

#### Section 4.4: Thabit ibn Qurra's Demonstration of Quadratic Equations

What is the difference between al-Khwarizmi's work on algebra and Thabit ibn Qurra's (836-901)?

We consider the cases  $x^2 + q = px$  and  $x^2 + px = q$  as illustrations of Ibn Qurra's approach to solving quadratic equations. He used two theorems from Euclid's *Elements*, and creatively applied them to solving these equations.

**Proposition 5 (Book II):** If a line segment AE is bisected at W and divided at B then the area of the rectangle AB x BE plus the square on BW is equal to the square on AW.

*Note that there are two cases to consider here. Draw a figure to illustrate each case.*

Here is how Ibn Qurra used the above theorem to solve  $x^2 + q = px$ .

Algebraically,

Purely algebraic way of completing the square:

**Proposition 6 (Book II):** If a line segment BH is bisected at W and extended in a straight line to BA then the area of the rectangle AH x AB plus the square on BW is equal to the square on AW.

*Draw a figure to illustrate this theorem.*

Here is how Ibn Qurra used the above theorem to solve  $x^2 + px = q$  (there is a typo in the book: in Fig 4.5, roots =  $px$ , not  $p$ ).

Algebraically,

Purely algebraic way of completing the square: