III.

## SQUARES AND RECTANGLES.

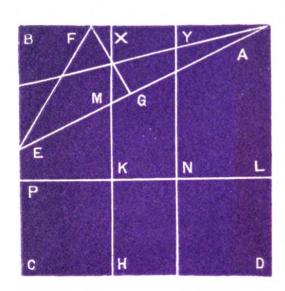
11

8. ABCD being the given square piece of paper, it is

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SQUARES AND RECTANGLES.

CHAP.



required to obtain by folding, the point X in AB, such that the rectangle under AB, BX is equal to the square on AX.

Double BC upon itself and take its midpoint E.

Fold through E, A.

Lay EB upon EA and fold so as to get EF, FG.

Take AX = AG

Then the rectangle under AB, BX=sq. on AX.

Complete the rect. BX HC and the square AXKL.

Let XH cut EA in M. Take FY=FB.

Then FB=FG=FY=XM

and  $XM = \frac{1}{2}AX$ .

Now, because BY is bisected in F and produced to A

Rect. under AB.AY + sq. on FY = sq. on AF

=sq. on AG+sq. on FG.

:. Rect. under AB, AY

=sq. on AG.

=sq. on AX.

But sq. on AX =four times sq. on XM =sq. on BY.

 $\therefore AX = BY$ 

and AY = BX.

∴ Rect. under AB, BX=sq. on AX.

AB is said to be divided in X in medial section.

Also

Rect. under AB, AY=sq. on BY,

i.e., AB is divided in medial section, also in Y.

- 9. A circle can be described with F as centre and passing through B, G and Y. It will touch EA at G, because FG is the shortest distance from F to the line EGA.
  - 10. Rect. XYNK=sq. CHKP,

i.e., Rect. under AX, XY=sq. on AY,

i.e., AX is divided in medial section in Y.

Similarly BY is divided in medial section in X.

- 11. Sq. on AB+sq. on BX=three times the rectangle under AB, BX.
- 12. Rectangles BH, and YD being each = rect. under AB, BX, rect. HY+sq. CK=rect. under AB, BX.
- 13. Rect. HY = Rect. BK, i.e., rect. under AX, BX = rect. under AB, XY.
  - 14. Rect. HN=Rect. under AX, BX-sq. on BX.
  - 15. Let AB=a, BX=x. Then  $(a-x)^2=ax$ .  $a^2+x^2=3ax$ .

Again,

$$x^{2}-3ax+a^{2}=0$$

$$x = \frac{a}{2} (3 - \sqrt{5})$$

$$x^{2} = \frac{a^{2}}{2} \cdot (7-3\sqrt{5})$$

$$a-x = \frac{a}{2} (\sqrt{5}-1) = a \times 6180.$$

$$(a-x)^{2} = \frac{a^{2}}{2} (3 - \sqrt{5}) = a^{2} \times 3819...$$
The rect. BXKP
$$= (a-x)x$$

$$= a^{2} (\sqrt{5}-2) = a^{2} \times 2360...$$

16.  $EA^2 = 5EB^2 = \frac{5}{4}AB^2$ .

$$EA = \frac{\sqrt{5}}{2}AB = a \times 1.1180...$$

17. In the language of proportion

AB : AX :: AX : BX.

The straight line AB is said to be divided "in extreme and mean ratio."