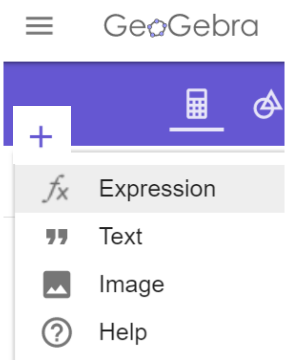
Curve Fitting Applied to Rapa Nui Stone Foundations

Introduction

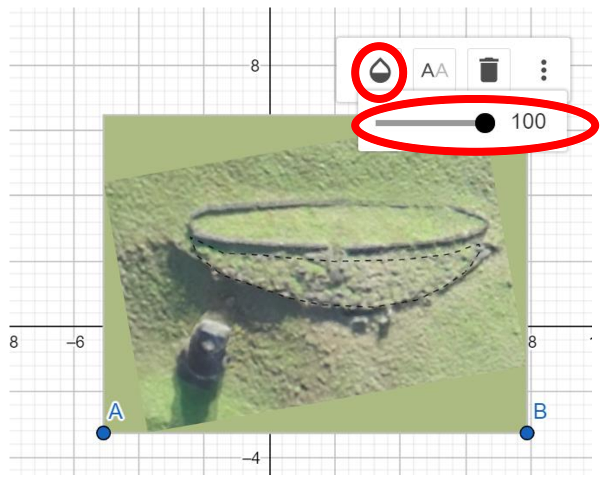
In this activity, curve fitting is applied to drone pictures of ruins of stone foundations of the traditional houses (hare paenga) on the island of Rapa Nui. The free mathematics application GeoGebra (geogebra.org) is used, but the activity can be adapted to other technology, such as Desmos (desmos.com). The activity can be used as a teacher demonstration or completed by students, individually or in small groups, with access to computers. (Imagery credit: Tukuh Technologies, LLC, www.tukuh.com)

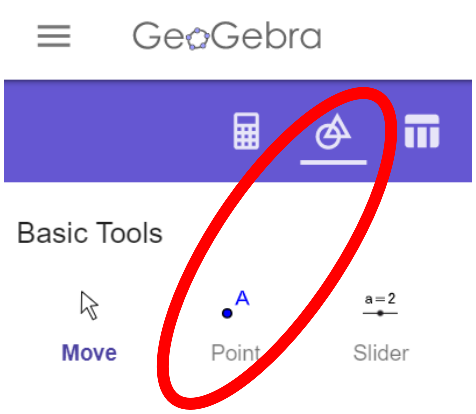
Step 1: Using a clipping tool, select one of the images below to screen capture and save as a graphics file, such as jpg, gif, bmp, etc., in order to be inserted into GeoGebra. Depending on the level of your students, you may want to use an image editing program and have the images already rotated so the major axis of the ellipse is vertical or horizontal. For more advanced students, you can have a tilted ellipse and allow them to rotate it themselves within GeoGebra.

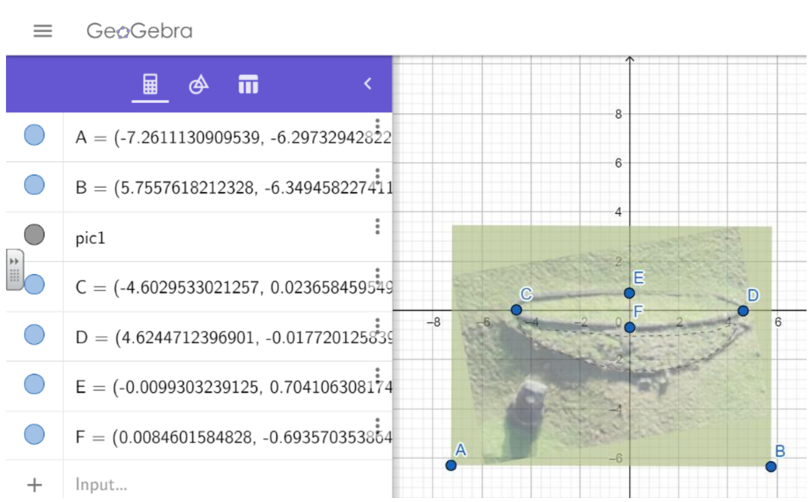


Step 2: Insert the image into GeoGebra by clicking on the + in the upper left corner, selecting image, and opening the file saved in Step 1.

