Robert Murphy: Mathematician and Physicist

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Abstract

Despite his short life, Robert Murphy (1806-1843) was a mathematician and physicist of "true genius," according to Augustus De Morgan [Venn 2009]. Murphy's research can be categorized into three areas: Algebraic Equations, Integral Equations, and Operator Calculus [Allaire 2002]. The majority of scholarship on Murphy is centered around his contributions to physics; however, historians such as Petrova [1978] and Bradley and Allaire [2002] have explored Murphy's linear operator theory. In the paper "Robert Murphy: Mathematician and Physicist," we provide a unified exposition in which we synthesize and expand on existing accounts of Murphy's life and mathematical contributions. Additionally, we give an overview of his mathematical papers and accomplishments in hopes of inspiring historians to examine and analyze his original works. This biography can be found in the online journal *Convergence,* available at the website of the Mathematical Association of America (www.maa.org).

As noted in our biography, all but one of Murphy's papers are available for download from Google Books or the Journal Storage Database (JSTOR). Murphy's first paper, *Refutation of a Pamphlet Written by the Rev. John Mackey Entitled "A Method of Making a Cube a Double of a Cube, Founded on the Principles of Elementary Geometry," wherein His Principles Are Proved Erroneous and the Required Solution Not Yet Obtained* [1824], has not been available in the United States. As a result, the authors here provide a transcript of the paper with commentary.

Appendix

Refutation of a Pamphlet Written by the Rev. John Mackey Entitled "A Method of Making a Cube a Double of a Cube, Founded on the Principles of Elementary Geometry," wherein His Principles Are Proved Erroneous and the Required Solution Not Yet Obtained.[1]

By Robert Murphy.

PREFACE

[iii][2] THERE are, in particular, three problems, which for many ages have exercised the mathematical world, and all attempts hitherto made for their solutions have been unsuccessful, – namely, the quadrature of the circle, the trisection of an angle, and the duplication of the cube. The quadrature, indeed, has been obtained to a degree of accuracy, far greater than what is necessary for all practical purposes; for Archimedes[3] having pointed out the path, Viète[4] and others have extended the numbers expressing the ratio of the diameter of a circle to its circumference, to a very great extent, and M. de Lagny[5] continued them to 127 places of Decimals,[6] but no solution strictly geometrical has ever yet been obtained.

If the equation expressing the length of the right line[7] which trisects the vertical angle of an isosceles triangle, in terms of the sides, be reduced to Cardano's[8] form, the value of the unknown quantity, though real, will be expressed by the sum of two surds[9] involving imaginary quantities. Yet this is the only case of cubics in which all the roots are real, as it manifest from the version of series, the expansion of a binomial, or the development [iv] of many algebraic expressions. See Wood's[10] and Bonnycastle's Algebra.[11]

Likewise, the ratio of the diameter of a circle to its circumference may be expressed by a series, as has been shewn by John Bernoulli[12] and the learned

[1]Published in Mallow, Ireland, by John Haynes, Printer, Spa-Walk, 1824.
[2]Numbers in square brackets represent the original page numbers of the article.
[3]Archimedes of Syracuse (ca. 287 BCE-212 BCE).
[4]François Viète (1540-1603). In [Murphy 1824] the spelling was given as "Vieta," which is the Latin version of his name.
[5]Thomas Fantet de Lagny (1660-1734).
[6]There does not appear to be a consensus on what the exact amount actually was, sources give anywhere from 112-127 decimal places.
[7]In other words, a straight line.
[8]Girolamo Cardano (1501-1576). In [Murphy 1824] the spelling was given as "Cardan," which is the English version of his name.
[9]This is a Latin word which means a root that is irrational. The origin of this word can be traced back to Al-Khowârizmî.
[10]See [Wood 1830].
[11]John Bonnycastle (1751-1821). See [Bonnycastle 1813].
[12]Johann Bernoulli (1667-1748).

Euler.[13] This also has been expressed by Dr. Brouncker[14] by a continued fraction.

The solution of the duplication, being dependent on the finding two mean proportionals, has also been attempted by various persons but without success. For some solutions, see Whiston's edition of Tacquet's Euclid,[15] and notes to Elrington's Euclid.[16]

This question has also been stiled the Delian problem from the following circumstance. When a Plague raged at Athens the citizens applied to the oracle of Apollo at Delphos, and the God assured them that when they would double an altar which was of a cubical form, still retaining that form, the plague would cease. Hereupon, the artizans thought they had occasion only to double all its sides but this rendered the altar eight times, instead of double the former. Wherefore they applied to the famous geometers of the age, whose names will be given hereafter.

But amongst all the attempts which have been made for the solution of the duplication, there has not been one more foolish or more erroneous, than [v] that of the Rev. John Mackey;[17] which being masked under the appearance of truth, consists of a collection of false propositions. The crime of deception has been aggravated by the pretensions which the REVEREND GENTLEMAN has made to the direction of Providence – afforded to such an 'humble individual' in his researches after 'unknown truths'.

To prevent despondency on the subject to the trisection, we are encouraged by Mr. Mackey's assuring us, that it can be obtained from his principles – the WELL-FOUNDED principles of the greatest series of triangles.

Mr. Mackey's Pamphlet, (a compound of false-hoods) has obtained the sanction of Maynooth College!!![18] We must either form a very low idea of the advancement of that seminary in mathematical learning, or be astonished at its connection in the enormity of this deception.

To prevent the continuance of this public imposition is the intention of this tract, which only intreats an impartial examination, though it is the first address of the author to the public.

R. M.

Mallow, October, 1824.

[13]Leonhard Euler (1707-1783).

[14]Lord William Brouncker (1620-1684). In [Murphy 1824], the name "Dr. Brounkley" was given. Long [1846] states that Murphy had a confusion between Dr. Brinkley and Lord Brounker.

[15]See [Whiston 1791].

[16]See [Elrington 1822].

[17]There appears to be no bibliographic information available on John Mackey; however, according to Barry [1999], Mackey was ordained in 1813 while at St. Patrick's College Maynooth. Interestingly, the archivist at Maynooth College claims to have no records on Mackey. What we do know is that there is a copy of his original paper located within the National Library of Ireland.

[18]This college was originally established as the Royal College of St. Patrick. It currently serves as the National Seminary for Ireland.

Refutation, &c.

[7] 1. The writer of the Pamphlet, which proposes to solve the duplication of the cube, speaks thus in his preface

> "Several Geometers have been of opinion that the solution of this problem and that of the Trisection are impossible by means of straight lines, and the circle. How they could arrive at such a conclusion, I am not able to say, unless they considered the work to surpass human ingenuity and talent, as the Solutions have not been discovered by any of the great men, &c".

Now as to the solution of the duplication, Mr. MACKEY'S success will be seen from the sequel. And as to the Trisection, the supposition that it cannot be solved by Euclidean Geometry, contained in the first six books is not unreasonable, since it appears by an Algebraic investigation, that the resulting cubic equation falls under Cardano's irreducible case.[19]

2.[20] Mr. Mackey proceeds to tell us that we have mechanical methods of solving these problems by means of straight lines, and the circle, and these solutions never could be obtained if Geometrical Solutions were impossible. He must here understand Geometry, either in the extensive signification of the doctrine of lines,[21] or in the confined sense of Euclidean Geometry. If in the former, we would remark that the Geometrical Solution is incontrovertibly [8] possible, since the actual solution of the trisection has been obtained by means of curve lines, amongst which may be reckoned my solution by means of the parabola, besides an elegant solution which I have obtained from the Quatuor Nodi,[22] a curve easily generated from the circle, by means of which the duplication also may be solved. But if he refers to Euclidean Geometry (as is more probable) let him be assured that his sacred assertion would not be received without demonstration.

3. The writer, then proceeding to show 'the untrodden path' which leads 'to the knowledge of unknown truths,' lays down three definitions, and an axiom. And proceeds to his first proposition, which theorem though undoubtedly true, Mr. Mackey endeavors to demonstrate by the quotation of a proposition utterly inapplicable.

4. Let particular attention be paid to his next proposition, See figure 1, in Mr. Mackey's pamphlet,

> 'if neither the segments (BD, CE) nor the parts (BF, FD, CH, HE,) into which they are divided be proportional to the sides of a

[19]This is when the discriminant of a cubic is imaginary, which indicates that all the roots are real and distinct.

[20]In [Murphy 1824] the article number, "2.," was missing. We have inserted it for clarity.

[21]The doctrine of lines refers to the portion of elementary geometry which is devoted to straight lines, i.e., one dimensional geometry.

[22]See Murphy's [1824] Note on Article the Second.

