Clark Atlanta University

### How to Create Periodic Functions from Geometric Shapes

The trigonometric sine and cosine functions are generated from the geometrical properties of the unit circle. We demonstrate that other periodic functions can be constructed by generalizing the methodology used to analyze the properties of the circular, i.e., trigonometric, functions. In particular, we investigate the elliptic, “square,” and “triangular” periodic functions, and derive a number of their critical mathematical features using only elementary trigonometry. At a somewhat more advanced level, we introduce the functional equation,  $f(t)^2 + g(t)^2 = 1$ , and show it has an unbounded set of periodic functions as solutions. An algorithm is given to explicitly calculate those periodic solutions possessing a second derivative. Finally, the following interesting and important result is obtained: the considered periodic functions always occur as a triplet of functions, rather than a pair.

## MAA James R.C. Leitzel Lecture

### Saturday, July 29, 8:30–9:20 AM, International Ballroom North

**Dan Meyer** Desmos

### Math’s Other Half

Whatever your job title, you are also an ambassador from the world of those who love math to the world of those who fear math. Your ambassadorship will either produce more people who love math or more people who fear math. Your effect will be non-zero. But the math that people fear is often just one half of math. Let’s discuss methods for helping fearful people encounter math’s other half.

## AWM-MAA Etta Z. Falconer Lecture

### Friday, July 28, 8:30–9:20 AM, International Ballroom North

**Talithia Williams** Harvey Mudd College

### Not So Hidden Figures: Unveiling Mathematical Talent

In the past few months, the movie “Hidden Figures” has brought visibility to the lives of African American women who served as NASA “human computers” in the 1960s. During that same time, Dr. Etta Falconer, the 11th African American woman to receive a Ph.D. in mathematics, began her tenure at Spelman College, motivating young women of color to be and do more than they dreamed possible in a field where their presence was lacking. I was fortunate to take her classes, engage her mathematical mind and dream of following in her footsteps. At Harvey Mudd College, I now find myself replicating those “Falconer moments” with my own students. I’ll share several of these strategies that you can use in and out of the classroom to encourage all students, particularly underrepresented students, to develop their mathematical talent and pursue mathematical sciences.

## MAA Chan Stanek Lecture for Students

**Thursday, July 27, 1:00–1:50 PM, International Ballroom South**

**David Richeson**    Dickinson College

### **Four Tales of Impossibility**

“Nothing is impossible!” It is comforting to believe this greeting card sentiment; it is the American dream. Yet there are impossible things, and it is possible to prove that they are so. In this talk we will look at some of the most famous impossibility theorems—the so-called “problems of antiquity.” The ancient Greek geometers and future generations of mathematicians tried and failed to square circles, trisect angles, double cubes, and construct regular polygons using only a compass and straightedge. It took two thousand years to prove conclusively that all four of these are mathematically impossible.

## Pi Mu Epsilon J. Sutherland Frame Lecture

**Wednesday, July 26, 8:00–8:50 PM, International Ballroom North**

**Ingrid Daubechies**    Duke University

### **Bones and Teeth: Analyzing Shapes for Evolutionary Biology**

For the last 8 years, several of my students and postdocs as well as myself have been collaborating with biologists to design mathematical approaches and tools that would help automate biological shape analysis. The talk will review this collaboration, sketching both the mathematics and chronicling the interaction with our biological colleagues.

## NAM David Harold Blackwell Lecture

**Friday, July 28, 1:00–1:50 PM, International Ballroom South**

**Rudy L. Horne**    Morehouse College

### **Hidden Figures: My Role as a Math Consultant for this Film**

In January 2017, the movie *Hidden Figures* was released by 20th Century Fox studios. This movie tells the story of three African-American women mathematicians and engineers (Katherine Johnson, Mary Jackson and Dorothy Vaughan) who would play a pivotal role towards the successful mission of John Glenn’s spacecraft orbit around the Earth and the NASA missions to the moon. For this talk, we give a brief review of the space race going on at the time between the United States of America and the former Soviet Union. We will discuss the lives and contributions that NASA mathematician Katherine Johnson and the NASA engineers Mary Jackson and Dorothy Vaughan made to the space race, particularly their work as it concerns John Glenn’s orbit around the Earth in 1962 and to the moon missions. Also, we will talk about the experiences of being a mathematical consultant for this film.

# Alder Awards

## Alder Award Session

**Friday, July 28, 2:30–3:20 PM, International Ballroom North**

The MAA established the Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member to honor beginning college or university faculty members whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond their own classrooms. Each year, at most three college or university teachers are honored with this national award. The awardees are invited to make a presentation in this session. The session is moderated by Deanna Haunsperger, Carleton College, MAA President.

This year's honorees are:

**Steven Klee** *Seattle University*

**Mary De Raeve Beisiegel** *Oregon State University*

**Steven Klee** *Seattle University*

### **Tell Me How You Got Here**

2:30–2:50 PM

Students sometimes believe they have failed to solve a problem without seeing the depth of what they have learned and discovered. In this talk, I will explore the importance of talking about math with our students and helping them find value in exploration, discovery, and even failure by encouraging them to share their ideas—even when they are incomplete and especially when they feel they are wrong. In doing so, we can help students see they know more than they think they know and they are not alone in their struggles.

**Mary De Raeve Beisiegel** *Oregon State University*

### **Teaching Mathematics as Though Their Lives Depend on It**

3:00–3:20 PM

How we teach mathematics has a significant impact on undergraduate learners. More traditional approaches to teaching often cause students to leave STEM fields, which has significant implications for their careers and daily lives. For example, students with STEM degrees are frequently earn higher incomes than students with non-STEM degrees. As another example, students with mathematical knowledge have greater capacity to interpret and understand numerical data and economic consequences of public policies and their own decisions. In contrast to traditional instruction, more engaging and active teaching practices result in better outcomes for learners. In this talk, I will share why and how I have restructured my teaching practices based on the knowledge that how I teach mathematics has long term implications for my students' lives.

# Invited Paper Sessions

## Spatial Graph Theory

Thursday, July 27, 1:00–5:00 PM, Continental Ballroom A

**Organizer: Erica Flapan** Pomona College

Spatial Graph Theory is a relatively young interdisciplinary field that brings together knot theory, low dimensional topology and geometry, combinatorics, and graph theory, and has applications in chemistry, molecular biology, and biophysics. In addition, because of its combinatorial nature, many problems in Spatial Graph Theory lend themselves well to undergraduate research. For these reasons, faculty at primarily undergraduate institutions as well as those at research universities may be interested in learning about Spatial Graph Theory.

**Emille Davie Lawrence** San Francisco University

### Topological Symmetry Groups of Möbius Ladders and the Petersen Graph in $R^3$

1:00–1:20 PM

The study of graphs embedded in  $S^3$  was originally motivated by chemists' need to predict molecular behavior. The symmetries of a molecule can explain many of its chemical properties, however we draw a distinction between rigid and flexible molecules. Flexible molecules may have symmetries that are not merely a combination of rotations and reflections. Such symmetries prompted the concept of the topological symmetry group of a graph embedded in  $S^3$ . We will discuss recent work on what groups are realizable as the topological symmetry group for several families of graphs, including the Petersen family and Möbius ladders.

**Hugh Howards** Wake Forest University

### Intrinsic Chirality of Graphs in $R^3$ and Other 3-Manifolds

1:30–1:50 PM

We say that a graph  $\Gamma$  embedded in a  $S^3$  is achiral, if there is an orientation reversing homeomorphism  $h$  of  $S^3$  leaving  $\Gamma$  setwise invariant. If no such homeomorphism exists, we say that the embedded graph  $\Gamma$  is chiral. There exist abstract graphs which have the property that all of their embeddings in  $S^3$  are chiral. Such a graph is said to be intrinsically chiral in  $S^3$ . This definition can easily be extended to graphs embedded in any 3-manifold, so it is natural to ask whether a graph which is intrinsically chiral in  $S^3$  would necessarily be intrinsically chiral in other 3-manifolds. We survey results about intrinsic chirality of graphs in  $S^3$  and other manifolds.

**Blake Mellor** Loyola Marymount University

### Alexander Polynomials of Spatial Graphs and Virtual Knots

2:00–2:20 PM

The Alexander polynomial is one of the oldest, and most studied, knot invariants. In this talk, we will briefly review the Alexander polynomial and extend it to two generalizations of classical knots: spatial graphs and virtual knots. In spatial graphs, as with knots, the Alexander polynomial is related to p-colorings of the graph, and can be used to determine whether the graph is p-colorable. In the realm of virtual knots, we will see how the Alexander polynomial is related to the odd writhe (and, more generally, the writhe polynomial) of a virtual knot.

**Kouki Taniyama** Waseda University

### Realization of Knots and Links in a Spatial Graph

2:30–2:50 PM

A  $\Theta n$  curve graph is a graph with two vertices and  $n$  edges joining them. Kinoshita showed the following. For any  $n(n-1)/2$  knots there exists an embedding of a  $\Theta n$  curve graph into space such that the knot types of the  $n(n-1)/2$  embedded cycles coincide with that of the given  $n(n-1)/2$  knots. We will consider the generalization of this result. It is closely related to the theory of Vassiliev invariants and local moves of knots.

**Ryo Nikkuni** Tokyo Woman's Christian University

### Conway-Gordon Type Theorems

3:00–3:20 PM

Some graphs have the property that no matter how they are embedded in Euclidean space they contain a knot or a link. In 1983, Conway and Gordon proved the following famous theorems: Every spatial complete graph on six vertices contains a two-component link with odd linking number, and every spatial complete graph on seven vertices contains a knot with non-zero Arf invariant. In this talk, we will present an overview of recent developments and ramifications related to the Conway-Gordon theorems in spatial graph theory.

**Danielle O'Donnol** Indiana University

### **Legendrian Spatial Graphs**

3:30–3:50 PM

This talk will give a brief introduction to contact structures, and focus on Legendrian graphs in the standard contact structure on  $R^3$ . A spatial graph is Legendrian if it is everywhere tangent to the contact structure. We will give an overview of results in this new area of research.

**Elena Pavelescu** University of South Alabama

### **Oriented Matroid Theory and Linear Embeddings of Spatial Graphs**

4:00–4:20 PM

Matroid theory is an abstract theory of independence introduced by Whitney in 1935. It is a natural generalization of linear independence. Oriented matroids can be thought of as combinatorial abstractions of point configurations over the reals. To every linear (straight-edge) embedding of a graph one can associate an oriented matroid, and the oriented matroid encodes the knotting and linking information in the embedded graph. In this talk, we introduce the basics of oriented matroids and we look at few graph theoretical results which use oriented matroids. In particular, we show that any linear embedding of  $K^9$ , the complete graph on nine vertices, contains a non-split link with three components.

**Kenji Kozai** Harvey Mudd College

### **Random Linear Embeddings of Spatial Graphs with Applications to Polymers**

4:30–4:50 PM

Random knots have been investigated extensively to model knotting behavior of linear polymers like DNA. In general, the complexity of knotting and linking increases as the polymer gets longer. After giving an overview of some of the random knot models that have been studied, we will discuss generalizations into random embeddings of graphs and overview the many questions that arise. As an example, random linear embeddings of a graph can be thought of as a model for the spatial configurations of non-linear molecules and polymers, and one might ask which configurations are typical. Leveraging known and new results about random knotting and linear embeddings of graphs, we show that certain “simple” graphs nearly always show up in their topologically simplest configurations.

## **Big Ideas About Big (and Less Than Big) Data**

**Thursday, July 27, 2:00–5:00 PM, Continental Ballroom B**

**Organizers: Tim Chartier** Davidson College

**Jennifer Galovich** St. John's University and the College of St. Benedict

Data analytics is a growing field, with graduate degrees, undergraduate majors and minors, and concentrations popping up at colleges and universities around the country. Data analysis impacts our lives broadly from predictions of movie rankings on Netflix to targeted marketing by retailers, to name two of many applications. The landscape of data science is broad. The ideas of the field can be applied using smaller datasets from a biometric device like a Fitbit or iWatch to large datasets in finance or health care. This session will sample areas of data science from a variety of applications, calling on various topics in mathematics such as graph theory and linear algebra, as well as statistical modelling. The session will also include presenters from government, academia and business demonstrating the inherent interdisciplinarity of studying big and less than big data.

**Talithia Williams** Harvey Mudd College

### **Know Thyself: Introspective Personal Data Mining**

2:00–2:20 PM

The leading edge breed of high-tech, wearable health technology is changing how we monitor personal data. We can quantify everything from heart rate and sleep patterns to body temperature and sex life. But, what is the average person to do with the massive amounts of data being collected? This talk makes a compelling case that all of us should be recording simple data about our bodies and will help you begin to analyze and understand your body's data. Surprisingly, your own data can reveal much more than even your doctors may know!

**Martin I. Meltzer** Centers for Disease Control and Prevention (CDC)

### Using Big and Less-Than-Big Data Sets in Public Health

2:30–2:50 PM

The use of different types of databases in public health to estimate the potential burden of disease and impact of interventions, will be illustrated by considering the following published examples: The use, and attendant problems, of large healthcare insurance databases to assess the risks of specific disease-related physician visits, hospitalizations and deaths. These databases will be contrasted by reviewing published papers that demonstrate the relative paucity of relevant epidemiological data during the 2009 H1N1 influenza pandemic and the 2014-2016 Ebola epidemic in West Africa. The overall conclusion that will be illustrated is that public health policy makers cannot assume that the relevant data will be available, requiring analysts to use a wide variety of data bases.

**Abhishek Mehta** Tresata

### Let Me See Your Papers: Using Real-Time Network Graph Traversal to Uncover Suspicious Offshore Activity

3:00–3:20 PM

As the biggest data leak in history, the release of the Panama Papers rocked the world in 2016, instigating a slew of criminal investigations and most notably leading to the resignation of Iceland's Prime Minister. The International Consortium of Investigative Journalists (ICIJ) made the database associated with the Panama Papers publicly available shortly thereafter. Using OPTIMUS, Tresata's Analytics Operating system, we decided to conduct some investigations of our own –scrutinizing entities within the dataset (in real-time) at a unique segment of one, discovering their associations, and seeing which interactions were above board.

**Michael Berry** University of Tennessee, Knoxville

### Toward Unsupervised Learning for Social Media Using Linear Algebra

3:30–3:50 PM

In large-scale text mining applications such as tweet classification there is need for fast yet robust techniques to summarize or track concepts without prior knowledge of the content. Linear algebra plays a very important role in the design and implementation of the underlying algorithms needed for the automated summarization of time-sensitive documents, especially those from social media. Matrix and tensor factorization methods can greatly facilitate the extraction of key documents (tweets) that can summarize a current stream and thereby reduce the exhaustive human effort that would be needed to read and synthesize an enormous number of documents. The long term goal of this research is to develop the core numerical algorithms and software needed for unsupervised learning when no prior labels or metadata is available.

**Dash Davidson** Tableau Software

### Finding and Telling Data Stories

4:00–4:20 PM

Hidden in any dataset, from the largest to the smallest, are stories –most often, many of them. In this session you will see how you can employ several different analytical techniques to draw these stories out of your data. Through combining visual analysis with storytelling, you will learn how to bring even the simplest of datasets to life in a compelling way.

**Michael Dorff** Brigham Young University

### Creating Partnerships with Industry and Finding Data Analytics Problems for Students

4:30–4:50 PM

Suppose you wanted to develop partnerships with people in business, industry, or governments (BIG) to get research problems, many of which are data analytics problems, for your students to work on and be better prepared for careers in BIG. How would you make these partnerships? How would you get research problems from industry? What would those problems look like? Answers to those questions can come through the PIC Math program. PIC Math is a MAA/SIAM supported program funded by NSF to prepare mathematical sciences undergraduate students for industrial careers by engaging them in research problems from industry. In this talk, we will discuss how faculty members like you (many of whom have no experience in applied math or in BIG) develop partnerships with people in industry, get data analytics research problems for their students as a result of these partnerships, and what these problems look like.



# Low Dimensional Symplectic and Contact Topology

**Friday, July 28, 1:00–4:00 PM, Continental Ballroom B**

**Organizers:** Dusa McDuff    Barnard College, Columbia University  
Whitney George    University of Wisconsin LaCrosse

The origins of symplectic and contact topology can be traced back to classical mechanical systems. Since then, both symplectic and contact topology have become very robust fields of study in their own right. The aim of this session will be to highlight techniques and recent results in the areas of low-dimensional symplectic and contact topology ranging from applications in knot theory to the theory of planar arrangements and singularities. Most of this work uses some version of Floer theory (such as contact homology or Heegaard Floer homology), which is an infinite-dimensional analog of Morse homology. We will aim to make this session understandable to nonexperts.

**Doug LaFountain**    Western Illinois University

## Constructing Interlocking Solid Tori in Contact 3-Manifolds

1:00–1:20 PM

This talk will be hands-on using models with which participants can experiment. We will see how to construct interlocking solid tori, which have interesting applications in contact topology, and demonstrate how every positive braid which is not an obvious stabilization supports interlocking solid tori. Applications and open questions will be described as well; anyone is welcome, no previous knowledge will be required.

**Bahar Acu**    University of Southern California and UCLA

## The Weinstein Conjecture

1:30–1:50 PM

The Weinstein conjecture asserts that the Reeb vector field of every contact form carries at least one closed orbit. The conjecture was proven for all closed 3-dimensional manifolds by Taubes. Despite considerable progress, it is still open in higher dimensions. In this talk, we will talk about its history and show that a  $(2n+1)(2n+1)$ -dimensional "iterated planar" contact manifolds satisfy the Weinstein conjecture.

**Jo Nelson**    Barnard College and Columbia University

## Contact Invariants and Reeb Dynamics

2:00–2:20 PM

Contact geometry is the study of certain geometric structures on odd dimensional smooth manifolds. A contact structure is a hyperplane field specified by a one form which satisfies a maximum nondegeneracy condition called complete non-integrability. The associated one form is called a contact form and uniquely determines a vector field called the Reeb vector field on the manifold. I will explain how to make use of J-holomorphic curves to obtain a Floer theoretic contact invariant whose chain complex is generated by closed Reeb orbits. In particular, I will explain the pitfalls in defining contact homology and discuss my work (in part joint with Michael Hutchings) which gives a rigorous construction of cylindrical contact homology via geometric methods. This talk will feature numerous graphics to acclimate people to the realm of contact geometry.

**Lisa Traynor**    Bryn Mawr College

## The Flexibility and Rigidity of Lagrangian Cobordisms

2:30–2:50 PM

Cobordisms are common objects of study in topology. I will discuss cobordisms that have additional geometric constraints imposed by symplectic and contact structures. These Lagrangian cobordisms between Legendrian submanifolds arise in a relative version of Symplectic Field Theory. I will discuss results that show that sometimes Lagrangian cobordisms are flexible, in that they behave like topological cobordisms, while at other times Lagrangian cobordisms are rigid, in that they have properties very different than those seen in the topological setting. This is joint work with Joshua M. Sabloff.

**Laura Starkston**    Stanford University

## A New Approach to the Symplectic Isotopy Problem

3:00–3:20 PM

One of the simplest closed symplectic manifolds is the complex projective plane, but we still have yet to answer one of the most basic questions about it: what is the classification of symplectic surfaces in  $CP^2$  up to symplectic isotopy? The adjunction formula determines the genus of such a symplectic surface from its homology class, and complex algebraic curves provide representatives of each of these homology classes. The symplectic isotopy problem asks if every symplectic surface is symplectically isotopic to one of these complex algebraic representatives. This problem has been solved affirmatively up to degree 17, but further progress has been halted by difficulties in the analysis of pseudoholomorphic curves. We present a new line of attack on this problem which translates it into a problem of finding certain Lagrangians with boundary.

# Mathematics and Democracy

**Friday, July 28, 2:00–5:00 PM, Continental Ballroom A**

**Organizer: Steven Brams** New York University

Democracy is fraught with different meanings that mathematics can help to make more precise. This session will include talks on the properties of voting systems that best reflect the will of the people in electing a single winner (e.g., for mayor or president), or best represent different factions in electing multiple winners (e.g., to a committee or council). Among other topics discussed will be different ways of apportioning representatives to states, or seats in a legislature to political parties; methodologies for drawing district lines to avoid gerrymandering; and the avoidance of different social-choice paradoxes.

**Paul H. Edelman** Vanderbilt University

## Political Hypotheses and Mathematical Conclusions

2:00–2:20 PM

When modeling or analyzing democratic processes, mathematicians may find themselves in unfamiliar territory: political philosophy. How we proceed mathematically may depend heavily on our conception of representative democracy and theory of government. I will give a number of illustrations to show how contestable political principles lead to differing mathematical analyses. Our mathematical conclusions are inherently governed by our political hypotheses.

**D. Marc Kilgour** Wilfrid Laurier University

## Multiwinner Approval Voting: An Apportionment Approach

2:30–2:50 PM

Approval voting is extended to the election of multiple winners—roughly proportional to their approval in the electorate—who may be either individual candidates, elected to a committee, or members of a political party, who fill one or more seats in a legislature. The sequential version of the divisor apportionment methods of Jefferson and Webster iteratively depreciate the approval votes of voters who have one or more of their approved candidates elected. The nonsequential versions of these methods, which are computationally complex but feasible to use in many elections, tend to elect more representative and diverse bodies than the sequential methods. Whereas the Webster method better satisfies representativeness and diversity than the Jefferson method, the latter, whose vote thresholds for winning seats duplicate those of cumulative voting in 2-party elections, seems fairer.

**Michael Orrison** Harvey Mudd College

## Voting and the Symmetric Group

3:00–3:20 PM

Suppose you are voting in an election that requires you to submit a complete ranking of the candidates, from your most preferred candidate all the way down to your least preferred candidate. If you enjoy thinking about abstract algebra, then you might be tempted to view your ranking as a permutation in the symmetric group on the set of candidates. In this talk, I will explain why doing so is worth your while, and how it can quickly lead to new insights and powerful techniques for wrestling with ideas in voting theory.

**Tommy Ratliff** Wheaton College

## Consistent Criteria, Problematic Outcomes, and the Hypercube

3:30–3:50 PM

Not all voting consists of selecting a winner from a set of candidates. For example, consider a tenure committee where the criterion is that a successful candidate must be excellent in both teaching and research. There are simple examples that lead to what is known as the discursive dilemma: The committee reaches one conclusion using the majority vote based on the recommendations of each member applying the criterion individually but obtains a different conclusion by first using the majority vote on each category and then applying the criterion. What happens if there are more categories or if the criterion is more complicated than a simple boolean AND? We can use the geometry of the hypercube and some graph theory to characterize all logical statements that lead to discursive-type dilemmas.

**Karen Saxe** Macalester College and AMS

## Ready for Redistricting 2020?

4:00–4:20 PM

Every ten years the seats of the US House of Representatives are reapportioned to the states and then each state commences to redraw its congressional district lines. In this talk we will give an overview of how the states do this and what changes (legal and procedural) have taken place since the last time we did this. We will highlight how mathematics is used to aid in the redistricting process and help detect when gerrymandering has taken place.

**William S. Zwicker** Union College

### **Orthogonal Decomposition and the Mathematics of Voting**

4:30–4:50 PM

Suppose several teachers are assessing the level of preparation of their common students, with the goal of splitting them into one group ready to tackle more abstract and challenging mathematical concepts, and a second group needing more review. An election is held, in which a ballot recommends a particular split, and the outcome is a collective decision on how to group. This seems quite different from an election in which a ballot is a ranking of candidates for President, and the outcome selects a winning candidate, but in both cases we are aggregating several binary relations of a specified type into a single binary relation (of a possibly different type). It turns out that there are “universal” rules for aggregating binary relations, which generate a surprising diversity of well-known aggregation rules as special cases. Differences between universal aggregators F and G can arise when an orthogonal decomposition separates ballot information into two components, with F using both and G discarding one of them. We’ll discuss two decompositions, related to the two types of elections mentioned above, and to a single voting rule proposed by John Kemeny.

## **No Longer Hidden Figures: Women Mathematicians Share Their Path to the Profession**

**Friday, July 28, 2:00–5:00 PM, Salon A-3**

**Organizers: Jacqueline Jensen-Vallin** Lamar University

**Talithia Williams** Harvey Mudd College

**Alissa Crans** Loyola Marymount University

The recent blockbuster hit, *Hidden Figures*, shines light on the talented group of African American women mathematicians that helped lead the United States in the race to become the first country to put a man on the moon. Their passion for mathematical excellence and desire to make meaningful contributions to the greater society allowed them to persevere in circumstances that were not always welcoming. In this series of talks, the speakers will take us on a journey from their budding mathematical interest to their individual paths to the profession, including any stumbling blocks along the way. Our hope is that these talks provide the audience with concrete experiences and ideas that can be implemented in and out of the classroom as we all seek to broaden the participation of women and underrepresented groups in mathematics.

**Talitha Washington** Howard University

### **The “Firsts” in STEM: Modern Day ‘Hidden Figures’**

2:00–2:20 PM

In a segregated courtroom, Mary Jackson had to petition a Virginia court to get permission to enroll in graduate-level courses in mathematics and physics at an all-white high school. In the movie, Jackson’s plea to the judge passionately conveyed the significance of being the “first”. Mary Jackson eventually became the first African American female aeronautical engineer at NASA and in the world. This presentation will highlight many “firsts” who are blazing trails in STEM fields, including the presenter, Dr. Washington, who was the first African American to earn a PhD in mathematics from the University of Connecticut in 2001.

**Shelby Wilson** Morehouse College

### **A Quest to Cure Cancer with Math**

2:30–2:50 PM

My mathematical journey has been shaped and molded by some incredible women in the mathematical sciences. This begins with my grandmother, mathematician and educator, Etta Falconer, and continues with many of the women included in this session. In this talk, I will highlight my journey to becoming a “Mathematical Biologist” and the women who helped me get here. I will discuss how my childhood love of mathematics came together with my interest in medicine to create a career path that I am passionate about.

**Candice Price** University of San Diego

### **Young, Mathematically Gifted, and Black**

3:00–3:20 PM

In February 2017, Erica Graham, Raegan Higgins, Shelby Wilson and myself created the website “Mathematically Gifted and Black” to highlight the contributions and service of Black Mathematicians to academia, education, government industry and society. This website hit on the importance of representation, knowing OUR history and knowing OUR impact. In this presentation, I will tell you about my mathematical journey through the lens of reflection of my participation in the creation of the website [www.mathematicallygiftedandblack.com](http://www.mathematicallygiftedandblack.com).

**Suzanne Weekes** Worcester Polytechnic Institute

**A Path, Thus Far...**

3:30–3:50 PM

Suzanne Weekes is a Professor of Mathematical Sciences at Worcester Polytechnic Institute, and co-director of the Mathematical Sciences Research Institute Undergraduate Program (MSRI-UP) and the Preparation for Industrial Careers in Mathematical Sciences (PIC Math) program. She will share a bit about her journey, thus far, as an academic mathematician.

**Emille Davie Lawrence** University of San Francisco

**Life Has Critical Points**

4:00–4:20 PM

As mathematicians, we are all familiar with what happens at critical points of a function. At a critical point, we either find ourselves reaching the top of a summit, the bottom of a valley, or perhaps a point where we could go up or down. I will share with you a few of the critical points in my life and how they have shaped who I am.

## **The Life and Legacy of J Ernest Wilkins (1923-2011)**

**Saturday, July 29, 1:00–4:00 PM, Salon A-3**

**Organizers: Ronald Mickens** Clark Atlanta University

**Talitha Washington** Howard University

**Ron Buckmire** National Science Foundation and Occidental College

J Ernest Wilkins earned a PhD in Mathematics at the age of 19 from the University of Chicago. In 1942 he became the seventh African American to earn a PhD in Mathematics. In 1976 he became the second African American to be elected to the National Academy of Engineering. Wilkins' career spanned academia, industry and government including the University of Chicago Met Lab during the Manhattan Project. He also helped establish the doctoral program in mathematics at Howard University. This session will share his impact in nuclear-reactor physics and optics, his plight of being a "negro genius", and his impact on the mathematical community.

**Ronald E. Mickens** Clark Atlanta University

**J Ernest Wilkins, Jr.: My Friend, Colleague, and Collaborator**

1:00–1:20 PM

This presentation reviews my experiences, both scientifically and socially, with Professor Wilkins during his tenure at Clark Atlanta University (CAU). In addition to being friends and colleagues, I will share stories of our lively interactions with other prominent African American scientists. I will also share insight on his engaging teaching style as well as the nature of our scientific collaborations. Finally, I will discuss how his prestigious career path led him to be my esteemed colleague at CAU.

**Carolyn Wilkins** Professor Berklee College of Music

**Sharon Wilkins Hill**

**The Remarkable Wilkins Family**

1:30–1:50 PM

We will discuss our Wilkins family, focusing on the achievements of J Ernest Wilkins Sr. as well as his wife Lucille and their three children. After placing these accomplishments within the larger historical context (black Chicago in the 1930s), Sharon Hill will offer some personal reflections on her father's legacy.

**Bob Fefferman** The University of Chicago

**J Ernest Wilkins at the University of Chicago**

2:00–2:20 PM

This will be a discussion of the extremely impressive career of J Ernest Wilkins as a student at the University of Chicago. I will also discuss some other examples of outstanding mathematicians connected with the University who are members of an underrepresented group.

**Cleo Bentley** Prairie View A&M University

**My PhD Dissertation Advisor –J Ernest Wilkins**

2:30–2:50 PM

In this talk, I will discuss my graduate experiences with Wilkins during my masters and doctoral programs at Howard University.

**Asamoah Nkwanta** Morgan State University

**Dr. J Ernest Wilkins, Jr.: The Man and His Works**

3:00–3:20 PM

This presentation is based on a research study conducted while the author was a member of the Mathematical Association of America (MAA) Institute in the History of Mathematics and Its Uses in Teaching (IHMT). In this presentation, we will highlight the significant contributions of the African-American educator and researcher in mathematics, physics and engineering - Dr. J Ernest Wilkins, Jr.

**Talitha Washington** Howard University

**The Scientific and Mathematical Impact of J Ernest Wilkins**

3:30–4:00 PM

At the age of 19, J Ernest Wilkins earned a doctorate degree in mathematics from the University of Chicago. Even though he was unable to gain employment at a research university due to segregation, he made great impacts in the applications of mathematics. This talk will provide an insight and overview to Wilkins' great contributions to solve problems in reactor theory and optics, as well as his work on the Manhattan Project.

# Contributed Paper Sessions with Themes

## My Favorite Math Circle Problem

**Part A: Thursday, July 27, 1:00–3:15 PM, Salon C-1 & C-2**

**Organizer: Bob Klein** Ohio University

A math circle is an enrichment experience that brings mathematics professionals in direct contact with pre-college students and/or their teachers. Circles foster passion and excitement for deep mathematics. Papers in this session highlight either a favorite problem from a math circle, or favorite collection of problems used together for one or two sessions of a math circle. Contributed papers should describe the launch of the problem, what happens during the circle, and ways of “wrapping up”, even if that doesn’t involve answering the problem.

**Diana White** University of Colorado Denver

### Superfactorials and perfect squares

Can you remove one of the terms of  $100! \cdot 99! \cdot 98! \cdot \dots \cdot 3! \cdot 2! \cdot 1!$ , so that what remains is a perfect square? Since first learning about this problem, it has been one of the author’s favorite problems to facilitate in Math Teachers’ Circles. In this talk, we provide an overview of how teachers commonly approach the problem, and the mathematics that often emerges along the way.

**David Crombecque** USC

### Mathematical Ciphers: An Math Teachers’ Circle day long workshop.

The Los Angeles MTC offers a two day workshop for local math teachers. One day is devoted to Applied Mathematics (like Mathematical Modelization) and the other day to General Mathematics (like Geometry). The activities presented here constitutes an entire day of the workshop but can also be split and used for monthly meetings. The theme of Mathematical Ciphers was chosen as it introduces the beautiful mathematics of Modular Arithmetic while dealing with mathematical concepts familiar and essential to middle-school and high school math teachers. Encryption methods are very much a hot topic in our era so it will peak the curiosity of many teachers. And of course, this can lead to a lot of fun activities!

**Mike Janssen** Dordt College

### Divide Your Cake (and Eat it, Too!)

Many people are familiar with a fair way of dividing a resource (such as a cookie) between two people; we often learn it as children. In this talk, I’ll describe a math teachers’ circle session I ran on the notion of fair division, which included an inquiry component exploring means of dividing a resource between more than two players. This was a particularly fun session, and illuminating for teachers, as it involves the use of mathematical thinking applied to a problem which does not fall within a traditional subject in school mathematics.

**Rebin Muhammad** Ohio University

### Islamic Geometric Pattern

An Islamic geometric pattern (IGP) is a 2D wallpaper that is created by using only a compass and a ruler. The history of IGPs date back to the 8th century and IGPs have been seen in most Islamic countries, where they have been used to decorate building walls and mosques. For the Math Circle session, we reconstruct one of these patterns, the “Seal of Solomon,” from scratch. Each student creates a tile and at the end of the session, we tile all of them into a single, decorated wall. Can we have different tilings? And why does the tiling work? What kind of symmetries do the tiles exhibit? Can you use the same construction line to create a different tile and are you sure that the lines between tiles are such that your design will tile the wall? In the second pattern, “Ten Chain,” students use modular arithmetic to construct the tile and to learn about  $Z_{10}$ . We ask questions like: Can you generalize this example? Can you create “5 Chain, 6 Chain, etc.”? What kind of chain cannot be constructed? This is followed by investigations of “interlacing” including when the strategy of interlacing works and the relationship between two neighboring chains.

**Parth Sarin** A&M Consolidated High School

**Philip B. Yasskin** Texas A&M University

### The Dissemination of Gossip

We will discuss a Math Circle on the dissemination of gossip. Each student in a group of  $n$  students (perhaps 4, 5 or 6) has a different pet. Everyone wants to know who has which pet. They communicate by gossiping. Initially each student only knows their own pet. When two students gossip, they exchange all the information they have with one another. In a group of  $n$  students, what is the minimum number of conversations it takes until everyone knows everything? This activity is based on the Gossip Protocol for computing as discussed at [https://en.wikipedia.org/wiki/Gossip\\_protocol](https://en.wikipedia.org/wiki/Gossip_protocol)



**Douglas O’Roark** Math Circles of Chicago

### **Bulgarian Solitaire**

Bulgarian Solitaire is a math circle activity that is accessible and hands-on, with a low floor and a high ceiling. It allows for children to engage in important mathematics: deciding on appropriate representations, extending problems and developing an inquiry mindset, employing problem solving heuristics, and making mathematical connections. It can be adapted to work with a range of age groups, and is an excellent activity for the first meeting of a math circle group in order to build buy in and set the stage for collaboration and expectations of engagement.

**Peter Tingley** Loyola University Chicago

### **Frogs and Toads**

I will discuss the frogs and toads problem from the summer 2013 MTCircular magazine (in the “Problem Circle” by Joshua Zucker): Frogs and Toads sit in 5 by 5 grid, with the middle square empty, in the configuration FFFTT FFFTT FF.TT FFFTT FFFTT. They want to switch places. Animals can move one step, or hop over one other animal, but only in the two directions they want to go. Is it possible? I love this as an exercise in pure, creative problem solving. It emphasizes what I think is one of the most important strategies: do a simpler problem first. The obvious simpler problem is the 3 by 3 case, and this is a good thing to do, but the nicest solutions don’t stop there: They find an even simpler problem! The 3 by 1 case! (Well, the simplest is the 1 by 1 case, but that is a bit too trivial). Then there is 5 by 1 case and the 7 by 1 case, and if you do these you really understand some things. Then, if you just squint right, you suddenly see the solution! And not just the solution, but also the solution to generalizations, such as the 9 by 9 case or even the 7 by 9 case. It isn’t so clear what to do with e.g. the 4 by 5 case, but that just means there are still things to talk about. I’ve done this with several different groups of teachers and students, and it has always been fun and led to interesting discussions. It is approachable by everyone, and most people leave feeling they’ve gained some insight. All of which is to say, it is one of my favorite math circle problems!

## **Part B: Friday, July 28, 1:00–3:55 PM, Salon C-1 & C-2**

**Brianna Donaldson** American Institute of Mathematics

### **Great Problems, Great Sessions, Great Circles**

The foundation of every great Math Circle community is inspiring mathematics problems, facilitated in an inspiring way. So, what makes a great Math Circle problem? Once you have a great problem to work with, how can you build a great session around it? Drawing on workshop surveys, discussions, a little bit of research, and plenty of examples and anecdotes, we explore some characteristics of good problems and good sessions in the context of Math Teachers’ Circles. We then consider the art and science of transforming great problems into great sessions, by examining the evolution of a few perennial favorites, including a session based on the card game SET.