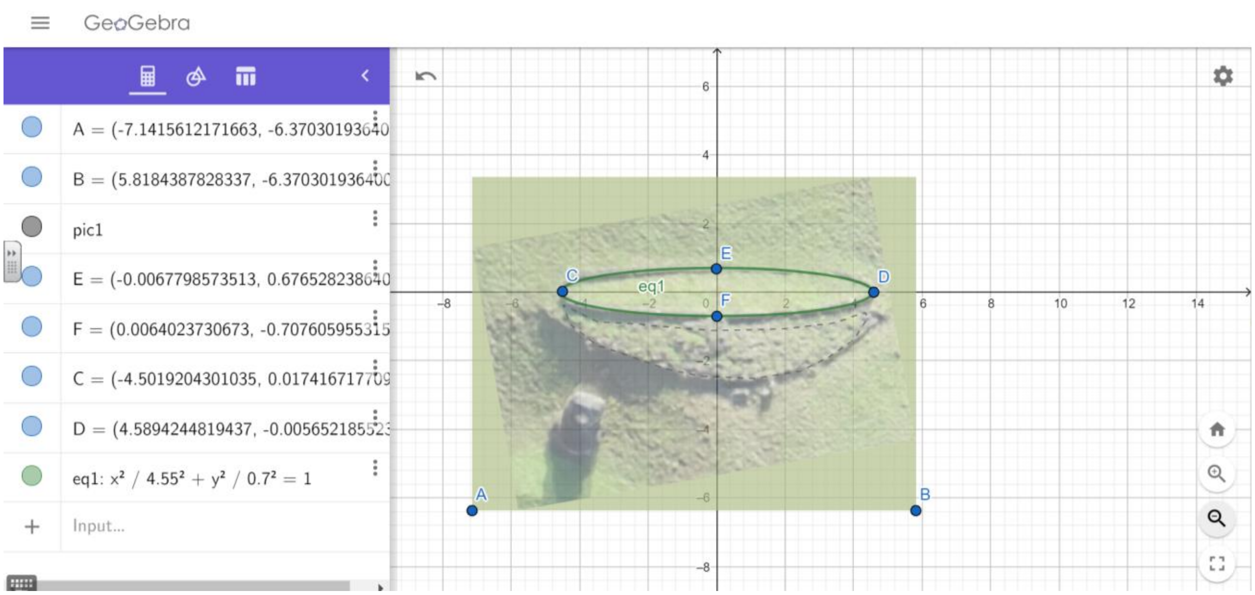
Step 3: Once the image loads in GeoGebra, click on the picture to select it. The first icon in the toolbar that appears will allow you to adjust the transparency, so that the coordinate system can be seen through the image. Clicking and holding the mouse on the image will allow you to move the picture, and moving the points A and B on the bottom corners of the image will allow you to rotate the image if necessary. The goal is to translate (and rotate) the image so that the center of the elliptical stone foundation is located at the origin of the coordinate system, and the foundation is oriented with the major axis either vertical or horizontal. The point tool in the Geometry Tools of GeoGebra can be helpful in centering the image, by observing the endpoints of the major and minor axes. (Open the Geometry tools, click on Point, return to the Calculator, click on Input, and then click on the image where you want to place the point.) By trial and error, translate and rotate the image so that the ruin of the foundation appears centered with the major axis either vertical or horizontal, like in the picture below.