> triangle (ADE,) the straight lines (BC, FH) which join the corresponding points of section, will not be parallel to the base DE, of the given triangle or to one another.'

In order to unravel the ambiguity of this enunciation we must have recourse to the demonstration in which he asserts that

> 'BF is not to CH as AD, to AE, (Ex-hypothesi,) therefore the hypothesis is that BD is not to CE, as AD to AE, that BF is not to CH, as AD to AE and that FD is not to HE, as AD to AE.'

He proposes to prove that of the right lines BC, FH, and DH, there are no two parallel.

[9] I intend, in the first place, to prove that this proposition is imperfectly demonstrated, and secondly that it is palpably false.

In the demonstration he says

> 'if BC, FH were parallel then BF would be to CH, as AB is to AC, but BF is not to CH as AB is to AC, because BF is not to CH as AD is to AE (Ex-hypothesi) and therefore BF is not to CH, as AB is to AC.'

For the support of which conclusion he quotes his Fifth Article, 'O full of all subtlety.' But what does his Fifth Article say?

> 'If the segments of the sides of a triangle be not proportional to the sides, they will not be proportional to the remaining parts of the sides.'

But what triangle does he refer to? Evidently, to the triangle ADE, since he says BF is not to CH as AD to AE. Therefore all the force of article Fifth, amounts to this: that BF is not to CH, as the sum of AB and FD is to the sum of AC and HE. Therefore his conclusion is unwarranted, and his demonstration is imperfect.

(See Fig. 1.) I next propose to prove that his proposition is false. For let ABC be any triangle, and in AB take any point L, draw LC. And in AC take any points F and H through which draw FE and GH parallel to LC, and HK parallel to AB. Produce GH to D. Now in the triangle ALC, since EF and GH are drawn parallel to the base LC, they divide the sides proportionally ([Elrington 1822, p. 121, Corollary to Euclid's Proposition 2, Book 6]). And since as one antecedent it to its consequents, so are all the antecedents to all the consequents ([Elrington 1822, p. 107, Euclid's Proposition 22, Book 5]). Hence, FH is to EG as AC is to AL, But AC has to AL, a greater ratio than it has to AB ([Elrington 1822, p. 105, Euclid's Proposition 16, Book 5]), therefore FH has to EG, a greater [10] ratio than AC has to AB. Again, since GB has to HC the ratio compounded of the ratios of GB to HK, and HK to HC, that is by similar triangles of GD to HD, and of AB to AC. But since GD is greater than HD, the ratio of GB to HC is greater than the ratio of AB to AC. In like manner, by producing EF it may be shown that EB has to FC a greater ratio than the ratio of AB to AC. Wherefore, if Mr. Mackey's proposition were true, EF and GH are not parallel to one another, but[23] since by construction they are both parallel to LC, they are parallel to one another ([Elrington 1822, p. 20, Euclid's Proposition 30, Book 1]. Wherefore, Mr. Mackey's proposition is absurd.

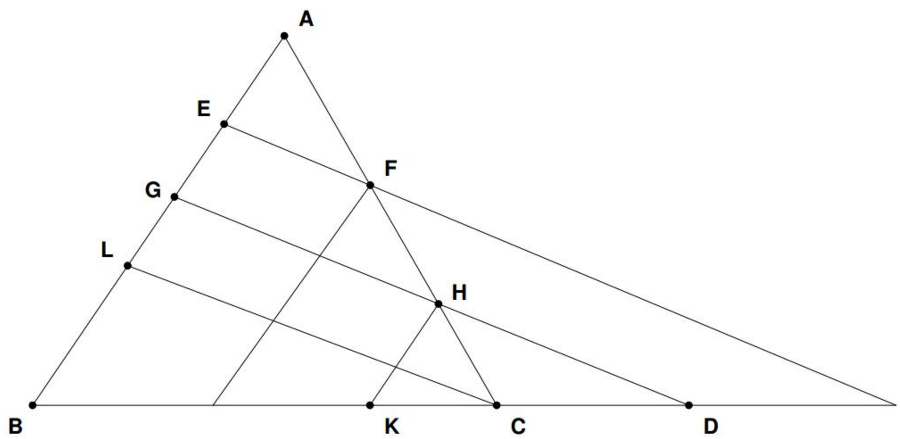

Figure 1: Murphy's Fig. 1.