**Jonas Meyer** Loras College

**Amanda Matson** Clarke University

### **Exploring Patterns with Technology**

In this talk we will present some of our favorite pattern finding problems in which online and offline computing tools provide ways to better experiment and visualize the problems. Our teacher participants enjoy learning new technology to bring to the classroom, and the technology offers more entry points for challenging problems where patterns are hard to find from limited “by hand” experimentation.

**Gabriella Pinter** University of Wisconsin-Milwaukee

### **Problems with a Twist**

At the UWM Math Circle we strive to engage middle and high school students in open-ended, collaborative problem solving activities. The problems presented here are some of our favorite start- or end-of-session problems that have a little twist, and are surprising or entertaining in some way. We find that these problems grab students’ attention and motivate lively discussions.

**Angela Antonou** University of St. Francis

**Mallory Johnson** University of St. Francis

### **Recognizing Group Structure in Shapes and Images**

In middle and high school mathematics, students have often been exposed to systems of numbers which satisfy certain properties. These properties generally satisfy the definition of a group. In addition, students study symmetries within geometry. In this talk, we present a math circle activity which relates geometry with the abstract concept of a group, resulting in artistic images which are derived from the mathematics.

**Monika Kiss** Saint Leo University  
**Rachel Cunio** Saint Leo University

### **Middle School Math Circle Problems**

Math Circle at Saint Leo University draws middle school students. We will discuss three different topics that are the most popular with students at our circle. The first activity involves M&M's and statistics. The second involves building Platonic as well as Archimedean solids using Magformers and discovering Euler's formula. Last, we will discuss Graph Theory problems that students of all ages enjoy. We will provide instructions and directions on how the activities are run as well as how we guide the students' discovery of the theories.

**J. Lyn Miller** Slippery Rock University

### **Roman Numeral Poker: Hilarity Did Ensue**

This talk describes an activity adapted by our undergraduate Math Club for hosting an on-campus visit by local gifted middle school students. The setting is that of playing a variation on the classic card game poker, with a deck featuring multiple occurrences of the individual Roman digits. Each player's goal is to create a legitimate Roman numeral of highest - or lowest - possible value, sometimes subject to constraints. Detailed rules of the game, implementation, selected feedback from the undergraduate hosts and the middle school participants, variations, and follow-up exercises will be presented.

**Cory Johnson** California State University, San Bernardino  
**Jeremy Aikin** California State University, San Bernardino

### **Mathematical Explorations of Musical Scales**

This spring, the Inland Empire Math Teachers' Circle explored the connections between musical scales and mathematics. In this session, participants constructed a mathematical model of a piano by utilizing the periodicity of the keyboard. Participants were then able to explore different types of musical scales by analyzing the symmetries that were highlighted by the model. Natural questions arose such as "What makes two musical scales the same or different?" "How many scales of a particular type exist?" "Can you find a musical scale in which there are a prescribed number of different scales?" This talk will describe the setup of the model and the discussions that transpired during the MTC session.

**Katie Haymaker** Villanova University

### **Quilting squares in a math circle**

This math circle problem deals with designing quilt-passing patterns for a quilting circle so that all participants can visit with each other in the process. The problem translates into a question about constructing row complete Latin squares. Small cases yield a rich exploration for math circle activities, and there are also several natural extensions.

**Bob Klein** Ohio University

### **Math Unbounded: Math Circles without Borders**

Math Unbounded is a global effort to work with communities to create sustainable math circles. Premised on the Navajo Math Circles Project model for remotely-supported circle mentorship, Math Unbounded works with local stakeholders to recruit and train circle leaders and participants to sustain math circles for students and teachers. Furthermore, Math Unbounded opens the spaces for mathematician collaborators to visit and contribute to the development of circles around the globe. Great and joyful mathematics belongs to all, and engaging communities in collaborative problem solving helps contribute to a more peaceful and just world. Focus areas for development have included Mexico, Panama, Guatemala, Nepal, and the Navajo Nation.

## **Innovative Approaches to Calculus Preparation**

### **Part A: Thursday, July 27, 1:00–3:55 PM, Salon A-3**

**Organizers: Benjamin V.C. Collins** University of Wisconsin-Platteville  
**Jennifer Good** University of Wisconsin-Platteville  
**Nathan Warnberg** University of Wisconsin-La Crosse

Success in most science, technology, engineering and mathematics (STEM) fields relies on calculus, and success in calculus relies on a good foundation in algebra and trigonometry. Although many students get that foundation in high school, many others –including many members of at-risk populations –arrive at college deficient in the basic skills they need for success in calculus. Most of these students have taken some variety of precalculus course, and repeating the same material, only faster, is often not an effective strategy. The aim of this session is for teachers and researchers to share ideas for how to improve the preparation of students for the study of calculus at the college level. We invite scholarly presentations of ideas to improve success in calculus among underprepared students. Such ideas may include, but are not limited to: research on student preparation or improvements in placement; research demonstrating effective instructional strategies, innovative classroom activities or pedagogies; redesigned prerequisite sequences or intervention strategies.

**Britney Hopkins** University of Central Oklahoma  
**Kristi Karber** University of Central Oklahoma

### **Rethinking Calculus 1 at a Metropolitan University**

Preliminary Report. As the number of underprepared Calculus 1 students in our department increases, we have been compelled to revise our traditional approach to placement. In this talk, we discuss our new methodology to place students into more appropriate courses in an effort to fit their evolving needs. This alternate approach involves incorporating a new placement system and adjusting the prerequisites of Calculus 1, which in turn has led to the creation of a new course.

**Teresa Woods** Michigan Technological University

### **Analysis of ALEKS Mathematics Placement Test Data**

In Fall 2014, Michigan Technological University began using an online mathematics placement test provided by ALEKS Corporation to determine the first mathematics course for the majority of incoming students. Three questions related to this implementation were recently investigated: (1) Do the cut scores being used for Precalculus and Calculus I result in the expected pass rates for those courses?, (2) Does the path by which a student becomes eligible for Calculus I –via a single placement test attempt, via multiple placement test attempts with online remediation in between, or by beginning in Precalculus at MTU –influence their likelihood of success in Calculus I?, and (3) Does a mathematical background weak in any particular curriculum area correlate with a lower chance of success in Calculus I? In this talk I will share the results of several statistical analyses aimed at answering these questions.

**Grace E. Cook** Bloomfield College

### **Hits and Misses While Preparing Students for Calculus and Physics**

Bloomfield College is a PBI (Predominantly Black Institute) located in Bloomfield, NJ. Ninety-three percent of the students receive financial aid and more than half of the students are first-generation college students. All Computer Science, Science, and Science/Math Education majors are required to take precalculus and calculus. In the fall of 2013, the faculty of the math department began an ongoing evaluation and reorganization of the first year mathematics program which includes precalculus. Since then, they have eliminated developmental courses, replaced one course, shifted topics between courses, and aligned the precalculus course with Calculus I and Physics I. Precalculus is offered as an enhanced and unenhanced course. The enhanced course meets five days a week, as opposed to three, with the additional two meetings being mandatory support with a mathematics specialist and an embedded student tutor. Fall 2016 was the first semester of the new alignment with Calculus I and Physics I. The following topics were added to the precalculus course based on feedback from calculus and physics instructors: polar coordinates, vectors, and limits. Basic functions and quadratics were shifted to the prerequisite course. In this session, I will share the successes and impediments that were faced in the process. I will discuss placement criteria, structure of the courses, and signs of improvement (or not!) in our calculus students.

**John Harland** Palomar College  
**Frank Savina** The Charles A. Dana Center at the University of Texas at Austin

### **Modernizing the Pathway to Calculus: Lessons Learned at Palomar College**

The Charles A. Dana Center at the University of Texas at Austin has designed a mathematics pathway that prepares students at the intermediate algebra level to succeed in college-level calculus. This innovative pathway to Calculus spans two semesters, and was designed by leading education researchers and veteran two-year college mathematics faculty. The curriculum takes into account the latest research on challenges students face in learning Calculus, and employs intentional strategies to develop students' skills as learners. This pathway has been taught over three semesters at several colleges and universities throughout the country. Palomar College in San Marcos, California is seeing promising initial results. Presenters will share content outcomes, design features, pedagogical techniques, and lessons learned from implementing this pathway at Palomar College.

**Emily Gismervig** University of Washington Bothell

### **Redesigning the Pathway to Calculus**

The University of Washington Bothell has been undergoing a three year long process of redesigning the prerequisite sequence to our calculus courses. We have developed a three quarter sequence leading from Quantway, a quantitative reasoning course containing intermediate algebra mathematics, to our two calculus courses. Our curriculum now emphasizes quantitative reasoning and application alongside a focus on developing an understanding of algebraic structure and skill in algebraic manipulation. We have woven growth mindset activities and other non-cognitive interventions into the curriculum and adopted active learning in every course. During this talk, I will outline our new curriculum, discuss how we trained full and part-time faculty in the new curriculum and pedagogy, present evidence of the effectiveness of the redesigned curriculum and talk about our current efforts to refine these courses.

**Ryan Stufflebeam** Transylvania University

### **Effects of a Change in Calculus Placement: A Four-Year Study**

In Fall 2013, the mathematics program at Transylvania University changed the prerequisites for Calculus I in an effort to improve retention in the course. At the same time, faculty members began to implement innovative pedagogical techniques - instructional videos, classroom flipping, and online tutorials - in Precalculus courses in hopes of better preparing students to tackle Calculus I. This talk will discuss the merits of the utilized pedagogical techniques and present the findings resulting from a study of student data in these courses since the change in placement occurred.

**Linda Becerra** UH-Downtown

**Jeong-Mi Yoon** UH-Downtown

### **Course Innovation Experiences for Calculus I Teaching at UHD Teaching at UHD**

Title: Course Innovation Experiences for Calculus I Teaching at UHD Calculus I is a substantial undergraduate course for all sciences and engineering majors. It is considered a barrier course because of a low student success rate each semester. We have implemented a university supported Course Innovation Proposal to improve student learning. The proposal initiated in the fall of 2014 and will finish on July of 2017. In this talk, I will present our program and make an integrated report based on our experience.

**Steven Schlicker** Grand Valley State University

**Ted Sundstrom** Grand Valley State University

### **Active Learning in Trigonometry**

Several studies provide extensive empirical support for active learning in STEM disciplines and mathematics. Active learning increases content knowledge, critical thinking and problem-solving abilities, and positive attitudes and enthusiasm towards learning in comparison to traditional lecture. There is also a growing body of evidence showing that active learning differentially benefits students of color and/or students from disadvantaged backgrounds and/or women in male-dominated fields. We have written a text, available as a free pdf file, whose goal is to actively involve students in their learning of trigonometry. The text incorporates Beginning Activities and Progress Checks in each section. Students complete Beginning Activities before class to review prior mathematical work or to introduce new concepts and definitions. Progress Checks are used in class to help students determine if they are understanding the material. In addition, the text contains links to Interactive GeoGebra applets that are designed to help students visually develop their intuition for concepts that appear in the course. We will describe how we use Beginning Activities, Progress Checks, and GeoGebra applets throughout the semester to help improve student success in trigonometry.

**Mile Krajcevski** University of South Florida

**Deniz Kardes** University of South Florida

### **The Use of Drawing as a Cognitive Tool in Undergraduate Mathematics**

In undergraduate mathematics, images and diagrams are tools that support conceptual understanding of mathematical ideas and they are used as a guide for students' intuition. Freehand sketching is a natural way to illustrate mathematical notions during regular classroom sessions. In a previous research we have investigated students' inclination to use visualization when solving problems in a Calculus III course. Our findings indicate that reluctance to use visualization as a tool in problem-solving strategies is not correlated with students' ability to sketch, but rather to the predominance of the analytic way of presenting mathematical statements in the curriculum. We argue that hesitancy towards using visual arguments adds to the perceived bias of visualizing mathematical statements, and deprives students of a powerful cognitive tool. Many elements of mathematical modeling are included in freehand drawing of mathematical objects (lines, polygons, curves, planes or spheres) and engaging students in these activities can lead to advanced conceptual understanding of the represented objects. We have developed visual strategies in problem solving situations for a variety of undergraduate mathematics courses, and with this presentation we will demonstrate how one can obtain new knowledge from an accurate mathematical drawing. We show how to demonstrate the advantages of visual thinking in mathematics through a set of appropriately chosen examples from undergraduate mathematics curriculum.

## **Part B: Friday, July 28, 8:30–11:05 AM, Salon A-3**

**Keith Carlson** University of Central Florida

### **Innovative Methods in the Teaching of Pre-Calculus in an Emporium Setup**

There is a direct correlation of student knowledge of pre-calculus algebra and trigonometry and their success in calculus. Failure in calculus essentially prevents the completion of studies in a wide variety of subjects. At the University of Central Florida we have been concerned with the DWF numbers each year. We have found that teaching certain topics in a more relaxed manner and emphasizing additional practice of problem solving in an Emporium Model of instruction in a computer lab with dedicated software has yielded a greater success rate. In this talk we will outline the details of our work.

**Gary Hagerty** Boise State University

### **Everyone's a Math Major Creates Optimal STEM Mindset**

Over the past decade Boise State University has increased its passing rates in courses leading to calculus from about 50% to 75%. This has doubled the number of students enrolling in calculus from precalculus and about doubled our calculus enrollment without a significant change in the number of entering first time full time freshman. Once enrolled in calculus, students from precalculus compare favorably to students enrolling directly into calculus from high school, resulting in 55% of Boise State Universities STEM graduates beginning their university experience in math courses leading to calculus. To achieve these results, several mindsets need to be reconstructed to a mindset which believes that if every student is given the opportunity to be successful in mathematics they can succeed. To create the successful mindset environment, we must change both the structure of the institution of mathematics education and the students understanding of the mindset of a successful mathematics student. This discussion will look at major mindset changes in regards to the institution of mathematics education and in regards to how students see themselves as being successful. This mindset change lays the groundwork necessary for optimal change in any reform in mathematics education environment.

**Daniel E. Otero** Xavier University

### **Preparing to study trigonometry through primary historical sources**

The author is one of seven PIs on an NSF grant project (grant no. 1523753), Collaborative Research: Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS). This talk will present a four-week classroom module, one of over a dozen Primary Source Projects (PSPs) currently available through TRIUMPHS, which the author designed to introduce and motivate the study of trigonometry in a precalculus course. Students are asked to read excerpts of historical documents (in English translation) and then work problems based on these readings; the goal is to provide a context and motivation for study of trigonometry. The PSP asks students to consider the origins of the subject in early mathematical astronomy and the ancient technology of timekeeping. Students encounter this development in works of the Greek astronomer-mathematicians Hipparchus and Ptolemy, the sixth century Hindu priest-astronomer Varāhamihira, the eleventh century Central Asian polymath al-Bīrūnī, and the sixteenth century European geometer Regiomontanus. Also to be discussed is the pedagogical benefits provided by direct immersion of students in these primary sources.

**Alan E. O'Bryan** Arizona State University

### **Developing Students' Understanding of Exponential Growth: A Research-Based Solution**

Understanding exponential growth and the prerequisite concepts of percent change and comparisons are critical to the study of calculus and differential equations as well as vocations that utilize practical applications of mathematics, such as nursing and business. In testing thousands of students using the validated Precalculus Concept Assessment (PCA) it is clear that most students completing precalculus courses have very weak understandings of exponential growth. For example, only 35% of college precalculus students at the end of their courses could identify the difference between the growth rates for the functions  $p(t) = 7(2)^t$  and  $p(t) = 7(3)^t$  (Research and Innovation in Mathematics and Science Education, 2007). In my presentation I will share learning trajectories, activities, and interactive applets we have designed to support and study students' learning of concepts related to exponential growth in college algebra and precalculus informed by mathematics education research on quantitative and covariational reasoning.

**Whitney George** University of Wisconsin-La Crosse

**Tushar Das** University of Wisconsin-La Crosse

**Nathan Warnberg** University of Wisconsin-La Crosse

### **Directionality of the Equals Sign**

The equality symbol is a fundamental object in all levels of mathematics. There is an extensive body of research literature investigating student misconceptions around both operational methods involving, and conceptual understanding of the equality symbol. Much of this research has focused on K-12 education. However, as students progress into college, many of their misconceptions of the equality sign persist. This is an especially unwelcome phenomenon in the case of remedial math classes like College Algebra. In this talk, we look at the directional flexibility of the equals sign in College Algebra students at a medium sized public university. In particular, we investigate how the directional flexibility of the equals sign effects how students work with logarithmic properties.

**Paul Howard** Oklahoma Christian University

### **A Unique Application of the Moore Method of Teaching College Precalculus**

Having been influenced by instruction that hides the messiness of doing mathematics, students enter college precalculus with a variety of philosophical beliefs about mathematics and what it means to do mathematics. Textbooks and instructors typically present polished mathematics and not the messiness that must occur behind the scenes. Yet doing mathematics is a messy process. Typical homework assignments are not designed to move students from the messy process of doing mathematics toward the clear presentation of mathematical ideas. Inspired by a Modified Moore Method of teaching I have developed a lab style workbook designed to encourage clear presentation of mathematics as well as reflection on mathematical ideas. A theoretical framework to examine the impact on philosophical belief systems will also be discussed.

**G. Gerard Wojnar** Frostburg State University

### **Mathematical Maturity: Dualities, Domination, Approximation, Fixed Points, Slopes, etc.– Concepts & Notations in Pre-Calculus**

Pre-Calculus offers many issues that may be framed to develop students' mathematical maturity, language, and frames of mind. Many dualities exist: The comparison of different monomial powers w.r.t. end behavior and w.r.t. behavior in a neighborhood of the y-intercept; this leads to attention to the linear & constant terms as providing the equation of the tangent line there. Issues of multiplicative “big zone” & “small zone” in the comparison of powers and roots, including limiting cases as the index approaches infinity. Fixed points are encountered by transforming a function by taking its absolute value, or taking its reciprocal (especially useful when addressing reciprocal trig functions and rational functions), or taking its square. These discussions again involve “big zone” & “small zone”, as well as issues of local approximation; e.g. the reciprocal graph enjoys a local minimum (resp. maximum) when the original graph had a local maximum (resp. minimum); e.g. the square of a multiplicatively “big” value is greater than its original value, while the square of a multiplicatively “small” value is less than its original value (especially useful when considering graphs of the squares of cosine & sine); e.g. a graph which starts out locally linear at a root will have its squared function be locally parabolic at said root (again useful when considering the squares of cosine & sine). Domination vocabulary is also appropriate (1) when contrasting exponential and polynomial functions, and (2) when analyzing details of end behavior of a rational function analyzed in terms of its quotient plus remainder fraction representation.

**Benjamin V.C. Collins** University of Wisconsin-Platteville

### **Flipping the Precalculus Classroom**

Can a flipped classroom improve student performance in Precalculus? Can a flipped classroom in Precalculus improve student performance in subsequent courses? In this project, students were assigned to watch expository videos created specifically for the class. The videos were based on the instructor's regular class notes. Class time was used to work on exercises in groups of 3-4, with the instructor available for help. Each worksheet started off with an in-class activity, which was normally somewhat more involved than a typical homework problem. When students were done with the in-class activity, they worked together on homework problems. Homework problems were standard for this type of course, drawn from a number of sources. At the end of the course, all students in Precalculus took a common final exam. Copies were made of the final for all students in the treatment group, as well as a similar number of students randomly selected from other sections of Precalculus. A student research assistant blindly scored a selection of problems from the finals. We also intend to track these students through calculus, to see if there is a difference in performance between the treatment and comparison groups. This is a preliminary report.

## **Enrichment, Experiences, and Examples with Modeling in Differential Equations Courses**

**Thursday, July 27, 1:00–4:15 PM, Salon A-4**

**Organizers: Brian Winkel** SIMIODE Director

**Ellen Swanson** Centre College

**Chris McCarthy** Borough of Manhattan Community College, CUNY

This session features talks in which colleagues who are using mathematical modeling to motivate the learning of differential equations share their experiences and mathematical offerings. Hopefully, others will be able to incorporate or build on these activities in their own course. We are interested in talks which feature real data (either collected or taken from the literature, or found online) and a full modeling process for students, i.e. stating assumptions, making identifications, creating a differential equation model, developing solution strategies, performing parameter estimations, rendering model validation, and iterating this process. Some evidence of the success of individual approaches should be offered. Presenters are encouraged to submit articles based on their presentation for consideration in a special issue of PRIMUS entitled, *A Modeling First Approach to Teaching Differential Equations*.

**Brian Winkel** Emeritus, US Military Academy, West Point NY

### **What Can You Learn from Doing Modeling in a Differential Equations Course?**

When a teacher attempts to enrich a differential equations course by introducing modeling to motivate and reinforce learning the teacher learns many, many things. First and foremost, comes new vocabulary, e.g., pharmacokinetics and glottochronology. Second, there is an introduction to whole new fields, such as retinal surgery, epidemiology, SCUBA diving, vibration analyses, paleobiology, and suspension systems. Third, one finds movement into new areas such as parameter estimation, methods for comparing models, and laboratories for collecting data, as well as connections with old areas, such as stochastic processes. We take the talk attendee on the journey we have experienced for over 45 years of using modeling in teaching differential equations and other subjects.



**Rosemary Farley**    Manhattan College  
**Patrice Tiffany**    Manhattan College

### **A Modeling First Approach in a Tradition Differential Equations Class**

Our differential equations course is required of every student in the School of Engineering and some students in the School of Science. As a required course, there is a syllabus with topics that have to be covered in preparation for a common cumulative final. All the traditional methods of solving differential equations by hand must be covered. This spring semester, there are nine differential equations classes and two of them are being taught with the modeling first approach advocated by SIMIODE. Our presentation will explain how we incorporate the modeling first approach into the differential equations classroom while still managing to cover the traditional topics required. We discovered that the modeling first approach works very well when students use a computer algebra system to solve previously inaccessible problems and get answers that make sense. We will provide several examples of modeling problems from the SIMIODE scenarios that we adapted to our needs and time restrictions. We will report on common final results and discuss some of the expected, and rather unexpected, benefits of this approach.

**Ellen Swanson**    Centre College

### **Inheritance: How Much Money did I Really Receive?**

An inherited Roth IRA requires the withdrawal of funds based on the age of the owner at the time of death and the current age of the beneficiary. There are online calculators that inform you of the amount of money you will receive each year. However, the factors that play into that amount are not readily available. Using the data from these online calculators, we create a differential equations model for the amount of money in the account. This activity can be used to motivate the method of integrating factors.

**Jean Marie Linhart**    Central Washington University

### **Human Population Modeling**

Two common models for populations are the exponential growth model and the logistic growth model. A third, related, differential equations model is the Gompertz growth model. All of these can be used to model human populations. Data on human populations is readily available on the web via the US Census, census data from other countries, and, additionally, historic world population data is available, allowing the question of overpopulation to be addressed. Modeling considerations for human populations are readily available for students to think of and think about. Students can start from the models and modeling assumptions and predict which should perform best, and then examine whether this is, in fact, what happens.  $R^2$  values give a measure of goodness of fit, and this transitions nicely into discussion of adjusted  $R^2$  which takes the number of parameters in the model into account. The carrying capacity or limiting population values gives another way to evaluate the performance of the models. Some datasets for cities or states are not well represented by any of these models. Students can consider different ways that model parameters can be estimated from the data, and why you might use one method over another. Students can also easily identify many reasons why these models are inaccurate, and offer substantive criticism. Students also can often see how historic events such as the 3/5ths Compromise and World War 2 affect human populations and the data we associate with them, and this transfer of knowledge is a rich experience for many.

**Danilo Diedrichs**    Wheaton College

### **Using Harvesting Models to Teach Modeling Techniques, Bifurcation Analysis, and Solution Methods in Ordinary Differential Equations**

Harvesting models based on ordinary differential equations are commonly used in fisheries and wildlife management to model the evolution of a population depleted by harvest mortality. We present a series of projects based on fishery harvesting models to teach the application of theoretical concepts learned in differential equations to scenarios encountered in real fisheries. These projects require a thorough understanding of simplifying assumptions inherent in various models, as well as a qualitative analysis of phase portraits, bifurcations, and stability of steady states. Parameters are estimated and equations are solved either analytically or numerically. Students learn to respond to a professional request from a fishery in the form of a scientific report, which requires organizing and communicating assumptions, models, solution methods, results, and a final recommendation with clarity and professionalism.

**Therese Shelton**    Southwestern University  
**Theresa Laurent**    St. Louis College of Pharmacy  
**Beulah Agyemang-Barimah**    Southwestern University

### **Pharmacokinetic Models for Active Learning**

Ordinary differential equations models in pharmacokinetics can range from very basic to much more complex. In this session we will discuss several modules with an emphasis on mathematical modeling. These activities are designed to be student centered, active and inquiry-oriented. Students start by exploring the development of a model for a zero order process of drug absorption and a model for order one drug elimination. Students can expand on this information to develop models that are used to describe drug concentrations when medications are administered by a patch, by intravenous drip, orally, or by injections.

**Chris McCarthy** Borough of Manhattan Community College  
**Jieying Li** Borough of Manhattan Community College  
**Jie Lan** Borough of Manhattan Community College

### **Simulation of Adsorption Models—Instantaneous & Non-Instantaneous Mixing**

Title: Simulation of Adsorption Models—Instantaneous and Non-Instantaneous Mixing Abstract: Our research involved developing and simulating models of adsorption-like processes. Adsorption is the chemical phenomena where molecules adhere to a surface. We developed a “particles in a box with one sticky side” toy-model of this, with the particles’ motion being represented as random walks on an  $N$  by  $M$  integer lattice. We investigated how instantaneous and non-instantaneous mixing effect the adhesion rate. We showed that this process can be modeled by Ordinary Differential Equations and Probability Generating Functions. However, in the non-instantaneous mixing case, a more accurate model makes use of the diffusion partial differential equation. We show how to calculate these models’ constants from theory (e.g., the diffusion coefficient) and how to verify these models by applying non-linear regression to data from their computer simulations.

**Michael Karls** Ball State University

### **Verifying Two-Dimensional Groundwater Flow Models**

In 2009 I began a series of student research projects aimed at validating classic groundwater flow models that involve the two-dimensional heat equation. The goals of these projects are to find ways to collect water well-head data, construct appropriate models, and compare the models to the data. I will report on the progress made, including both the challenges encountered and the results obtained.

**James S. Sochacki** James Madison University  
**Roger J. Thelwell** James Madison University  
**Anthony Tongen** James Madison University

### **An Unstoppable Force Meets an Immovable Object**

The spring-mass system is commonly used not only to discuss the theoretical solution of second order differential equations, but also to understand the underlying physics. This article will expose students to solving second order differential equations using three common solution techniques; characteristic equation, Laplace transform, and power series. The method of variation of parameters will also be demonstrated when external forces are applied to the system, which provides solutions that are interesting and non-intuitive. Finally, the students will learn how these differential equations with their corresponding initial conditions yield conserved quantities. Since simple electric circuits also obey a similar second order differential equation, students will be asked to use the mathematics they learned for the spring-mass system to explain these circuits, including the nonhomogeneous case with external voltages.

**Reza O. Abbasian** Texas Lutheran University  
**John T. Sieben** Texas Lutheran University

### **Fine Tuning Torricelli’s Law: Multiple Approaches to Estimating the Discharge Coefficient**

The application of Torricelli’s Law in determining the theoretical flow rate of water being discharged from a tank is a typical topic in an introductory differential equations course. The theoretical discharge rate generally is considerably different than the observed discharged rate. The ratio of the two rates is expressed as the discharge coefficient which is a function of several variables. Common practice is to have students gather data and use their experimental set up to establish initial conditions for the differential equation and to build a particular solution using a given value of the discharge coefficient. In our presentation, will examine two elementary methods for determining the discharge coefficient, and two more advanced methods, the second of which models the discharge coefficient as a function of pressure as well as the radius of the orifice. We compare our results to the established analytical results. The procedure and results can easily be and be used as a teaching tool in a differential equations class introducing modeling, error considerations, and use of technology. Key Words: Torricelli’s Law, Differential Equations, Optimization, Intended audience: Undergraduate mathematics and statistics instructors , Themes: Technology- based Teaching, use of multiple technology tools.

## **Encouraging Effective Teaching Innovation**

**Part A: Thursday, July 27, 1:00–4:55 PM, Salon C-4**

**Organizers: Susan B. Crook** Loras College  
**David Failing** Lewis University  
**Russ Goodman** Central College

Faculty are eager to offer activities in the classroom that foster student success, but many are not formally trained in pedagogy. This session will consist of presentations of demonstrably effective and innovative classroom techniques. Talks will address the reasoning behind, design, and implementation of resources or activities. While these activities may be whole course techniques, we also seek presentations of drop-in activities to bolster student learning and reflection in any course. Techniques do not have to be original to the presenter, but sources

must be credited and evidence of success (or failure and redesign) is expected. To maximize the session's usefulness, a Google Drive folder will be created and shared as a repository for the speakers' slides and supplementary materials.

**Mary Shepherd** Northwest Missouri State University

### **Reading Guides Plus Active Learning**

Instead of having students watch video(s) for a flipped classroom, students in my classes have been asked to read the assigned textbook section and complete a reading guide before coming to class. I have done this in Precalculus, Calculus 1 and 2. Class time starts with a Quickie Quiz that reviews (or introduces to those who have not read beforehand) the basic ideas from the section to be covered. Class time is then spent in 3-5 activities that (hopefully) allow students to wrestle in groups or pairs with the concepts and "usual" points of confusion of the material. During the Fall 2016 semester, I worked with another faculty member to implement my reading guides and activities in her precalculus class at the same time I was using them in my own precalculus class. We had a new textbook and the reading guides were all rewritten and "new" activities were designed. A pre and post concept tests (the older PCA) were given in both classes. You will experience a little of what this type of class is like, hear about some successful and some less than successful activities, and see the results of the pre/post concept test.

**Jenna P. Carpenter** Campbell University, Buses Creek, NC

### **Using Think-Pair-Share to Generate Insightful Student Questions**

Think-Pair-Share (TPS) activities are an example of classroom assessment techniques (CATs), a collection of formative assessments that are quick and easy to use in the classroom and which provide valuable feedback to both student and instructor regarding student mastery of course content. This talk will examine an approach for using the TPS format to generate student questions about challenging content. The TPS structure provides a "safe" environment for students to ask questions; requires no preparation, resources or classroom set-up; leads students to generate strong and thoughtful questions; helps instructors quickly zoom in on misconceptions; and can be done in 5-10 minutes of class time, from start to finish. Research on CATs show that they increase student engagement, have a particularly positive impact on low achievers and students with learning challenges, reduce time required for students to grasp content, develop critical analysis skills in students, and provide valuable information on student learning with minimal time investment on the part of faculty.

**Rebecca Coulson** Rutgers University  
**Alejandro Ginory** Rutgers University

### **Visualizing Mathematical Reasoning: A Diagrammatic Approach**

Getting students accustomed to reading, understanding, and writing proofs is one of the more challenging aspects in college-level mathematics education. A major reason is that the student must learn a precise language (with accompanying symbols) and, at the same time, try to develop formal mathematical reasoning. Over the last year, we have been working to address this problem by employing a tool we are calling "proof maps," in which the chain of reasoning of a proof is broken down diagrammatically. The exercise of creating a proof map forces the writer to examine the inferences used in the proof, and makes apparent the non-linear structure of many proofs which can often get obscured when written. We introduced this concept in an "Introduction to Mathematical Reasoning" course at Rutgers, and in this talk we will discuss our observations.

**Emilie Hancock** University of Northern Colorado  
**Gulden Karakok** University of Northern Colorado

### **Using Portfolio Problems to Develop Metacognitive Thinking During Problem Solving**

Research in mathematical problem solving has long identified metacognition as an essential component of the problem-solving process. As students learn new mathematical concepts and problem-solving strategies, they should also learn how to manage and regulate the application of this new knowledge. Thus, providing students authentic problem-solving experiences necessitates a focus on metacognitive thinking. In particular, shifting, at least to some degree, the responsibility of monitoring and control from teacher to student. In this talk, I discuss the use of portfolio problems to foster a mathematical "point of view" and metacognitive tendencies in a first-year undergraduate math course. On multiple occasions, students worked together in small groups on a problem typically more open-ended than usual course work, with key mathematical ideas related to current content. During class, each group member used a different colored pen to identify individual contributions to the group's collective scratch work. Students submitted individual portfolio write-ups, providing a detailed documentation of their judgement and decision-making processes during the problem-solving attempt. Using classroom examples and sample student work, I describe how the classroom culture, problem-solving sessions, and portfolio write-ups contributed to metacognitive thinking specifically, and a mathematical point of view more generally.

**Lewis Ludwig** Denison University

### **Applying Cognitive Psychology in the Mathematics Classroom**

In this presentation, we will discuss three low-cost techniques based on studies from cognitive psychology that can help improve student learning in the classroom. The three general principles considered are 1. distributed practice and interleaving of material 2. testing to improve retention of knowledge 3. exploratory questioning Each technique will be explained via a concrete classroom example along with the studies supporting these findings. This presentation is intended for a general audience.

**John McGowan** Texthelp

### **UDL Math, What is it and Why I Need to Know to Engage and Empower my Students**

Creating digital math used to be a difficult process. It can be complicated to type and frustrating for students and teachers alike. Universal Design for Learning (UDL) moves beyond one method to respond and empowers students to create and consume math in the method that they prefer (not just the method the teacher prefers). We will focus on how you can bring UDL into your classroom by making math digital. I will showcase UDL techniques using EquatIO and other apps, enhancing ways students can respond to math problems digitally and collaboratively.

**Ksenyia Fuhrman** Milwaukee School of Engineering  
**Anthony van Groningen** Milwaukee School of Engineering

### **Can Typesetting Mathematical Notation Improve Student Learning?**

We believe the use of appropriate mathematical notation and style positively impacts student learning and conceptual understanding. To encourage proper use of notation, we required students in introductory calculus to typeset their solutions to finding a derivative using the limit definition. Our intention was to examine how the constraints imposed on syntax by the software may help develop a critical eye to the validity of their written work, particularly when using the equal sign or the limit operator. This presentation will report on the results of our study and discuss some of the beneficial and negative consequences of using typesetting software such as Microsoft Word's Equation Editor.

**Trefor Bazett** University of Cincinnati

### **TA for the Day: Student Leadership in Flipped Calculus Classes**

A key feature in our flipped calculus classes is a "TA for the day" experience. A couple students are expected to become content experts on the material of the day, working with the professor prior to class to ensure mastery and confidence. In class, they work much as a teaching assistant would, helping groups master the content. TA for the day accomplishes several goals. It gives students a leadership opportunity that helps develop their professional skills. It aids the logistical problem of flipped classrooms for larger class sizes and frees up the instructor to spend more time interacting with small groups. It creates a sense of community and a spirit of collaboration in the classroom which has many spillover benefits. It helps to demonstrate to students the intricacies in learning mathematics, and makes them more effective learners. Finally, the preparation stage means the instructor gets the opportunity to have a discussion with every student in the class, which can lead to a more positive relationship with the students. Student feedback has been overwhelmingly positive, and the 'flipped class with TA for the day' model substantially outperforms traditional lecture sections at the University of Cincinnati.

**Angie Hodge** University of Nebraska Omaha

### **TACTivities: Learning Tools for the Math Classroom**

This session will describe how tactile learning activities, namely TACTivities, can be designed in a manner that they help both teach new concepts to students and practice newly learned concepts. Series are used as an example to demonstrate how this is done; however, the teaching ideas can be modified to most algebra and calculus concepts. Participants will also learn how to create their own TACTivities and tips for using them successfully in a course. These activities are most often done in groups of 2-4 students, but they can also be done individually. Modifications for different class sizes and types of classrooms will also be provided.

**Roger Wolbert** Edinboro University of Pennsylvania

### **The Teaching and Learning of Inverses**

Students in introductory mathematics classes often struggle with the concept of inverse relationships and functions (domains, ranges, one-to-oneness, interchanging of x- and y-values, etc.). In this session, participants will be engaged in a classroom activity found to be brief and effective at addressing the common misconceptions students have with inverses and at bolstering their confidence in finding inverses both graphically and algebraically. Following the activity, participants will be led through a discussion on how to reinforce the concept of inverses, and how the ideas from this activity can carry over to more advanced functions and their inverses.

**William Gryc** Muhlenberg College

### **Pedagogy for Poets: Guided-Inquiry for Groups in a Course of Mathematics for the Liberal Arts**

On the first day of a mathematics for the liberal arts course there is a lot of anxiety in the room. There is the anxiety of students, many of whom would rather take a course in any other subject. But the professor also has to worry about students who are uninterested in the subject matter of the course and whose instinctual reaction to any math problem is “I don’t get it.” In this paper we will discuss the author’s approach to teaching such a class by centering all in-class learning around guided-inquiry group-work. In particular, we will discuss the implementation of guided-inquiry in class, how guided-inquiry furthered the course learning goals, and how students responded to its use and performed in the course.

