8. ABCD being the given square piece of paper, it is

required to obtain by fold-
ing, the point **X** in **AB,**
such that the rectangle
under **AB, BX** is equal to
the square on **AX.**

Double BC upon itself
and take its midpoint **E.**

Fold through **E, A.**

Lay **EB** upon **EA** and
fold so as to get **EF, FG.**

Take **AX=AG**

Then the rectangle un-
der **AB, BX**=sq. ou **AX.**

Complete the rect. **BX
HC** and the square **AXKL.**

Let **XH** cut **EA** in **M.**   Take **FY=FB.**

Then **FB=FG=FY=XM**

and $XM = \frac{1}{2}AX.$

Now, because **BY** is bisected in **F** and produced to **A**

Rect. under **AB.AY**+ sq. on **FY**=sq. on **AF**

$\qquad\qquad\qquad$ =sq. on **AG**+sq. on **FG.**

∴ Rect. under **AB, AY**

$\qquad\qquad$ =sq. on **AG.**

$\qquad\qquad$ =sq. on **AX.**

But sq. on **AX**=four times sq. on **XM**=sq. on **BY.**

∴ **AX=BY**

  and **AY=BX.**

∴ Rect. under **AB, BX**=sq. on **AX.**

AB is said to be divided in **X** in medial section.

Also

  Rect. under **AB, AY**=sq. on **BY,**

i.e., **AB** is divided in medial section, also in **Y.**

9. A circle can be described with **F** as centre and passing through B, G and Y. It will touch **EA** at G, because FG is the shortest distance from F to the line EGA.

10. Rect. XYNK=sq. CHKP,

i.e., Rect. under AX, XY=sq. on AY,

i.e., AX is divided in medial section in Y.

Similarly BY is divided in medial section in X.

11. Sq. on AB+sq. on BX=three times the rectangle under AB, BX.

12. Rectangles BH, and YD being each = rect. under AB, BX, rect. HY+sq. CK=rect. under AB, BX.

13. Rect. HY = Rect. BK, *i.e.*, rect. under AX, BX = rect. under AB, XY.

14. Rect. HN= Rect. under AX, BX−sq. on BX.

15. Let $AB=a$, $BX=x$.

Then $(a-x)^2=ax$.

$$a^2+x^2=3ax.$$

Again,

$$x^2-3ax+a^2=0$$

$$x = \frac{a}{2}(3-\sqrt{5})$$

$$x^2 = \frac{a^2}{2}\cdot(7-3\sqrt{5})$$

$$a-x = \frac{a}{2}(\sqrt{5}-1) = a \times \cdot6180.$$

$$(a-x)^2 = \frac{a^2}{2}(3-\sqrt{5}) = a^2 \times \cdot3819\ldots\ldots$$

The rect. BXKP

$$=(a-x)x$$

$$=a^2(\sqrt{5}-2) = a^2 \times \cdot2360\ldots\ldots$$

16. $EA^2 = 5EB^2 = \frac{5}{4}AB^2.$

$$EA = \frac{\sqrt{5}}{2}AB = a \times 1\cdot1180\ldots\ldots$$

2

17.   In the language of proportion

$$AB : AX :: AX : BX.$$

The straight line AB is said to be divided "in extreme and mean ratio."