5. Therefore, his Ninth Article requires demonstration, since it is supported by an untrue theorem. Now, his Tenth Article is founded on this Ninth and the Eleventh, on the Tenth, &c. Therefore, his cobweb structure falls to the ground.

The learned Divine assures us in the conclusion of his preface

> 'that the trisection can be had by means of the principles contained in this work.'

Surely we must admire his sagacity, who through the mist of error can trace away to unknown regions.

He tells us also that his method of solution contains the principle upon which, in his opinion, Plato's[24] Mechanical Solution[25] was constructed. However he appears to be mistaken, for Plato's was founded on truth.

6.[26] But not to detain the reader on this part of the subject, we proceed to show that he has not solved the required problem (See the Seventh Figure in Mr. Mackey's Pamphlet.). In order to do which, [11] we will show algebraically

[23] In [Murphy 1824] the word "but" was repeated twice.

[24] Plato (ca. 427 BCE-ca. 347 BCE).

[25] This is in reference to Plato's solution of the problem of doubling the cube by means of Plato's machine; however, it seems unlikely that Plato would have given a mechanical solution [O'Connor and Robertson 1999].

[26] In [Murphy 1824] the article number, "6.," was missing. We have inserted it for clarity.

that in his solution of finding two mean proportionals, the two given right lines are connected by a certain law. And, therefore, his solution extends only to a particular case and is not general.

7. Let the two given right lines AB and BR be represented by a and b, respectively. Now if four quantities be in continual proportion, the common ratio is equal to the cube root of the quotient arising by dividing the last term by the first.

Demonstration.

Let m be the first term in any geometrical progression, y the common ratio, and n the last term, therefore the progression is m, my, my^2, my^3 when the progression consists of four terms. Therefore $my^3 = n$, wherefore $y^3 = \frac{n}{m}$ whence we have $y = \frac{n}{m}\Big|^{\frac{1}{3}}$.[27]

8. Now since a and b in this progression are the extremes, the common ratio is $\frac{b}{a}\Big|^{\frac{1}{3}}$. Therefore the means are $a \times \frac{b}{a}\Big|^{\frac{1}{3}}$ and[28] $a \times \frac{b}{a}\Big|^{\frac{2}{3}}$, and the progression is a, $a \times \frac{b}{a}\Big|^{\frac{1}{3}}$, $a \times \frac{b}{a}\Big|^{\frac{2}{3}}$, b, or a, $\overline{a^2b}\Big|^{\frac{1}{3}}$, $\overline{ab^2}\Big|^{\frac{1}{3}}$, b.[29] Therefore [12] $BD = \overline{a^2b}\Big|^{\frac{1}{3}}$ and $BL = \overline{ab^2}\Big|^{\frac{1}{3}}$. Hence $BM = b$ [see 17th article in Mr. Mackey's pamphlet] and $BE = \overline{ab}\big|^{\frac{1}{2}}$ and since $b^2 = BS \times \overline{ab}\big|^{\frac{1}{2}}$ therefore $BS = \frac{b^3}{a}\Big|^{\frac{1}{2}}$. Again since $BK = BE = \overline{ab}\big|^{\frac{1}{2}}$, therefore $BC = \overline{a^3b}\Big|^{\frac{1}{4}}$ or $\overline{\overline{ab}\big|^{\frac{1}{2}} \times a}\Big|^{\frac{1}{2}}$, but BC is to BK as BK is to BQ, therefore $BQ = \frac{ab}{\overline{a^3b}\big|^{\frac{1}{4}}} = \overline{\frac{a^4b^4}{a^3b}}\Big|^{\frac{1}{4}} = \overline{ab^3}\Big|^{\frac{1}{4}}$. Hence the three ranks of the proportionals are[30]

$$\begin{array}{lllllllll}
AB, & BE, & BM, & BS, & \text{or} & a, & \overline{ab}\big|^{\frac{1}{2}}, & b, & \frac{b^3}{a}\Big|^{\frac{1}{2}}; \\
AB, & BD, & BL, & BR, & \text{or} & a, & \overline{a^2b}\Big|^{\frac{1}{3}}, & \overline{ab^2}\Big|^{\frac{1}{3}}, & b; \\
AB, & BC, & BK, & BQ, & \text{or} & a, & \overline{a^3b}\Big|^{\frac{1}{4}}, & \overline{ab}\big|^{\frac{1}{2}}, & \overline{ab^3}\Big|^{\frac{1}{4}}.
\end{array}$$