**Meri Hughes** University of Mary Hardin-Baylor

### **History of Mathematics via London, Paris, and CERN**

Partly because it provides an appropriate avenue for study abroad, History of Mathematics remains a part of the Mary Hardin-Baylor math curriculum. After having run the course twice, a blueprint is presented for designing a short-term global experience appropriate for math, engineering, computer science, and mid-level math education majors. Successes and challenges are discussed, including using guided-inquiry learning to reach a broad level of mathematical abilities, and integrating into the course objectives sites of historical mathematical interest around Paris, London, and CERN.

## **Part B: Friday, July 28, 8:30–11:45 AM, Salon C-4**

**Lina Wu** Borough of Manhattan Community College

### **Using The Maple Technology To Connect Mathematics And Art**

The presenter is interested in sharing her research experiences on how to use an educational technology of Maple Software as a bridge to connect mathematics and art in her cross- disciplinary pedagogy. A sequence of Maple projects in geometric art designs such as “2017 Olympic Sports Art” , “2016 Culture Art” , “2015 Geometric Abstract Art” , “2014 Funny Faces Images” , and “2013 Polar Art” has been completed in pilot Calculus teaching at Borough of Manhattan Community College. Visualizing beauty of math-related artwork created by computer-generated images in Maple has ignited students’ passion to learn. Demonstrating animated graphic images with simplified symbolic calculation on the use of Maple has enhanced students’ conceptual understanding ability. Practicing on Maple projects in Math Labs has increased students’ problem-solving skill. Projects descriptions and students’ work will be presented. These pilot Calculus sequence courses were supported by Grant of Minority Science Engineering Improvement Program through the U.S. Department of Education and Teaching Fund of Enhanced Learning in Classroom.

**Ranthyony A.C. Edmonds** University of Iowa

### **A Partially Flipped Model for a College Trigonometry Course**

Flipped instruction has garnered a lot of attention as an alternative to lecture based instruction. Common pitfalls include resistance from instructors due to the perceived amount of time to create instructional videos and materials, and from students due the amount of time learning outside of class. Partially flipped instruction addresses these concerns by incorporating both traditional and out of class instruction. It can also alleviate the amount of time spent on additional materials by instructors, while still holding students accountable for their own learning outside of class. This talk describes a partially flipped model developed for a college trigonometry course in the Spring of 2017. Motivation and implementation of the design will be described in detail, but it initially stemmed from the desire to incorporate Universal Design for Learning Principles (UDL) into a mathematical setting. The main features of this model are instructional videos, created with Doceri for iPad, viewed outside of class once a week by students, coupled with a short assessment based on that instruction. The following ‘flipped’ period involved individual and/or group activities expanding upon concepts introduced in the videos. Activities ranged from worksheets to team inquiry-based learning to games. Canvas by Instructure was used heavily throughout the course. Quantitative data with regards to assessments will be shared as well as the results of a qualitative survey given to students about their experience in the course. Possible extensions of this model for singular activities and/or other introductory courses will also be discussed, along with reflections on the successes and shortcomings of the model.

**Liz Bouzarth** Furman University  
**John Harris** Furman University  
**Kevin Hutson** Furman University

### **Back to the Drawing Board: Reshaping a Math Course in Walt Disney World**

In an effort to create a course that captured an authentic, real-world experience for students, we created a three-week, May-semester course called Math and the Mouse: Explorations of Mathematics and Science in Walt Disney World. In this class, students experience Walt Disney World through an immersive mathematical lens by spending three weeks in Florida investigating problems typical of managing a large-scale resort and theme park, often involving scheduling, logistics, and queuing. The course incorporates material from a variety of areas including math modeling, optimization, probability, and statistics, and it faces challenges similar to many courses that are project-based, lack a textbook, and serve students with diverse mathematical backgrounds. In this talk, we give examples of projects and experiences in the course that have been intentionally redesigned between our first and second course offerings to bolster students' participation and understanding in this nontraditional learning environment. We highlight the introduction of Lightboard lectures and the redesign of two projects to make the interactions with professionals from Disney and from [touringplans.com](http://touringplans.com) more meaningful. The course involves a reflective written component, and we give evidence of improvement based on students' reflective feedback, feedback from the professionals with whom they interact, and our professional observations.

**Senan Hayes** Western CT State University  
**Alicia Maloney** Western CT State University

### **Using Exam Wrappers to Improve Student Outcomes Exam Wrappers to Improve Student Outcomes**

In this talk the results of a study using Exam Wrappers in a first year math course will be discussed. Exam Wrappers were created to help students use returned exams as a tool to identify their own strengths and weaknesses, reflect on their study strategies, consider the time they used to prepare for an exam, and characterize their errors made during an exam. The study was designed to determine if Exam Wrappers would give insight to student behaviors regarding exam results and if a student used an exam outside of knowing the actual result. The research methods used to gather data for this study were quantitative and qualitative to give a holistic view. Details of the study's design and data collection protocol will be shared and the results of the Exam Wrapper study will be given at the presentation.

**Matthew J. Prudente** Saint Vincent College

### **A Team-Based Learning Approach to Linear Algebra**

Team-Based Learning (TBL) is a collaborative learning strategy developed in the late 1970s by Larry Michaelsen as a way to engage large classes. Here, the author introduces the main components of TBL and adaptations made in its implementation to accommodate a Linear Algebra class. Then, the author discusses how this enhances his college's liberal arts mission and compares the results of a TBL class against a lecture-majority version of Linear Algebra.

**Feryal Alayont** Grand Valley State University

### **Using Case Scenarios in Teaching Discrete Mathematics**

The various counting techniques seen in Discrete Mathematics provide the perfect background for problems with multiple solution methods and also for problem solutions with difficult to identify issues. This presentation will describe how to use case scenarios of analyzing and comparing multiple student solutions as an instruction method to improve student understanding of counting techniques and flexibility in using these techniques.

**Andrew J. Miller** Belmont University

### **The Pythagorean Theorem: A Gateway to Proofs**

Reading, evaluating, and appreciating proofs is central to a deep understanding of mathematics. Most of our students are introduced to proofs in high school geometry classes, but this experience often leaves them intimidated or, worse, bored by the experience. We present a unit on the Pythagorean Theorem intended to reintroduce general mathematics students to the joys and importance of proofs. The design of the unit was informed by science on effective learning practices, including active learning, connecting new information to old, creating a "need to know," and the value of reflection to absorb new ideas.

**Jacqueline Jensen-Vallin** Lamar University

### **Encouraging Metacognition Through Reflection Papers**

Students who think about how they are learning and forge connections between topics they are studying will understand material better. We can encourage (force?) them to reflect on what they are learning and to make connections through weekly directed writing assignments. We will discuss a possible prompt for these assignments, provide some examples, and discuss upcoming changes to implementation.



**Amanda Harsy**    Lewis University

### **Comparing Mastery-based and Traditional Assessment in Calculus II Courses**

As educators, it is important to recognize that our assessment methods affect student attitudes. If we want students to learn from their mistakes and counteract a fixed-mindset of learning, perhaps we should look at what we incentivize in the classroom. One way that professors are attempting to counteract math anxiety, poor STEM retention, and a fixed-mindset of learning is through using and researching a new assessment model called “mastery-based testing” (MBT). In MBT, students are given problems in which they can only receive full credit for the problem after they demonstrate mastery of the concept being tested. Each test includes similar questions over the same concepts from previous tests which allows students who have not mastered an idea to retest and reevaluate old concepts. In this talk, we will present the results of qualitative and quantitative data from a two-year study comparing MBT and traditional assessments in six Calculus II classes.

**Austin Mohr**    Nebraska Wesleyan University

### **Using Points-Free Grading to Promote Perseverance**

In a points-free grading scheme, all assignments are regarded as either acceptable or unacceptable as opposed to receiving a numerical score corresponding to partial credit. By setting a clear, discrete level at which credit is obtained, students are encouraged to persevere through challenging work in instances where they might have otherwise accepted partial credit. The safety net of partial credit is replaced with opportunities for revision of unacceptable work, allowing students to respond to instructor feedback on the topics where it is most useful. I will describe the framework I used for my most recent points-free course (second-semester Calculus) and offer reflections on its effectiveness.

## **Recreational Mathematics: Puzzles, Card Tricks, Games, Gambling and Sports**

**Part A: Thursday, July 27, 2:00–4:35 PM, Salon A-1**

**Organizers: Paul R. Coe**    Dominican University

**Sara B. Quinn**    Dominican University

**Kristen Schemmerhorn**    Concordia University Chicago

Puzzles, card tricks, board games, game shows, gambling, and sports provide an excellent laboratory for testing mathematical strategy, probability, and enumeration. The analysis of such diversions is fertile ground for the application of mathematical and statistical theory. Solutions to new problems as well as novel solutions to old problems are welcome. Submissions by undergraduates or examples of the use of the solutions of these problems in the undergraduate classroom are encouraged.

**Arthur Benjamin**    Harvey Mudd College

### **The BINGO Paradox**

Suppose we walk past a crowded BINGO parlor. There are hundreds, perhaps thousands of cards being played. Is it more likely that the winning card will be a horizontal win or a vertical win? Surprisingly, a horizontal win is more than twice as likely as a vertical win. The proof uses elementary combinatorics and probability, but if we dig deeper, there is a beautiful connection to integer partitions and q-binomial coefficients.

**Mark Bollman**    Albion College

### **Lucky Lines, Lucky links, And The Probability Of Crushing Disappointment**

Lucky Lines, a game offered by the Oregon Lottery, and Lucky Links, a Connecticut Lottery offering, share the common unusual features of combining lotto-like drawings with elements of bingo and tic-tac-toe. Since winning combinations are based on the number of tic-tac-toe lines completed on a ticket rather than the number of numbers matched in the state’s drawing, it is possible to win money with few matches or to match many numbers—as many as 4 out of 8—and win nothing. This latter event qualifies as a crushing disappointment, and this talk will describe the two games and explore how likely this unfortunate event is.

**Robert W. Vallin**    Lamar University

### **Some New Problems from Two Old Sources**

In one hand a harried waiter holds a sloppily plated order of  $N$  pancakes. In the other hand the waiter holds a spatula and in a sequence of flips tries to reorder the pancakes, largest on the bottom to smallest on the top. This is the Pancake Problem. We use the integers  $1$  (smallest) through  $N$  (largest) to notate size of the cakes. Then the orders of the pancakes after any flip form a permutation of  $1, 2, 3, \dots, N$ . Gilbreath permutations arise from a magic trick first published by Norman Gilbreath in 1958 and have been applied to objects such as continued fractions, knots, and quasicrystals. In this talk we present a few results and a stack of problems that arise when we put Gilbreath permutations and pancake flipping together.

**John Harris** Furman University

### **Logs and Explorations: Investigating a Classic of Magic**

“Mental Logs” — a classic of magic — is an effect that typically involves four rectangular rods (or logs) on which lots of digits are printed. A spectator can mix up and arrange the logs as desired, using the digits to form several four digit numbers. With lightning speed, the magician is able to state the sum of the selected numbers — easily beating those using calculators. Whether this is presented as a magic trick or simply as a puzzle to solve, the math behind the trick is clever and is quite accessible to undergraduates. This talk will discuss the theory behind Mental Logs and ways that further student exploration can lead to enhancements and improvements to the effect. Volunteer assistants will be given their own sets of mental logs to keep!

**Jason Rosenhouse** James Madison University

### **Question Puzzles**

There is a famous logic puzzle that goes like this: You come to a fork in the road. One path leads to your destination, the other leads nowhere helpful. There are two people by the side of the road. One only makes true statements, while the other only makes false statements. You do not know which is which. You may ask one question of one of the people, after which you must make your selection. What should you ask, assuming you want to be certain of reaching your destination? This is an example of a “question puzzle,” a genre of logic puzzle in which the goal is to elicit useful information by asking questions of liars and truth-tellers. We shall survey some of the possibilities inherent in such puzzles. In particular, we shall discuss the “hardest logic puzzle ever,” and some recent attempts to extend such puzzles to non-classical and multi-valued logics.

**David Nacin** William Paterson University

### **Finite Group KenKen**

KenKen is a popular type of puzzle where the goal is to fill the entries of a grid with numbers to produce a Latin square. Cells are grouped together by darkened boundaries called cages. Each cage is assigned an operation and target number. When the operation is applied to all entries in a cage, the output must equal this number. One can also construct these puzzles over a finite group, where the associativity and existence of inverses allow for some very nice structure. Many questions arise. Which cage patterns on our grid allow for the possibility of unique solutions? When are two cage patterns essentially equivalent for constructing puzzles? For a given cage pattern, which clues result in solvable puzzles and when is the solution unique? We answer all of these questions for both the cyclic group  $C_4$  and Klein-four group  $V$ . For  $C_4$  the set of possible numbers of solutions is exactly  $\{0, 1, 2, 3, 4, 8, 10, 12\}$ . We discuss this classification and give a necessary and sufficient condition for a puzzle to have a unique solution. We then discuss what happens over the Klein-four group  $V$ , discussing the proof that there are no puzzles there with unique solutions. It can be shown that any solution comes as part of a group with three others, giving us  $\{4, 8, 12\}$  as the set of numbers of solutions. We conclude with the order five case and end with several open problems related to the theory we’ve developed.

**Eric Eager** University of Wisconsin - La Crosse

### **NFL Team Ranking Methods and Their Abilities to Predict Games**

In this talk I will discuss various ways to measure the strength of teams in the National Football League. I will compare these rankings, highlighting where and why discrepancies arise, and show how they can be used to predict who will win and/or cover the Vegas spread in various matchups week-to-week during the course of a season.

**Jay L. Schiffman** Rowan University

### **Systematic Counting, Binomial Coefficients, World Series Scenarios and the 2016 World Champion Chicago Cubs**

Systematic Counting, Binomial Coefficients, World Series Scenarios and the 2016 World Champion Chicago Cubs Abstract: The stage was set for an amazing historical moment in our national pastime. Around half past midnight on Thursday, November 3, 2016 at Progressive Field in Cleveland, OH in the last of the tenth inning, Cleveland pinch hitter Michael Martinez bounced a ball delivered by CUBS relief pitcher Mike Montgomery to third baseman Kris Bryant who fielded the ball and made an accurate throw to first baseman Anthony Rizzo who completed the play for the final out. Hence the moment finally arrived and the CUBS ended the longest drought in sports history between World Championships lasting 108 years. No more waiting until next year. Next year had finally arrived and Chicagoland rejoiced! This paper examines all the possible outcomes occurring in a best of seven World Series and historically considers those that have actually transpired. Systematic enumeration will be employed and diagonals in Pascal’s triangle play a role in the solution. The CUBS became only the second National League team to overcome a 3-1 Series deficit and win. Do we know the other ball club in NL history who actually accomplished the feat twice and in one instance won the deciding game at home? Which series length has been the most frequent throughout the years? How have the leagues fared and what is the dichotomy between the home and road teams? There have been series that have lasted longer than seven games. Do we know when this happened and why? These and many other fascinating questions will be discussed and resolved. Participants will see the marriage of sports and recreational mathematics in full view as we commemorate an historic event for baseball and the great city of Chicago.

## Part B: Friday, July 28, 2:00–4:55 PM, Salon A-1

**Anne Quinn** Edinboro University of PA

### A Mathematical Analysis of Social Math

Although surfing social media sites is a favorite recreational pastime for many teenagers and adults, I found analyzing the mathematics behind social media to be even more interesting. A variety of mathematical problems will be discussed for social media users at all levels. Mathematics used to solve these problems will range from simple multiplication rules to logarithms and exponents to network-encoded matrices. The idea of “6 degrees of separation” will be discussed in several contexts. Analysis of big data sets for Facebook, LinkedIn, and Twitter will be discussed.

**Jie Mei** University of Rhode Island  
**Edmund A. Lamagna** University of Rhode Island

### Transamerica and the Shortest Path Problem on Triangular Grids

Players in the board game TransAmerica build a railroad network linking five cities on a map that is a triangular grid. Since only two tracks are placed per turn, it is advantageous for players to know the shortest path connecting their cities to increase the chances of winning. In building networks, players can introduce junction points where their tracks fork in different directions. These junctions are called Steiner points, and the resulting connected network is a Steiner tree. Historically, the problem of building minimal Steiner trees has been studied for the Euclidean plane and for rectilinear grids. In both cases, the problem is NP-hard. There are interesting differences between locating Steiner points on a triangular grid and these other geometries. For example, when connecting three terminal vertices on a triangular grid, there may be multiple choices for the Steiner point, whereas there is only one choice for the Euclidean and rectangular problems. A spanning tree is another way to connect the terminal vertices that does not involve adding extra points. The optimal solutions for the two strategies can be compared using the Steiner ratio, the ratio of the minimal length Steiner tree to the length of the minimal spanning tree. The presentation includes some new results about Steiner trees on triangular grids and a conjecture about the Steiner ratio.

**Doug Chatham** Morehead State University

### The $n+k$ Kings Problem

The “ $n$ -kings problem” asks for arrangements of  $n$  pieces on an  $n \times n$  board so that no two pieces attack each other as kings or as rooks. We add pawns to the problem and show that for  $n \geq k + 5$ , we can place  $k$  pawns and  $n + k$  kings on a  $n \times n$  board so that no two kings attack each other as kings or as rooks.

**Dana P. Rowland** Merrimack College

### Mathematical explorations with Swish

In the ThinkFun card game Swish<sup>TM</sup>, each card contains one hoop (annulus) and one ball (disk) positioned as elements in a  $4 \times 3$  array. The cards are transparent, and the goal is to stack a subset of between two and 12 cards so that each hoop is filled with a ball, resulting in a *swish*. If we interpret the stacking of cards as addition in  $\mathbb{Z}^{12}$ , then a swish is a particular type of subset of  $\mathbb{Z}^{12}$  whose elements sum to zero. Cards can be flipped or rotated prior to stacking, however, which introduces additional structure. We construct a set of seven cards so that a swish can be formed using any card in the deck together with some of these cards—similar to writing an element in a vector space as a linear combination of basis vectors. We examine various other mathematical questions inspired by playing this game, with connections to discrete probability, combinatorics, linear algebra, and abstract algebra.

**Steve Bacinski** Davenport University  
**Tim Pennings** Davenport University  
**Stefana Rusu** Davenport University

### How to Win at Tenzi

We will work through a Markov chain analysis of the simple dice game Tenzi to find the probability of winning in  $k$  rolls, the advantage of rolling a speed  $x$  faster than your opponent, and ultimately how to win at Tenzi. Along the way, we will discover a function with some interesting properties including predictable jumps at every rational number, and continuous at the irrationals.

**Mark R. Snively** Carthage College

### The Expected Length of a Game of Tenzi

In the dice game Tenzi, a player has ten six-sided dice and repeatedly rolls any subsets of the dice in order to have all ten show the same number with as few rolls as possible. A simple strategy is to choose a number at the start of the game and always re-roll any dice not showing that number. For this strategy we find that the expected number of rolls to win the game is 16.6. We then derive the expected number of rolls when playing the game with any number of dice.

**Brian Heinold** Mount St. Mary's University

**Probability Questions from the Game Pickomino**

Pickomino is a simple and fun dice game. It has some similarities with Yahtzee, and nearly every move involves some probability, making it a nice game for teaching probability. This talk will cover a wealth of interesting problems that arise from Pickomino for all levels of students, mathematicians, and math enthusiasts.

**Stacy L. Hoehn** Franklin College

**Computer Simulations as a Lens into the Mathematics of Crazy Eights and Farkle**

Card and dice games like Crazy Eights and Farkle often have fairly straightforward rules for gameplay, but can be surprisingly challenging to analyze mathematically. Computer simulations of these games can be useful when trying to recognize trends and develop conjectures about in-game probabilities and optimal strategies. In this talk, I will describe how computer simulations of these games can be used in mathematics courses to teach students about the experiment-conjecture-proof process, as well as some insights into these two games that have come from creating and analyzing the computer simulations.

**Cherith Tucker** Oklahoma Baptist University

**Card Games in an Undergraduate Geometry Course**

This presentation will introduce the games SET and Spot It!, which can be used in an undergraduate college geometry classroom to challenge student intuition, illustrate the power and limitations of models in an axiomatic system, and add a little fun to the class.

## Novel Introductions to Number Theory

**Thursday, July 27, 3:00–4:35 PM, Salon C-6**

**Organizer: Sarah L. Mabrouk** Framingham State University

This session invites presenters to share interesting ways in which to introduce undergraduate students to topics in number theory. These “tastes” of number theory may be demonstrations, in-class activities, projects, proofs, or ways in which to guide undergraduates to explore and learn about areas of number theory while improving their ability to write proofs. Those discussing demonstrations or in-class activities are encouraged to share key portions. Presenters are welcome to share their first experiences teaching topics in number theory or how they have modified their approaches over time. Presentations related to teaching topics with which students experience difficulty and student reaction as well as information about successes and failures are encouraged. Abstracts should provide a glimpse of the demonstration, in-class activity, project, or proof to be discussed and information about the related topics in number theory in addition to the software or application, if any, used. Those whose presentations are dependent upon software or tablet explorations must provide their own laptop or tablet.

**Stefan Erickson** Colorado College

**A Group Activities Approach to Number Theory**

Number theory has long served as our “introduction to proofs” course at Colorado College. Our innovative block schedule provides extended periods of classroom time and intensive focus on one subject at a time. This flexibility has encouraged our professors to create an active learning environment. Over the years, I have developed handouts that guide students to find patterns in numbers. Topics include Pythagorean triples, linear Diophantine equations, Euler’s Theorem and totient function, and quadratic reciprocity. Through working in small groups in class, students discover the natural beauty of number theory and stoke their interest in theoretical mathematics. I will begin by briefly presenting my general philosophy about teaching number theory under Colorado College’s Block Plan. I will also share a sample of my in-class worksheets. My hope is that the audience will take away some new ideas of how to make number theory fun and engaging to their students.

**Susan H. Marshall** Monmouth University

**Presenting MAA Articles on Number Theory**

In this presentation, we’ll talk about an assignment given to my junior Number Theory class last year. The course carries a general education requirement entitled “reasoned oral discourse.” Students must “orally present mathematical ideas and information in a reasoned and effective manner with attention to elements of vocal and nonverbal quality.” This requirement gave rise to our final project, where students were assigned an article to read and then present the results to the rest of the class. Articles in MAA publications such as *College Mathematics Journal* and *Math Horizons* (especially those that had won writing awards) were chosen to ensure the students were able to understand the content. A happy side effect was the range of topics all students were exposed to during the presentations, including typical topics such as the digits of  $\pi$  and innovative topics such as applying number theory to video games. Equally satisfying was their increased enthusiasm for the subject. We’ll discuss the particulars of the assignment, how it went, and how it might be improved in future semesters, with attention to elements of vocal and nonverbal quality.

**Everette L. May** Salisbury University

### The Wehmueller Conjecture

This talk is the story of a student's attempt to build on the ideas of Fermat's Last Theorem. In the spring of 2010, after watching *The Proof*, the NOVA/BBC video of Andrew Wiles's successful quest to prove the theorem, Kara Wehmueller, one of my students in discrete mathematics, stated that, instead of trying to use three integers  $a$ ,  $b$ , and  $c$  such that  $a^3 + b^3 = c^3$ , one should look for four integers  $a$ ,  $b$ ,  $c$ , and  $d$  (a "Wehmueller Quadruple") such that  $a^3 + b^3 + c^3 = d^3$ . In general, she said, for any integer  $n > 1$ , one should search for  $n + 1$  integers  $x(1), x(2), \dots, x(n), x(n + 1)$  such that  $x(1)^n + x(2)^n + \dots + x(n)^n = x(n + 1)^n$ . Additionally, she exhibited the Wehmueller Quadruple (3,4,5,6). Finally, she stated the following: The Wehmueller Conjecture. For each integer  $n > 1$  there is a "Wehmueller  $n$ -tuple." Thus began a new "Fermatian" quest. The talk will detail the (unfinished) history of that quest, suggest some directions in which it might proceed, and seek the help of the audience in resolving the Wehmueller Conjecture.

**David Terr** UC Berkeley

### Some Interesting Infinite Families of Primitive Pythagorean Triples

In this paper we investigate families of primitive Pythagorean triples of the form  $(a, b, c)$ , where  $mc - nb = t$ ,  $mc - na = t$  or  $mb - na = t$  for some fixed positive coprime integers  $m$  and  $n$ , and  $t$  a fixed nonzero integer. A few of these cases are especially interesting since the solutions may be simply written in terms of Fibonacci and Lucas numbers.

**Darren Glass** Gettysburg College

### Arithmetical Structures on Graphs

In this talk, we will discuss the concept of an arithmetical structure on a finite connected graph, first introduced by Dino Lorenzini. While these structures are typically defined in terms of linear algebra and have interesting applications in algebraic geometry and algebraic combinatorics, they also can be framed in terms of elementary number theory in a way that provides an entrapment to students to think about open questions related to divisibility and congruence.

## Inquiry-Based Teaching and Learning

**Part A: Friday, July 28, 8:30–11:05 AM, Salon A-2**

**Organizers: Brian P. Katz** Augustana College

**Victor I. Piercey** Ferris State University

The goal of Inquiry-Based Learning (IBL) is to transform students from consumers to producers of mathematics. Inquiry-based methods aim to help students develop a deep understanding of mathematical concepts and the processes of doing mathematics by putting those students in direct contact with mathematical phenomena, questions, and communities. Within this context, IBL methods exhibit great variety. Activities can take place in single class meetings or span entire curricula for students of any age; students can be guided to re-invent mathematical concepts, to explore definitions and observe patterns, to justify core results, and to take the lead in asking new questions. There is a growing body of evidence that IBL methods are effective and important for teaching mathematics and for fostering positive attitudes toward the subject. This session invites scholarly presentations on the use of inquiry-based methods for teaching and learning. We especially invite presentations that include successful IBL activities or assignments, that support observations about student outcomes with evidence, or that could help instructors who are new to IBL to try new methods.

**Mona Mocanasu** MSU Denver

### An IBL Approach to Abstract Algebra

An IBL Approach to Abstract Algebra In this presentation we discuss several IBL activities designed to introduce to the students main topics in Abstract Algebra, such as the idea of an operation table, and later in the semester the notion of permutations and the nature of the permutation group. Compared to an introductory math course, in Abstract Algebra one of the IBL challenges is when and how we introduce standard definitions and notations—for example, how do we cross the bridge between allowing students to use their own permutation descriptions, and explaining the traditional transposition or cycle notations. We also discuss the progress of students' attitude towards the IBL methods, and the change in expectations as the course advances. Finally, we would like to address assessment methods specific to IBL activities, and how these compare to the traditional classroom.

**Justin Dunmyre** Frostburg State University

### **An IBL Introduction to Proofs Class with Specifications / Standards Grading and Group Work**

Specifications / standards based grading is a grading scheme wherein points are eschewed in favor of a holistic approach. A key feature of such a system is that it encourages resubmission instead of fixed grades. This fits an introduction to proof class nicely; reworking proofs to improve them often leads to deeper understanding of the proof writing process. This grading scheme also paired well with an IBL and group work approach to the course. Students attempted 5 or so proofs before coming to class. When class began, students were placed into groups randomly and discussed those proofs. From each group, a presenter was randomly chosen to put the group's proof on the board. Following a question / answer period, the class voted to approve the proof. Approved proofs were candidates for a recall quiz administered at the end of the class. The students also submitted proofs, typeset in LaTeX, of up to 4 new statements and 4 resubmissions each week. Typeset proofs and quiz proofs were graded on specifications according to the EMRN rubric. To earn an A, a student needed at least 50 typeset proofs with the E grade (In this case, E means "Excellent Example" or "Exceeds Expectations"). In this talk, I focus on the logistics - what did the classroom look like, what did grading look like, and what tweaks I plan to make for the next time I teach the course. My hope is that this will help instructors new to IBL to try it out while also providing some interesting twists for seasoned instructors.

**May Mei** Denison University

### **All In: My First Experience with Guided Student Presentations**

This talk is a reflection of my first experience teaching a course entirely through guided student presentations. I've dabbled in an inquiry-based approach before at the calculus level, but this was senior-level course in Topology in which the content was to be conveyed almost exclusively through presentations by students. I'll discuss resources that I've used, challenges that I faced, and the benefits that this approach has yielded. I will also share qualitative results of essay-based assessments of student attitude towards this approach.

**John Ross** Southwestern University

### **Lessons Learned Creating IBL Course Notes**

In this talk we discuss the process of creating IBL course notes for a Real Analysis class in Spring 2017. The experience was a positive one, allowing us to shape course notes tailored to suit the needs of the class and the students. However, as junior faculty with limited IBL experience, the process was not without its difficulties. We discuss our process, successes, failures, and lessons learned in hope that others can learn from the experience.

**Adrian P. Gentle** University of Southern Indiana  
**Yalcin Sarol** University of Southern Indiana

### **Reanimating the Dinosaurs: A Reflection on Inquiry-Based Learning and Faculty Renewal**

We reflect on our experiences as mid-career faculty members making the transition from more traditional lecture-based classrooms to inquiry-based learning (IBL), a form of active learning with a long history in the US mathematics community (Coppin, et al. 2009). We find that the increased engagement of students leads to an increased connection with students, which in turn has led to a renewed energy in the classroom. Above and beyond the well-established advantages for student learning (Freeman et al. 2014, Laursen et al.), the implementation of evidence-based teaching practices can greatly improve the instructor's classroom experience. We argue that the rejuvenating effects on faculty create a compelling message that can help drive professional development efforts as the profession moves towards evidence-based teaching practices.

**Sarah Dumnich** Frostburg State University

### **An Inquiry-Based Approach to IBL**

In an IBL classroom, it is important to maintain a classroom culture which values productive failure. The person in the class who might recognize and appreciate this the most is the instructor. Through my experiments with IBL in my probability course, I found many methods which were effective, and many others which were less so. From changing the group structure a month into the course, to changing the schedule for the semester, I found that teaching an inquiry class is a learning experience in itself. By encountering common difficulties to those new to IBL, I can share my perspective and help others avoid these roadblocks. I will include student feedback, online resources, and helpful pointers from lessons learned along the way.

**Elizabeth Thoren** Pepperdine University

### **Mathematical Inquiry for Liberal Arts Students**

All students should have access to a mathematical inquiry experience; however, supporting meaningful inquiry necessarily looks different in different contexts. In this talk I will discuss some challenges and strategies for supporting this work in a Mathematics for Liberal Arts course.

**William T. Mahavier** Lamar University

### Writing in an IBL Calculus Course

Pre-abstract quiz. . . . “Real Analysis is the place for theorems –Calculus is the place for \_\_\_\_\_.” We believe that Calculus can be a place of definitions, precise writing and therefore a gateway for attracting students to higher mathematics. We will describe a set of IBL notes for Calculus I, II & III that is freely available, includes practice problems for homework and covers the traditional topics. We describe how we use the course to train mathematical writing, mathematical thinking, mathematical maturity and well, other calculus stuff like derivatives and integration!

## Part B: Saturday, July 29, 8:30–11:45 AM, Salon A-2

**Jonathan Weisbrod** Rowan College at Burlington County

### Competing to Learn: An In-Class Playing Card Competition Where Students Explore Set Theory Operations

The students in a math for liberal arts class often are not intrinsically motivated to explore mathematics. While many students are sharp in their chosen fields of study, many have not reached their full potential of using their critical thinking skills in a mathematical context. The group playing card game presented in this paper eliminates grades as motivators and uses the students’ natural competitiveness as a drive to dig deeper into their mathematical thinking. In order to win, students must optimize, logically predict other groups’ hands, and stretch their set theory skills to score as many points as possible in the game. This activity serves as a low-risk formative assessment as it allows students to make mistakes and learn from them without grade penalties. The classroom becomes a natural low-threshold high-ceiling environment as every student regardless of skill level can participate. Since students compete in groups, the more-prepared students assist the less-prepared students for the benefit of the entire group. Finally, the instructor is able to reference this game to introduce new units in the course, such as logic and probability.

**Kayla B. Dwelle** Ouachita Baptist University

### Level Up: A Continued Experiment in “Gamifying” an Active Learning Classroom

I believe in active learning. In fact, I believe in active *scholarship*, which includes students taking responsibility for their performance in my courses and, hopefully, ownership over their successes as well as their failures. Too often, students see themselves as victims of the pedagogical choices of their educators. In an attempt to “speak truth into powerlessness,” last fall I did an experiment: I called it ‘gamifying’ my active learning classroom. There was class currency, game boards, and ways to use currency. My idea was based on giving students ownership of their grades and encouraging ‘right’ behavior like attendance, study time, and reflection. It also gave them ways to recover from setbacks like an unexcused absence or missing a deadline. I learned a lot from my experiment and reported on my implementation and assessment of results for that first semester in a talk at JMM Atlanta. For the spring semester, I adapted my experiment for a new semester based upon both my evaluation and student feedback on the previous term. In this talk, I will discuss the adjustments made in my second attempt and my interpretation of the results together with an appraisal of the entire year.

**Robert Sachs** George Mason University

### Student Guided Reinvention of Green’s Theorem

With a mild reordering of topics within a traditional multivariable calculus course, students can be guided towards rediscovering Green’s theorem in its simplest setting. The important precursor is the notion of line integral and its use in finding circulation and flux of a vector field in 2-D. A student attitude towards productive exploration, dubbed “math hacking”, leads to Green’s theorem in either its circulation or flux form, although students have on occasion made an alternative valuable discovery instead. Even when unsuccessful, the attempt at discovery has pedagogical value for students.

**Amanda H. Matson** Clarke University

### Finding Mathematics in Chaos - Invite Your Students to Persevere

One of the beauties of mathematics is the ability to see structure amid chaos. Using this as a theme encourages students to persevere when they would usually throw in the towel. In this talk, I will give examples of using this theme in Foundations of Mathematics (for preservice elementary school teachers) and in Number Theory (which doubles as our introduction to proof course).

**Amy Ksir** U.S. Naval Academy

### Coordinating IBL and non-IBL Calculus II

This year at the Naval Academy, half of the sections of Calculus II, both fall and spring, were taught using IBL and half were not. We will report on final exam results; results of other assessments; the joys of several IBL instructors working as a team, and the challenges of coordination.

**Patrick X. Rault** University of Arizona

### Hybrid IBL

The speaker will discuss a variety of technological tools for use in a synchronous online classroom or for use in a videoconferencing “Interactive TeleVision” (ITV) classroom which connects multiple locations. Pros and cons of using Lifesize Video or Zoom, Adobe Connect, and Dropbox for a highly active electronic classroom will be discussed. The speaker will describe how his own background with using Inquiry-Based learning (IBL) at a traditional liberal arts college (SUNY Geneseo) has influenced his decisions in this new distance-learning environment. Tools for use in standard face-to-face classes will also be discussed, such as Dropbox for between-class assignments and Adobe Connect for electronic office hours.

**Marshall Gordon** Park School of Baltimore

### Mathematical Habits of Mind - The Essential Dimension for Learning Mathematics

That problem solving is thought of as the “heart of mathematics” (Halmos, 1980) surely makes sense. And with that understanding, mathematical habits of mind would be the heart of the mathematics curriculum. Made available as content would make doing mathematics - creating and solving mathematics problems, all the more possible for every student. In that direction, some practices may well come naturally to mind, such as *looking for patterns*, or *guessing*, perhaps even *arguing by counter-example*. However, others that shed light on complex mathematical situations may need to be uncovered with the mathematics teacher helping take part in the question-asking. And as students come to see how *tinkering*, *taking a problem apart*, *visualizing* and other heuristics are critically valuable in making headway, they naturally become disposed to including those problem-clarifying strategies as part of how they think. As a consequence, all students require less and less direction in engaging mathematics problems. This presentation would include various instances of applying mathematical habits of mind.

**Adam J. Castillo** The University of Texas at Austin

### Understanding Two-Year College Mathematics Faculty Perceptions and Use of Cooperative Learning

Cooperative learning, or the instructional use of small groups so that students actively work together to increase their own and each other’s learning, is a well-documented pedagogical approach to promote student learning. However, despite ample research on cooperative learning in the K-12 setting, there is little research on two-year college math faculty perceptions and their reported use in math courses. I conducted a pilot study on two-year college math faculty at four Texas community colleges involved in major math reform initiatives. These colleges were purposefully selected to identify math faculty familiar with, and with nominal administrative support for, reform teaching strategies such as cooperative learning. The purposive sample was intended to identify participants familiar with cooperative learning, but not necessarily employing it with fidelity, enabling me to characterize: (1) perceptions of two-year college math faculty regarding cooperative learning and its use and (2) what the implementation of cooperative learning might look like in two-year college math courses. This talk will focus on pilot study results on faculty perceptions of cooperative learning and how it is used in developmental and college-level math courses. I will also discuss the process for testing and modifying research instruments for future research on two-year college math faculty at different colleges.