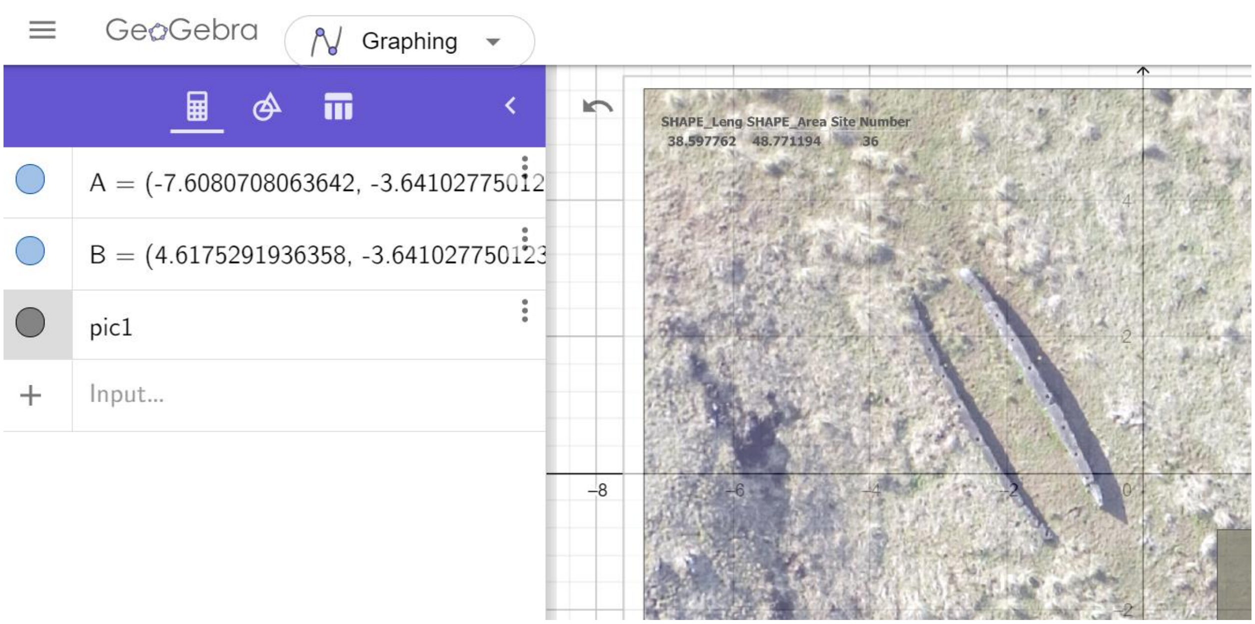




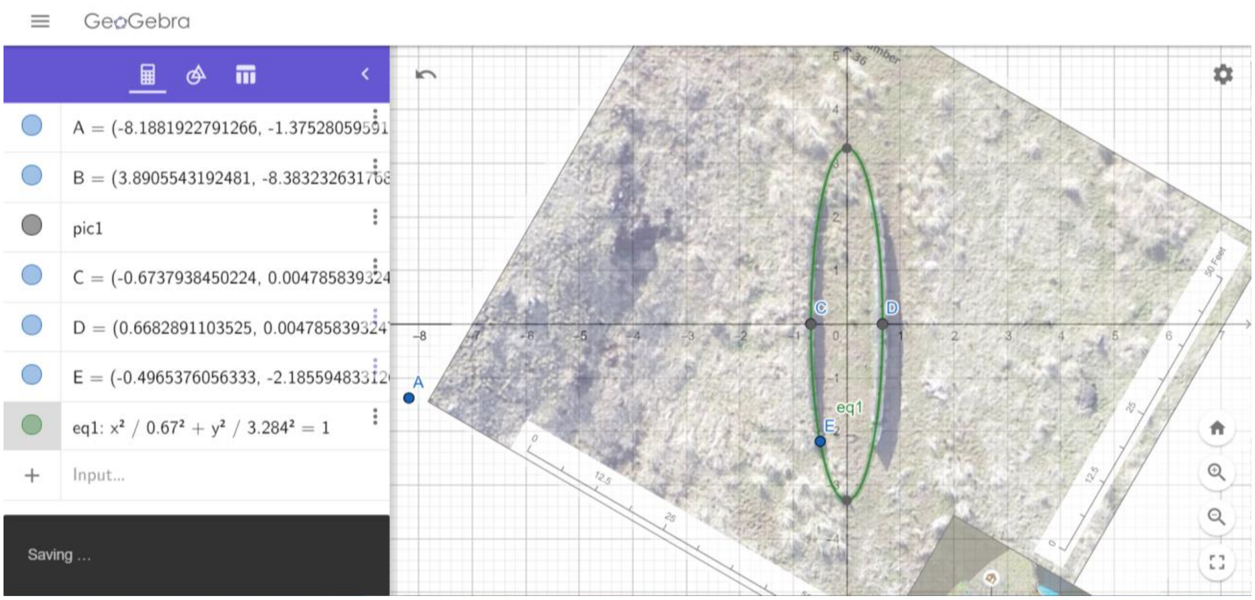
Step 4: Next we are going to fit an ellipse to the picture. If the endpoints $(\pm a, 0), (0, \pm b)$ of the axes are known, the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The ellipse can be easily plotted in GeoGebra to see how well it fits over the foundation stones, as in the screenshot below.