Now, it is evident from the 22nd article in Mr. Mackey's pamphlet, that his solution extends only to those cases in which QS is to QR as KM is to KL. That is, wherein $QR \times KM = QS \times KL$ but QR is the excess of BR above BQ, therefore $QR = b - \overline{ab^3}\Big|^{\frac{1}{4}}$ in like manner. $KM = b - \overline{ab}\big|^{\frac{1}{2}}$, $QS =$

[27] In [Murphy 1824] the notation $\overline{x}\big|^{\frac{1}{n}}$ is used to denote $\sqrt[n]{x}$.
[28] In [Murphy 1824] the word "and" was repeated twice.
[29] In [Murphy 1824] the comma between the terms $\overline{ab^2}\Big|^{\frac{1}{3}}$ and b was omitted.
[30] In [Murphy 1824] majority of the commas were omitted in the following proportions.

$\left.\overline{\tfrac{b^3}{a}}\right|^{\frac{1}{2}} - \left.\overline{ab^3}\right|^{\frac{1}{4}}$, and [13] $KL = \left.\overline{ab^2}\right|^{\frac{1}{3}} - \left.\overline{ab}\right|^{\frac{1}{2}}$ wherefore[31] $\overline{b - \left.\overline{ab^3}\right|^{\frac{1}{4}}} \times \overline{b - \left.\overline{ab}\right|^{\frac{1}{2}}} = \overline{\left.\overline{\tfrac{b^3}{a}}\right|^{\frac{1}{2}} - \left.\overline{ab^3}\right|^{\frac{1}{4}}} \times \overline{\left.\overline{ab^2}\right|^{\frac{1}{3}} - \left.\overline{ab}\right|^{\frac{1}{2}}}$, that is, by actual multiplication, transposition, and reduction $2 - \left.\overline{\tfrac{a}{b}}\right|^{\frac{1}{4}} - \left.\overline{\tfrac{a}{b}}\right|^{\frac{1}{2}} = \left.\overline{\tfrac{b}{a}}\right|^{\frac{1}{6}} - \left.\overline{\tfrac{a^7}{b^7}}\right|^{\frac{1}{12}}$, which being a property extending only to certain cases and values of a and b. We conclude that unless a and b have this relative value his solution fails.[32]

9. It must be next proved that the case requisite for the duplication of the cube is not one of those cases denoted by the equation $2 - \left.\overline{\tfrac{a}{b}}\right|^{\frac{1}{4}} - \left.\overline{\tfrac{a}{b}}\right|^{\frac{1}{2}} = \left.\overline{\tfrac{b}{a}}\right|^{\frac{1}{6}} - \left.\overline{\tfrac{a}{b}}\right|^{\frac{7}{12}}$. For in the duplication, $b = 2a$. Hence by substitution and reduction $2 - \left.\overline{\tfrac{1}{2}}\right|^{\frac{1}{4}} - \left.\overline{\tfrac{1}{2}}\right|^{\frac{1}{2}} = \left.\overline{2}\right|^{\frac{1}{6}} - \left.\overline{\tfrac{1}{128}}\right|^{\frac{1}{12}}$ if in this present case a and b had the proper relative values.[33] But this equation is not true as is evident from the logarithmic operation.

10.[34] Perhaps Mr. Mackey, having seen that this equation is nearly true, many have thought that this solution could be more easily imposed on the public. Because if the solution which he gives were tried [14] experimentally, the error would be imputed rather to the inaccuracy of the experiment, than to the fallacy of his doctrine.

Since the duplication of the cube is dependent on the finding two mean proportionals between two given right lines, we will pay particular attention to this problem (See figure 7, in Mr. Mackey's Pamphlet.).

11. Let $AB = a$ and $BM = b$. Hence $BE = \left.\overline{ab}\right|^{\frac{1}{2}} = BK$ and $BC = \left.\overline{a \times \left.\overline{ab}\right|^{\frac{1}{2}}}\right|^{\frac{1}{2}}$ therefore $CE = \left.\overline{ab}\right|^{\frac{1}{2}} - \left.\overline{a \times \left.\overline{ab}\right|^{\frac{1}{2}}}\right|^{\frac{1}{2}}$ and $CD = \overline{\left.\overline{ab}\right|^{\frac{1}{2}} - \left.\overline{a \times \left.\overline{ab}\right|^{\frac{1}{2}}}\right|^{\frac{1}{2}}} \times \frac{m}{n}$. If CE be to CD as 'n' to 'm' and as KM to KL , also $KM = b - \left.\overline{ab}\right|^{\frac{1}{2}}$ therefore $KL = b - \left.\overline{ab}\right|^{\frac{1}{2}} \times \frac{m}{n}$. Hence[35]

$$BD = \frac{\left.\overline{\overline{n-m} \times \overline{a \times \left.\overline{ab}\right|^{\frac{1}{2}}}}\right|^{\frac{1}{2}} + m \times \left.\overline{ab}\right|^{\frac{1}{2}}}{n}.$$