**Victor Piercey** Ferris State University

### Pedagogy of the Oppressed: Lessons for Inquiry-Based Learning in Mathematics

In *Pedagogy of the Oppressed*, Paul Freire describes “dialogics” as a teaching strategy to guide the oppressed to liberate themselves from oppression. Freire’s distinguishes his pedagogy (based on dialog) from what he calls the “banking” model of education: knowledge is “deposited” into the learners account. He argues that the banking model replicates the conditions of oppression and as such, fails to lead to liberation. Freire’s analysis has a great deal in common with the insights informing inquiry-based learning in mathematics, but also has much to add. I will share my comparison, including a mapping that applies Freire’s work to populations of general education math students in colleges and universities.

**Jessica Ellis** Colorado State University

**Brian P. Katz** Augustana College

### Equitable Practices in IBL

Research on inquiry-based learning (IBL) suggests that it has potential to “level the playing field” for certain groups of students. However, the data that support these claims are not sufficient to warrant the tempting generalization that IBL pedagogies address equity issues in mathematics classrooms in general, which leaves us with many questions. What new equity challenges are present in IBL classrooms that might be dormant in teacher-centered classrooms? How do students of color experience IBL courses? How do majority and minority memberships impact students’ experiences in IBL courses? How do intersectional identities relate to and evolve in the context of inquiry? What specific practices can make IBL classrooms equitable, and how do those practice function to achieve this goal? How are questions of equity related to inquiry-based course structures in general? In this talk, we will integrate published research, new data, and our own experiences to open up the discussion of these questions and equitable practices in the IBL community.



## Part C: Saturday, July 29, 1:00–3:15 PM, Salon A-2

**Annie Han** BMCC-The City University of New York  
**Margaret Dean** BMCC-The City University of New York  
**DanPing Zhong** BMCC-The City University of New York

### Put Students in the Driver Seat on Their Mathematics Learning Journey

In recent decades, inquiry-based learning has risen in the literature as an active learning, student centered, and innovative method to teach college mathematics. One historical version called the Moore Method (1920) was applied to theory and proof courses, and now the method has been applied to a vast number of mathematics courses and in many levels of mathematics topics. After teaching the IBL Mathematical Literacy course for 5 years, the authors recently implemented the IBL method in a Calculus I course. This hands-on inquiry based calculus course gives students the opportunity to discover and explore mathematics concepts. This course puts students in the driver seat on their mathematics-learning journey during the semester. The presentation will share the results of a one semester IBL Calculus course's success, productive failure, and measured outcomes focused on the assessment of student engagement in the course. Be prepared, this presentation will be Inquiry Based Learning in action - you are in the driver seat!

**Alessandra Pantano** University of California, Irvine

### Visualizing Ideas from Calculus

We present a series of activities aimed at developing an intuition for mathematical ideas. Using manipulatives, like laser-cut wooden shapes, 3D-printed quadrics or styrofoam shapes, students (re)discover some of most interesting concepts from calculus. During the talk, the audience will have a chance to 'play' with these activities and experience - first hand - the pleasure of visualizing mathematical concepts.

**Lisa Driskell** Colorado Mesa University

### Technology, 3D Printing, and Brochures in a Liberal Arts Course

In a liberal arts course designed to explore the mathematics found among patterns in nature, students discovered connections via course activities. Technology was sometimes used to observe relationships and deepen understanding of the topics. One endeavor with technology allowed students to study recursion by investigating, constructing, and 3D printing fractal-like patterns. The details of this activity will be discussed including the interfaces and open source programs used. In addition, the final brochure project will be reviewed as an alternative to standard student presentations.

**Thomas Clark** Dordt College

### Splines from Scratch using SageMathCloud

Splines are a common topic in Numerical Analysis, though many standard texts avoid the issue of constructing them, focusing on applications and properties only. Other texts that do provide an algorithm use a method of construction layered with clever algebraic manipulation that divorces the basic properties of the spline from its construction. However, with good leading questions, students can build splines from scratch using standard linear algebra and piecewise functions. SageMathCloud (SMC) provides an excellent (and free) software environment in which to do this work as well as a platform from which to disseminate handouts and track student progress. To accomplish this students work to determine the desired properties the spline should possess, mathematize these conditions into sets of equations, and then convert these equation sets into a matrix-vector problem that is easily solved. Finally, the 'solution' is converted to a piecewise function. The novelty is that students can do this with minimal direction from the instructor. Students derive the necessary properties of progressively complex splines, first linear, then quadratic, then culminating in beautiful cubic splines. They also write the code that builds the matrix-vector problem needed to create each spline. I will also demonstrate how SMC proves to be an excellent tool for teaching, not just numerical analysis, but almost any mathematics class.

**Allison Henrich** Seattle University  
**Inga Johnson** Willamette University

### Teaching an Interactive Introduction to Knot Theory

Knot theory is one of the most popular areas for undergraduate research, and inquiry-based courses are designed, in part, to cultivate in students behaviors and thought processes needed for research. For these reasons, we had a hunch that an IBL introduction to knot theory textbook could be a resource not only for professors looking to teach inquiry-based undergraduate topics courses, but also for mentors who teach short crash courses to prepare undergraduates for research in knot theory. We have now taught with our new book, "An Interactive Introduction to Knot Theory," both in the classroom and in the summer REU setting. We are excited to share the book and our tips for using it with others!

**Jonathan Hulgan** Oxford College of Emory University

### **Inquiry-Based Graph Theory for Non-Majors**

The curse and blessing of teaching graph theory to undergraduates is that nobody has heard of it. This makes it an ideal topic of study for an inquiry-based learning course. In this talk, I will briefly share my experiences with teaching such a course to non-majors. The aim of the course is to let students glimpse an answer to the question, “How does one do research in mathematics?” In-class team investigations introduced to basic concepts and definitions in graph theory and gradually acclimated students to wrestle with difficult problems in the field. From these guided examples, students developed their own questions to investigate on their own; these investigations were kept in a research journal, and helped students revise and refine their questions throughout the semester. I will share comments and examples from students concerning this process.

**Jonathan White** Coe College

### **The Peano Axioms: Natural Numbers by IBL**

Abstract: I offer a small problem sequence introducing the Peano axioms as a possible introduction (both for instructors and students) to IBL approaches. We regularly use this sequence over about a week with students in their first or second years, so it provides an ideal “trial size” first exposure to IBL. I will explain the structure of the sequence and share some student feedback.

## **Data Science: Big Data, Big Questions**

**Friday, July 28, 8:30 AM–12:05 PM, Salon A-1**

**Organizers: Jacci White** Saint Leo University

**Monika Kiss** Saint Leo University

**Brian Camp** Saint Leo University

Data Science. What is it? Why is it important? Who is doing it? How are you using it? Is it only Business Analytics? Come and share your experience as you learn from others about the innovative and quickly growing interest in Data Science. We encourage the submission of scholarly work, including, but not limited to, original research, innovative ideas, demonstrations, problems, applications, projects, curricular materials, single class descriptions, whole course outlines, and whole programs. Proposals will be selected that show innovation, detail, and evidence based results with a primary focus on pedagogy and curriculum related to data science.

**Brian Camp** Saint Leo University

**Monika Kiss** Saint Leo University

**Jacchi White** Saint Leo University

### **Building the Data Science Path Science Path**

Mathematics majors are generally very broad allowing for the preparation of students for a variety of career paths. More recently, Data Science has become an intriguing option for mathematics majors. In this talk, we discuss what Data Science is and how it can be included in your curriculum. There is no one correct answer at this point as there is no current standard for what a Data Science curriculum looks like. Some topics included in our discussion will be what some schools are currently doing, how data science can be included as a minor or specialization of a more traditional mathematics major and finally what an undergraduate Data Science major itself might look like.

**James Quinlan** University of New England

### **Developing an Academic Data Science Program**

When universities spend more than 100 million dollars on a data science initiative, you can rest assure data science is more than just en vogue. This talk will focus on the development and proposal of a new academic program in data science at a small liberal arts university including motivation, courses, equipment, and fostering corporate partnerships. The program is designed to provide students with foundational mathematical, statistical, and computing skills, reinforced with project-based learning experiences, that prepare them for the wide variety of data-driven careers in a range of fields, including emerging fields and those where data science approaches have not been traditionally used.

**Marcus L. Walker** University of Minnesota Duluth

**Learning by Doing: Data Science for Mathematics and Statistics Undergraduates through Experiential Learning Collaboration with Industry Clients**

The Harvard Business Review famously hailed “Data Scientist” as the sexiest job of the 21st century, but even that might be selling it short. Reality is a puzzle that is beautiful beyond compare, and people will dump money all over you if you can solve it. Great strides are being made, but academic curriculum in general has been slow to adjust to the needs of mathematics and statistics undergraduates hoping to work in modern data science. In this presentation I will talk about what is being done at the University of Minnesota Duluth to encourage mathematics and statistics undergraduates to develop the technical skills, experience, and modes of creative thinking they will require by giving them the opportunity to move beyond regression assignments and into working with business, industry, and government clients on projects involving exactly the kind of data and methods they will see in the field. I also offer a personal account of what it is like both to learn and to mentor in that context.

**Christopher Thron** Texas A&M University-Central Texas  
**Lucas Brandt** Texas A&M University-Central Texas

**Modeling Expenses and Liabilities in an Army Accounting System**

This study analyzed a particular U.S. Army unit’s financial and supply structure to optimize accurate spending, budgeting, and accounting. The accounting structure for the unit is complicated, with credits and debits posting from many sources. When the unit orders parts, there is a variable time delay between “commitments” (submitted orders) and their corresponding “obligation” (funds exiting the unit’s account). While it is possible to check the system for individual transactions, the unit produces thousands of lines of orders each day, so it is not feasible to check the status of individual transactions. As a result, it is nearly impossible to know the total outstanding commitments that have not obligated. This liability tracking is vital to the successful execution of the budget at the end of each fiscal year, when overspending has serious legal implications. A probabilistic model of commitment processing delay was developed using iterative pattern matching based on four months of data. Several characteristics of the model agreed with known facts about each supply system, and their expected delays. The model produced also identified several accounting errors and mistakes for individual transactions, indicating its usefulness as a tool for auditing recorded data and tracked expenses. Results showed that delay profiles were consistent across subunits, indicating that the study’s methodology and model can be usefully applied to other units in other budgeting scenarios. Additional data is being assembled to further verify and extend the model. We conclude that our methodology can assist brigade-level comptrollers in tracking funds and forecasting outstanding liabilities.

**Joan E. DeBello** St. John’s University

**Big Data Bigger Mathematics: How Math Impacts Big Data and How College Curriculum Reflects This**

Big Data Bigger Mathematics: How Math Impacts Big Data and How College Curriculum Reflects This This paper will discuss how mathematics has impacted the age of Big Data and Data Analytics. It will address the impact mathematics has on Big Data and how courses are being created across program curriculum due to its influence and popularity. With Big Data being interpreted and analyzed in all areas from Computer Science, Mathematics, Business, Sports and Healthcare, It will discuss ways to incorporate big data into mathematics courses for all majors and sample problems that are being used to motivate and teach mathematics to all majors along with simple computer applications and programs designed to help teach big data in mathematics courses.

**Russ Goodman** Central College

**Ingredients for a Worthwhile Data Analytics In-Class Project: a comedian, some software, and lots of curiosity!**

This talk will share the details of an intriguing and appealing in-class project for students in an introductory Data Analytics class for advanced mathematics majors. The project originated with a joke from a popular comedian, produced a pair of relevant data sets for the small groups of students to explore, and took the groups in a variety of interesting directions.

**Aihua Li** Montclair State University

**Class Projects Dealing with Big Data**

Class Projects Dealing with Big Data Abstract In this talk, I will share my class projects involving big data. In my recent PIC Math class (Preparation for Industrial Careers in Mathematics), I organized 4 student groups each of them working on an industry problem given by our industry conduct. All projects dealt with data analysis and 3 of the 4 worked on large data. It is evident that data science techniques played an important role in solving these problems, which can be seen from the project titles: 1. Search for Nearest Correlation Matrices to Guide Risk Management of Stock Activities 2. Mathematical Modeling of Upper Tolerance Limits for Metal Concentrations in Certain Regions 3. Mathematical Modeling of Likelihood of Customers’ Buying at Staples 4. Group Marketing Analysis of the Influence of Email Responses on DeBoer’s Auto Sales and Services When working on the projects, students must learn a software requested by the industry representatives and advanced methods to analyze large data. Python, R, and Matlab, were heavily used throughout. I will introduce the organization of the class, benefits students obtained and challenges students faced when working on the projects. Summary of results of the projects will be discussed.

**Andy Keck** Western State Colorado University

### **Robust PCA and the Analysis of Surveillance Video**

Robust PCA (Candes et al. 2009) provides an unsupervised method for separating a matrix into the sum of a low rank matrix and a sparse matrix holding large, localized differences. We apply this technique to surveillance video to differentiate between background components and moving foreground components. This provides a powerful tool for detecting interesting anomalies.

**John P. Boardman** Franklin College

### **Big Data and the Professional Workplace**

To address the needs of the professional workplace and their increasing reliance on data science, our department introduced a new course titled *Big Data and the Professional Workplace* open to students of all majors. The course combines an introduction to techniques used in the analysis of big data with elements of the professional workplace. Students work in teams under the guidance of an alumnus coach from the private sector to analyze a big data set. In the first half of the course, the students are introduced to their data sets while learning statistical techniques and R programming. The second half of the course is devoted to working with their teams to answer questions about their data. The course also addresses the ethical challenges associated with big data and emphasizes the attitudes and dispositions attractive to employers. The course culminates with presentations made to a general audience. This course is built into a curriculum that combines traditional mathematics and statistics courses with other experiential project-based courses.

**Boyan Kostadinov** City Tech, CUNY

### **The Data Science of Fitting a Dinosaur**

We digitally sample a hand-drawn dinosaur with 69 planar points along its boundary, and using linear least squares, we fit finite Fourier models to the data provided by the Cartesian coordinates of the sampled points, considered as functions of their normalized index. The fitted coordinate parametrization is then coded in R, visualized and compared with the original sample for a different number of harmonic components in the finite Fourier model, which offers not only least squares approximation but also interpolation options. We provide a convenient approach to digitally sample an image using the R programming environment. We believe that this fun application could offer some educational value in the data science classroom in the context of fitting models to data generated from images, in addition to exploring least squares techniques, which are the backbone of many machine learning algorithms in data science. Last but not least, we provide the R code for all computational components in this project, which should add more educational value given that the R programming language is one of the most popular computational platforms for doing data science.

**Amanda J. Mangum** Niagara University

**Mansoor Haider** North Carolina State University

### **Clustering Algorithms and an Application to Acoustic Radiation Force Impulse (ARFI) Imaging**

One application of Big Data is medical imaging. Acoustic Radiation Force Impulse (ARFI) is an ultrasound imaging technique in which acoustic waves are focused at a point, causing displacement of the tissue that is then tracked over time to measure elastic and viscoelastic material properties from the imaging data. The data set chosen for this work is rather large (roughly 2000 pixels by 60 pixels), and can be treated as a high dimensional vector for clustering algorithms. Specifically, this data imaged areas of porcine atherosclerotic plaques. Atherosclerosis is a cardiovascular disease in which plaque accumulates along the wall of an artery, altering blood flow and increasing the risk for heart attack or stroke. In this original research project, we investigate the application of data clustering algorithms, namely K-Means, Self-Organizing Maps (SOMs), and Relational SOMs, to ARFI imaging for early detection and characterization of atherosclerotic plaques. In this context, we hope to cluster images based on similar patterns in the data set. Based on the dimension, size and scope of image patterns considered in this work, the clustering configuration used for each clustering algorithm considered was a 3x3 lattice of nine neurons. We discuss the data, the three algorithms, the clusterings, and metrics to determine which clusterings are ideal. Pedagogically speaking, this talk will present an introduction to two clustering algorithms and a variation which could be implemented in a course on data or clustering.

## **Writing Across the Curriculum in Mathematics**

**Part A: Friday, July 28, 11:10–11:45 AM, Salon A-2**

**Organizers: Anil Venkatesh** Ferris State University

**Benjamin Gaines** Iona College

**Victor Piercey** Ferris State University

Many institutions have adopted “Writing Across the Curriculum” programs and implemented first-year writing seminars. Even when such programs are not in place, instructors are becoming increasingly aware of research that has identified writing as a high impact practice for enhancing student learning. In particular, writing-based assessments help students to shift focus from grades to deep learning and to develop skills that transcend any one subject area. In all levels of math courses, writing assignments can be used to develop critical thinking skills,

provide a better understanding of logical argument, and engage students who may otherwise be left behind. This session invites talks on all aspects of writing in mathematics, especially those pertaining to Writing Across the Curriculum programs. We also welcome presentations on the implementation of Writing to Learn principles in math courses, training of students in discipline-specific skills such as proof writing, and interdisciplinary writing initiatives.

**Katharine Shultis** Gonzaga University

#### **(Writing Enriched) Introduction to Proofs**

In Gonzaga University's newly implemented Core Curriculum, students must take several "writing enriched" courses and are encouraged to take at least some of these courses in their major. This talk attempts to prove the conjecture that an introductory proofs course is a natural course to satisfy the learning outcomes of a writing intensive course at any institution and proposes that we should be meeting these learning outcomes. Along the way, we'll look at the process at Gonzaga, what I did in my Spring 2017 course, and how well the course actually met the writing focused learning outcomes.

**Visala R. Satyam** Michigan State University

**Mariana Levin** Western Michigan University

**John P. Smith** Michigan State University

**YoungGon Bae** Michigan State University

**Kevin Voogt** Michigan State University

#### **Students' Experiences in a Transition to Proof Course**

The transition from computational work to learning how to write proofs is known to be difficult for undergraduates. We know some of the difficulties they encounter, but we also need to understand students' experience, on how they make sense of this change in what mathematics is and what it means to be successful at math. We report on students' experiences of a transition to proof course focused on proof writing, based on interviews with students after completing the course. We focus on how they perceive this work as different from past courses, what they learned about mathematical writing, and their sense of success. We find that students go through a long period of adjusting to the task of writing but come away with new understandings of what mathematics is as a result.

### **Part B: Friday, July 28, 1:00–4:55 PM, Salon A-2**

**Maria Fung** Worcester State University

#### **Writing to Learn Journals in a Math for Elementary Teachers Course**

This talk will describe a series of journal writing assignments from a Mathematics for Elementary Teachers course. Prompts are used at first, and then later students complete more complex assignments as a vehicle to reflect on what they have learned, to connect concepts and to consider different contexts for their content knowledge.

**Sarah Wolff** Denison University

#### **Student Writing as a Tool for Synthesizing Ideas in Calculus**

Writing is an important strategy to help students understand their own thought processes and develop their own questions as they tackle complex problems. In order to have my calculus students start to understand mathematics as a web of interrelated ideas, rather than a bunch of formulas with steps to be memorized, I introduced 'write-ups' in my fall multivariable calculus class. The assignment was a weekly mini-essay that required students to fully explain the 'why' behind their solution before providing the explained solution itself. In this talk, I will describe the general assignment, show some examples of student work, reflect on the growth I saw in the students, and also talk about how to get students on board with the idea.

**Jason Moliterno** Sacred Heart University

#### **Do Students *Really* Understand What They're Doing?**

In math classes, students often do many calculations but are unaware of why the calculations work the way they do. This often leads to mindlessly following an algorithm, but having no real understanding of the mathematical concepts at hand. In this talk, I discuss several writing assignments that I have given in classes such as First Year Seminar, Introductory Statistics, Calculus I, Linear Algebra, and Differential Equations. These writing assignments range from essay questions on quizzes or exams to out-of-class writing assignments. The purpose of these writing assignments is to go beyond just doing rote calculations and to have a deeper understanding of the underlying mathematical ideas. These assignments give students the opportunity to organize their thoughts and to become better writers and thinkers.

**Chris Oehrlein** Oklahoma City CC

### **Reflection Writing in Applied Calculus and Statistics Courses**

As available technology has changed, students are being asked to memorize rules and procedures less and are being asked to become better problem solvers and users of that technology. Lost in the transition can be a real understanding of the underlying concepts. Students can develop that deeper level of comprehension through writing. The presenter asks students to write short paragraphs in response to prompts about conceptual aspects of applied calculus and introductory statistics courses. After an initial assessment of and comments about the responses, the students may rewrite their responses. The presenter will show examples of the types of questions and methods of assessment used to help students learn from the writing-rewriting process.

**Sarah Hanusch** SUNY Oswego

### **Summative Portfolios in Mathematics Courses**

One way to incorporate writing into a mathematics classroom is by assigning a summative portfolio at the end of the semester. Creating a portfolio requires the student to synthesize key ideas, select artifacts, and reflect on the course. In this presentation I will share descriptions of the portfolio projects I have assigned, my perceptions of the assignment, and some excerpts of student writing from completed assignments.

**Yelena Vaynberg** LTU

### **Writing in Geometry Course**

This talk will focus on how the use of writing was integrated into the Geometry in Arts course. While the primary purpose was to motivate mathematical thinking, I also wanted to teach students how to communicate mathematical ideas clearly to those outside of their classroom. I will describe some of the projects that students have completed, using specific examples. Sample writing assignments will be described, along with background information that students can be given as a motivation for their work. In addition to exploring the mathematical examples, I will report on student feedback regarding these assignments.

**Edwin P. Herman** University of Wisconsin-Stevens Point

### **Using Discussion Boards and Papers in an Introductory Statistics Course**

A decade ago I modified my Introductory Statistics course to include online discussions and written papers. Students would have online readings and would discuss their opinions, using statistics to inform and support those opinions. These discussions culminated in written papers, typically three per semester. I used topics such as global climate change and the state of the economy –controversial topics with large amounts of associated data –and required students to use statistical arguments to support their paper positions. In this presentation I will discuss my method for using discussion boards and papers, as well as my grading rubrics. A comparison of class grades will be made to illustrate that paper-writing assesses a different skill set than exam-taking, allowing students to better display a deep understanding of certain core concepts. Additionally, I will discuss changes in my paper topics –for example, given our current administration’s views on climate change, many of my data sets and background readings are no longer as certain as they once were.

**Magdalena Luca** Mass College of Pharmacy

### **Scientific Writing? What Is That?**

In all degree programs offered at our university, strong verbal and written communication skills are absolutely essential for students’ communication with patients, prescription of drug treatments, and understanding of public health issues. All humanities and many of our English courses are designated as writing intensive (WI), and all students must meet writing proficiency (WP) standards through our Writing Across the Curriculum programs. Students are rarely required to write in math courses, however, and they do not see the purpose of writing in such courses. In this presentation I will address the extensive use of scientific writing in an upper-level Biostatistics course. In this course, emphasis is placed on scientific reasoning: reading, writing, and interpreting statistical analyses found in journal articles. These skills prepare our students well for capstone seminars, the MCAT, and in-depth understanding of scientific studies. Students use scientific writing for all assignments and presentations. I will present specific examples in which writing is an essential component.

**Dan Kemp** South Dakota State University

### **Mathematical Writing Through Group Projects**

Calculus students are introduced to writing mathematics by word processing, including mathematical symbols, solutions to projects done outside of class in assigned groups. The projects are mathematical exercises that are typically not found in calculus texts that involve mathematics related to concurrent syllabus topics. The best projects have a historical basis. For example, after studying convergence of series, students prove that Euler’s Basel series converges by working through a recently published elementary proof. The write up is to be done in narrative style using the MSWord Equation Editor for mathematical symbols. The papers, one per group, are graded primarily for content, but 20% of the grade is for writing style and mathematical clarity. Mathematical writing and use of Equation Editor is not a skill brought to college by students, but definitely improves throughout the semester. Composite solutions are constructed and used to help teach mathematical writing. Students turn in an evaluation form of group activity and behavior to help insure group members make equal contributions.

**Daniel Kiteck** Indiana Wesleyan University

### **Writing on the Mathematics of a Great Mathematician from History**

Three times I have taught a History of Mathematics course, and each time the biggest student project is writing a 2500-word paper on the mathematics of an historical mathematician of choice. I have developed the course to help students grow in their writing ability leading up to this final paper. They write multiple essays, and I give them feedback in a detailed rubric of areas in which to grow. Each student also gives feedback to another student's essay and draft of the final paper to consider the perspective of assessing someone else's paper. I have incorporated the book *Journey Through Genius* and point to its chapters as quality examples of writing for students to emulate in their own papers. I have watched students grow in their writing on mathematics.

**Emlee Nicholson** Millsaps College

### **Making Writing Meaningful in an Entry Level Math Course**

**Abstract:** For decades, Millsaps College has followed a Writing Across the Curriculum model. Millsaps launched a new general education curriculum 2 years ago. It included a skills based freshman seminar tasked with developing students' problem solving and collaboration skills. The seminar is discipline specific but skill development is the primary goal. I have taught this course twice on the topic of cryptography. The writing in this course was born of necessity- assessment. A test can assess what students know. In math, what students know and what they can do are intrinsically intertwined. However, tests are better at assessing content knowledge than skill level. In this talk, I will discuss the implementation of process reports. My students provide written explanations of their problem solving process. This introduces students to the sort of general thinking and writing that goes into a proof without the knowledge of mathematics and rigor involved in proof writing. Students include everything they tried, why they tried it, if it did not work, how they decided it did not work, what they chose as the next strategy and why, etc. Furthermore, their explanation of process must be well written as thought of in the discipline of mathematics: clear, effective and concise, and as easy as possible to understand. The talk will include both successes and failures of these assignments.

**Adam Giambrone** University of Connecticut

### **Incorporating Reading/Writing Assignments into a Liberal Arts Mathematics Course**

At many institutions, a number of students take a liberal arts math course to satisfy a general education requirement. Consequently, students may come into such a course with the mindset that the course is simply a box to be checked off on a checklist of graduation requirements. In an effort to increase student engagement and help students see how such a course fits into their college education, reading/writing assignments about intelligence, learning, and thinking have been woven into the Elementary Discrete Mathematics courses at the University of Connecticut, starting in the Fall 2015 semester. In this talk, we will discuss the creation and implementation of these reading/writing assignments, using student feedback and responses to highlight some benefits and challenges to incorporating this type of assessment into a liberal arts math course.

## **Math Potluck: A Student Swap Session**

**Saturday, July 29, 1:00–2:55 PM, Salon A-4**

**Organizers:** **Alissa Crans** Loyola Marymount University

**Jacqueline Jensen-Vallin** Lamar University

**Candice Price** University of San Diego

**Alejandra Alvarado** Eastern Illinois University

**Dora Ahmadi** Morehead State University

**Timothy Fest** SIAM

**Angela Spalsbury** Youngstown State University

Calling all undergrads and faculty advisors! Does your department have (or want!) a Math Club or student chapter of the AWM, MAA, PME, or SIAM? This session will provide a forum for sharing your favorite or most successful student activity. The presenter(s) will provide a "how-to" for a single math event that a math club or student chapter has held. Together, we will build a toolbox of successful activities to take back to each of our campuses! Following the morning presentations, a free lunch will be held for all presenters and attendees of this session to promote continued discussion and collaboration amongst participants. Please indicate in your abstract submission whether your group is a Math Club or student chapter of AWM, MAA, PME, or SIAM.

**Katie Anders** University of Texas at Tyler

### **Job Wanted: Building a Math Club Talk About Internships and Career Opportunities**

We will discuss how to build a math club talk about internships and career opportunities outside academia. We will present techniques for discovering local, regional, and national opportunities.

**Monica E. Busser** Youngstown State University

### **“Bigs and Littles” Mentoring Program: Confronting the “Leaky Pipeline Effect”**

Our Youngstown State AWM chapter started a mentoring program called “Bigs and Littles” in Spring of 2017. Through this program first and second year female math majors are assigned to a third or fourth year female math major who has exhibited excellence in her early mathematical career. In its first semester it included nine littles and seven bigs who were assigned to one another based on career plans, interests, and any preferences the little specified. The program was created in hopes of correcting the “leaky pipeline” effect by giving students a sense of belonging to the Department and providing them with someone to give advice and support. Bigs have proven to be a great resource for information on opportunities such as research, conferences, REUs, and graduate school, and the program caused a dramatic increase in first year female math majors wanting to become involved in a research project.

**Gabbie Van Scoy** Youngstown State University

**Natalie Halavick** Youngstown State University

### **The PME Ohio Xi Chapter Regional Conference: A How-To Guide**

The Ohio Xi Chapter of Pi Mu Epsilon at Youngstown State University has held an annual PME Regional Conference for nearly the past thirty years. Both college and high school students from across the region are invited to attend and present their research, as well as have the opportunity to interact with others with an interest in mathematics. We will discuss planning, organizing, funding, and running this annual event.

**Linda Beverly** CSU East Bay

### **Open Source Hack Day at CSU East Bay**

We will discuss an activity that was successful due to the collaboration of four different clubs from CSU East Bay. We have had several activities held between the Recreational Math and Computer Science Club, AWM, ACM-W, and SciTalk. We will focus on our experience with Open Source Hack Day.

**Leonida Ljumanovic** UW-Platteville

### **Mathematical Modeling Contests for College Students**

Mathematical modeling contests for college students I have been faculty member at UW-Platteville for more than eight years. At UW-Platteville Mathematics Department we do not have applied math program, and yet we have successfully organized mathematical modeling contest for UW-Platteville students since 2013. My interest in applied math started after learning about some of the existing math modeling contests for college students in 2009. In the beginning, I often could not even organize one team to participate at these contests. Students seemed to be unwilling to sign up for something they were unfamiliar with, and in order to participate at some of these contests students had to travel about two hours from our campus. So group of faculty members at UW-Platteville started organizing annual mathematical modeling contest only for UW-Platteville students. I will talk about our mathematical modeling contest, how our students benefit from it, and how the contest led to organizing a separate student club from already existing Math Club, which is called the Mathematical Modeling club. I am the adviser for this club, so I will also talk about the UW-Platteville Mathematical Modeling club, how it differs from UW-Platteville Math Club, and what are the future plans for the club.

**Paul Fonstad** Franklin College

### **So You Think You Know Math: The Math Club Game Show!**

Trivia isn't just good for improving your memory or helping your brain make new and unexpected connections; it's fun too! A trivia game show consisting of categories like “Math in Disney Songs,” “Punny Math,” and “Mathematicians You Never Knew You Knew” makes for an ideal math club activity that will excite both participants and audience members alike. This talk will discuss how to best format your game show to keep students engaged, and give ideas for ways to add your school's own unique twist to the contest. Two tested versions of the game show will be provided to all attendees: one meant for a more general college audience, and one designed to challenge even the savviest of math majors!



## Undergraduate Research Activities in Mathematical and Computational Biology

**Friday, July 28, 1:00–3:35 PM, Salon A-4**

**Organizer: Timothy D. Comar**    Benedictine University

This session is dedicated to aspects of undergraduate research in mathematical and computational biology. First and foremost, this session would like to highlight research results of projects that either were conducted by undergraduates or were collaborations between undergraduates and their faculty mentors. Of particular interest are those collaborations that involve students and faculty from both mathematics and biology. Secondly, as many institutions have started undergraduate research programs in this area, frequently with the help of initial external funding, the session is interested in the process and logistics of starting a program and maintaining a program even after the initial funding expires. Important issues include faculty development and interdisciplinary collaboration, student preparation and selection, the structure of research programs, the acquisition of resources to support the program, and the subsequent achievements of students who participate in undergraduate research in mathematical and computational biology. Finally, the session also welcomes the presentation of materials and project ideas that can be used to help get students started in research in mathematical and computational biology.

**Ada N. Morse**    University of Vermont

### **Tiered Mentoring in REUs: DNA Nanostructure Self-Assembly case study**

We discuss a tiered mentoring approach to the inclusion of graduate students in summer REU programs, using the DNA nanostructure self-assembly REU at St. Michael's College as a case study. Mathematics graduate students Ada Morse (Summers 2015-1016) and Margherita Ferrari (Summer 2016) were introduced to the project at an intermediate level, mentoring groups of undergraduate students (including math, computer science, and biology majors) as well as being mentored (both in mentorship as well as their own research) by the faculty advisors. We review potential advantages of this system for both undergraduates and graduates as well as possible issues to be aware of and avoid. Included in this discussion will be an overview of the research projects and results during the summers in question.

**Timothy D. Comar**    Benedictine University

### **An Age-Structured Pulse Vaccination Model for HPV**

This talk focuses on results we have obtained with undergraduate students researchers on developing and analyzing an age-structured pulse vaccination model for HPV. The model partitions the population into classes based on age, level of sexual activity, and disease status. Our model assumes that three doses of the vaccine are needed to become fully vaccinated. We establish the conditions for which the disease free solution is globally attractive and for which the infective population remains permanent. We discuss the impacts on the spread of HPV in the the population if some individuals receive less than the full three-dose vaccination regimen. We also discuss how the students are prepared to conduct similar projects and provide avenues for future projects appropriate for undergraduates.

**Pengcheng Xiao**    University of Evansville

**Adam Lonnberg**    University of Evansville

### **A Simplification and Quantitative Analysis of Stress Reaction System- HPA Axis**

The human stress response is controlled largely by the hypothalamic pituitary adrenal (HPA) axis. Models predicting the levels of the hormones involved very often not analytically solvable because of complexity. Many of these models predict a bistability in this axis. Said bistability results in two steady states, a normal high-glucocorticoid receptor (GR) state, and an alternate low-GR steady state. In this research, we proposed one simplified HPA axis model in order to solve it more easily while maintaining key features such as the bistability thereof. This allows for a similar model that retains usefulness providing more analytic insight into the equilibrium stability of the system.

**Hwayeon Ryu**    University of Hartford

**Quinton Neville**    St. Olaf College

### **Feedback-Mediated Dynamics in a Model of a Long-Looped Nephron**

This talk starts with presenting results of the 2016 summer project on a feedback mechanism in the kidney, tubuloglomerular feedback (TGF) system, which regulates the tubular fluid pressure, flow, and its NaCl concentration at the nephron level. In particular, we are interested in studying how the feedback-mediated dynamics in the long-looped nephron differ from those for the short-looped nephron, which could help to better understand the formation of highly concentrated urine in the desert mammals. Model development, and analytical and numerical study results will be presented and compared with experimental data. Finally, this talk will end with a couple of different directions for future investigation based on our modeling results.

**Barry C. Husowitz**     Wentworth Institute of Technology

### **Artificial Neural Networks (ANN) Applied to Monte Carlo and Molecular Dynamic Simulations**

The computational expense to accurately simulate large atomistic systems is enormous. In order to overcome this computational cost empirical potentials are used, which rely on parameters that are adjusted to reproduce either experimental or first principle references. Furthermore, careful parameterization of such potentials can be challenging and has to be validated for each system. Artificial Neural Networks (ANN) approaches such as those proposed by Behler and Parrinello circumvent this parameterization problem by employing machine learning techniques to calculate the potentials. The Behler-Parrinello approach calculates the potential energy surface based off a set of reference calculations. This approach has the advantage that it can be well automated, new potentials can be constructed easily, permits application of potentials to arbitrary structures, and reaches accuracies comparable to first principle methods with a much lower computational cost. I am interested in using ANN's to calculate potential energy surfaces to use in Monte Carlo simulations and Molecular dynamic simulations of polymeric systems and crystal structures. In particular, I am interested in simulating the crystal structure of Perovskite to obtain x-ray scattering data that we can compare to experimental x-ray scattering data.

**Courtney L. Davis**     Pepperdine University

**Timothy A. Lucas**     Pepperdine University

### **Undergraduate Research on Discrete Models of the Impact of Drought and Invasive Crayfish on California Newt Populations**

In collaboration with undergraduate researchers we have developed discrete mathematical models for studying the population dynamics of the California newt (*Taricha torosa*), a species of special concern in California. Recent declines and local extinctions of native California newt populations in Santa Monica Mountain (SMM) streams motivate our investigation of the impact of drought and invasive crayfish (*Procambarus clarkii*) on newt population sizes. Multiple studies predict the recent severe drought in California will persist and even increase in duration and severity. In addition, invasive crayfish have decimated native newt reproduction in SMM streams. Our models incorporate demographic parameters such as survival rates for newt life stages and egg production, which depend upon habitat availability and rainfall. We estimate these parameters using 15 years of local stream survey data. Our models capture the observed decline of the studied newt population and replicate crayfish trapping data. The first model makes predictions about how the length and severity of drought can affect the likelihood of persistence and the time to critical endangerment of a newt population. The second model evaluates the persistence or the time to extinction for newt populations under crayfish trapping regimes when varying the trapping resources, frequency of trapping, and susceptibility of the crayfish population to trapping. Predictions made with both models inform restorative efforts and crayfish management. In addition to the research, we will discuss the recruitment and training of mathematics and biology undergraduates and their contributions to the statistics, modeling and programming.