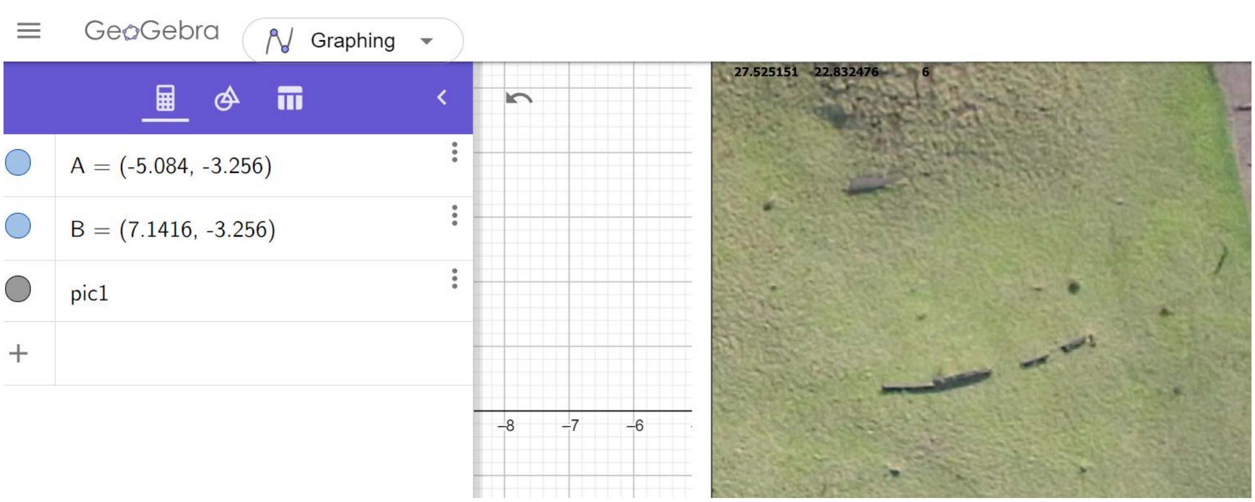
In cases where the paenga stones are missing from the ends of the major axis, as in the image below, a little algebra can be used to find the equation of the ellipse.