[31] In [Murphy 1824] the overline is used for grouping terms, much as modern mathematicians use parentheses. However, Murphy never uses parentheses for this purpose. Additionally, in [Murphy 1824] the overlines for the next two groups terms, $\overline{b - \left.\overline{ab^3}\right|^{\frac{1}{4}}}$ and $\overline{b - \left.\overline{ab}\right|^{\frac{1}{2}}}$, were omitted.

[32] See Murphy's [1824] Note on Article the Eighth.

[33] In [Murphy 1824] the minus sign between the terms $\left.\overline{2}\right|^{\frac{1}{6}}$ and $\left.\overline{\tfrac{1}{128}}\right|^{\frac{1}{12}}$ was omitted.

[34] In [Murphy 1824] the article number, "10.," was missing. We have inserted it for clarity.

[35] In [Murphy 1824] the outer most overline for the first grouped term in the numerator of BD, $\left.\overline{\overline{n-m} \times \overline{a \times \left.\overline{ab}\right|^{\frac{1}{2}}}}\right|^{\frac{1}{2}}$, was omitted. However, Murphy inserts it when he uses the equation later.

Likewise,[36] $BL = \frac{\overline{n-m}\times \overline{ab}\big|^{\frac{1}{2}}+mb}{n}$. Now if AB were to BD as BD to BL, (article 22 in Mr. Mackey's pamphlet) hence [15]

$$a : \frac{\overline{\overline{n-m}\times\overline{a\times\overline{ab}\big|^{\frac{1}{2}}}}\Big|^{\frac{1}{2}}+\overline{m\times\overline{ab}\big|^{\frac{1}{2}}}}{n} :: \frac{\overline{\overline{n-m}\times\overline{a\times\overline{ab}\big|^{\frac{1}{2}}}}\Big|^{\frac{1}{2}}+\overline{m\times\overline{ab}\big|^{\frac{1}{2}}}}{n} : BL = \frac{\overline{n-m}\times\overline{ab}\big|^{\frac{1}{2}}+mb}{n}.$$

Therefore $\overline{n-m}\times\overline{ab}\big|^{\frac{1}{2}}+mb = \frac{\overline{n-m}\big|^{\frac{1}{2}}\times\overline{ab}\big|^{\frac{1}{2}}+m^2b}{n}+\frac{2m\times\overline{n-m}\times\overline{b\times\overline{ab}\big|^{\frac{1}{2}}}\Big|^{\frac{1}{2}}}{n}$.[37] Consequently, $\overline{m^2-mn}\times\overline{ab}\big|^{\frac{1}{2}}+m^2b-mnb+2m\times\overline{n-m}\times\overline{b\times\overline{ab}\big|^{\frac{1}{2}}}\Big|^{\frac{1}{2}}=0$, and by reduction $\overline{ab}\big|^{\frac{1}{2}}+b-2\times\overline{b\times\overline{ab}\big|^{\frac{1}{2}}}\Big|^{\frac{1}{2}}=0$, whence by transposition and squaring $ab+b^2+2b\times\overline{ab}\big|^{\frac{1}{2}}=4b\times\overline{ab}\big|^{\frac{1}{2}}$. Therefore $\overline{ab}\big|^{\frac{1}{2}}=b$, hence $a=b$, which therefore is the only case to which his solution extends.

So after a great number of preparatory propositions and corollaries &c. &c., he shews a method of finding a mean proportional between two right lines which (as I have proved) must be equal if his solution be true. *Wonderful Discovery!!* What an honour to our age!! But the solution of the duplication requires one line to be double of the other, wherefore Mr. Mackey's attempts is fruitless.