## **Connecting Introductory Mathematics Courses to Students' Intended Majors and Careers**

**Friday, July 28, 1:00–4:55 PM, Salon C-4**

**Organizers: Rebecca Hartzler**     University of Texas-Austin

**Suzanne I. Dorée**     Augsburg College

**Susan Ganter**     Virginia Polytechnic Institute and State University

**Thomas A. Hoft**     University of St. Thomas

This session explores the many ways in which introductory mathematics courses can be created or renewed to meet the needs of the partner disciplines and lay the groundwork for students' future careers. For example, talks may share novel activities, examples, or projects suitable for introductory mathematics courses that showcase how mathematics is used in the partner disciplines or in specific careers. Presentations may describe curricular innovations, such as courses or pathways, which were designed or revised to support students from specific majors or on specific career paths. Talks may describe successful course-embedded strategies that help first-year students discern their major or career path. Presentations may report on models for collaboration between mathematics faculty and faculty from other departments or people from industry on the introductory mathematics curriculum. Each talk should address some aspect of how introductory mathematics courses can be aligned with external needs of students' intended majors or careers.

**Lisa Holden** Northern Kentucky University  
**Brooke Buckley** Northern Kentucky University  
**Dhanuja Kasturiratna** Northern Kentucky University

### **A 3-Pronged Approach for Helping Students Discover Careers in the Mathematical Sciences**

Newly declared majors are introduced to our department and careers within the mathematical sciences during a required, 1-credit hour seminar that is often team-taught by mathematicians and statisticians. Successful class strategies have included lively alumni panel discussions, speakers from local companies, and visits from current upper-level majors. Next, early undergraduate summer research experiences are encouraged through a unique University-sponsored program known as UR-STEM. Students, particularly those designated as at-risk, are funded to work on research experiences accessible to freshmen- and sophomore-level students. Finally, in partnership with the PIC Math Program, a course was offered this spring to provide research opportunities that partnered teams of students with local businesses and organizations. This multi-step approach has not only made students aware of current paths to careers in the mathematical sciences but has also educated faculty and strengthened ties to department alumni.

**Jenna Reis** Fitchburg State University

### **Freshman Seminar in Applied Math: A Lesson in Adaptive Teaching**

This presentation will outline the design of the Freshman Seminar in Applied Mathematics offered at Fitchburg State University. It is relatively new, and is required of all math majors within the Applied Math concentration. Classwork supplements guest speakers and a career panel, and topics include linear algebra, graph theory, differential equations, and operations research. The presentation will also cover the adaptive approach to content as a result of nontraditional students in the course and lessons learned.

**Brandy S. Wieggers** Central Washington University

### **Math 299s: Introduction to the Math Major**

Math 299s was designed to ensure that students don't enter the math major as sophomores and then discover during their senior year that they aren't actually interested in the breadth of mathematics beyond calculus and computation. Bringing the problem solving, professional development, communication and writing experiences into the sophomore course and introducing mathematical topic areas with specific focus on the research problems explored by our faculty increased students' knowledge of mathematics and our program. In addition, we have had students more actively pursuing REU experiences, scholarships, and conferences. This presentation will focus on the professional exposure and development component where students create resumes, look for dream jobs they'd like to do five years post graduation, and think about the experiences and skills that they need to be competitive in applying for those positions. Recommendations for developing your own course and ideas for bringing components of this course into math club meetings and seminar experiences will also be provided.

**Timothy Yusun** Simon Fraser University  
**Tamon Stephen** Simon Fraser University

### **Teaching Simulation in an Introductory Operations Research Course**

The MATH208W (Introduction to Operations Research) at Simon Fraser University is a gentle introduction to mathematical modeling in operations research, with an emphasis on applications and computation for analysis. The only prerequisite for MATH208W is a first calculus course; this is because the course is designed as a recruiting tool for students who are considering an operations research major or minor. For many students it is the first course where they are exposed to larger, more realistic cases that can be solved using mathematical modeling, giving them some idea of the possible careers a major in OR allows them to pursue. In the Spring 2017 semester we decided to include an experimental aspect to the course, by adding a module on discrete-event simulation. Our goal in doing this was to introduce them to simulation as a problem-solving tool, and encourage them to pursue further studies in mathematics. This was not easy, as the mathematical backgrounds of the students varied dramatically. In this talk, we discuss what we did to incorporate simulation into the MATH 208W curriculum; this involved writing course notes from scratch, creating exercises, and incorporating a software implementation aspect (using Microsoft Excel).

**Phong Le** Goucher College

### **Community Based Learning in an Introduction to Data Course**

Local community organizations with data and statistical needs provide beneficial authenticity and context in the classrooms. Students can provide creative, broad perspectives to problems and issues articulated by those organizations. The students learn about the work of data consultants and small nonprofits. This paper describes characteristics of equitable partnerships. This also includes an account of a partnership with a nonprofit health service clinic.

**Anders Hendrickson** St. Norbert College

### **Customizing a Statistics Course for Business Majors**

At St. Norbert College in Wisconsin, the recent reorganization of the business school provided an opportunity to redesign the quantitative component of the undergraduate business major. Cooperating with the business faculty, the mathematics department replaced a business calculus course that had outlived its purpose with an introductory statistics course tailored to the needs of our college's business students. The new course emphasizes probability and inferential statistics while still including a short, highly practical calculus module targeted at economics. We will relate the process of creating this course, discuss some of the challenges and benefits of cooperating with another department, and look back on how well the course succeeded in its first year.

**Helen E. Burn** Highline College

### **Incorporating Partner Discipline Voices in Retooling Introductory Statistics in Washington State**

This session details the process currently underway and preliminary results related to aligning Introductory Statistics to specific programs of study in post-secondary institutions in Washington State. The session begins with background context situating Washington State within the larger math pathways effort and describing the state-level process that identified the need to seek input from partner disciplines in examining student learning outcomes for introductory statistics. Following the background context, the session shifts to focus on the approach taken to seeking meaningful input from the partner disciplines. This includes drawing from existing research conducted by the professional associations (e.g., MAA and ASA) and collecting primary data through interviews with faculty in the partner disciplines, including faculty in business, nursing, and STEM fields. Session participants will gain insights from the preliminary results.

**Francisco Savina** The Charles A. Dana Center at The University of Texas at Austin

### **The Right Math for the Right Student at the Right Time: Facilitating Multidisciplinary Discussions**

The Charles A. Dana Center at the University of Texas at Austin seeks to ensure that all students in higher education will be prepared to use mathematical and quantitative reasoning skills in their careers and personal lives. To achieve this, it takes coordinated action across all sectors to align mathematics courses to programs of study, and requires mathematics departments to shift from a service-oriented approach to being an engaged partner discipline. It is essential that math departments take a leadership role in having multidisciplinary discussions with partner disciplines to better align mathematics courses to programs of study. To facilitate these discussions, the Dana Center has developed a suite of multidisciplinary discussion tools that faculty and administrators can use to prepare for, and implement, multidisciplinary discussions focused on identifying a default mathematics course requirement that is most relevant for each program of study. Presenters will share these tools and provide background into their design and development. Participants will gain insight into how to customize these tools for use at their own institutions.

**Suzanne I. Doree** Augsburg College

**Pavel Belik** Augsburg College

**Jody Sorensen** Augsburg College

### **Renovating Calculus Through Conversations with the Partner Disciplines**

As part of the 10-institution *Collaborative Research: A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships* (SUMMIT-P), faculty at Augsburg College are renovating the 3-semester calculus sequence to better meet the needs of the partner disciplines. In this talk, we will describe the findings of earlier research from the MAA CRAFTY Curriculum Foundations Project and the process we used in this new project to connect with faculty from Biology, Business, Chemistry, Computer Science, Economics, Environmental Science, and Physics both on our campus and through the national consortium about how calculus is used by students majoring in these fields. We will present the partner's "wish-lists" of what their students need to know and be able to do coming out of the calculus sequence, and a summary of changes planned in response for Augsburg's courses. Changes will include increasing the relevance and frequency of applications in the courses; adapting and developing materials to make quantitative labs the primary focus of the courses; and re-sequencing the topics to better mesh with the timing needed by the partner disciplines.

**Stuart Boersma** Central Washington University

### **Pre-calculus Student Activities for STEM Intending Students**

The Charles A. Dana Center at the University of Texas at Austin has developed an innovative new curriculum for STEM intending students on the path to Calculus. This curriculum provides students a meaningful precalculus experience that values and honors their future STEM courses by providing extensive opportunities to explore mathematics within authentic STEM contexts. In order to provide authentic contexts, many student activities were designed in collaboration with faculty members from other disciplines. This paper will describe these collaborations and provide specific examples of student activities that are tied to STEM disciplines.

**Robert G. Root** Lafayette College

### **Winning Students from Several Disciplines with a Course in Evolutionary Game Theory**

Evolutionary game theory has broad appeal to students in biology, neuroscience, economics, political science, business, government, and even engineering. An introduction to the field requires very little math knowledge. High school algebra 1 and discrete probability is enough to support a semester-long investigation that takes students from the elements of game theory to Hamilton's rule and multilevel selection using examples that appeal to a broad swath of students, and demonstrate the usefulness of game theory as a modeling tool in a wide variety of situations, including some that seem to be everyday to many. While not a modeling course per se, the breadth of applications for approachable mathematical models can draw students further into the mathematics curriculum.

**Gregory V. Bard** University of Wisconsin—Stout

### **Realistic Examples of Bayes's Rule from Cybersecurity**

The words "Bayesian Reasoning" have been used in a great deal of computer science research articles over the past decade, especially for computer security, and even in articles appearing in ordinary non-technical magazines. However, the applications—when modeled realistically—require much more complex models than the primitive models appearing in a typical textbook for a course in Discrete Mathematics or Finite Mathematics. This has the unfortunate side-effect of leaving students stranded when their coursework prepares them insufficiently for modeling realistic situations (in their full complexity) as they would occur in applications. The author teaches both Discrete Mathematics and Finite Mathematics, as well as advanced courses in Computer Security and Cryptography. Drawing from the author's computer security experiences at the National Security Agency (NSA), the author has prepared three case studies of Bayesian Reasoning: anomaly-based intrusion detection in computer networks; filtering emails to remove "spam" messages; and scanning compiled software to detect viruses or other malware. These will be presented with no assumption of knowledge of computer science beyond the elementary level. It is hoped that these case studies will motivate cybersecurity students in Discrete Mathematics, and excite them about the more technical skills that they will acquire in the 300-level and higher courses. Moreover, there is some hope that even a few top business students in Finite Mathematics will gain an appreciation for the capacity of mathematics to address society's most pressing needs in computer security.

## **Exploring Zeros of Polynomials**

**Friday, July 28, 1:00–5:35 PM, Salon C-6**

**Organizers: Michael Brilleslyper** U. S. Air Force Academy

**Beth Schaubroeck** U. S. Air Force Academy

Mathematics Magazine (June 2016) have focused on intriguing properties of polynomials and their zeros. Given the long, rich history of this topic in mathematics and the wide array of techniques utilized, we believe talks in this area would appeal to a variety of mathematicians and students. Possible topics include extensions of standard results such as Descartes' rule of signs or the rational roots theorem, dynamics of numerical root finding schemes, interesting graphical properties of sets of zeros, properties of zeros for specific families of polynomials (e.g., cyclotomic polynomials), novel proofs of standard results, and illuminating talks on well-known theorems and conjectures such as the Gauss-Lucas Theorem, Siebeck's Theorem, and the Sendov Conjecture. Talks that utilize technology to visualize particular phenomena or results are particularly encouraged, as are talks that provide a historical perspective. We are also interested in talks that present open problems suitable for undergraduate research or independent study. Talks in this session should be accessible to advanced undergraduate students.

**Pavel Belik** Augsburg College

### **When and Why does Laguerre's Method Misbehave?**

One of the top Google search results on Laguerre's method for finding roots of polynomials starts with "A root-finding algorithm which converges to a complex root from any starting position." We discuss this method and try to understand why one would make such an incredible claim. Along the way, we encounter a beautiful numerical method, simple polynomials, complicated basins of attraction, cool analysis, loss of significance, effects of symmetry, and quite a few unanswered questions.

**Craig J. George** University of Houston

### Reimagining Polynomial Root Solutions via Taylor Polynomial Transformations

In the history of solving for the roots of polynomials, several standard methods have been constructed through geometric reasoning. In this talk, I will show the Taylor expansion of a polynomial about a point can transform the polynomial from an “unsolvable” form to solvable and make the process arguably simpler. This talk will go in detail on the method of using Taylor expansions for polynomials. A concrete example of this method: in the case of a 2nd degree polynomial  $f(x) = ax^2 + bx + c$ , a general formula can be found for the roots of all such  $f$  simply by examining the single root of  $f'$ , say  $z = -\frac{b}{2a}$ . Then, looking at the Taylor expansion of  $f$  about  $z$  gives the ‘New Quadratic Formula’:  $x_{\pm} = z \pm \sqrt{\frac{-f(z)}{a}}$  a much nicer formula. The derivation of this quadratic formula is straight-forward, but I plan on presenting it to demonstrate the appeal of using Taylor in this context. Though the cubic formula is a bit longer, it too can be drastically simplified from the standard formula. As for  $n = 4$ , there is a general ‘algorithm’ for finding the roots. Lastly, since degrees  $i$  greater than or equal to 5 do not have general formulas for their roots, this talk will examine how to use the technique mentioned above to construct conditions on the coefficients that (when satisfied) imply that the polynomial can be transformed to an easier-to-solve form, e.g., for  $n = 5$ ,  $a + bx + x^5$ .

**Paul Zorn** Saint Olaf College

### Descartes’ (and Budan’s and Fourier’s) Rules of Signs

Descartes’ Rule of Signs, dating from around 1637, bounds the number of positive roots of a real polynomial based on the sign pattern of the polynomial’s coefficients. Over the last 380 years a surprising number of proofs, generalizations, and reinterpretations have appeared, including one partly attributed to Fourier in the early 1800s. The talk will review the Rule’s classical form, mention some proofs, and point to some (more) modern extensions.

**Harry Richman** University of Michigan

### Looking for a “Local” Gauss-Lucas Theorem

The Gauss-Lucas theorem is a statement that constrains the zeros of the derivative  $f'(z)$  of a complex polynomial, based on the zeros of the original polynomial  $f(z)$ . It is “global” in the sense that the constraint depends on knowing where all zeros of  $f(z)$  are. If all zeros of  $f(z)$  are on the real axis, then Rolle’s theorem implies that between any two zeros of  $f(z)$  there must be a zero of  $f'(z)$ . This statement is “local” because you only need to know two zeros of  $f(z)$  to have a constraint on  $f'(z)$ . This motivates the question: is there some “local” strengthening of the Gauss-Lucas theorem? This appears to be an open problem. In this talk we discuss some obstacles to finding such a statement.

**Christopher Frayer** University of Wisconsin-Platteville

### Geometry of Generalized Cubic Polynomials

Given a complex-valued polynomial of the form  $p(z) = (z - 1)^k(z - r_1)^m(z - r_2)^n$  with  $|r_1| = |r_2| = 1$  and  $k, m, n \in \mathbb{N}$ , where are the critical points? The Gauss-Lucas Theorem guarantees that the critical points of such a polynomial will lie within the unit disk. We will further explore the location and structure of these critical points. Surprisingly, when  $m \neq n$ , the unit disk contains two ‘desert’ regions in which critical points cannot occur, and each  $c$  inside the unit disk and outside of the desert regions is the critical point of exactly two such polynomials. Special attention will be given to the development of geometric intuition and using GeoGebra to provide graphical illustrations.

**Michael Brilleslyper** U. S. Air Force Academy  
**Beth Schauboreck** U. S. Air Force Academy

### Trajectories of Critical Points

The Gauss-Lucas theorem states that the critical points of a polynomial of a single complex variable all lie within the convex hull its zeros. If all the zeros lie on the unit circle, then the critical points all occur inside the unit disk. The exception is when the polynomial has repeated roots, in which case critical points lie on the unit circle. In this talk we consider a polynomial of degree  $n+1$  with all its zeros on the unit circle. We fix  $n$  of the zeros and move the other zero through a complete revolution along the circle. As this zero moves, we examine the trajectories inside the unit circle traced out by the  $n$  critical points. We show several animations that illustrate the loci of critical points may have 1 to  $n$  connected components. We also describe a simple class of polynomials for which the trajectories form a family of circles.

**Barbara Margolius** Cleveland State University

### Asymptotics of Random Processes and Zeros of Polynomials

In this talk, we explore a collection of applications for which identifying zeros of polynomials is important. For quasi-birth-death processes (QBDs) with time-varying periodic transition rates, we can find asymptotic estimates of transition probabilities by expressing the generating function for the QBD as a ratio of two functions with the denominator function a polynomial. With that setup, the level probabilities can be approximated with a geometric distribution for which the ratio,  $r$  is determined by the zeros of the polynomial in the denominator. If the probability generating function is given by

$$P(z, t) = \frac{f(z, t)}{g(z)}$$

then

$$p_n(t) \approx r^n f(r, t)$$

where  $p_n(t)$  is the probability of being in level  $n$  at time  $t$ ,  $r$  is such that  $g(r^{-1}) = 0$ , and  $f(r, t)$  is periodic. A similar approach is possible for bounded random walks in the plane. We use visualization software developed by Elias Wegert to explore the zeros of the denominator polynomials.

**Jeff Johannes** SUNY Geneseo

### Galois Theory for High School Students

This spring I taught a semester-long Galois theory course to advanced undergraduates. Part of their final experience was to write an introduction to Galois theory suitable for high school students. In this talk we will consider this attempt to bridge the abstract work of a course following group theory to the pre-calculus level of finding roots of polynomials (or not). We will also consider Galois' own infamous attempts to communicate the work he devised before the age of 21. By considering Galois' ideas before the time of group theory, and undergraduates' perceptions upon freshly learning the material, we will strive to present sophisticated material in a way that is broadly accessible.

**Edward Early** St. Edward's University

### Decent Polynomials

A *nice* polynomial is a polynomial that has the maximum possible number of distinct integer roots between the original polynomial and all of its derivatives. Such polynomials are completely classified up to degree 3, with no examples known in higher degrees. We therefore look in higher degrees for *decent* polynomials, where only the original polynomial and its first derivative must have distinct integer roots. This relaxed condition leads to examples in some higher degrees and avenues for undergraduate research projects.

**Jonathan Martin** Lexington, KY

**Andy Martin** Kentucky State University

### The "Look and Say" Polynomial

Puzzling over the sequence 1, 11, 21, 1211, 111221, . . . led John Conway to discover his eponymous constant 1.3035772... as the positive real zero of a 71st degree polynomial with integer coefficients. This talk will discuss this magical result.

**Bahman Kalantari** Rutgers University

### An Invitation to Polynomiography via Exponential Series

Polynomiography stands for algorithmic visualization of polynomial equations, a subject with many potential applications in STEM and art. Here we consider partial sums of the exponential series. While the exponential series is taught in standard calculus courses, it is unlikely that properties of zeros of its partial sums are considered in such courses, let alone their visualization as science or art. The Monthly article by Zemyan discusses some mathematical properties of these zeros. Here we exhibit some polynomiographs of the partial sums and present a brief introduction of the underlying concepts. Polynomiography establishes a different kind of appreciation of the significance of polynomials in STEM, as well as in art. It helps in the teaching of distinct topics at many levels. It also leads into new discoveries on polynomials and inspires new applications. We also present a link for the educator to get access to a demo polynomiography software together with a module that helps teach basic topics to middle and high school students, as well as undergraduates.

**Axel Brandt** Davidson College

### Avoiding Conflict using Zeros of Polynomials

We discuss two questions related to scheduling committee meetings for the U.S. House of Representatives so that no representative is conflicted by trying to be in two meetings at the same time. This problem will be approached by constructing a polynomial for which every committee schedule resulting in a conflict is a zero of the polynomial. After providing some intuition, we will discuss Alon's Combinatorial Nullstellensatz and how it can be used to guarantee the existence of a committee schedule without conflicts. The talk will conclude with ideas for undergraduate research projects related to the Combinatorial Nullstellensatz.

## Mathematics in Video Games

**Saturday, July 29, 9:30–10:35 AM, Salon A-5**

**Organizers:** Heidi Hulsizer    Benedictine College  
Nickolas Hein    Benedictine College

Video games are a ubiquitous part of popular culture. While it is generally accepted that developing a video game often requires the application of mathematics, many neglect the fact that mathematical principles may appear in how one plays the game. This session seeks presentations of mathematical problems and solutions that may appear in the development or play of modern games. Presenters are encouraged to show college-level mathematics that might appear in a range of courses. We broadly interpret video games to range from single-player to massively multiplayer and to include games played on various types of devices (console, mobile, etc.). This session will be of interest to gamers and instructors looking for innovative examples to use in their classes.

**Andrew Sward**    Augustana College  
**Dat Tran**    Augustana College  
**Lan Dang**    Augustana College

### The Skyrim Problem

In the game of Skyrim, there are 93 herbs, each containing exactly 4 traits from a list of 52 total traits. These traits are initially hidden from the player (in the game, but not outside of the game). A player may mix three herbs together at a time, and any herbs having traits in common will reveal these traits on these herbs. Given the trait distribution of these herbs is always the same with every play-through: How many mixes does it take to reveal every trait on every herb? We show that every trait can be revealed with 61 mixes. We show how to formulate this problem into a binary integer linear program, and present a greedy probabilistic algorithm for solving it.

**Kevin Murphy**    Saint Leo University

### The Mathematics Behind The Hands of Time From Final Fantasy

One of the reasons for the huge success of the Final Fantasy franchise is the variety and detail of the mini-games that support the main story. Many of these mini-games are highly mathematical in nature. This talk centers on “The Hands of Time” game from Final Fantasy XIII-2, which is versatile enough to be accessible to students in a Liberal Arts Mathematics course or challenging for a Senior research project.

**Anil Venkatesh**    Ferris State University

### Partitions by Harmonic Means and the Damage-Per-Second Indicator

In many video games, players defeat opponents by making a series of damaging attacks. Accordingly, average damage dealt per second (DPS) is one of the most important indicators of player strength. This indicator is typically given by a product of terms including damage per attack, rate of attacking, and often several other factors as well. Furthermore, players are generally able to obtain multiplicative increases (or bonuses) to each of the factors that determine DPS. Is there a well-principled additive decomposition of the DPS indicator? Such a decomposition would allow game developers to authoritatively assess the relative value of bonuses to each of the constituent factors of DPS. In this talk, we present a general construction of such an additive decomposition and explore the consequences of this result for DPS indicators in several contemporary games.

## Online Assessment: Where We Have Been, Where We Are and Where We Are Going

**Saturday, July 29, 1:00–3:35 PM, Salon C-6**

**Organizers:** Barbara Margolius    Cleveland State University  
John Travis    Mississippi College

Online assessment is now a common part of the academic experience for faculty and students. The technology has been around long enough to evolve substantially from early implementations. The purpose of this session is to allow faculty to share what is new, what they are hoping for in the future, and what have we learned from present and past implementations of the systems. We also invite contributions regarding pedagogical issues surrounding the use of these resources. We are seeking expository talks on what resources are available, demonstrations, and innovative ideas as well as scholarly talks about the effectiveness of online assessment resources. Talks on online homework, placement testing, just in time resources, and other forms of online assessment are welcome.



**Alison Reddy**    Univeristy of Illinois

### **A Decade of Online Assessment at The University of Illinois.**

The University of Illinois Math Placement Program began in 2007 and College Algebra was redesigned with an online co-requisite remediation component in 2009. We will discuss the need to evaluate preparedness precisely and the challenges and goals of implementing online co-requisite remediation to maximize student outcomes within the context of a single course. Both implementations of online assessments have been successful in addressing the individual needs of our students and we have seen improved success rates, student satisfaction, and increased pathways to graduation. Getting students started, and retaining them, in the appropriate math class is important for their mathematical success and success on campus in general. Collected data will be shared.

**Brooke Buckley**    Northern Kentucky University

### **Building Online Assessments for Introductory Statistics**

IMath is a free, open-source homework and course management system specifically designed for mathematics courses. It supports randomized, algorithmic questions, has built-in support for mathematical symbols, and includes a simple graphics editor. The system is completely web-based which means it runs on any operating system using any of the major browsers without compatibility problems. There are banks of pre-written questions for common undergraduate mathematics topics, and there are some questions available covering statistical content. To allow for complete customization to accommodate instructor needs, the platform allows original questions to be written using simplified LaTeX syntax. Beginning in fall 2016, I transitioned all homework in a mass lecture introductory statistics course to IMath and have seen marked improvement. Prior to this implementation, 45.71% of students were not successful in completing the course in the large lecture format. Since fall 2016, the percentage of students earning a D, F, or W has dropped to 32.17%, which is on par with traditional-sized lecture sections of this course at our university. IMath seems to be relatively unknown in the mathematics community, and its ease-of-use and cost make it a tool worth sharing.

**Michael B. Scott**    California State University, Monterey Bay

### **Using Online Technology to Improve Student Performance in Mathematics Courses**

California State University, Monterey Bay (CSUMB) uses a web-based homework system to supplement pre-calculus, calculus and other mathematics courses. The online homework platform has been expanded into an integrated learning management system (OtterMath) that coordinates every aspect of the course. Expansion of the system makes possible the collection of rich data streams about individual students. In addition to course grade data such as exam, homework and quiz scores, items such as college entrance exam scores, GPA, course resources accessed are also included in the data. We will discuss the measurement, collection, analysis and reporting of these data for the purposes of understanding and optimizing student performance. Additionally, we will discuss how the OtterMath system will be integrated into CSUMB's Calculus Redesign. This will include a description of how OtterMath will monitor student performance in real-time to target interventions for lower achieving students in order to increase their likelihood for success in the course.

**K. Andrew Parker**    NYC College of Technology

### **Asynchronous Online Office Hours with WeBWork**

Asynchronous Online Office Hours with WeBWork: (*or how I learned to let go of the "Email Instructor" button*) Assessment is not limited to the realm of homework, quizzes, and tests. As instructors, we frequently assess our students through simple interactions such as addressing the questions students have. This kind of assessment is typically of the face-to-face variety, which can put non-traditional students at somewhat of a disadvantage. At New York City College of Technology, a commuter college in the CUNY system, we have high rates of students who work full- or part-time in addition to their studies. We also have high rates of students caregiver responsibilities, or other demands on their time. In order to facilitate student engagement with faculty as well as their peers, the Opening Gateways team has developed a "bridge" between WeBWork (an open-source online homework system) and our WordPress-based learning platform, OpenLab. "Opening Gateways to Completion: Open Digital Pedagogies for Student Success in STEM" is a large, cross-institutional collaboration between the mathematics departments at New York City College of Technology and Borough of Manhattan Community College aimed at supporting student success in gateway math courses. This talk will showcase a new "Ask for Help" feature that replaces the "Email Instructor" option in WeBWork. It is designed to take what used to be a private (and often repetitive) interaction and instead place it in a semi-public setting. We will share lessons learned from our initial rollout this Spring, feedback we've received from students and faculty, and also future directions for development and release to the public under an open-source license.

**Grant Sander**    Arizona State University

### **Shifting to Conceptual Mathematics Teaching: What is Needed from an Online Homework System**

Mathematicians and math educators have recognized a need for higher quality mathematical experiences in undergraduate courses. However, most mainstream curricula and textbooks are still focused on traditional approaches to mathematics that are procedural in nature - and consequently, most online homework sets within major online homework systems are procedural in nature. As a consequence, teachers who are interested in teaching conceptually have a relatively small set of ready-made resources at their disposal. In my presentation I will share how we have leveraged a certain online homework system (IMathAS) to efficiently create conceptually-oriented online homework questions

to accompany a conceptual precalculus curriculum, and design a conceptually-oriented online college algebra course. I will share aspects of the system that were necessary for our design of conceptually-oriented curricula, and how others can use this system to leverage modern technology to support conceptual teaching.

**Kristin Lassonde** Contra Costa College

### **Scaffolding Online Math Homework for Effective Assessment**

Are you students getting A's on the homework but still managing to fail in-class assessments? Do you feel that online homework does not give enough information to know if your students are understanding the material or not? Students frequently utilize numerous resources while working on homework including tutors, computational math software, the solution's manual, peer study groups, and even their professor. While these are great resources to use while learning the material, if all of a student's study time follows this model, then the student often does not develop the skills needed to perform independently. In this talk I will discuss some techniques that I have found useful for scaffolding online math homework, for both face-to-face and online classes. By implementing these strategies, I have found that students' homework scores more closely represent their content knowledge. Additionally, as part of this scaffolding, students spend time self-assessing, which ultimately raises their scores during in-class assessments. I have personally effectively used these strategies in developmental math, college-level algebra, trigonometry, precalculus, and calculus.

**Aparna Higgins** University of Dayton

### **Gratitude to Online Assessment – from COW to Now**

As a slow grader at an institution where calculus sections routinely had forty or more students, I was delighted to discover Calculus on the Web (or COW, an online system for practicing and learning calculus, based at Temple University) about fifteen years ago. I started using this free online homework assessment tool in my calculus courses, and I have never regretted the decision. Now I use the publisher's online homework system associated with the textbook we have selected. I will describe some features of MyMathLab that fit my philosophy of homework in those classes. At the University of Dayton, we use MyMathLab for skills tests in our first and second semesters of calculus, and I will provide a quick sketch of how well that works.

**Jennifer Szczesniak** Hagerstown Community College

### **A Three Year Journey With Online Homework**

Three years ago I decided to use online tools for the Precalculus and Calculus I courses at Hagerstown Community College. My primary goal was to improve student outcomes and consistency across classes taught by different instructors. I began with an online system from a major publisher. Having already transitioned to open-source materials in other courses I talked myself into transitioning the online homework to open-source tool. This came with some success, a lot of failure, and many unexpected bumps. I will share my experiences with different systems, detailing what worked, what did not work, hopes for the future and where I am currently in my journey.

## **Euclid and the Mathematics of Antiquity in the 21st Century**

**Saturday, July 29, 1:00–4:15 PM, Salon A-1**

**Organizers: Elizabeth T. Brown** James Madison University  
**Edwin O'Shea** James Madison University

Euclid's *Elements* is a fundamental text of mathematics in the western tradition. Geometry, number theory, logic, and the axiomatic method: all bear Euclid's stamp. Moreover, the *Elements* was considered a central text of every liberal arts education well into the nineteenth century, more than two millennia after its writing. The recent centennial of the MAA provides a fitting occasion on which to revisit the influence of mathematics's past on future mathematics and culture. We seek contributions that relate the work of Euclid or other mathematicians of antiquity to modern mathematics or the modern undergraduate curriculum. Original research, unique expositions, descriptions of courses with a significant integration of the mathematics of antiquity, and curricular materials are all welcome.

**Andrew Leahy**    Knox College

### **Bring back the Pappus-Guldin Theorems**

The most powerful geometrical proving technique of antiquity was the theory of proportions introduced in Books V and VI of Euclid's *Elements*. Archimedes was undoubtedly the most successful practitioner of this theory, using it to compute areas, centroids, and volumes and surfaces of revolution—key applications of today's integral. 600 years after Archimedes, Pappus of Alexandria demonstrated a pair of theorems—today called the Pappus-Guldin theorems—which showed that given any two of these quantities the third was easily determined. These theorems still appear in many calculus textbooks today, but often only on the periphery. In this talk, we will show how putting the Pappus-Guldin theorems up front in calculus results in an intuitive and unified presentation of volumes and surfaces of revolution that at the same time extends the formulas we teach in calculus today.

**Jerry Lodder**    New Mexico State University

### **A Course in Geometry based on Historical Sources**

A course in plane geometry at the undergraduate level is taught today from a system of highly polished axioms going well beyond the postulates and common notions of Euclid's *ELEMENTS*. We outline the contents of a historical curricular module that examines the proof of the Pythagorean Theorem from Book I of the *ELEMENTS*. With the ancient Greek view of area in mind, we see how the Pythagorean Theorem becomes transparent by finding parallelograms that are on the same base and contained between the same parallel lines. Of course, the Pythagorean Theorem is used heavily today for the distance formula in Euclidean space, and depends on the parallel postulate (Postulate V, Book I). Should this postulate fail, what would replace the distance formula? What would replace area measure? The former question finds resolution in Riemannian geometry and the latter in symplectic geometry, both being examples of non-Euclidean geometry. We close with a brief excerpt from a curricular module based on the work of N. Lobachevsky and F. Klein.

**Viktor Blasjo**    Utrecht University

### **Euclid's Geometry is Physical, not Abstract**

Euclid's geometry is often taken to be eminently pure and abstract theory. This point of view is congenial to modern conceptions of mathematics, and was popular historically among philosophers such as Plato. I argue, however, that view was rejected by not only Euclid himself but also by virtually all major geometers from antiquity to Leibniz. These mathematicians instead saw the very foundations of geometry as anchored in concrete figures constructed by instruments such as ruler and compass in the physical world. I use this perspective to argue that several so-called flaws in the *Elements* are not flaws at all. I also refute various arguments and alleged evidence that Euclid held the Platonic view. The physicalist interpretation of classical geometry goes well with hands-on trends in modern pedagogy and vindicates it against snobbish and one-sided emphasis on an abstract and purely logical conception of mathematics.

**Charlie Smith**    Park University

### **My Big Fat Greek Course**

MA 350 History of Mathematics is specifically designed for mathematics and mathematics education majors at Park University. It uses the achievements of the great Greeks of antiquity as a springboard to more recent developments. For example, the construction problems which originated between 500 and 400 B.C.E. provide an opportunity to introduce the concepts of algebraic, transcendental and constructible numbers, setting the stage for the decisive work of Euler, Gauss, Wantzel and Lindemann centuries later. The course structure consists of the following units: The Pythagoreans, Classic Construction Problems, Euclid, Archimedes, Diophantus, Fibonacci, and The Cubic Controversy. Emphasis is placed on both fact and legend. A field trip to the Linda Hall Library is required; it has a wonderful collection of classic mathematics books dating back to 1482! Films and maps are utilized to add variety and detail that would otherwise be lacking.

**Jeffrey Clark**    Elon University

### **Rationals, Irrationals, and Commensurable Magnitudes: Euclid and the Real Numbers**

In Book V of Euclid's *The Elements* he discusses the theory of ratios of magnitudes as attributed to Eudoxus. In dealing with these ratios he describes their properties in comparison with commensurable ratios, i.e., rational numbers. In essence this discussion describes the positive real numbers in terms of their relationship to the rational numbers, a process that we now refer to as Dedekind cuts. This talk will summarize Euclid's exposition in comparison with how we approach the real numbers today in a typical Analysis class.

**J Christopher Tweddle**    Governors State University

### **Solving Quadratic Equations with Geometric Algebra**

The Pythagoreans utilized the “application of areas” to solve algebraic expressions. Book VI of Euclid’s *Elements* includes results in proportionality that are essential to the application of geometric algebra to solve quadratic equations. In particular, propositions 28 and 29 give constructions that lead to solutions of equations of the form  $x(a * x) = b^2$  and  $x(x + a) = b^2$ , where  $a$ ,  $b$  and  $c$  are positive numbers, respectively. The propositions will be presented and translated into modern mathematical language. The construction procedure will be demonstrated by example. Finally, connections between Euclidean geometric solutions and modern techniques for solving quadratic equations will be drawn.

**Ian Pierce**    US Air Force Academy

**Kurt Herzinger**    US Air Force Academy

**Courtney Kunselman**    US Air Force Academy

### **Climbing Greek Ladders to Reach for Eigenvectors**

Theon’s ladder (also known as a Greek ladder) is an ancient method for approximating irrational roots of integers. One can also modify a Greek ladder to approximate roots of certain polynomials. In particular, one can find previous results in the literature concerned with approximating one of the roots of a quadratic polynomial subject to certain constraints on its coefficients. We will address how one can extend these ideas to the consideration of any quadratic polynomial; in addition we will show how to adjust the Greek ladder so that it will approximate either of the roots (rather than just one). We may also explain those situations in which a Greek ladder fails to converge and discuss some ongoing related work with polynomials of higher degree. Understanding why and how these Greek ladders behave as they do requires only some basic ideas from linear algebra and a basic understanding of limits.

**Kathi Crow**    Salem State University

### **FYS: Math of the Middle East and North Africa**

First year seminars provide an introduction to college as well as an opportunity to delve into topics outside of the typical math curriculum. In this talk I will describe a course which touches on themes from the ancient world and uses them to introduce students to the modern university. We will discuss how the library at Alexandria was a center for learning that attracted mathematicians such as Euclid, Eratosthenes, and Hypatia just as the university library is a hub for learning on campus. As we delve into islamic tilings and connections with modern theorems we will discuss citations, plagiarism, and academic honesty. The course will end with a survey of modern mathematicians from the Middle East and North Africa to show students that not only is mathematics a vibrant subject with many avenues of research but also that math research is a global endeavor with people from all over the world contributing their ideas.