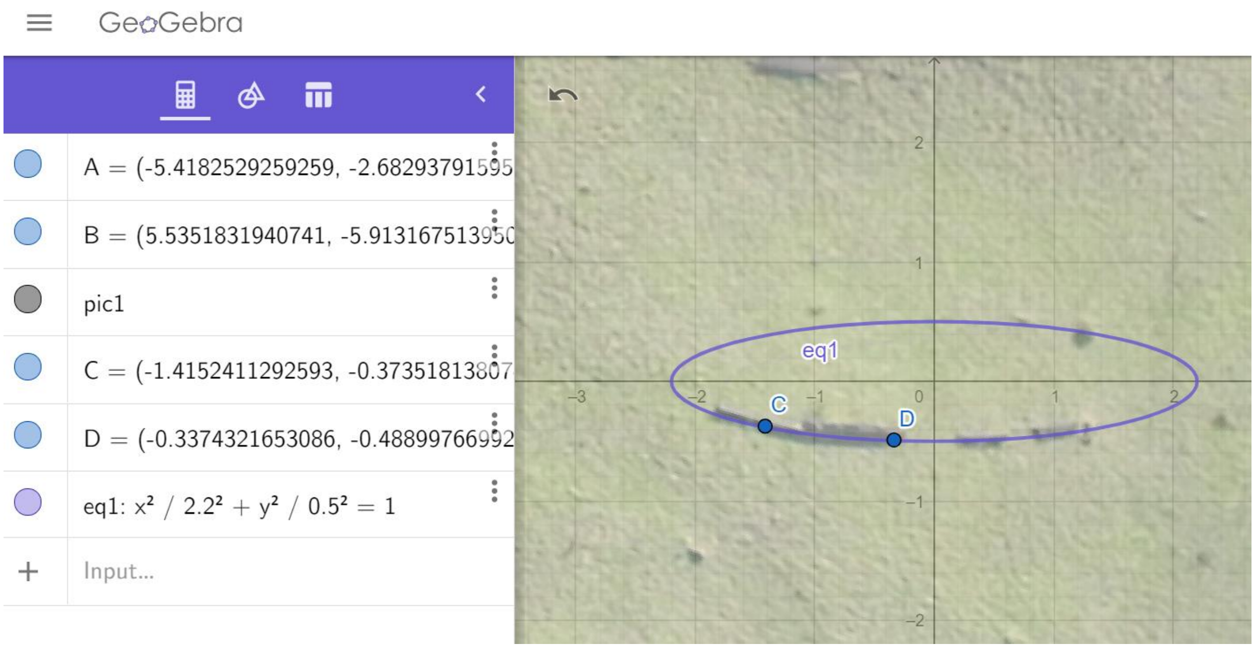
The image can still be rotated and translated to center the elliptical ruin at the origin. (The center will occur at the widest part of the ruin.) In this case, we can use the point tool to find one set of endpoints of an axis. So, we know either a or b in the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Then the point tool can also be used to find another point on the foundation. Substituting that point into the equation for x and y and then solving for the unknown a or b, will result in the equation for the ellipse. (If the attempt at centering was off a bit, adjustments can be made by moving the image and adjusting the equation of the ellipse.) Below is the result of this process on the image above.



The process can also be applied to images with more missing stones, such as the one below; the difference is it's a little more challenging to estimate where the center of the ellipse is. However, adjustments can be made during the process to improve the fit.



Assuming the equation has the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, only 2 points need to be selected and substituted into the equation to solve. (There are actually 4 solutions $(\pm a, \pm b)$, but we only need one of them.) Using the Zoom tool on GeoGebra can help in getting better estimates of the two points being used to fit the ellipse. Below is the result of applying the process to the above image.



Step 5: The demonstration/activity can be concluded with a discussion in which the students reflect on fitting an ellipse to the hare paenga ruins. Some possible questions include:

- Are archaeologists correct in saying that the paenga hare foundations are elliptical?
- Is an ellipse the best curve to fit to the images? Would some other curve fit better?
- Are there other ways to fit an ellipse to the images?
- What are some challenges to analyzing archaeological ruins?
- Why were the foundations centered at the origin and aligned vertically or horizontally?
- What if the ruins were analyzed as they appear in the images (not centered, tilted)?
- What are some challenges to fitting an ellipse when all we have is a photograph of a hare paenga that is not shot directly overhead from a drone?
- In some of the figures, the entrance platform can be seen quite clearly. Discuss the probable shape of the exterior boundary of the entrance platform.
- The figures provided by Tukuh Technologies, present different amounts of data, which depend on the number and location of the stones. Discuss what would be the minimum number, and location, of the stones needed to be able to construct the ellipse that originates the base. For example: What if all the stones left were all on one side?
- Discuss the hare paengas' disadvantages that would arise if the axes of the elliptical stone bases were increased.