12. In conclusion, we shall endeavor to investigate the source of fundamental error in Mr. Mackey's [16] pamphlet, and it seems to arise from his doctrine of the greatest series of triangles dissimilar to each other, and to a given triangle. And which have the greater angles at their bases on the same side, with the greater angle at the base in the given triangle, and his argument is to this effect: that if the segments of the sides of a triangle as described in his eleventh article be subdivided into proportional parts of an indefinite number, and the corresponding points of division be connected, they will constitute a series of triangles dissimilar to each other, and to the given triangle. And therefore concludes that if a series of triangles be formed which have their greater angles at the base on the same side with that in the original triangle, and are dissimilar to it, they are produced from the same genesis as is evident from his twentieth and twenty first articles.

13. Now this conclusion is unjust, and the thirteenth article untrue. See Fig. 2. For let ADE be any triangle which has the angle at D greater than the angle at E. Produce all the three sides of the triangle, in DE produced, take any point, C make DP equal to EC, draw CR parallel to DE, the point R is between P and D. Since AE is to AD, as EC that is DP is to DR, but since AE is greater than AD ([Elrington 1822, p. 13, Euclid's Proposition 19, Book 1]) therefore DP is greater than DR, and R falls between D and P. In the right line RD between the points R and D take any point B. Join

[36]In [Murphy 1824] the overline for the first grouped term in the numerator of BL, $\overline{n-m}$, was omitted. However, Murphy inserts it when he uses the equation later.

[37]See Murphy's [1824] Note on Article the Eleventh.

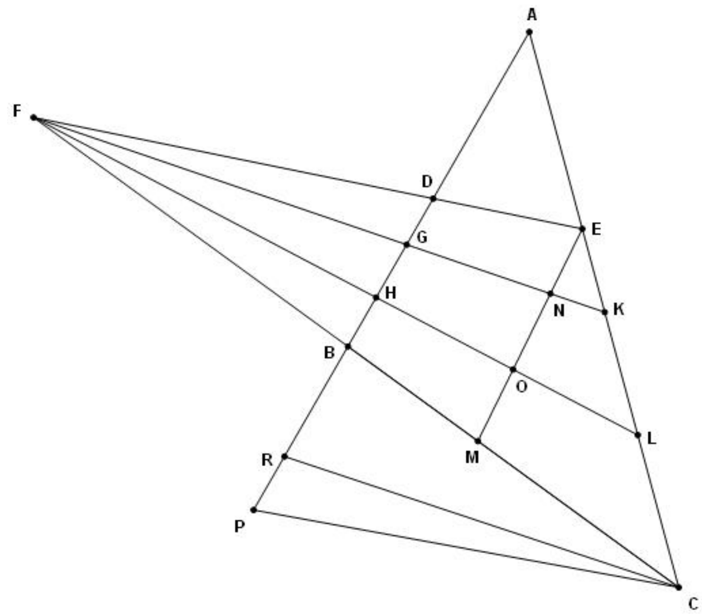

Figure 2: Murphy's Fig. 2.

CB and produce it, and since the angles DEC and RCE taken together are equal to two right angles, ([Elrington 1822, p. 19, Euclid's Proposition 29, Book 1].) therefore CB when produced will cut ED produced ([Elrington 1822, p. 3, Euclid's Axiom 12, Book 1],). [17] Let them meet in F and in DB, take any number of points G and H through which from F draw FG and FH. These evidently when produced will meet CE in as many points K and L. Through E draw EM parallel to DB. Now since the angle ADE is greater than the angle AED by hypothesis, therefore the angle FDB is greater than the angle AED ([Elrington 1822, p. 11, Euclid's Proposition 15, Book 1]). But the angles DGK, DHL, and DBC are each greater than the angle FDG and consequently greater than the angle AED. But the angle AED is greater than each of the angles EKG, ELH, and ECB, ([Elrington 1822, p. 12, Euclid's Proposition 16, Book 1],) much more then are the angles, AGK, ALH, and ACB respectively (that is when the greater number of points taken in DB). Right lines are drawn from several points of the segment, DB (as G and H) of the side (AB) of a triangle (ABC) to as many points (K and L) in the other segment (EC) which is the greater segment and a part of the greater side (AC) ([Elrington 1822, p. 13, Euclid's Proposition 19, Book 1],) of the triangle ABC. And these straight lines become the base in the greatest series of triangles which are neither similar to one another, or to the triangle ABC and which have the greater angle at the base in each triangle on the same side with the greater angle, (ABC) at the base in the triangle ABC. Now if Mr. Mackey's 13th Article were true these straight lines (DE, GK, and HL) would divide the segments (DB and CE) into proportional parts but they do not. For since FD is to DG as FE is to EN (by similar triangles) and for the same reason FD is to DB as EF to EM. Therefore DG is to DB as EN to EM in the like manner it [18] may be shewn that GH and HB have to DB the same ratios that NO and OM have to EM respectively. But EN, NO, and OM have not individually to EM, the same ratios that EK, KL, and LC, have to EO since NK, OL, and MC are not parallel because they meet in F. Therefore DG, GH, and HB have not respectively to DB the same ratios that EK, KL, and LC have to EC, respectively, wherefore the 13th article is false, and therefore also the 21st and consequently the 22nd[38] Article is not solved. Wherefore the duplication of the cube is not solved and the principles of Mr. Mackey's pamphlet are false.