**Marshall A. Whittlesey**    California State University San Marcos

### **The mathematics of the Sphaerica of Menelaus**

The *Sphaerica* of Menelaus of Alexandria is an ancient text on spherical geometry and trigonometry. Originally written in Greek, the text was translated and heavily reworked in later centuries by Arabic mathematicians. This text is not as well known as Euclid’s *Elements* partly because spherical geometry has not been a standard part of the mathematics curriculum for many decades, but also because the rise of trigonometry has rendered many of Menelaus’ results and techniques less important. However, the text contains a number of intriguing results not well known to modern mathematicians that illustrate the talent of the ancient writers. We discuss some of these theorems, their proofs, and their applications.

**Maureen Carroll**    University of Scranton

**Elyn Rykken**    Muhlenberg College

### **Geometry: It’s *Element*-ary**

The *Elements* is the most natural entry point into the study of axiomatic geometry, and provides the perfect scaffolding for building a compelling narrative for a self-contained course, that is, one that provides, rather than requires, a good understanding of Euclidean geometry. With this in mind, we discuss a junior/senior level geometry course developed over two decades of teaching geometry and history of mathematics courses. The course begins with Heath’s translation of the first four books of the *Elements* and places a heavy emphasis on active participation. Specifically, our students present updated versions of many of the propositions in Books I and III. Along the way, after establishing a firm foundation of neutral geometry, we consider the behavior of the postulates beyond the tried-and-true Euclidean plane. This investigation leads students to discover the insufficiency of Euclid’s set of axioms, and thus, presents an organic introduction to the work of Hilbert. In addition to discussing the course, we will detail our companion book project.

# General Contributed Paper Sessions

**Organizers: Feryal Alayont** Grand Valley State University  
**Holly Zullo** Westminster College

## Analysis

**Thursday, July 27, 8:30–9:40 AM, Salon C-6**

**Hudson Akewe** University Of Lagos

### Stability of Implicit Jungck-Kirk-multistep iterations for Generalized Contractive-type Mappings

Stability of Implicit Jungck-Kirk-multistep Iterations for Generalized Contractive-type Operators Hudson Akewe and Adesanmi Mogbademu  
 Department of Mathematics, University of Lagos, Lagos, Nigeria Abstract: In this paper, we establish stability results of implicit Jungck-Kirk-multistep iterations when dealing with a pair of weakly compatible generalized contractive-like operators defined on a normed linear space. The stability for implicit Jungck-Kirk-Noor, Jungck-Kirk-Ishikawa and Jungck-Kirk-Mann iterations are obtained as corollaries. Our results generalize and improve most stability results in the literature.

**Buthinah Bin Dehaish** King Abdullaziz University

### Some Fixed Point Theorems for Monotone Lipschitzian mappings

Through this talk we will discuss the existence and uniqueness of the fixed points for a new class of Lipschitzian mappings which are monotone.

**Adam Coffman** Indiana - Purdue Fort Wayne

### $J$ -Holomorphic Curves in Rough Almost Complex Structures

A pseudoholomorphic curve, or  $J$ -holomorphic curve, is a differentiable map from a Riemann surface to a manifold with almost complex structure  $J$ , that satisfies an analogue of the Cauchy-Riemann equations. When  $J$  is smooth, pseudoholomorphic curves have well-known regularity and uniqueness properties. I will survey what can happen when  $J$  is only continuous or satisfies a Hölder condition.

**Alexander Kastner** Williams College

**Johann Gaebler** Harvard University

**Cesar Silva** Williams College

**Xiaoyu Xu** Princeton University

**Zirui Zhou** University of California, Berkeley

### On the Isomorphism and Centralizer Problems for Infinite Rank-One Transformations

In ergodic theory, the isomorphism problem asks when two measure-preserving transformations are isomorphic, while the centralizer problem asks which transformations commute with a given measure-preserving transformation. We investigate these questions for the newly defined class of *partially bounded transformations* which encompasses many well-known infinite rank-one transformations with bounded cuts such as the infinite Chacón transformation and the Hajian-Kakutani transformation. In particular, we prove that the only transformations that commute with a partially bounded transformation  $T$  are the powers  $T^n$  for  $n \in \mathbb{Z}$ . Further, we characterize exactly when a partially bounded transformation is isomorphic to its inverse.

**Julia St. Goar** Merrimack College

### Caputo Boundary Value Problems in Nabla Fractional Calculus

We will consider right focal boundary value problems involving the Caputo difference operator in the context of the area of Nabla Fractional Calculus. In some cases, this consideration will lead to finding the existence of solutions to nonlinear versions of the boundary value problems. For each problem, we will define a Green's function and consider behavior changes as the order of the Caputo operator increases.

# History and the Philosophy of Mathematics

Thursday, July 27, 8:30–9:40 AM, Salon C-8

**Johannes C. Familton** BMCC

## Olinde Rodrigues' Contribution to Catalan Numbers

Title Olinde Rodrigues' contribution to Catalan numbers. Abstract Benjamin Olinde Rodrigues was born into an Iberian Jewish family in 1795 residing in France. Most mathematicians who heard of Rodrigues connect him with Rodrigues rotations, which were based in his PhD thesis, and a later paper that he wrote in 1840. Few mathematicians know that he also wrote three 'notes' on combinatorics. He specifically wrote two notes on Catalan numbers, and one on an elementary derivation, without using the Taylor series, of the expansion of binomial series.. This presentation will focus on Rodrigues' 1838 contribution to Catalan numbers. This presentation will put this fascinating man's life in context with the two notes that he wrote about Catalan numbers. A brief history about what lead to the problem that Rodrigues tackled will be included. This will be followed by what Rodrigues pointed out and showed in his notes. This will include a summary of his first observation followed by a summary of a 'shorter proof' he later wrote of his original observation.

**Joel Haack** University of Northern Iowa

## From the Smithsonian Institution Exhibit for the MAA Centenary: Founding Member Richard P. Baker

Artifacts from Richard P. Baker, a founding member of the MAA from the Iowa section, were on display at the National Museum of American History for the MAA centenary. This talk will feature details of his life and work, with a focus on the mathematical models he created while at the University of Iowa.

**Ishraq Al-Awamleh** New Mexico State University

## A Line by line English Translation of a 1402 AD Arabic Poem about Algebra

We will present a line-by-line translation into English of an Arabic poem concerning arithmetic and algebra, by Ibn Al-Ha'im (1356-1412 AD). The poem is titled Al Mknifi'l-jabr wa'l-muqabala, and to our knowledge, it has not been translated into English before. The poem, written in rhyming Arabic verse, describes state-of-the-art algebraic (al-jabr wa'l-muqabala in Arabic) concepts, definitions, and techniques of the early 15th century. It was composed in 1402 AD and copied in Arabic by Abdel Fattah in 1882 AD. It consists of 59 lines divided into 6 sections: introduction, definitions, addition and subtraction, multiplication and division, six canonical equations, and a wrap-up. It is short but covers a large amount of mathematics in a very compact form, making it difficult to understand even to those who know Arabic. We will give an English commentary of the poem's six sections, together with a line-by-line English translation. Our work has been aided by two main sources: first, Zakariah Al-Ansari's (1888 AD) interpretation and analysis in Arabic of the same poem; and second, Mahdi Abdeljaouad's work on the English analysis and interpretation of another poem from the 12th century, Urjūzafi'l-jabr wa'l-muqābala (which means Poem about Algebra), by Ibn al-Yāsamin. We have presented a general interpretation of the poem in MathFest'16. Base on feedback from the audience, we have translated each line of the poem into English.

**Shigeru Masuda** Long-Term Researcher, RIMS, Kyoto University

## The Mathematical Theories of the Capillary Action by Laplace, Gauss and Poisson

We discuss historical development of classical theory of capillary action from the viewpoint of physical mathematics, in particular, the differences of the equations and theories since Laplace 1806-07, Gauss 1830 and Poisson 1831. Poisson issues the last book 1831 in rivalry to Laplace in which he criticizes the preceding works : Laplace' and Gauss' methods of the handling of molecular action. The former two physics are introduced by Laplace. Gauss also treads on the Laplace' physics. Poisson discusses the newness in his book entitled : "New theory of the capillary action" These situations of mathematical physics owe to the arrival of continuum, on which we summarize at first the trailblazers of the capillary action : Laplace, Gauss and Poisson. Poisson proposes his academic paradigm of mathematical physics in rivalry to Laplace and Gauss, in which he discusses the essential theories of capillary action based on the mechanics, fluid mechanics and heat theory. We discuss also the two constants  $K$  and  $H$  of Laplace,  $\Phi$  and  $\Psi$  of Gauss,  $F$  and  $H$  of Poisson, which are used to make their equations standing on the essential conceptions. In the other hand, Poisson cites his book on the capillary action, discussing the same theme of the development and example of the two attractive and repulsive forces. Although deducing into the same results of the fundamental formula respectively as follows, Poisson asserts his own discussion on the attractive and repulsive forces, whose method comes from the essential conception among the hydrodynamics, hydrostatics and heat theory.

**Colm Mulcahy** Spelman College

## The Library of Irish Mathematics

The free online Annals of Irish Mathematics and Mathematicians documents over 2700 mathematical people who are either Irish, work in Ireland, or had Irish doctoral advisors. It also documents over 700 books, mostly from the last two centuries. We will survey the latter aspects, including some noteworthy works from before 1800.

# Algebra

Thursday, July 27, 8:30–10:10 AM, Salon C-7

**Irawati Irawati** Institut Teknologi Bandung

## Fully Prime, Almost Fully Prime and Fully Weakly Prime Ring

In this paper we will show the relation between Fully Prime Ring, Almost Fully Prime Ring and Fully Weakly Prime Ring. We will observe the condition should be given to a fully weakly prime to become an almost fully prime ring and to become a fully prime ring.

**Chad R. Mangum** Niagara University  
**Kailash C. Misra** North Carolina State University  
**Naihuan Jing** North Carolina State University

## Fermionic Representations of Twisted Toroidal Lie Algebras

Lie algebra representation theory has been significant in various areas of mathematics and physics for several decades. In this talk, we will discuss one instance of this theory, namely certain representations of twisted (2-)toroidal (Lie) algebras, which we view as universal central extensions of twisted multi-loop algebras. The usual loop algebra realization generalizes the familiar realization of affine Kac-Moody algebras. To facilitate our study of the representation theory, however, we will discuss a new realization given by generators and relations; this is similar to a realization by Moody, Rao, and Yokonuma in the untwisted case. Subsequently, we will discuss an application, namely fermionic free field representations, which are similar to those of Feingold and Frenkel in the case of affine algebras.

**Margaret Rahmoeller** Roanoke College

## Using Crystal Base Theory to Learn About Certain Demazure Crystals

In 1968, Victor Kac and Robert Moody defined a class of infinite dimensional Lie algebras called affine Lie algebras. An affine Lie algebra can be viewed as the universal central extension of the Lie algebra of polynomial maps from the unity circle to a finite dimensional simple Lie algebra. Kashiwara showed that irreducible modules for the  $q$ -deformed universal enveloping algebra of an affine Lie algebra admit crystal bases. Kang, Kashiwara, Misra, Miwa, Nakashima and Nakayashiki gave the path realizations of affine crystals as a semi-infinite tensor product of some finite crystals called perfect crystals in 1991. In this talk, we describe crystal base theory in general and how to use this path realization to better understand certain Demazure crystals for the quantum affine algebra  $U_q(A_n^{(1)})$ .

**Chad Awtrey** Elon University

## When is a Polynomial Isomorphic to an Even Polynomial?

For an irreducible polynomial  $f(x)$  with integer coefficients, let  $K^f$  denote the stem field of  $f$ . That is,  $K^f$  is the smallest subfield of the complex numbers that contains both the rational numbers and a root of  $f$ . If  $g(x)$  is another irreducible polynomial with integer coefficients, call  $f$  and  $g$  isomorphic if  $K^f$  and  $K^g$  are isomorphic as fields. In this talk, we focus on the question: if the degree of  $f$  is even, when does there exist an even polynomial  $g$  such that  $f$  and  $g$  are isomorphic? We discuss a simple method that both answers this question and constructs such a polynomial  $g$  if it exists. We end with an application to computing Galois groups of polynomials of even degree.

**Thomas Q. Sibley** St. John's University

## Is Equity Unusual ? (At least in Total Products)

The product of all the elements of a non-Abelian group depends on the order. These possible *total products* have been known for over 30 years. We will make a foray into a related but previously unexplored question: When do the  $n!$  possible orderings split equitably among the possible total products?

**Chandra Kethi-Reddy** University of Central Florida

## Graphing the Relationship of an Operation with the Relationships of Numbers being Operated on

Operations tend to be used as 'vanishing mediators' when we visually coordinate numbers together (whether in the Cartesian coordinate system, or even an elementary multiplication table). I claim that we can derive significant mathematical vision out of visually coordinating coordinates with the operations that coordinate them. We can show this by drawing iteratively generated operations (e.g., +, \*, ^, ...) on an axis or by fixing one of the coordinates as a vanishing mediator. Using tables like these is phenomenological evidence of performing mathematical analysis on the very relationship between functions and the numbers that flow through them. Ultimately, I make the case that we should study iteratively generated operations in a similar fashion to how we study the natural numbers, where possible.

**Jennifer F. Vasquez**    The University of Scranton  
**Michael P. Allocca**    Muhlenburg College  
**Steven T. Dougherty**    The University of Scranton

### Symmetries in Permutations

Using a visual representation of permutations, we investigate various geometric subgroups. The ideas here are related to topics in topology and abstract algebra. Time permitting, we will discuss connections to sorting and biological processes.

## Geometry

**Thursday, July 27, 1:00–3:25 PM, Salon A-2**

**Sarah L. Mabrouk**    Framingham State University

### Reflections on Teaching Geometry Courses

During the past five years, I taught five sections of a Euclidean geometry course, a directed study course in higher geometry, and a graduate-level geometry course for middle school and high school teachers. Teaching these geometry courses has provided me with opportunities to create a variety of assignments, to explore different approaches for helping students to learn how to write proofs, and to experience challenges which are different from those encountered when teaching service courses and the calculus sequence. In this presentation, I will discuss some of the frustrations and delights of teaching these geometry courses, share in-class activities and explorations that I used while teaching geometry courses, and showcase some of the assignments that I created as well as describe my approach for teaching geometry courses and student results.

**Christopher R. Lee**    University of Portland

### Folded Symplectic Four Manifolds

A folded symplectic manifold is a manifold equipped with a differential two-form that degenerates in a controlled way along a smooth hypersurface. We show that two orientable, four-dimensional folded symplectic toric manifolds are isomorphic provided that their orbit spaces have trivial degree-two integral cohomology and there exists a diffeomorphism of the orbit spaces (as manifolds with corners) preserving orbital moment maps.

**Aldo Cruz Cota**    Texas Wesleyan University

### An Introduction to Persistent Homology

We introduce the notion of Persistent Homology, an algebraic tool that allow us to extract the qualitative (topological) features from data. We start by reviewing the concept of simplicial homology of a simplicial complex and how it can be modified to define the persistent homology of a point cloud. Following Carlsson et al, we describe the classical example showing how a set of  $3 \times 3$  high-contrast patches sampled from a collection of natural images has the topology of the Klein Bottle. If time permits, we will also discuss other applications of persistent homology.

**Boyan Kostadinov**    City Tech, CUNY

### From a High-Dimensional Random Polygon to an Ellipse: A Fourier Analysis of Iterated Circular Convolutions

We consider random,  $m$ -dimensional polygons with  $N$  vertices, having zero centroids. We investigate a normalized, midpoint averaging transformation, repeatedly applied to the original polygon, which we identify with a normalized circular convolution modulo  $N$ . We prove that if the normalized convolution is repeated sufficiently many times, then the limiting shape is that of a planar affine-regular polygon, inscribed in an ellipse, embedded in a 2-dimensional subspace of the  $m$ -dimensional space, spanned by a basis specified by the principal harmonic components of the discrete Fourier transforms of the initial vectors of vertex coordinates. We provide a compact analytical description of the limiting elliptical form, as well as the limiting plane, using the language of circular convolutions and the tools of discrete Fourier analysis. In this paper, we generalize to the higher-dimensional case, previous results obtained in the planar case. This paper is based upon work supported by a PSC-CUNY Traditional A Research Grant #60190-00-48.

**Kevin Sonnanburg**    The University of Tennessee

### Blow-up Continuity in Mean Curvature Flow

Under mean curvature flow, a closed, embedded hypersurface  $M(t)$  becomes singular in finite time. For certain classes of mean-convex mean curvature flows, we show the continuity of the first singular time  $T$  and the limit set " $M(T)$ ", with respect to initial data. We employ an Angenent-like neckpinching argument to force singularities in nearby flows. However, since we cannot prescribe initial data, we combine Andrews'  $\alpha$ -non-collapsed condition and Colding and Minicozzi's uniqueness of tangent flows to place appropriately sized spheres in the region inside the hypersurface.



**Shirley L. Yap** California State University East Bay

### Anchor Triangles in a Delaunay Triangulation

Anchor simplices are elements in the Delaunay triangulation of a set of points  $P$  which are local minima of the distance function  $d$  which measures the distance between a given point  $x$  and the nearest point to it which is in  $P$ . In this talk, I will discuss recent results about the density of the set of anchored triangles in the Delaunay triangulation of a set  $P$  of points in the plane under certain sampling, boundary, and packing conditions. Such triangles are useful for reconstructing two dimensional surfaces.

**Ryan Hood** Austin College

**J. Mealy** Austin College

### Timelike Spiraling Geodesics in Staircase Metric Geometries

Further results in the category, *staircase metric geometry*. After providing a brief introduction to our category of geometric systems, we report on the continued investigation of geodesics in these systems that exhibit a spiraling characteristic. Specifically, we report on the adaptation of one of our earlier construction schemes to the space-time category. Prerequisite to this was the establishment of an *angle change law* (analogous to Snell's law of optics) in systems with signature (1,1) that additionally feature a very general boundary quotient scheme. This was then utilized to complete the construction of a system featuring a family of timelike spiraling geodesics. Finally, on a broader note, we discuss the state of our ongoing work in *staircase metric geometry*.

**Mu-Ling Chang** University of Wisconsin-Platteville

### Evaluation of Pi by Nested Radicals

It is well known that  $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$  by Mathematicians. Does the irrational number  $\pi$  have such an unexpected limit representation, which can be proved by using only undergraduate mathematical skills? In this article, we will use geometry, trigonometry, mathematical induction, and the concept of limits to prove the existence of such a limit. The content of this talk can be found in the December 2016 issue of Mathematics Magazine.

**Ollie Nanyes**

### Everywhere Wild Knots

Classical knot theory deals with embeddings of a smooth circle into 3 space called a knot. Two knots are equivalent if there is a homeomorphism of 3 space taking one knot to the other knot. Wild knot theory deals with knots that have at least one point at which they are not differentiable. There are examples of knots which are nowhere differentiable. In this talk I discuss some open problems and some work on one major open problem: are there knots so wildly embedded that there is no continuous deformation of the knot itself to the standard unknot? 1

**Edward Aboufadel** Grand Valley State University

### 3D Printing of Eugenia Cheng's "Associahedron"

In her first book, Eugenia Cheng discusses a three-dimensional object known as the "Associahedron", derived from an application of category theory to cooking. To describe the object, a two-dimensional net of regular polygons is presented that is to be folded into a three-dimensional object. However, she warns that the polygons that make up the net don't quite fit together, so that a few of the faces are "wonky". In this talk, we explore how to 3D-print this "wonky" object, which leads to a discussion of Platonic solids, Archimedean solids, and Johnson solids. The topics in this talk can be incorporated into undergraduate courses such as geometry or a capstone course.

## Applied Mathematics

**Thursday, July 27, 1:00–3:55 PM, Salon C-7**

**Victoria Powers** Emory University

### Power in the U.S. Legislature

We use a well-known model of the U.S. legislative system as a monotonic simple game (a yes-no voting system) with four types of voters – the president, the vice-president, senators, and representatives. Power indices such as the Banzhaf index and the Shapley-Shubik index were formulated as a way to measure the power of voters in a simple game. We apply the theory of power indices to our model and show that for any power index the president has more power than all other voters and a senator has more power than a representative. We also show that for "reasonable" power indices a senator has more power than the vice-president and the vice-president has more power than a representative, although there exist power indices for which a senator and the vice-president have the same amount of power as well as power indices for which a representative has more power than the vice-president.

**Joseph F. Kolacinski** Elmira College  
**Brandon Payne** Elmira College

### **A Mathematical Comparison of Open and Closed Primaries through Fairness Criteria**

This talk explores the differences between open, closed, and semi-closed primary elections, from a theoretical standpoint. We define five of the most fundamental “fairness criteria,” a set of desirable characteristics “fair” election systems should satisfy. Using these definitions, we determine that by allowing voters to choose which primary election in which to vote, open primaries satisfy four out of the five fairness criteria while closed primaries do not: anonymity, neutrality, majority, and Pareto. We also determined that closed primaries satisfy monotonicity while open primaries do not. We use these results to argue that open primaries are superior to closed primaries in selecting candidates that represent the interests of the greatest number of citizens.

**Sarah S. King** University of Louisville

### **Extending Difference of Votes Rules**

May’s Theorem (1952) characterized simple majority rule as an aggregation function that was strictly monotone, anonymous, and neutral. Llamazares (2006) characterized a new class of functions known as difference of votes rules with similar axioms and a new one called cancellation. We drop neutrality in order to completely characterize our extended difference of votes rules. These rules have an obvious application when there is one alternative that is favored as a its the “status quo” or otherwise. We then drop anonymity to show that the characterization only changes slightly and is heavily dependent on just two axioms, but these slightly modified difference of votes rules turn out to be so contrived that they are almost impossible to show up in real world scenarios.

**Prasad Senesi** The Catholic University of America

### **The Root System and Combinatorics of Linear Voting Methods**

Recent algebraic and geometric approaches in social choice theory provide have provided new perspectives and insights into the behavior of voting methods. In particular, one can construct a vector space of ballot results and view a positional voting method as a linear map on this space which is equivariant with respect to a natural symmetric group action. In this talk, we show how type-A root systems naturally describe the geometry of a positional voting method. We use the geometry and combinatorics of this root system, and its associated partition of the profile space into Weyl chambers, for several applications, including a generalization of the reversal symmetry criterion.

**Collins Bekoe** Prince of Songkla University

### **Non-standard Finite Difference Scheme for Vector Disease Models**

Epidemic models are normally solved using standard numerical methods like the Euler and Runge- Kutta numerical integrators and other finite difference methods. These standard methods fail to preserve the dynamics of the nonlinear differential equations and this can lead to spurious solutions. This is because of numerical instabilities that arise for certain values of the model parameters. This paper applies a nonstandard finite difference method to solve a vector disease model that preserves the positivity as well as maintaining the qualitative stability of the nonlinear differential equations. In particular, the Mickens’ rule is used to approximate the nonlinear terms in the system of differential equations in a non-local way thereby considering some other terms in an implicit way. The results show that the nonstandard finite difference scheme is ideal in solving epidemic models as it preserves and obeys the dynamics of the nonlinear differential equations.

**TJ Leiterman** St. Norbert College

### **Modeling Phytoplankton Population Dynamics**

Phytoplankton are microscopic plants, diverse in shape, and form the basis of aquatic ecosystems. In this talk, we introduce a mathematical model that predicts the vertical distribution of phytoplankton in freshwater, inland ecosystems. The growth of phytoplankton is intimately connected to nutrient and light availability. Quantifying the growth due to light availability requires quantifying and modeling the seasonal settling velocity of the phytoplankton as particles. To accurately model settling velocity, the low Reynolds nature of the fluid-particle system is exploited and added to an experimental component. In this talk, careful mathematical consideration is paid to the interaction between the forces of buoyancy, gravity, and drag. This work has additionally required geometrical considerations. Specifically, the model described in this talk is guided by the characterization of particle separation and settling velocity for sparse phytoplankton populations of both spherical and slender shape. The interactions that exist between biology, physics, and mathematics will be the focus of this talk.

**Jillian Stupiansky** University of North Alabama  
**Jessica Stovall** University of North Alabama

### **Modeling the Devastation of Hemlock Trees in the Great Smoky Mountains**

Recently there has been a great decline in the health and number of Eastern hemlock trees found in the Great Smoky Mountains National Park. This is due to the presence of the Hemlock Woolly Adelgid, an insect which prevents the spread of nutrients through a tree by feeding on the tree's sap. We have created a mathematical model that represents the spread of the devastation. We will discuss the biological reasoning behind the development of the model, as well as the analysis that has been conducted, including the equilibria of the model and the basic reproduction number  $R_0$ . Simulations will accompany the analysis. Ultimately, the goal of this research is that it can be used to help find a solution that will revive the hemlock population.

**Rachel Grotheer** Goucher College

### **Reconstructing the Cheshire Cat in Technicolor: The Problem of Hyperspectral Medical Imaging**

Reconstructing an image from the set of data coming from a medical imaging device is a notoriously ill-posed and difficult problem. Recently, hyperspectral imaging (using hundreds of wavelengths in the imaging process) has been proposed as a way to improve the quality of the image by increasing the amount of data. But increasing the data set leads to its own set of problems, namely the escalation of the computational cost required to solve the reconstruction problem. We give an overview of the mathematical foundation of a modality known as hyperspectral diffuse optical tomography (hyDOT), and of the model reduction technique proposed to help make working with such a large data set more computationally feasible.

**John C. Wierman** Johns Hopkins University

### **Strict Inequalities Between Bond Percolation Thresholds of Archimedean Lattices.**

A percolation model is an infinite random graph model for phase transitions and critical phenomena. The percolation threshold corresponds to a phase transition point, such as a melting or freezing temperature. The exact value of the percolation threshold is known for only a few two-dimensional percolation models. An Archimedean lattice is an infinite vertex-transitive graph that tiles the plane with regular polygons. There are eleven Archimedean lattices, which are of interest to physicists studying percolation models. However, most information about their percolation thresholds has been obtained by simulations. Recent computational advances in the substitution method produce new rigorous bounds for the bond percolation thresholds of Archimedean lattices, which prove several new strict inequalities. The substitution method is based on stochastic ordering of probability measures on partition lattices, and translates an exact value or bound for the percolation threshold of one lattice into a bound for the percolation threshold of another lattice.

**Dimplekumar N. Chalisehajar** Virginia Military Institute

### **Trajectory Controllability of Nonlinear Systems: An Analytical and a Numerical Approach**

**Abstract:** In this short article we have discussed trajectory controllability of the second order nonlinear integro-differential system by using theory of monotone operators and tools of cosine operators. We will then describe a general method for computing numerical approximations to control functions. We will illustrate this method with a nonlinear integro-differential equations and a nonlinear hyperbolic equation in one/two spatial dimensions.

**Keywords:** Trajectory controllability, Second order Integro-differential system, Monotone operator theory, Cosine operators, Finite difference technique

**Satyanand Singh** New York City College of Technology

### **A Sinusoidal Twist with Exponential Influences**

The under damped oscillating spring mass dynamical system with displacement shaped like  $u(t) = Pe^{-\gamma t/2m} \cos(\mu t - \delta)$  and no external force is most interesting as a discrete mathematical model with applications in quantum dynamics and mechanical systems. If we let  $m$  be the mass,  $\gamma$  the damping constant,  $c$  the spring constant and  $t = \tau_1 > 0$  the instant where the first extreme value occurs, we will show that the distance  $S$  traveled by a particle modeled in this system results in the unexpectedly nice closed form expression given below.

$$S = P \left[ |\cos(\phi)| (1 + \coth(\gamma\pi/4m\mu)) e^{-\gamma\tau_1/2m} - |\cos(\delta)| \operatorname{sgn}(u(0)u(\tau_1)) \right].$$

**Sijie Liu** University of Alabama

### **Twin Interval Arithmetic and Its Application in Optimization Problems**

By introducing the sampled point into the twin interval arithmetic, it significantly improved the quality of its result by the guarantee of the “non-empty” property for its inner interval component. With the new advantage, we applied the twin interval arithmetic idea into the interval optimization algorithms, and results showed it would do great help on the efficiency and accuracy of the current algorithms.

## Outreach, Mentoring and Assessment

Thursday, July 27, 1:00–4:40 PM, Salon C-8

**John T. Ajai** Taraba State University, Jalingo, Nigeria

**Herbert R. Hanawa** Yola Electricity Distribution Company, Jalingo, Nigeria

### Gender, School Location, Age, and Subject Combination as Predictors of Secondary School Students' Achievement in Mathematics

The study examined gender, school location, age, and subject combination as predictors of secondary school students' achievement in mathematics in Taraba State, Nigeria using the descriptive ex post facto design. The sample of the study is made of 420 (224 male and 196 female) out of 3,966 students in 12 senior secondary schools in Jalingo education zone of Taraba state (2016/2017 academic session). Mathematics Achievement Test (MAT) with KR-20 reliability index of .86 was used for data collection. Mean, t-test, and multiple regression were the statistical tools for data analyses. Finding of the study revealed no significant effect of gender, age, school location and subject combination on students' achievement in mathematics. The study also found weak correlation for all the predictor variables. Even though school location and sex are negatively correlated, subject combination and age group are positively correlated with achievement in mathematics. All the correlations are not statistically significant at .05 confidence level. The implication is that achievement in mathematics is not predictive on any of these variables. Therefore, it is recommended that teachers, government, parents and all stakeholders in education should discard negative beliefs and practices that tend inhibit good performance, but ensure that students are encouraged (irrespective of gender, age, school location and subject combination) to study mathematics. This will go a long way to reduce the prevalent poor performance in mathematics. Key words: gender, age, school location, subject combination, achievement.

**Frank Morgan** Williams College

### AMS Notices: Math and Diversity

We discuss various issues for *Notices of the American Mathematical Society*, including our focus on mathematics and our attention to underrepresented groups, from the perspective of the Editor-in-Chief.

**Charles P. Funkhouser** California State University Fullerton

**Miles R. Pfahl** Turtle Mountain Community College

### American Indian-based Mathematics Materials for Undergraduate Courses

This project has developed and researched undergraduate mathematics materials based in the culture and mathematics of American Indian Peoples for integration into undergraduate courses. Mathematics topics include probability and statistics, number theory, transformational geometry, calculus, and pre-service elementary and secondary education-related content. These materials—both paper and electronic—are classroom ready, and are developed and assessed in consultation with Tribes in the Rocky Mountains, the Plains, the Pacific Northwest, and the Southwest. This work is an NSF DUE funded project.

**Nicole M. Panza** Francis Marion University

### Project Math T.R.A.C.K.

Project Math T.R.A.C.K. is an outreach program funded by a grant from the State of South Carolina in an effort to increase student math test scores from grades K-12. The program is a partnership between Lee and Sumter County School Districts and Francis Marion University. The goal of the program is to support student learning by refreshing and supplementing the mathematical knowledge of their teachers. Teachers participate in monthly intensive workshops throughout the school year and a two-week Summer Institute. An overview of the program and results are discussed.

**Violeta Vasilevska** Utah Valley University

### Math-Cyber Security Conference for High School Students

The *Math, Cryptography, and Cyber Security Conference* (<http://www.uvu.edu/conferences/mathcybersecurity/index.html>) is a one day conference at Utah Valley University aimed at high school students of the surrounding area. The main objective of the conference is to show high school students, their teachers, as well as the undergraduate students involved in it, the importance of cyber security and the application of math in cryptography. In addition, the conference gives students the opportunity to encrypt/decrypt secret messages and provides opportunities for them to learn about various cyber security careers. Moreover, students have opportunity to talk to cyber security scientists and learn how they use math in their everyday jobs. In this talk we discuss the goals, the structure, and the impact of the program on all of the involved participants.

**Anant Godbole** ETSU  
**Ryan Andrew Nivens** ETSU

### **Mathematics Outreach via the ETSU STEM Center**

I will describe a variety of summer initiatives housed at the ETSU Center of Excellence in Mathematics and Science Education. These include the Eastman MathElites Program; the Niswonger Mathletes Program; Improving Teacher Quality Projects; the Tennessee Governor's School; and State MSP Projects. Most importantly, though, I will describe efforts, via Battelle and State funding, to enable HS students to incorporate open source coding into their math classes.

**Martha Lee H. Kilpack** Brigham Young University

### **Math Camp for Junior High Students: Keep It motivated and Fun**

My first year teaching math camp for 12- to 14-year-olds I was excited to share some fun math topics but scared the students would be bored instead of have fun—as a camp should be. I will talk about some tactics to keep the classroom moving. This will include some specific activities that went over well with the students.

**Neeti Sinha** Johns Hopkins University (past), currently independent (author, speaker)

### **Exposing General Audience to the Voice of Mathematics**

Exposing general audience to the voice of mathematics Under the theme of “Pursuit of Truth” at Saint Louis University I tried to shape up a TEDx talk on the subject of mathematics. From my perspective there isn't a better subject to address reality than mathematics. Catching me off-guard, a facilitator in the rehearsal round frustratingly snapped for not to be able to follow anything. I scrambled to revamp the talk starting with plain and basic, such as squared and cubed number depictions, then moving to formulations of reality—first simpler of classical mechanics then more complex renderings, such as Dirac equation—to notice the audience cheerfully draw in into the farther intricacies of mathematics as detailed as the expressions in general relativity and quantum field. Foundational concepts and fitting analogies seems to be the key to garner enthusiasm.

**Viveka Brown** Spelman College

### **Going from “Hidden Figures” to “Visible Figures” : Mentoring African American Females in Mathematics and oOther STEM Fields.**

With the recent release of the movie Hidden Figures, discussions around Black women mathematicians have occurred throughout the nation with two questions in mind: 1)Why did it take so long for these women's stories to be heard? and 2)Are there other stories like the women in Hidden Figures? This session examines the K-12 science and mathematics experiences of ten African American women who participated in a project funded by the National Science Foundation. Specifically, the women were asked about the characteristics of their mentor(s) and/or role models. The goal of this session will be to explore those positive characteristics deemed by these women and to find ways towards reducing the negative characteristics that almost deterred them from pursuing a STEM degree. Although this study only looked at African American women in STEM, this session can also be useful for anyone wanting to mentor students, especially underrepresented students in STEM.

**Paul R. Bialek** Trinity International University

### **What can Students do with a Math Major? Five Basic Career Fields for Math Majors**

Often, students ask, “What can I do with a mathematics major?” We will talk about the five basic career fields for mathematics majors, so that you will be well prepared to answer this question.

**Leonida Ljumanovic** UW-Platteville

### **Mathematical Modeling Contest for College Students**

Mathematical modeling contests for college students I have been faculty member at UW-Platteville for more than eight years. At UW-Platteville Mathematics Department we do not have applied math program, and yet we have successfully organized mathematical modeling contest for UW-Platteville students since 2013. My interest in applied math started after learning about some of the existing math modeling contests for college students in 2009. In the beginning, I often could not even organize one team to participate at these contests. Students seemed to be unwilling to sign up for something they were unfamiliar with, and in order to participate at some of these contests students had to travel about two hours from our campus. So group of faculty members at UW-Platteville started organizing annual mathematical modeling contest only for UW-Platteville students. I will talk about our mathematical modeling contest, how our students benefit from it, and how the contest led to organizing a separate student club from already existing Math Club, which is called the Mathematical Modeling club. I am the adviser for this club, so I will also talk about the UW-Platteville Mathematical Modeling club, how it differs from UW-Platteville Math Club, and what are the future plans for the club.

**Alan Alewine**    McKendree University

### **Assessment in 10 Minutes!**

Don't know where to start with assessment? Why, start here! Come to this high-energy talk, and learn the ropes. I will share what my university and mathematics program are doing assessment-wise. Feel free to take what you want, and leave the rest.

**Rachel Frankel**    University of Cincinnati, Blue Ash College

### **The Efficacy of Post Assessment Extra Credit Quizzes**

When the class average on an exam is very low an instructor is faced with two issues: poor grades (leading to resentful atmosphere in the class room and perhaps lack of motivation to do better) and lack of student knowledge of the material (in math, new material builds on previous material so students will continue to do badly). An extra credit quiz based on the test questions can address both of these issues. While an extra credit opportunity is always welcomed by students, the question is: Does it actually fulfill its purpose of improving students' knowledge of the material being assessed? Do students who did badly on the test also do badly on the quiz meaning that they still aren't motivated to study and they still don't know the concepts? This talk will include a description of the extra credit quizzes based on test questions, the unique assignment of extra credit points on a sliding scale designed to motivate poor and excellent students, and the results on all ranges of student scores to determine the efficacy of the method.

**James R. Valles**    Prairie View A&M University

**Teresa Hughes**    Cypress-Fairbanks Independent School District

### **Student-Led Error Analysis as Training for Pre-Service Teachers**

Many students in mathematics classes are familiar with information presented to them in similar manners: the use of a standard lecture or flipped class to present material and then practice on problems similar to presented examples, or inquiry-based learning is used to foster deductive learning. The second author developed and refined a method of having students examine incorrect solutions to example problems, determine the type of error made in the work, and provide a correction to the work. Through this the students were then expected to apply this error coding to their own work on assessments. The speaker will discuss how he incorporated this error analysis into his math content course for pre-service teachers as well as the resulting student participation and observations gathered from its implementation.

## **Number Theory**

**Friday, July 28, 8:30–10:40 AM, Salon C-8**

**Ji Young Choi**    Shippensburg University of PA

### **Dividing by 9**

This talk will present a new short way to find the quotient and remainder when an integer is divided by 9. Then, it will show how to find the sum of digits in a nonnegative integer using the digits in the quotient and the remainder when divided by 9.

**Zoe Cramer**    Central Connecticut State University

**Frederic Latour**    Central Connecticut State University

### **Properties of Rational Base Representations of Positive Integers**

Every positive integer has a unique representation in every integer base  $b \geq 2$ . It turns out that this result remains true in every rational base  $b > 1$ . We investigate properties of rational base representations of positive integers. For instance, in integer base  $b$ , the number of digits in the representation of  $n$  is  $\lfloor \log_b n \rfloor + 1$ . This formula does not work in non-integer bases; we will show how to modify the formula to make it valid in every rational base. We will also study the properties of the initial digit strings and final digit strings of a rational base representation of an integer, and whether the sum of the digits in a representation of  $n$  tells us anything about divisors of  $n$ .

## Frederick D. Chichester

### Generating Larger Near-Isosceles Primitive Pythagorean Triples Using Pell-Type Sequences

If the integers of the Pythagorean triple  $(a, b, c)$  are arranged in ascending order, then  $a$  represents the length of the shorter leg,  $b$  the length of the longer leg, and  $c$  the length of the hypotenuse of the corresponding Pythagorean triangle. If such a triangle approximates an isosceles triangle, its corresponding triple is said to be near-isosceles. In the case in which the sides  $a$  and  $b$  are to be made nearly equal,  $b-a = d$ , and the percent error of approximation is expressed as  $100(b-a)/a = 100d/a$ . In order to generate near-isosceles primitive Pythagorean triples (PPTs) it is necessary to find values of the Euclid integers,  $m$  and  $n$ , such that the error of approximation is sufficiently small for a given value of  $d$ . This problem has been solved in earlier work, for the case in which  $d = 1$  by using the Pell sequence:  $0, 1, 2, 5, 12, 29, 70, \dots$ . However, specifying  $d = 1$  leads to increasingly larger gaps between consecutive near-isosceles PPTs. In an earlier paper, presented in the 2016 MathFest, a Pell-type sequence was introduced to generate three near-isosceles PPTs with  $d = 7$  and 17 between consecutive near-isosceles PPTs with  $d = 1$ :  $(119, 120, 696)$  and  $(696, 697, 85)$  with error of approximation of less than five percent. In the present paper the same approach is used to introduce more near-isosceles PPTs with  $d$  greater than one and error of approximation lower than five percent between consecutive near-isosceles PPTs with  $d = 1$ :  $(696, 697, 985)$  and  $(4059, 4060, 5741)$ .

**Andy Martin**     Kentucky State University

### A Tale of Two Constants

There are many little-known number curiosities with fascinating back stories, such as the Euler-Mascheroni, Brun, and Conway “Look and Say” Constants. This talk will focus on two other Real constants, Mills’ Constant and Khinchin’s Constant.

**Timothy B. Flowers**     Indiana University of Pennsylvania

### Exploring Restricted $m$ -ary Partitions

An integer partition of  $n$  is a way of expressing  $n$  as a sum of positive integers called parts. An  $m$ -ary partition is an integer partition where each part is a power of  $m$ . Consider counting these  $m$ -ary partitions when restrictions are placed on the number of times each part may be used. We will overview our recent work and some of the known results related to these counting questions.

**Steven Edwards**     Kennesaw State University  
**William Griffiths**     Kennesaw State University

### Generalizations of Delannoy Numbers

The Delannoy numbers,  $D(m, n)$ , which count the number of lattice paths from  $(0, 0)$  to  $(m, n)$ , are a doubly-recursive sequence satisfying  $D(m, 0) = D(0, n) = 1$ , and  $D(m, n) = D(m-1, n) + D(m, n-1) + D(m-1, n-1)$ . We introduce a family of doubly-recursive sequences which are generalizations of the Delannoy numbers. The properties of these sequences produce interesting binomial identities.

**Robert Styer**     Villanova University  
**Colin Lubner**     Villanova University

### Multiplicative Persistence of Nonzero Fixed Point Digits

If  $f(n)$  is the product of the digits of  $n$ , we define the multiplicative persistence of  $n$  as the number of iterations  $k$  needed so  $f^k(n)$  is a single digit. Almost all positive integers have a fixed point zero; we investigate those cases whose fixed point is a nonzero digit. In bases 4, 6, 10, and 12, if the fixed point digit is relatively prime to the base, the maximum persistence is one.

**Emerald T. Stacy**     Oregon State University

### The Smallest Nontrivial Height of Abelian Totally $p$ -adic Numbers

Given an algebraic number  $*$ , the height of  $*$  gives a measure of how arithmetically complicated  $*$  is. We say an algebraic number is totally real (or totally  $p$ -adic) if its minimal polynomial splits completely over  $\mathbb{R}$  (or  $\mathbb{Q}_p$ ). We say  $*$  is abelian if  $\mathbb{Q}(*)$  is Galois with abelian Galois group over  $\mathbb{Q}$ . Let  $d$ ,  $\xi$ , and  $p$  be a prime. Then there exists a smallest nontrivial height of an abelian totally  $p$ -adic algebraic number of degree  $d$ , which we will call  $h_{d,p}^{ab}$ . This talk will describe aspects of the set  $fh_{d,p}^{ab}$ :  $p$  is a prime  $g$ , and why it can be determined by a congruence condition on  $p$ .

**Patrick Bray**     Rowan University  
**Hieu D. Nguyen**     Rowan University

### On a Series Formula for the Area of the Mandelbrot Set and Zagier’s Conjecture

The Mandelbrot set is one of the most well known fractals in all of mathematics because of its exotic self-similarity. However, it remains an open problem to determine its exact area, which may shed light on the dimension of its boundary. Efforts to calculate this area have mostly relied on Ewing and Schober’s series formula, whose terms  $b_m$  are rational and have powers of 2 as their denominators. Closed formulas for the 2-adic valuation of  $b_m$  are known for the case where  $m$  is odd. We discuss results on Zagier’s conjectured formula for the case where  $m = 2 \bmod 4$  by extending  $p$ -adic methods used by Shimauchi.

# Teaching and Learning Advanced Mathematics

Friday, July 28, 8:30–11:40 AM, Salon C-7

**M. Leigh Lunsford** Longwood University

## Transitioning from the Finite to the Infinite in $\ell^2$ .

Compactness in  $\mathbb{R}^n$  guarantees that one can pack at most a finite number of balls of radius  $r < 1$  into the closed unit ball. *This is true no matter how small the value of  $r$ .* In the classic Hilbert space  $\ell^2$  this does not apply. Interestingly, it is not hard to show that you can pack at most two balls of radius  $1/2$  into the closed unit ball of  $\ell^2$ , but if the radius is small enough, for instance  $1/4$ , then you can pack infinitely many balls into the unit ball. Thus, as you decrease from a radius of  $1/2$  to a radius of  $1/4$  you will reach a value where the packings radically change. This value is a boundary for an abrupt transition from the finite to the infinite in  $\ell^2$ . What is this number and can we prove it has this property? Attend this talk to find out!

**Victoria Krupnik** Rutgers University

**Keith Weber** Rutgers University

**Timothy Fukawa-Connelly** Temple University

## Students' Epistemological Frames and their Interpretation of Advanced Mathematics

In this paper, we present a comparative case study of two students with different epistemological frames watching the same real analysis lectures. We show that students with different epistemological frames can interpret the same lecture in different ways. These results illustrate how a student's interpretation of a lecture is not inherently tied to the lecture, but rather depend on the student and her perspective on mathematics. Thus, improving student learning may depend on more than improving the quality of the lectures, but also changing student's beliefs and orientations about mathematics and mathematics learning. *Key words:* epistemic frames, student understanding of lecture, real analysis

**Timothy A. Lucas** Pepperdine University

## Slopes: An Interactive App for Exploring Differential Equations

A proper study of differential equations requires that students visualize solutions and analyze plots related to the structure of the equations. Although there is mathematical software such as Maple, Matlab, or Mathematica that performs these functions, these programs are expensive and students must invest a significant amount of time to learn the functions and proper syntax. *Slopes* is an interactive mobile app that I have developed with a research team of faculty and students at Pepperdine University that allows students to explore numerical methods and graphical solutions to differential equations. The name of the app originates from the fact that the derivative can be interpreted as the slope of the function and most activities revolve around plotting slopes. The advantage of using a mobile application is that iPhones and iPads are highly portable and feature larger touch screens that allow students to view and manipulate content easily. Research based on observations of mathematics courses at Pepperdine University has shown that students are more willing to collaborate and share their results when using a tablet such as the iPad (Fisher, Lucas & Galstyan 2013). The intuitive interface of *Slopes* invites students to fully immerse themselves in the world of differential equations so that they can understand the concepts from not only algebraic, but also graphical and numerical perspectives.

**Heidi Hulsizer** Benedictine College

## Round Robin Proofs

Learning how to create proofs can be a difficult process for many undergraduate students. This session illustrates a method in which students sit in a circle and take turns adding to various proofs. This forces students to think about several different proofs in one setting and also forces them to consider alternative methods of proof provided by classmates. This method was used in an Honors Set Theory class but could easily be extended to a Discrete Math or Proofs class. The benefits and disadvantages of this method will be discussed.

**Karin R. Saoub** Roanoke College

## Bridging the Gap: Mid-level Proofs Courses and their Effect on Student Learning and Outcomes

Students in upper-level theoretical courses are faced with complex and abstract concepts, higher expectations, and more focus on proofs and writing skills. The difficulty of these courses is compounded without adequate preparation in proof writing and proof techniques. Roanoke College recently adjusted the mathematics major to include mid-level proofs courses to better prepare students for their upper-level theoretical courses. In particular, these transition courses are designed to provide additional practice in proof writing in an environment with more accessible material and moderate expectations. This talk will discuss the format of these courses and results on student learning and outcomes, from both the grade data and assessment outcomes.



**Miranda Bowie** University of North Alabama

### **Flipping an Introduction to Proofs Course**

In the past I have taught my introduction to proofs (discrete mathematics) course in a lecture format. However, the outcomes each semester were not what I desired. The students struggled with the material and never fully engaged in class discussions. Fortunately, at MathFest last summer I heard Mike Janssen from Dordt College explain the use of reading guides in his proofs class. Although it was only a few weeks before my class was to begin, I decided to give it a try. In this talk, I will discuss how I flipped my introduction to proofs course. I will also include student reactions and modifications for future semesters.

**Donna Beers** Simmons College

### **Community Detection with Hierarchical Clustering Algorithms: Connecting Graph and Network Theory to Analyzing Social Networks**

In this presentation we will describe a new course on graph and network theory that we developed and offered last fall to mathematics majors. We will focus on community detection, a special topic within the course. Communities, also called clusters or modules, are important organizational structures within complex networks, and community detection is of intense interest to researchers in sociology, biology, computer science, and organizational management. We present a commonly used definition for “community” along with a method for measuring the modularity of a proposed partition of a given network into clusters. We describe a class of algorithms, called hierarchical clustering methods, whose goal is to maximize modularity. We illustrate how to carry out agglomerative hierarchical clustering, and how to produce their associated nested diagrams and dendrograms. We will provide examples of projects where students may carry out community detection by applying open source software tools to analyze data that they generate or that they gather from social media.

**Paul von Dohlen** William Paterson University

### **Coding and GUI Use in the Teaching of Undergraduate Numerical Analysis**

We will investigate different approaches to using coding and graphical user interfaces (GUIs) in the teaching of an undergraduate numerical analysis class. When learning about numerical methods, it is essential that students experience applications of the methods to problems for which hand calculations would be excessively tedious. In order to do so, an instructor can use various levels of coding based on, amongst other factors, the computing proficiency of the intended students. Thus, the instructor can use approaches ranging from pre-programmed graphical user interfaces (GUIs) to complete code creation and implementation. The instructor must decide what level of coding is appropriate for the course so as to use the computing experiences most effectively.

**Timothy Trujillo** Sam Houston State University

### **Triangular Ramsey Numbers: An Undergraduate Research Project**

Undergraduate research projects provide students with the tools and skills they need to make informed decisions about pursuing a career in research mathematics. We discuss the details of an undergraduate research project with a group of three students at the Colorado School of Mines during the 2015-2016 academic year. In particular, we will discuss: the planning of the project, its implementation, the research results, the submission of the research paper, and the talks given by the student researchers.

**Jane McDougall** Colorado College

### **In-Class Exercises in Complex Analysis**

In the process of learning abstract mathematics, it can be helpful to revisit new ideas, and to encounter them in different contexts. It can also be helpful to have hands on models to work with. We will present two exercises using simple paper models, that we have used now in several senior-level complex analysis courses, and that allow students to revisit ideas about continuity and connectedness in analysis.

**Kryssa C. Goodhart** Rowan University

**Jay L. Schiffman** Rowan University

### **Utilizing Truth Tables to Furnish Some Neat Mathematical Properties**

Students during their undergraduate careers in mathematics are exposed to operations that may or may not enjoy the commutative, associative, or distributive properties. What is not as well-known involves operations which may anti-commute, anti-associate, or anti-distribute with respect to themselves or to other operations. If one considers two propositional variables and attempts to form all possible truth functions, they arrive at sixteen possibilities including the obvious truth tables such as conjunction, conditional, and biconditional. Our focus here is to demonstrate that basic truth-table logic serves as fertile ground for obtaining some surprisingly fascinating outcomes.

**Bill Linderman** King University

### **A Novel Idea: Teaching Mathematics using Apostolos Doxiadis's *Uncle Petros and Goldbach's Conjecture***

We discuss how Apostolos Doxiadis's gem of a novel, *Uncle Petros and Goldbach's Conjecture*, can be used as required reading in an upper-level mathematics course as a way to explore several important themes in higher mathematics - the nature of proof, creativity, truth, and obsession. A work of fiction about a mathematical genius, the novel is rich with historical mathematical references and includes cameos by Ramanujan, Gödel, and Turing.

**Mike Krebs** California State University, Los Angeles

### **Is there a Topology on $\mathbb{Q}$ that Detects Continuous Extensions to $\mathbb{R}$ ?**

Let  $\mathbb{Q}$  be the set of rational numbers, and let  $\mathbb{R}$  be the set of real numbers in the usual topology. Consider the function  $f$  from  $\mathbb{Q}$  to  $\mathbb{Q}$  where  $f(x) = 1$  if  $x$  is greater than the square root of 2, and  $f(x) = 0$  otherwise. With the usual topology on  $\mathbb{Q}$ , we have that  $f$  is a continuous function. However,  $f$  does not extend to a continuous function from  $\mathbb{R}$  to  $\mathbb{R}$ . How awful! Can we remedy this situation by changing the topology on  $\mathbb{Q}$ ? In other words, does there exist a topology on  $\mathbb{Q}$  so that a function from  $\mathbb{Q}$  to  $\mathbb{Q}$  is continuous if and only if it extends to a continuous function from  $\mathbb{R}$  to  $\mathbb{R}$ ? In this talk, we answer that question. The solution uses only basic definitions and theorems that appear early in a first course in point-set topology, so this question can be used as a challenge problem or a portfolio problem for an introductory Topology class. One can generalize the question to subsets of  $\mathbb{R}$  other than  $\mathbb{Q}$ . We conclude the talk by formulating a conjecture as to precisely which subsets of  $\mathbb{R}$  possess a topology of the desired form.

## **Teaching and Learning Calculus, and Mathematics and Technology**

**Friday, July 28, 1:00–3:40 PM, Salon C-8**

**Robert R. Rogers** SUNY Fredonia  
**Eugene Boman** Penn State - Harrisburg

### **Solving Problems with Calculus, Not Calculus Problems**

Nearly all mathematics exists because somebody wanted to solve a specific problem. But when we teach mathematics, we often turn this basic fact on its head. For example, we frequently teach the theory and techniques of calculus first, and then have students solve “calculus problems” - problems designed to reinforce students’ skill with each technique. We propose that instead of having students solve “calculus problems,” we should initially have them solve “problems using calculus” (and, of course, any other applicable tools) to motivate the techniques. That is, they should be taught to solve problems of interest which require calculus, but not just calculus, and which emphasize problem solving using the entirety of the mathematics they have learned. The presenters will provide problems they have used in University Calculus I and II to implement this idea.

**Ken Collins** Charlotte Latin School

### **Investigations To Improve Student Learning In Calculus**

This session will share several investigations we have used in the classroom to help strengthen our students’ understanding of calculus. These investigations will focus on limits and derivatives. We will discuss some ideas for developing these investigations and offer reproducible copies to use in your classes.

**Rachel Weir** Allegheny College

### **I Can't Lecture!**

Motivated by research that shows that students from traditionally underrepresented groups in the sciences can benefit from the use of active learning techniques, I have been moving away from my traditional lecture-style approach over the past few years. Most recently, in my Fall 2016 Calculus I course, I found myself redesigning my content delivery during the course of the semester. In this talk, I will share some of the activities that I used, an overview of the successes and challenges of this redesign, and my plans for future calculus courses.

**John H. Johnson** The Ohio State University

### **Technology Mediated Active Learning in a Large Lecture Calculus Class**

The active calculus pilots are part of the two-year course redesign project at the Ohio State University's Mathematics Department. One component of this pilot is the use of technology both in and out of class. Some of these technologies included Top Hat, Piazza, Remind, GeoGebra, Desmos, and the occasional GIF. Overall, these technologies helped facilitate student engagement, attendance, and participation, but some of them also contributed to in-class distractions. I'll outline some of the opportunities and challenges I encountered while using technology to facilitate active learning, and, present some qualitative survey data on students' perspective on the in-class experience.

**Gerard Ornas**     McNeese State University

### **Transitioning to IBL Teaching**

Being assigned two sections of Calculus I for the spring semester, it was decided to teach one with a semi-Inquiry Based Learning approach as part of a transition in the author's teaching philosophy. The second was taught with a traditional lecture based approach as a control. We discuss the courses, the results, issues encountered, lessons learned, and plans for the future.

**John A. Kerrigan**     Rutgers University

### **Teaching and Learning Mathematics in an Active Learning Classroom**

Recently, Rutgers University converted several sections of its traditional "Topics in Mathematics for Liberal Arts" course to flipped hybrid sections. In addition, Rutgers University has also recently created "active learning classrooms" designed to foster student collaboration, peer teaching, and active instructor presence. In this talk, I will describe how I've transformed my teaching to make learning undergraduate mathematics a highly active and scaffolded process in the exciting new active learning space. Participants will take away both flipped classroom instructional strategies and active learning strategies that can be implemented in the classroom immediately.

**Robert Jacobson**     Roger Williams University

### **Teaching Complex Analysis With A Spherical Camera**

A spherical camera, sometimes referred to as a 360-degree camera, is a camera that captures all or most of the spherical image of its surroundings and produces a single image file. Consumer grade spherical cameras like the Ricoh Theta S can be purchased for just a couple of hundred dollars putting them within reach of the adventuresome undergraduate complex analysis professor. This talk describes one professor's experience incorporating the use of a spherical camera in teaching complex analysis by assigning projects that apply the Riemann sphere and stereographic projection to spherical photos in order to build physical objects that illustrate mathematical concepts.

**Peter Olszewski**     Penn State Behrend

### **Teaching College Algebra with Knewton**

In the summer of 2016, the leaders of Knewton asked me to pilot a new digital learning program. From the company who powers MyMathLab study plan, Knewton is a real-time, adaptive, and algorithmic-based program that focuses on drill and practice. It is a mastery-based software designed to personally guide students on their individual strengths and weaknesses to understand concepts. This talk will present how Math 21: College Algebra I was taught to students in a hybrid format. In addition, this presentation will outline the positive outcomes, the struggles, and lessons learned from using Knewton.

**Philip B. Yasskin**     Texas A&M University

**Andrew Crenwelge**     Texas A&M University

**Joseph Martinsen**     Texas A&M University

**Matthew WeiHING**     Texas A&M University

**Eikagra Sharma**     Texas A&M University

**Shiva Saravanan**     A&M Consolidated High School

**Matthew Barry**     Texas Center for Applied Technology, Texas A&M Engineering Experiment S

### **Continuing Development of MYMathApps Calculus**

MYMathApps Calculus is an online text for calculus 2 and 3, with more topics planned. The text includes many animations in addition to the standard static graphics. Examples in the text have full solutions. Exercises in the text have hints, answers and solutions which gradually appear after the students have a chance to try it themselves. There are external links for background information, internal links to additional examples, and pop-up notes for small comments and proofs, that do not obstruct the flow of the text but are available to those students who wish to learn more. Exercise pages have links back to the pages where the material for each exercise is discussed. Many topics have randomly generated tutorials and exercises based on the Maplets for Calculus. MYMathApps has grown out of the WebCalc project involving Yasskin, Don Allen and Mike Stecher at Texas A&M University, and the Maplets for Calculus project involving Yasskin and Doug Meade at the University of South Carolina. The text is being written using modern web technologies so it is available on all devices. Graphics are made using the Maple computer algebra system. The work is supported in part by NSF DUE TUES-2 grant 1123255.

**Ashley Johnson**     University of North Alabama

### **A GeoGebra Project for Future Teachers**

In the last decade, many school districts have invested in technology for their students (chrome books, ipads, etc), and many teachers feel pressured to use these resources in their classrooms. In my College Geometry course, which is largely aimed at future teachers, I have students complete a project which involves creating IBL style applets in the program GeoGebra and worksheets to accompany them that they could use in their own classrooms. This talk will include the instructions for the project and examples of student submissions.

**Azar Khosravani** Columbia College Chicago

### **Fibonacci and Lucas Identities with Excel**

In the mathematics curriculum, the Fibonacci sequence is used as a convenient example for illustrating new concepts to mathematics undergraduates and even non-majors. There are numerous easily-stated algebraic identities and divisibility rules involving the Fibonacci numbers (and Lucas numbers) that make this sequence particularly interesting. We will review some of these identities and divisibility properties and show how Excel can be used to motivate and illustrate these relationships in a classroom setting.

## **Modeling or Applications**

**Friday, July 28, 1:00–4:40 PM, Salon C-7**

**Mami Wentworth** Wentworth Institute of Technology

### **Wildfire Modeling in a Project-Based Course**

We recently offered a catastrophe modeling class, in which students simulated flood, wildfire and disease spread. The class was developed in collaboration with AIR Worldwide, a local risk analysis company, to motivate students with real life applications as well as to teach them the skill set desired for careers in industry. In this talk, I will focus on the development of a wildfire model and simulation of wildfire spread using real soil data. Monte Carlo simulation results are used to estimate damage and cost of insurance for a hypothetical building using regression. I will also discuss the overall course development, including the process of collaboration and areas of improvement.

**Barry C. Husowitz** Wentworth Institute of Technology

### **Project Base Learning in Numerical Analysis via Artificial Neural Networks**

Project-based learning gives students a chance to apply what they are learning in class to real-world problems and challenges. This dynamical approach gives the students a broader perspective of the subjecting being taught and helps them acquire deeper knowledge of the subject. Numerical analysis is a subject of mathematics that can be readily applied to real world problems. In this talk I will present a project that was investigated by students in my numerical analysis class. I will present how these students used the stochastic gradient approach for a simple Artificial Neural Network to analysis the Iris data on the UCI Machine Learning Repository. Furthermore, I will talk about how these students are planning on working further with Artificial Neuron networks and the back propagation algorithm to analysis toxicity data for the fathead minnow. Our goal is to try to design guidelines for chemicals that are safer for aquatic life by Artificial Neural Networks.

**Michele L. Joyner** East Tennessee State University

### **Experiences from Implementing an Industrial Project-Based Course in the Curriculum**

We have implemented an industrial project-based course at East Tennessee University as part of the NSF funded Preparation for Industrial Careers in Mathematical Sciences program jointly sponsored by MAA and SIAM. In this talk, we will discuss how we developed industrial partnerships, found projects, recruited students and developed a writing-, oral-, and computationally-intensive course as part of our curriculum. We will discuss the difficulties of such a course and how we overcame or are trying to overcome some of those difficulties in the second-year implementation. We will also talk about the benefits for the students from both a faculty and student perspective as well as the industrial liaison prospective. Furthermore, we will discuss an implemented syllabus which focuses on not only the industrial project but also many different activities implemented to aid in bridging the gap between finishing college and starting in the workforce.

**Malgorzata A. Marciniak** CUNY

### **Efficiency of Geometrical Designs in Engineering and Biology**

Creative research projects for students are a part of Active Learning and Inquiry Based Learning teaching methodologies. Student projects can represent various difficulty levels, lengths and scopes. They can be blended together with the course material or stand alone. During the talk, I will present few topics for creative research projects that touch upon mathematical aspects of designing efficient shapes in engineering and biology with possible solutions. Since the required mathematical knowledge belongs to the curriculum of Calculus III course, the projects are entirely within the scope of students who completed that course. However, after looking more carefully at the topics, one can observe a potential for faculty research in optimization theory.

**Jeong-Mi Yoon** UH-Downtown

### **An Interdisciplinary Undergraduate Research Project for Wind Turbine Modeling**

For the last few decades wind power is one of the most cost effective energy resources as it is renewable and pollution-free. A variety of research has been motivated on the development of wind turbine design to make optimal energy conversion system. I will develop an undergraduate interdisciplinary research project which explores modeling and control of wind turbine dynamics based on Calculus and Differential Equations concepts.

**Dennis G. Collins** UPR-Mayaguez

### Peace movement paradox in generalized gravity

This talk follows up on talks by the Author “The peace movement paradox” at the SIDIM 2017 Conference in Ponce, Puerto Rico on March 3, 2017, “A conjectured hypercube invariant in generalized gravity” at the Indiana MAA Section Meeting at Purdue University, West Lafayette, IN on Oct. 8, 2016 and a talk “Toward generalized gravity” at the MAA Mathfest in Columbus, OH on Aug. 4, 2016. In certain cases the following problems become equivalent: 1) minimize Fisher information, 2) maximize covered sides, 3) maximize entropy, 4) maximize energy, and 5) solve a Newton dynamical system based on a potential, where in the last case maximum entropy flow can lead to fluctuations in a potential well. Peace movements put more energy into the well, which can lead to points being pulled together more (more peace) but ALSO, with pulsing, greater excursions away from unity (more war). R.J. Rummel (The Conflict Helix) tried to explain this situation by catastrophe theory with war and peace being equilibrium points, whereas here war and peace are considered as turning points on either side of an intermediate equilibrium scale point. Case studies are considered.

**L. Jeneva Clark** University of Tennessee

**Peggy Bertrand** University of Tennessee

### Let's Get Physical! Teaching Math through the Lens of Physics

A professional development project for secondary teachers was funded by the Tennessee Higher Education Commission (THEC), through an Improving Teacher Quality (ITQ) grant, to equip teachers to teach mathematics using physics experiments. This presentation will describe the modeling content of the PD, providing pedagogical ideas for teaching introductory mathematics courses and for teaching preservice mathematics teachers. One example activity involves the physics of falling stacks of coffee filters that attain terminal velocity. This motivates students to use math to model natural phenomena. They collect data and analyze it using logarithms, linearization, and graphing. The equation  $a = g - (b/m)vn$  represents the acceleration of a falling object that experiences drag, where  $b$  and  $n$  are the free parameters of the model,  $m$  is the mass of the falling body, and  $g$  is acceleration due to gravity. When terminal velocity is reached,  $a = 0$ . Using this information, students experiment to determine the free parameters. In the process, they review logarithms, linearization, uncertainty, and error. Other similar classroom lab activities, from this workshop, will be described.

**Hwayeon Ryu** University of Hartford

### Dynamics of Neuronal Networks with Coupling Delays: A Modeling Study

Mathematical modeling tools have been recently used to study various complex biological systems. In particular, due to technical limitations of direct experiments on a real brain system, a series of mathematical models have been developed to help biologists to uncover important features of its dynamics, mostly on a small-network or neuronal-network level. A neuronal network consists of many coupled neurons and their interaction involves coupling (or time) delays arising from the transmission of signals between coupled neurons. In this study, we examine the impact of coupling delays within biologically realistic neuronal networks on their pattern formation using mathematical modeling. The model results will provide a solution that will improve understanding of how the rhythmic behaviors of the brain are connected to pattern generation and propagation at the small-network level. More broadly, the project has implications for how abnormal brain functioning, such as epileptic seizures, migraine and hallucinations, occur as these behaviors are believed to closely link to the synchronization and traveling waves in neuronal networks.

**Cassia Smith** University of the Virgin Islands

**Robert Stolz** University of the Virgin Islands

**Jonathan Jossart** University of the Virgin Islands

### Machine Learning Approach for the Prediction of Dissolved Oxygen Concentration

Dissolved oxygen (DO) is essential to many forms of life in aquatic ecosystems as most living organisms require oxygen for their basic metabolic processes. DO concentration is one of the main indicators used in assessing water quality and the health of an aquatic environment. This project seeks to construct an accurate model for predicting DO concentrations in a lagoon area within Brewers Bay, St. Thomas VI and also to explore the influences on DO fluctuations. The ability to forecast oxygen ( $O_2$ ) levels will be invaluable in monitoring the health of local marine areas and analyzing the effect of bio phenomenon and human interference on those areas. Similarly, the model will also be useful for future research that might investigate any correlation between animal behavior patterns and oxygen concentration. A machine learning approach via MATLAB software will enable the development of several types of regression models. Though research is still ongoing, it has been possible so far to develop meaningful stepwise linear regression and decision tree models. Preliminary results have indicated the influence of several statistically significant predictors (air/water temperature, wind speed, etc.) on DO. Once a highly accurate model is developed, we can apply the working model to other oceanic regions.

**Tim Antonelli** Worcester State University

### **The Importance of Population Dynamics in Modeling the Control of Disease-Carrying Mosquito Populations.**

Mosquito-borne diseases like malaria, dengue, and Zika affect hundreds of millions of people each year. While vaccines and treatments for some mosquito-borne diseases exist, they are unavailable or ineffective for others. Control efforts for these diseases rely on controlling the mosquito population, typically via larval habitat destruction or insecticide spraying. However, these approaches have largely failed to decrease disease incidence. Thus, there has been interest in developing novel approaches to control mosquito populations that incorporate recent developments in genetic engineering. These include genes that suppress the population and genes that can spread and make the population less able to transmit disease. Mathematical models are an integral part of developing such tools, as they can predict outcomes prior to the release of genetically modified mosquitoes, which can be costly or met with public and regulatory opposition. Most models take a population genetics approach, modeling changes in gene frequency in the population over multiple generations. We show how the incorporation of density-dependent population dynamics in an ordinary differential equation model can lead to different results than population genetic-only models. Because changes in gene frequency may happen quickly following the release of genetically modified mosquitoes, it is important to consider the simultaneous effect of population dynamics, such as forces that act to return the population to carrying capacity. We discuss a variety of differences between population genetic-only models and models that include population dynamics for genetic engineering strategies such as *Wolbachia*, underdominance, and CRISPR/Cas9.

**Suzanne M. O'Regan** North Carolina A&T State University

### **Tipping Points in Epidemics**

Infectious diseases are among the most visible and costly threats to individual and public health. Quantitative tools that forecast disease emergence and document progress towards disease elimination would lead to tremendous economic benefits. Non-parametric tools that take advantage of the fact that infectious disease systems can exhibit sudden changes in state ("tipping points") would advance infectious disease forecasting significantly. In this talk, I will introduce theory for anticipating tipping points in infectious disease systems. I consider epidemiological SIS and SIR models that are slowly forced through a tipping point (a transcritical bifurcation). I develop expressions for the behavior of several candidate indicators during the approach to emergence or elimination. I show how the candidate indicators may be used for anticipating critical transitions in infectious disease systems. It is hoped that these results, which show the anticipation of tipping points in infectious disease systems to be theoretically possible, may be used to guide the construction of online algorithms for processing surveillance data.

**Morteza Shafii-Mousavi** Indiana University South Bend

### **Using Market Data for Markov Chain Prediction**

Modeling Markov Process for Market Analysis. I show how to use historical index prices DOW 30, S&P 500, etc. to find probabilities and use these to build up Markov Chain transitional matrices. Further, we model spreadsheets to use the computer to find conditional probabilities and steady state probabilities of market performances in the future. The techniques are applied in Markov Decision Analysis.

## **Teaching and Learning Developmental Mathematics**

**Saturday, July 29, 8:30–9:55 AM, Salon C-6**

**Radieah Banihani** New Mexico State University

### **Rafeef Begins to Learn Arithmetic in a Jordanian Kindergarten**

Children in Jordan begin kindergarten at the age of four. In this paper four-year-old Rafeef will take us through her beginning arithmetic lessons. Along with 19 classmates and one teacher, she attends school from 7:30 to 12:30, and after some preliminaries, she starts learning about numbers in Arabic, and shapes and measurement. We will give a brief history of education in Jordan, describe the physical setting of her kindergarten, and the supplies and materials used, and her typical daily activities, with the stress on methods of introducing numbers in Arabic and in standard numerals which are also read in English. We will see many similarities, but also significant differences, with kindergartens in the United States. The talk will be illustrated with authentic photographs from a Jordanian school with children doing a variety of tasks.

**Carmel Schettino** Avenues: The World School

### **Problem-Based Learning: Encouraging Girls in Secondary Mathematics**

Although the Gender Achievement Gap is closing in mathematics, the "interest gap" in pursuing STEM fields is not. One recommendation for encouraging young women to continue on in mathematical studies is to find instructional methods that allow them to feel more included in the learning process - fostering attitudes such as self-confidence, empowerment and agency. In this study, the journeys of five adolescent girls who were currently studying secondary mathematics with a pedagogical approach that sought to encourage such attitudes named relational Problem-Based Learning (PBL) were explored and analyzed to describe the relationship between the pedagogical approach and the girls' attitudes.

**Eugenia Cheng** School of the Art Institute of Chicago

### **Gender vs Character: Designing Inclusive Math Activities**

In this talk I will share what I have learned from teaching abstract mathematics to art students at the School of the Art Institute of Chicago. I have taught ten such classes, contrasted with years of teaching math majors in more usual academic institutions. First I will describe the art students' accounts of how they were put off math in previous parts of their education, including memorization, formulae, lack of creativity, and being told they are wrong. I will discuss character traits, comparing art students' experiences with those of math majors who were not put off in high school. This difference is exemplified by comparing those who like being right with those who dislike being wrong, and I will propose new terminology to describe these character differences. This new terminology also helps to clarify a possible character difference between male and female students, without prescribing the difference or pigeonholing students by gender. The words are "ingressive", to replace what might previously have been thought of as masculine, and "congressive" to replace feminine. I will then describe how these insights have helped me design classroom activities that successfully appeal to congressive students, who were previously put off by the ingressive presentation of math. I will propose that if we are inclusive towards congressive students this could result in greater inclusion of female students as well.

**Nnenna K. Uka** Department of Mathematics, Abia State Polytechnic, Aba, Nigeria

### **Using Factorization to Effectively Teach Quadratic Equations in Tertiary Institutions in Nigeria**

There are many mathematical concepts which, while solving, result to quadratic equations/expressions. Among these are indices, logarithm, permutation, integration by partial fractions, evaluating limits of functions and finding their existence. The resultant quadratic equations are usually factorable. This, therefore, needs to be handled in an effective manner to make its teaching and learning easy to understand. The aim of this paper therefore, is to acquaint the students and mathematics teachers with effective strategies of teaching quadratic equations by factoring. The paper provides step by step strategies mathematics teachers could use in teaching and applying quadratic equations using factorization. This invariably will make teaching and learning of mathematics interesting and enjoyable. A recommendation among others is that mathematics teachers should be encouraged to adopt this step-wise approach while teaching mathematical concepts that involve quadratic equations.