14. Nor need we be surprised that Mr. Mackey's attempt was unsuccessful for this problem has exercised the talents of Plato, Philo of Byzantium[39]. Archytas[40], Hero[41], Apollonius[42], Pappus[43], Sporus[44], Gregory[45], Descartes[46], and the most learned of our moderns without success.

15. If b represent any arch we have the well-known formula $\overline{\cos b + 3\cos\frac{1}{3}b \times \frac{1}{2}Rad.}\Big|^2 = \cos^3\frac{1}{3}b$,[47] whence appears the dependence of the duplication on the trisection.

The solution of either of these problems (the duplication of the cube or the trisection of an angle) is of the greatest importance both to Algebra and Geometry. For the construction of those equations which fall under Cardano's irreducible case is impracticable by pure geometry, (that of the right line and circle) so long as the trisection remains unsolved as is evident from trigonometrical principles.

[19] 17. To enter into the subject of the trisection would be to digress from the purport of this tract. We shall conclude with hoping that Mr. Mackey's next attempt will be more successful.

[38] In [Murphy 1824] the "nd" was missing.

[39] Philo of Byzantium (ca. 280 BCE-ca. 220 BCE). In [Murphy 1824] this was given as "Phil the Byzantine."

[40] Archytas of Tarentum (ca. 428 BCE-ca. 350 BCE). In [Murphy 1824] the spelling was given as "Architas."

[41] Heron of Alexandria (First Century CE).

[42] Apollonius of Perga (ca. 262 BCE-ca. 190 BCE).

[43] Pappus of Alexandria (ca. 290-ca. 350).

[44] Sporus of Nicaea (ca. 240 BCE-ca. 300 BCE). In [Murphy 1824] the spelling was given as "Sporius."

[45] It is reasonable to conjecture that Murphy is referring to James Gregory (1638-1675).

[46] René Descartes (1596-1650). In [Murphy 1824] this was given as "Des Cartes."

[47] In [Murphy 1824], *Rad.* represents the radius. Most know this expression in the following form: $\cos(3x) = 4\cos^3(x) - 3\cos(x)$. Murphy is considering a general case, where the radius of the circle considered may not be 1. However, one can easily get the modern version of the formula by letting the radius be 1 and $b = 3x$.

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NOTES.

ARTICLE THE SECOND.

The curve of the Quatuor Nodi, not being treated by any writer heretofore, it may appear necessary to give a concise description concerning it. If from the extremity of a diameter of any circle as center with any portions of the diameter as radii, arcs be described intercepted between the circumference, and diameter, and those arcs be bisected, the bisecting points will determine the locus of the curve. The equation of the curve is bicubic.

ARTICLE THE EIGHTH.

From the final equation here given we may easily deduce $\frac{a}{b} = 1$. Consequently, $a = b$.

ARTICLE THE ELEVENTH.

The process of reducing the equation

$$\overline{n-m} \times \overline{ab}\big|^{\frac{1}{2}} + mb = \frac{\overline{n-m}\big|^{\frac{1}{2}} \times \overline{ab}\big|^{\frac{1}{2}} + m^2 b + 2m \times \overline{n-m} \times \overline{b \times \overline{ab}\big|^{\frac{1}{2}}}\Big|^{\frac{1}{2}}}{n}$$

is this: multiply by n and develop the expression $\overline{n-m^2}$ and $2m \cdot \overline{n-m.}$ Transpose to the left side of the equation the term $m^2 b$ and divide by $\overline{m^2 - mn}$. And the equation is reduced.

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