**Janet St.Clair** Alabama State University

### **Aesthetic Computing - Order of Operations Understanding Rescue**

Order of operation conventions were not needed until algebra became fully symbolic. Long ago, after algebra became symbolic, mathematicians would list their conventions at the beginning of their work. The rise of the mathematics textbook industry and computer programming might have influenced the rigid teaching of order of operations in schools. Even with order of operation conventions, simplifying certain expressions such as  $6 \div 2(1 + 2)$  and  $48/2(9 + 3)$  is controversial. Some students in developmental mathematics courses and mathematics courses for non-mathematics majors do not fully understand the order of operations. Paul Fishwick's aesthetic computing in algebra was used to help two students who struggled with simplifying algebraic expressions. Pre-questions given to one student showed that he did not know what the order of operation conventions were. Aesthetic computing in algebra involves changing an algebraic expression from implicit notation usually seen in mathematics textbooks ultimately to an expression tree with words. One expands or goes beyond the tree with words by writing a story, creating a structure (e.g., configuration of objects in a room), or creating a space (layout of rooms in a dormitory). Students chose interesting mediums to use to inspire their assignment of words to trees and went beyond their tree with words to write creative stories. There was evidence that aesthetic computing helped students better understand word classes, order of operations, and structure of algebraic expressions. It offered a way to connect their interests with mathematics. It would be interesting to examine how aesthetic computing influences student discussion about simplifying controversial expressions.

**Mary B. Walkins** The Community College of Baltimore County

### **Encouraging Mindfulness and a Growth Mindset in Developmental Mathematics Classrooms**

Respectively, in spring 2016 and in winter 2017, I enrolled in a three-session Mindfulness Workshop and a 10-day Culturally Responsive Teaching (CRT) Seminar at the Community College of Baltimore County (CCBC). My participation in these two activities was for my own professional growth. However, in spring 2016 we were challenged to share at least one contemplative practice with our current students. Starting in the middle of that semester, I encouraged my students to be more mindful and present in the developmental mathematics classroom. Then, in spring 2017, I decided to immediately implement, in my classrooms, one area of CRT that I had recently learned about the growth mindset. Certainly, I didn't want my students to be stuck with a fixed mindset, and I have been intentionally encouraging a growth mindset in students to help them be more confident about their engagement in developmental mathematics exercises. I will share the Mindfulness and CRT practices used and give examples of students' responses on a Mindfulness Questionnaire and to Attribution Questions.

# Probability and Statistics

Saturday, July 29, 8:30–10:40 AM, Salon C-7

**Song Chen** University of Wisconsin La Crosse  
**Chad Vidden** University of Wisconsin La Crosse  
**Marco Vriens** University of Wisconsin La Crosse

## Use Simple Math to Extract Business Information from Internet Data: A Study on Volkswagen's Emission Scandal

There are huge interests among companies to extract actionable business information from the internet data but the enormous volume and noisy nature of online data make finding meaningful messages a very difficult challenge. This talk will walk you through a commercial project on how to use simple math and statistical ideas such as simple probability, linear projection, graph theory, etc. to analyze 875,000 online comments from Volkswagen's car forum and apply the findings to assess the Volkswagen's Emission Scandal in 2015.

**Deborah Jean Gougeon** University of Scranton

## What Would You Give For That: Grade Expectations vs. Performance in an Introductory Business Statistics Course

Over a period of five years, this study surveyed a total of 386 undergraduate students on the first day of a required *Introductory Business Statistics* course. Since all 386 students in this course filled out the survey, there was a one-hundred percent response rate. Students were asked to respond to eleven questions that measured their perceptions and expectations regarding this course. For example, students were asked whether the course would be relevant or not to their overall business education, how much time they anticipated spending on homework assignments as well as on general study for the course, and what their anticipated final grade would be. Since all of these factors can have a significant impact on both pedagogy and outcomes when planning and teaching an *Introductory Business Statistics* course, this study takes a closer look at these attitudes and perceptions by making comparisons with students' previous experience with quantitative courses, their reported time spent on studying *Introductory Business Statistics*, and their overall expectations regarding their final grade. The goal is to demonstrate empirically the impact of student attitudes on learning outcomes in *Introductory Business Statistics* courses.

**Nancy A. Hernandez** TAMUCT-Department of Mathematics  
**Christopher Thron** TAMUCT-Department of Mathematics

## Type I Error: Conditional or Unconditional?

We show that Internet statistics resources sometimes represent type I error as a conditional event (conditioned on the null hypothesis being true) and sometimes as an unconditional event. The original 1933 definition by Neyman and Pearson treats type I error as unconditional: on the other hand, many current textbooks define "probability of type I error" as the probability of a conditional event. In popular usage, it appears that there is no clear consensus on what the "correct" definition of type I error is. Furthermore, confusions between the two definitions often lead to misinterpretations of the results of hypothesis tests.

**Steven Kim** California State University, Monterey Bay

## An Alternative Perspective on Consensus Priors with Applications to Phase I Clinical Trials

We occasionally need to make a decision or a series of decisions based on a small sample. In some cases, an investigator is knowledgeable about a parameter of interest in some degrees or is accessible to various sources of prior information. Yet, two or more experts cannot have an identical prior distribution for the parameter. In this talk, we discuss the use of a consensus prior (compromised prior specification) and compare two classes of Bayes estimators. In the first class of Bayes estimators, the contribution of each prior opinion is determined before observing data. In the second class, the contribution of each prior opinion is determined after observing data. We compare these two approaches with applications to Phase I clinical trials, and we illustrate the usefulness of a consensus prior when we make a series of decisions for patients.

**Niharika Yennum** Texas A & M University - Kingsville  
**Stephen Sedory** Texas A & M University - Kingsville  
**Sarjinder Singh** Texas A & M University - Kingsville

## Improved Randomized Response Strategies for Collecting Sensitive Data

Obtaining truthful answers to sensitive questions on a survey can be problematic. By using randomized response techniques a respondent can protect their privacy yet provide honest answers to these types of questions. While the status of the individual respondent remains unknown it is possible to make estimates of the proportion of persons within a population who belong to the sensitive group. In this talk we review a bit on how these techniques work, look at two recent methods (that of Kuk (1990) and that of Singh and Grewal (2013)), and finally introduce a new method that improves on these. We discuss some of the advantages of the method and report on the results of an investigation into those situations where the method outperforms the other methods in terms of increased efficiency and better protection.



**Alex Plyukhin** Saint Anselm College

### **Random Walks with Fractally Correlated Traps**

We consider the survival probability  $f(t)$  of a random walk on a host (fractal) lattice with spatially correlated traps. The traps form a sublattice with a fractal dimension smaller than that of the host lattice. For weakly absorbing traps we find that  $f(t)$  first decays according to stretched exponential kinetics followed by a transition to power-law decay at longer time scale. The duration of the initial stretched exponential regime decreases with an increasing absorption rate, and for perfect traps the decay of  $f(t)$  follows the power law for all times.

**Leila Setayeshgar** Providence College

### **Bayes' Rule and the Law**

Bayesian inference is an approach in mathematical statistics where the probability of a hypothesis is updated as more evidence and data become available. It has wide applications in many areas such as machine learning, evolutionary biology, medicine and even in the judicial system. This talk will explore how Bayesian inference can be used in a specific court case to assist jurors in the process of legal decision making, demonstrating the power of mathematics in the court room.

## **Graph Theory**

**Saturday, July 29, 1:00–3:10 PM, Salon C-8**

**Anthony Shaheen** CSU Los Angeles

### **Isoperimetric Constants of Paley graphs**

In this talk we will give asymptotic bounds for the isoperimetric constant (or expansion constant) of a generalized Paley graph. The isoperimetric constant of a graph is a way to measure how “good” a graph is as a communications network. This is work that I did with undergraduate students at CSULA.

**Eric Landquist** Kutztown University

### **The Generalized Steiner Cable-Trench Problem with Application to Error Correction in Vascular Image Analysis**

The Cable-Trench Problem (CTP) is the problem of connecting buildings on a campus to a building housing the central server so that each building is connected directly to the server via a dedicated underground cable. This problem is modeled by a weighted graph in which the vertices represent buildings and the edges represent the only allowable routes for digging trenches and laying cables between two buildings. Edge weights typically represent distance. A Steiner version of the CTP considers the possibility in which some subset of the buildings is connected to the central server. In this talk, we define the Generalized Steiner CTP (GSCTP), which considers the situation in which, even for the same distance, the cost of digging a trench is more costly for some edges versus others because of soil composition or physical obstacles, for example. The GSCTP has several natural applications, but we will focus on its nontrivial and novel application to the problem of digitally connecting micro-CT scan data of a vascular network and eliminating false-positive results. The CTP and its variants are NP-hard, so determining exact solutions to very large instances of the GSCTP are computationally infeasible. However, we show that straightforward modifications to Prim’s algorithm find very good approximations to exact solutions to the GSCTP efficiently. This solution strategy allows us to fully automate the error-correction process in our application to vascular image analysis.

**Mohammad Javaheri** Siena College, School of Science

### **Cycle Double Covers of Infinite Planar Graphs**

A cycle double cover of a graph is a collection of cycles that cover every edge of the graph exactly twice. It is conjectured that a bridgeless graph always admits a cycle double cover. This conjecture is trivial for a finite planar graph, since the collection of faces gives such a cover. We explore cycle double covers of infinite planar graphs.

**Kirsten Stor** Fort Lewis College

### **Drawing Graphs as Superthrackles**

A *superthackle* is a drawing of a (simple) graph in the plane in which every pair of edges cross exactly once. This is a variant of the previously studied *thackle*, a drawing in which every pair of non-adjacent edges cross exactly once, but adjacent edges do not cross. Our main theorem is that a graph can be drawn as a superthackle in the plane if and only if it is either bipartite and planar or it has a parity embedding on the projective plane. In this talk we will discuss this characterization of superthackable graphs, as well as some interesting corollaries.

**Samuel Iselin** Valparaiso University  
**Hector Reyes-Figueroa** Valparaiso University  
**Zsuzsanna Szaniszló** Valparaiso University

#### **L(4)-Labelings of Trees**

An  $L(4)$ -, or  $L(4, 3, 2, 1)$ -labeling is a simplified model for the radio channel assignment problem. It is a natural generalization of the widely studied  $L(2, 1)$ -labeling. An  $L(4)$ -labeling of a graph  $G$  is a function  $f$  from the vertex set of the graph to the set of non-negative integers, such that for any two vertices,  $x, y$ ,  $d(x, y) + |f(x) - f(y)| > 4$ . The  $L(4)$ -labeling number,  $k(G)$  of  $G$  is the smallest positive integer  $k$  such that  $G$  has an  $L(4)$ -labeling with  $k$  as the maximum label. In this paper we determine the  $L(4)$ -labeling number for trees of small width, subdivisions of stars and spiders. We also present an improved bound on the  $L(4)$ -labeling number of caterpillars.

**Karolyne Fogel** California Lutheran University  
**Aparna Higgins** University of Dayton  
**William Higgins** Wittenberg University  
**John Villalpando** California Lutheran University

#### **Invariants of $L(2, 1)$ -Colorings**

An  $L(2, 1)$ -coloring of a graph is a labeling of the vertices using non-negative integers such that adjacent vertices differ in label by at least 2 and distance two vertices differ in label. The invariant span, denoted by  $\lambda$ , is the smallest integer such that for a given graph there exists an  $L(2, 1)$ -coloring of the graph using only non-negative integers less than or equal to  $\lambda$ . The invariant caps is the least number of color classes required to create an  $L(2, 1)$ -coloring on a given graph. We examine the differences between the span, caps, and chromatic number. In particular we show the value of caps for trees.

**Lauren Keough** Grand Valley State University  
**Darren Parker** Grand Valley State University

#### **Winning the Lights Out Game with the Most Edges**

The original Lights Out Game released by Tiger Electronics was played on a 5x5 grid of buttons where some of the buttons start with their lights on and others with lights off. Pressing a button toggles that button and all of its neighbors (right, left, above, and below). The goal is to turn all of the lights off. This can be generalized to playing the game on a graph. Here pressing a vertex means the vertex and each of its neighbors in the graph toggle. We can further generalize by playing the game with  $r$  colors (rather than two: “on and off”). We ask for the maximum number of edges a graph on  $n$  vertices can have and still be always winnable no matter the number of colors or initial configuration. To answer this question we explore the game’s connection to linear algebra.

**Katie Anders** University of Texas at Tyler  
**Kassie Archer** University of Texas at Tyler

#### **Enumerating Unimodal Rooted Forests Avoiding the Pattern 123**

We say that an unordered rooted labeled forest avoids the pattern  $\pi \in S_n$  if the sequence obtained from the labels along the path from the root to any vertex does not contain a subsequence that is in the same relative order as  $\pi$ . Forests that avoid  $\{312, 213\}$  are unimodal forests. We enumerate the class of unimodal forests avoiding the pattern 123 via a bijection with the set of partitioned cycle decompositions of permutations in  $S_n$ .

**Rob Thompson** Carleton College

#### **How Many Ways Can You Slice a Donut?**

What kinds of cuts can you make on a donut that will separate it into exactly two pieces with no unnecessary cutting? More precisely, what graphs admit an embedding on a torus whose removal results in two components, but removing any subset of the embedding leaves a connected surface? In this talk we’ll discuss this interesting problem at the intersection of graph theory, algebraic topology and junk food.

## **Teaching and Learning Introductory Mathematics**

**Saturday, July 29, 1:00–4:10 PM, Salon C-7**

**Sara Jensen** Carthage College

#### **Teaching and Learning Mathematics with Knitting**

This talk will be centered around a course recently taught by the presenter called *The Mathematics of Knitting*. We will discuss the aims and student learning outcomes of the course as well as some of the techniques used for measuring student success. A significant portion of the talk will be devoted to the various projects and concepts covered in the course (e.g. least common multiples, groups of symmetries, and manifolds). Examples of knitted projects and props will be provided. Finally, we give an example to show that not only can knitting be used to teach mathematics, but knitting can create and inspire new mathematics.

**Erin Moss**     Millersville University of Pennsylvania

### **Mathematics in a Feminist Theory Course**

Mathematics can be used in powerful ways to advance social justice. It can help us interrogate and understand inequities, and then formulate persuasive arguments. Similarly, topics involving social justice can stir up genuine passion and provide an incentive to learn and value mathematics. This spring, I co-taught an upper-class Feminist Theory course, something I never thought a mathematics professor would be given the opportunity to do. We frequently used mathematics to explore issues related to gender inequities and draw conclusions regarding their existence and scope. In this presentation, I will describe several of the ways we used mathematics in the Feminist Theory course. Issues we explored include the cumulative impact of the wage gap, representations of girls and women in children's literature, the "pink tax," and the bodily proportions of dolls frequently given to children. The contexts addressed as a class were motivating to the students, and they were more relaxed about the mathematics they encountered than students typically are during a traditional mathematics course. A unit on social justice would be appropriate to integrate into a course on liberal arts mathematics, or it could function as a stand-alone course itself. Alternatively, the activities could be integrated piecemeal into a developmental or introductory mathematics course as a compelling way to introduce particular mathematics concepts already part of the curriculum.

**Jeremiah Hower**     Florida International University

### **College Algebra Redesign: Shaping Institutional Change**

We will discuss work done over the past five years in reforming some gateway mathematics courses at Florida International University. This will include enhanced pedagogical structures, new student support systems, and the deployment of student learning assistants.

**Claire Gibbons**     Oregon State University

### **Examining the Variation of Mathematical Content Presented During College Algebra Instruction**

College Algebra is a prerequisite for calculus and is thus an important stepping stone in the careers of STEM-intending undergraduates. However, College Algebra has low pass rates across the United States, interrupting students' pathways to success. Furthermore, instructors' presentation of mathematical content has an impact on student achievement. To examine the mathematical content presented by one university's College Algebra instructors, the Mathematical Quality of Instruction (MQI) observation protocol was applied to video clips of direct instruction. Lesson clips were chosen that featured similar examples of identifying asymptotes and end behavior of rational functions. Despite having a common curriculum, wide variation was observed in the mathematical content offered during instruction. This talk will describe the variation observed in the mathematical content and discuss the effectiveness of the MQI protocol in capturing this variation.

**Marianna Bonanome**     New York City College of Technology

### **A Gateway Math Course Re-Imagined and the Faculty Seminar Developed to Support It**

"Opening Gateways to Completion: Open Digital Pedagogies for Student Success in STEM" is a large, cross-institutional collaboration between the mathematics departments at New York City College of Technology and Borough of Manhattan Community College aimed at supporting student success in gateway math courses (supported by the U.S. DOE). We discuss how this project has introduced open-source digital technologies, open educational resources, and active learning strategies into the sequence of high-enrollment mathematics courses required for STEM disciplines at each college. The creation of an intensive seminar for faculty participants offered by the Opening Gateways team in the fall 2016 semester, followed by the implementation of a class structure for a gateway Algebra and Trigonometry course which includes active learning pedagogies, the utilization of WeBWorK (an open-source, online homework system) and OpenLab (City Tech's online platform) is discussed.

**Sarah Wolff**     Denison University

### **Assignments to Help Students Reflect on their Learning**

In a linear algebra class or early proofs class, it is sometimes a challenge to help students recognize that they learn from every part of the class, including the homework assignments. In this talk, I will describe several exercises I have used to help students reflect on what they have learned from homework and problem sessions. I will also discuss how I have encouraged students to read and learn from feedback on their assignments.

**Haley A. Yapple**     Carthage College

### **Incorporating Network Science into a Discrete Structures Course**

In this talk I will share my experience teaching Discrete Structures, as an applied mathematician with research interests in network science. This undergraduate course typically includes an introduction to Graph Theory from the perspective of pure mathematics (properties of simple structures and graph isomorphism) and/or computer science (trees and algorithms for path finding). When teaching this course myself, I refocus this unit from the perspective of applied mathematics: framing graphs as networks in social, technological, or biological settings, introducing empirical graphs and large random networks, and studying measures of node centrality and community structure and their interpretation. I will discuss the benefits and drawbacks of this treatment for undergraduates in mathematics and computer science and hope to start a discussion among attendees about their experiences.

**Ryan Savitz**    Neumann University  
**David DiMarco**    Neumann University  
**Fred Savitz**    Neumann University

### **The Mtile Means: An Instructional Tool for Teaching Introductory Statistics**

This paper discusses a tool that can be used in the instruction of descriptive statistics in introductory statistics courses. This tool is the mtile means. An m-tile mean is the arithmetic mean of data values that break a data set into m-1 equal parts. The mtile means is an interesting way for students to practice the calculation of measures of location and measures of central tendency. It is also a method for engaging students' critical thinking skills while demonstrating the concepts underlying the aforementioned measures. The mtile means also provide an opportunity to utilize technology, such as the mathematical software package Maple, in the classroom.

**Sarah Ann Fleming**    Belmont University

### **Tools for Teaching Logic**

This talk will explore creative problems and puzzles that can be used in teaching symbolic logic. From Lewis Carroll's humorous sorities to questions from the LSAT and other standardized tests, there are many tools available for teaching logic. Students can have fun putting their logic skills to work and see practical applications to learning logic in the process.

**Opeyemi Adekunle Kayode**    University of South Africa

### **A Critical Reflection on Constructivist-Learning Environment in Mathematics at the Further Education and Training Phase, using Six Selected Citadel of Learning in South Africa**

This paper outlines how radical constructivist theory and research has facilitated the development of an authentic learning model in promoting constructivist-learning environment in Mathematics at the F.E.T phase. The paper traces how radical constructivist theory has developed and employed within the South African contexts with special focus on teaching and learning of mathematics. It went further to examine the optimal engendering of more viable models that could help to address existing complexities around the contemporary teaching and learning mathematics at the F.E.T phase of learning in South African Schools. Based on existing evidences from the literature and published journal the paper argued that authentic learning Model in action research promote constructivist learning , as it developed and progresses, supports facilitator to be more effective. The paper concludes with a critical overview of the Implications of radical constructivism as the potential to influence practice. Key Words: constructivist learning theory and Authentic Learning Model.

**Itai Seggev**    Wolfram Research

### **Physical Variables: An Introduction for Mathematicians**

It is common to discuss variables as mere placeholders, waiting to be replaced with values so that computations can occur. Any other symbol would do as just as well, as long they are used consistently. While de-mystifying letters x and y as well as teaching students the proper scope of variables is crucial to their advancement, telling them that variables are mere placeholders flies in the face of how they are used partner disciplines (and, indeed, certain mathematical fields). Physicists and chemists routinely write equations that cannot fit in the placeholder model, for example  $V(x, y) = x^2 + y^2 = r^2 = V(r, \theta)$ . I will describe the conceptual model such formulae, how they can be made mathematically precise, and how we might prepare students to understand them.

**Joseph F. Kolacinski**    Elmira College

### **A Mathematician Looks at American History**

The Quantitative Reasoning requirement at Elmira College is designed for students to develop strategies to solve problems in a structured, logical and analytical way and to apply these strategies to specific questions in specific disciplines. This creates the opportunity for our Mathematics Faculty to reconfigure introductory material in particularly interdisciplinary ways. This talk will address one course addressing this requirement.

# Graduate Student Session

## Great Talks for a General Audience: Coached Presentations by Graduate Students

**Saturday, July 29, 1:00–5:00 PM, Boulevard A**

**Vanessa Rivera-Quinones** University of Illinois at Urbana-Champaign

### Survival of the Fittest: Modeling Adaptation and its Role in Disease Ecology

Have you ever wondered about the meaning of the phrase “survival of the fittest?” To an evolutionary biologist, fitness simply means reproductive success and reflects how well an organism is adapted to its environment. Using Adaptive Dynamics we can describe the long term evolution of a population by considering small mutations. This process can lead to diverse populations which in turn can shape the spread of a disease. It has been shown that highly diverse host populations exhibit lower disease prevalence. However, interactions between host, pathogen and their environment can be complex and lead to unexpected outcomes. Through mathematical models we can study adaptation as an evolutionary process and its relationship to creating and preserving diversity. As a motivating example, we focus on the zooplankter *Daphnia dentifera*, commonly known as “water flea”, which experiences epidemics of the fungal parasite *Metschnikowia bicuspidata*. In this talk, we will derive a mathematical measure of fitness and address the connections between this theory and disease modeling.

**Ann Rogers** DePaul University

### The Jacobian and Dixmier Conjectures

One of the most tantalizing open problems in mathematics is the Jacobian (Keller) Conjecture, a version of which posits that any locally injective polynomial map of two-dimensional complex affine space is globally invertible. Although the conjecture is commonly stated in terms of algebraic geometry, the strategies aimed at proving it have varied widely. These efforts have included approaches from analysis, algebra, combinatorics, and birational geometry, to name a few. We focus on summarizing the history of the algebraic approach; specifically, the precise conditions under which the Jacobian Conjecture can be shown to be a consequence of the generalized form of Dixmier’s conjecture, which claims that in characteristic zero, any endomorphism of the Weyl algebra  $A_n(k)$  must be an automorphism.

**Abdullah Abu-Rqayiq** New Mexico State University

$BIUX^2X_2$

Mathematical books that are written in Arabic in the 19<sup>th</sup> century are scarce. There are several reasons for this including cultural, geopolitical, and cognitive reasons. A mathematical manuscript “Al-Bahjah Al-Saniyah fi Mabda’ Al-Uloulum Al-Riyadiyah” (in English: Fundamentals of Mathematical Sciences), written by Ahmad At-Tabbakh, is one of the rare and significantly important mathematical products of that time. This manuscript can help us understand some math education aspects in Egypt and surrounding regions in the first quarter of the 19<sup>th</sup> century. The manuscript is about 145 pages and each page consists of 17 lines. In my talk I give a brief historical background of the manuscript and analyze the content. I also show how this manuscript serves as a cross-sectional dialogue between the Arabic and the western civilizations in the early 19<sup>th</sup> century.

**Derek J. Sturgill** Ohio University

### Teachers’ Enactment of Classroom Inquiry and The Outcomes of This Enactment

Findings are described from a multiple case study containing 4 Grades 4-6 teachers, their classroom inquiry projects, and their project outcomes. These teachers participated in a yearlong K-8 professional development (PD) program about classroom inquiry (CI), action research (AR), and the teaching of mathematics and science. Details are presented on why they choose their individual projects, how they enacted their projects, and what they learned as a result of their enactment. Data was collected during the program (e.g., meeting notes) and via six interviews, three observations, and several project materials. The first interview focused on participants’ demographics, why they chose their projects, and what they have learned or would like to learn from their projects. Other interviews, which included post-observation interviews, also highlighted project outcomes, but included questions regarding their enactment. At the start of the project, participants were unsure how CI and AR differ, and throughout the program, they experienced challenges enacting their projects. However, from the support of their “critical friends,” they persevered. Common reported outcomes included positive impacts on their pedagogy, knowledge of student learning, and knowledge of CI and AR.

**Kevin Sonnanburg** The University of Tennessee

### **Geometric Analysis and Mean Curvature Flow**

A major focus of geometric analysis is the study of PDEs applied to manifolds or surfaces, which might change their shape as a result. It affords opportunities for both challenging problems in mathematics and a variety applications. You'll get a short description of the required background, to illustrate its many layers, then rely on pictures, because the ideas are very intuitive. The equation I am working with, mean curvature flow (related to the heat equation), has applications in image processing, material science, and cell biology. Finally, I'll show some of my specific a little of my specific problem dealing with when the equation breaks down, and what makes it dynamic and engaging.

## **Saturday, July 29, 1:00–5:00 PM, Boulevard B**

**Peter Lewis** University of Kansas

### **Why Study *Stochastic* Differential Equations?**

It is easy for mathematicians to convince the general public the importance of studying differential equations: “differential equations are mathematical models for many natural phenomena.” While this claim is accurate to a certain extent, these models are an overly idealized interpretation of reality. Nature is inherently messy. Nature is noisy. Nature is *stochastic*. In this sense, stochastic models are more natural descriptions than their non-random counterparts. This is why we study stochastic differential equations. In this talk I will first briefly describe Brownian motion and the noise it induces. This so-called *white noise* is the most commonly used random perturbation of deterministic systems. Next, I will give a basic (Calculus 1) example of an ordinary differential equation which models population growth and show how solutions of this equation behave with and without white noise. Lastly, I will make some remarks about established and ongoing research of models used in the study of traffic flow and fluid mechanics, among other disciplines.

**Harry Richman** University of Michigan

### **Descartes' Rule of Signs and Beyond**

Finding an exact formula for the roots of a polynomial is a hard problem. Descartes found a method for solving an easier problem: bounding the number of positive roots, based on the coefficients. We will discuss how and why this rule works, and how it generalizes to higher dimensions.

**Nancy A. Hernandez** TAMUCT-Department of Mathematics

**Christopher Thron** TAMUCT-Department of Mathematics

### **Application of Probability in the Field of Health Informatics of Probability in the Field of Health Informatics**

Health Informatics is an interdisciplinary field that spans the design, development, adoption and application of information technology (IT) based innovations in healthcare services. Mathematics and statistics play an important role in interpreting the wealth of data. In this talk we describe two important examples of the use of probability and statistics in improving medical treatment. We first explain VLAD charts, which are used in real-time continuous monitoring of performance that is important not only for intervention, but for policy making and program planning. We also talk about conditional probability and its use in deciding between alternate treatments. Both of these materials explained statistical methods that have direct applications in the medical field. This research was conducted as part of a project (supported by the U.S. State Department's Fulbright Program) to promote health informatics in Sudan.

**Arielle Gaudiello** University of Central Florida

### **A Mathematical Model of the Human Papillomavirus (HPV) with a Case Study in Japan**

The human papillomavirus (HPV) is a sexually transmitted infection prominent among young adults in most countries. The disease typically clears in two years naturally, but causes increased risk of development of cancerous cells post-infection. We develop an ordinary differential equation model including vaccination of the female classes and investigate existence and stability of the disease-free equilibrium and endemic equilibrium. We also discuss applications to recent issues arising in Japan, where government-based vaccination programs have terminated and vaccination has drastically decreased.

# PosterFest 2017

## PosterFest 2017

**Friday, July 28, 3:30–5:00 PM, Salon D**

**Rasitha Jayasekare** Butler University

### Learning Statistics Through Applications to Community

The elementary statistics course offered by the department of mathematics and actuarial science is a service course intended to meet the needs of students pursuing studies in many different areas within the college as well as for those who will later pursue postgraduate studies in social and natural sciences or professional programs in medicine. The topics include data analysis, descriptive statistics, linear regression, chi-square tests, analysis of variance, and tests and confidence intervals for means and proportions. In this course, the students usually undertake two group projects to analyze data on various topics. The first project is describing and learning from data and the second is an application of statistical procedures to employ inferential statistics. Last summer, the focus was on community related topics. Data on trash, recycling and waste water treatments from the community were used in the class. The student experience was changed from merely learning statistics to learning how it relates to the world outside the classroom. Students' feedback showed that they obtained a greater awareness of their community. This poster will describe the tools and activities that were implemented for the community based learning approach in statistics.

**Qingxia Li** Fisk University

**Xinyao Yang** Xi'an Jiaotong Liverpool University

### Supremum of a set in a Dedikind Complete Topological Space

The supremum for a set in a multi-dimensional, Dedikind complete topological space is determined. The example is given to illustrate that the condition of Dedilind complete is necessary for the existence of supremum.

**David N. Pham** QCC CUNY

### G-Quasi-Frobenius Lie Algebras

For a finite group  $G$ , V. Turaev showed that crossed  $G$ -algebras (or  $G$ -Frobenius algebras) are the algebraic structures which classify 2-dimensional homotopy quantum field theories where the target space is a  $K(G, 1)$ -space. In this talk, we propose a Lie version of a  $G$ -Frobenius algebra which is motivated by a somewhat recent categorical formulation of  $G$ -Frobenius algebras as commutative Frobenius objects in the braided monoidal category of left  $D(k[G])$ -modules, where  $D(k[G])$  is the Drinfeld double of the group algebra  $k[G]$  with its standard Hopf structure. We call the aforementioned structures  $\mathfrak{g}$ -quasi-Frobenius Lie algebras (for  $\mathfrak{g}$  a finite dimensional Lie algebra). We discuss the geometry of these structures and give some examples.

**Timmy Ma** University of California, Irvine

### Admission Predictors for PhD Success: Preliminary Results

There are many factors that can be collected on students in a mathematics PhD program: bachelor's GPA, bachelor's major, GRE scores, gender, URM status, institution tier, etc. We are in the process of analyzing and studying data collected from students entering a mathematics program at a southern California institution over the course of 9 years. In this poster we will present demographic and academic data as well as some preliminary results of our study, in which we seek to identify which factors, if any, significantly contribute to the probability of a student succeeding in obtaining a PhD through the mathematics program.

**Amanda H. Matson** Clarke University

### Journey to Tenure

At one year post-tenure and promotion, it is time to share this mathematician's journey to tenure with those who are starting out. It involves balancing involvement on campus, regionally, and nationally.

**Jeffrey C. Lagarias** University of Michigan  
**Takumi Murayama** University of Michigan  
**Harry Richman** University of Michigan

### Composing and commuting dilated floor functions

The floor function  $\lfloor x \rfloor$  is a simple and easy-to-understand function on the real line. When the floor function is combined with arbitrary polynomials through iterative compositions, the resulting behavior can be erratic and unpredictable. In this work we focus on studying in detail one simple case: the dilated floor function  $f_\alpha(x) = \lfloor \alpha x \rfloor$  and compositions thereof. We classify all real parameters  $(\alpha, \beta)$  such that the dilated floor functions  $f_\alpha(x)$  and  $f_\beta(x)$  have non-negative commutator, i.e.

$$[f_\alpha, f_\beta](x) = \lfloor \alpha \lfloor \beta x \rfloor \rfloor - \lfloor \beta \lfloor \alpha x \rfloor \rfloor \geq 0$$

for all real  $x$ . The problem is connected with the theory of Beatty sequences and with the Diophantine Frobenius problem in two dimensions.

**Ron Buckmire** National Science Foundation

### Programs and Funding Opportunities at the National Science Foundation

The Division of Undergraduate Education at the National Science Foundation has a number of programs and funding opportunities to support the improvement of the teaching and learning of undergraduate mathematics. This poster will summarize some of the opportunities that are most applicable to early-career mathematics faculty. These include the Improving Undergraduate STEM Education (IUSE), Scholarships in STEM (S-STEM), Research Experiences for Undergraduates (REU), Faculty Early Career Development (CAREER) and more. Other opportunities at NSF include serving as a reviewer for proposals and working as a program officer for a limited term (1-3 years).

**Axel Brandt** Davidson College

### Assessing Attitudes in K-12 Math Enrichment: A Preliminary Report

The aim of this study is to evaluate the relative impact of afternoon mathematics enrichment activities on mathematics-related attitudes of middle school students in a 6-week summer program. This poster will discuss the program, enrichment activities, methods, and preliminary observations.

**Ann E. Rogers** DePaul University

### The Jacobian and Dixmier Conjectures

One of the most tantalizing open problems in mathematics is the Jacobian (Keller) Conjecture, a version of which posits that any locally injective polynomial map of two-dimensional complex affine space is globally invertible. Although the conjecture is commonly stated in terms of algebraic geometry, the strategies aimed at proving it have varied widely. These efforts have included approaches from analysis, algebra, combinatorics, and birational geometry, to name a few. We focus on summarizing the history of the algebraic approach; specifically, the precise conditions under which the Jacobian Conjecture can be shown to be a consequence of the generalized form of Dixmier's conjecture, which claims that in characteristic zero, any endomorphism of the Weyl algebra  $A_n(k)$  must be an automorphism.

**Nancy A. Hernandez** TAMUCT-Department of Mathematics  
**Christopher Thron** TAMUCT-Department of Mathematics

### Development of Health Informatics at Alzaeim Alazhari University in Khartoum, Sudan

Health Informatics is an interdisciplinary field that spans the design, development, adoption and application of information technology (IT) based innovations in healthcare services. Mathematics and statistics play an important role in interpreting the wealth of data. In this poster, we describe some aspects of a project (supported by the U.S. State Department's Fulbright Program) to promote health informatics in Sudan. In addition to being a developing country, Sudan's progress has been slowed due to economic and technological sanctions imposed by the U.S. in 1993. We describe our activities in improving the status of health informatics at the medical school of Alzaeim Alazhari University in Khartoum, Sudan. In particular, we describe curriculum development (including improved mathematics and statistics content), as well as implementation of learning management and electronic medical records systems.

**Amanda E. Redlich** Bowdoin College

### Grocery Shopping and Cookie Recipes: Making Good Random Choices

Often we need to make decisions quickly, with limited information. Which of 20 supermarket checkout lines to choose? Which of 20,000 internet cookie recipes to bake? One way to solve this problem is to use the "power of two choices", a way to incorporate randomness as part of the decision-making process. This controlled randomness is effective in many applications, from grocery shopping to server farms. Here, two different processes using this idea are presented with mathematical analysis and some natural real-world examples.



**Ben Hartlage** US Air Force Academy  
**David Higginbotham** US Air Force Academy  
**Ian Pierce** US Air Force Academy  
**Beth Schaubroeck** US Air Force Academy

### **Grit and Diligence as Predictors of Success**

All students at the United States Air Force Academy (USAFA) must complete a two-course sequence in calculus as a graduation requirement; for many students this is a significant hurdle that they must clear at the beginning of their course of study. Previous investigations at USAFA examined what factors predict success in our lowest-level mathematics course, Math 130 (Algebra and Trigonometry) and the follow-on mathematics course, Math 141 (Calculus I). Academic measures such as scores on the mathematics placement test and incoming ACT/SAT scores did not do well at predicting success in either Math 130 or Math 141. Anecdotal evidence indicated that student success was much more determined by non-academic factors. We are currently investigating the contributions of the personality traits of Grit and Diligence (Duckworth et al, 2007; Galla et al, 2014) to academic success. Are either Grit or Diligence useful predictors for student retention at USAFA (do the students complete their first, second, and fourth semesters at USAFA)? Are Grit and Diligence useful predictors for student success in mathematics courses and other courses that build upon these mathematics courses?

**Meredith G. Anderson** Adams State University

### **Euclid Abandoned**

As is widely known throughout the mathematical community and beyond, Euclid served as the primary geometry text in universities for generations before falling out of use in the 18th and 19th centuries, and at times was the sole such text. This decline in the use of Euclid was fairly rapid, which seems perplexing given the longevity of its use as an educational text. Common reasons cited for this decline are the discovery of non-Euclidean geometries alongside the advent of the calculus accompanying the Industrial Revolution. With this came an increased focus on the practical usefulness of geometry over the rigor of an axiomatic study. I think there is yet more substance undergirding this trend. The difficulty in accurately illustrating this decline is the number of vast educational and social changes occurring at the time. Changes in the rate and composition of publications, and in teaching methods and the structure of schools all make it challenging to collect and analyze representative sources. I hope to assemble a corpus of texts to bound the time span during which Euclid fell out of favor as well as find citations indicating the precise reasons. The inhomogeneous nature of these texts will require a detailed analysis in order to determine both the endpoints and length of period of decline.

**Na Yu** Lawrence Technological University

### **Mathematical Modeling of Electroreception of Weakly Electric Fish**

Similar to the five senses humans and most other vertebrates experience, the weakly electric fish use electroreception to perceive their environment (like radar) and to communicate with other weakly electric fish. We will see how mathematical modeling can discover the electroreception based on experimental recordings.

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