• HISTORICALLY SPEAKING,—

Did you ever want to tell your mathematics class how the Romans wrote "large" numbers? Phillip S. Jones, University of Michigan, Ann Arbor, Michigan, supplies some usable and interesting information concerning Roman numerals. He also tells why dentists "extract teeth" and mathematicians "extract roots."

"Large" Roman numerals

George Janicki of Elm School, Elmwood Park, Illinois, commented at a recent meeting that his junior high-school students had become excited about writing their home addresses and phone numbers in Roman numerals. However, they had become a little frustrated by the large numbers to be found in a big city's house and telephone numbers.

This suggested that a few comments on Roman numerals, especially devices for writing large numbers with them, might be helpful to teachers, particularly to those without library resources.[1]

The ultimate origins of Roman numerals are actually unknown, and the theories concerning them, though interesting, are so numerous that to retell all of them would take more space than is available here. Further, use of Roman numerals continued with many modifications for centuries after the fall of Rome so that one can always quibble over what was a "Roman numeral" and what was a medieval modification of the symbolism of the ancient Romans.

The story of M = 1000 illustrates all this and relates to our original problem of how to write large numbers with Roman numerals.

In pre-Roman or Etruscan writing the symbol for 1000 was 8 which probably was derived from a similar symbol in the Etruscan alphabet, but the nature of the connection between the alphabet and Etruscan numbers is uncertain.

In old Roman times ⦶ was used for 1000. This may have come from turning the Etruscan symbol on its side or from the use of an early form of the Greek letter Φ.

In later Roman times ∞, CIↃ, and M were all used. The first of these may have been derived from the symbol of our previous paragraph but it has also been thought to be either a cursive form of the second symbol, CIↃ, or derived by adding arcs at both sides of the Greek X, thus, (X), to distinguish its use to represent 1000 from the X representing 10.

The symbol CIↃ was used most commonly in the middle ages. It also may have been derived from Φ or it may originally have been merely I enclosed in parentheses. In print, in the middle ages, it appeared to be merely I enclosed between two *c's*.

By adding additional pairs of parentheses or *C's* large numbers were formed thus: CCIↃↃ = 10,000, CCCIↃↃↃ = 100,000 and even CCCCIↃↃↃↃ = 1,000,000. Halves of these numbers were represented by omitting the parentheses on the left of the *I*, e.g., IↃ = 500. This last symbol probably

[1] This note is based largely on D. E. Smith, *History of Mathematics* (Boston: Ginn and Co., 1925), Vol. II, pp. 654–64, a work which could well be in every high-school library, and Florian Cajori, *A History of Mathematical Notations* (Chicago: Open Court Publishing Co., 1928), Vol. I, pp. 30–37. Also used were L. C. Karpinski, *The History of Arithmetic* (Chicago: Rand McNally & Co., 1925), and G. Friedlein, *Die Zahlzeichen und das elementare Rechnen der Griechen und Römer* (Erlangen: 1869). It is assumed that all students and teachers have available D. E. Smith and Jekuthiel Ginsburg, *Numbers and Numerals*, currently obtainable from the Washington office of the National Council of Teachers of Mathematics.

led to the later use of the letter *D* for 500.

A bar above the numerals was at times used to represent a thousand times the value of the numerals, thus: $\overline{\text{CXX}} = 120{,}000$. Vertical bars represented multiplication by 100, thus: $|\text{X}|\,\text{DCXC} = 1690$. A double bar over the numeral represented 1000×1000, thus: $\overline{\overline{\text{V}}} = 5{,}000{,}000$, while a single bar above together with vertical lines were used to represent 100×1000, thus: $|\overline{\text{X}}|\,\overline{\text{CLXXX}}\text{DC} = 1{,}180{,}600$.

We are all familiar with the additive and subtractive principles used in Roman numerals although not everyone knows that the subtractive principle was seldom used by the old Romans and that it did not come into popularity until about the middle of the fifteenth century. Likewise few know that, in later years, a multiplicative principle was also used occasionally, thus: XXXM = 30,000. However, Smith contends that in this case the *M* was not thought of as a numeral but as merely an abbreviation for *mille, a thousand.* Variations on this multiplicative idea are to be found when IIII^{xx} was written for 80 (i.e., 4

M C

times 20) and IIII IIII LXXIII = 4473 where again the M could be thought of as an abbreviation rather than as a number symbol. C too has been explained both as an abbreviation for *centum,* hundred, and as a shortened form of the old Roman symbol for 100, Θ.

The pointing out of the connection of *centum* with cent, century, per cent, is of course illustrative of the way historical materials may be an aid in teaching for meaning and interrelationships. This use of history is seen again in the pointing out of our inheritances from Roman fractions. They made much use of multiples of 12 for the denominators of their fractions. Twelfths were called *uncia* as was the sub unit of their coinage. Their basic coin, the *as,* corresponding to the pound, was divided into 12 unciae. From these facts come our words *ounce* and *inch* as well as the 12 inches in a foot and 12 ounces in a pound, troy.

Thus, though the Romans contributed little to the development of real mathematics, we see that, quite appropriately, we owe them much of our language and system of weights and measures.

Word origins

Mathematicians are likely to think that they are in the same position as Alice's Humpty Dumpty who said, you remember, "When I use a word it means just what I choose it to mean—neither more nor less. . . . The question is which is to be the master—that's all." Mathematicians are inclined to think that they are the complete masters; that, irrespective of what they may mean elsewhere, in mathematics words do as they are told, mean exactly and only that which they are defined to mean. In fact, one mathematician took some self-righteous pleasure in pointing out that as a group we use simple common words such as *group, ring, ideal, radical,* (heavens! they sound subversive!) for complicated abstract ideas while other sciences use complicated Greek and Latin derivatives for simple objects and ideas.

Logically it is true that mathematical terms are perfectly arbitrary, but in another sense the mathematician is not the complete master either, for some of his words come to him from the past. He can not easily change them and still communicate clearly. Further, even when he makes up his own words he usually has some reason for choosing and defining them as he does.

These reasons, historical or psychological, are worth knowing and may actually help students when first meeting a term to understand the related concepts as well as to remember the term. For these reasons as well as because they add interest to instruction, we will mention a few facts and some sources of further information about word origins.

In recent notes in this department of THE MATHEMATICS TEACHER we have provided historical background for *real, imaginary, radian, mil.* We might add that the angular unit in the metric system, the *grad*, one one-hundredth of a right angle, is derived from the same source as our word *degree;* namely, the Latin word *gradus* meaning step, pace, or rank. *Gradus* was originally used for the sexagesimal angular unit when geometrical, astronomical, and trigonometric works were first translated from Arabic into Latin in the eleventh to fourteenth centuries.

The excursions into mathematical etymology scattered through the excellent recent *An Introduction to the History of Mathematics*, by Howard Eves,[2] are interesting. On pages 195 to 196 he tells the well-known stories of *algebra, algorism,* and *sine*. Earlier the tales of *cipher* and *zero* appeared. He did not choose to include accounts of *root, radical,* and *extract.* The Arabic word for *root* was used by Al-Khowarizmi (*c.* 825) for what we would call the first degree term of a quadratic equation. The first chapter of his algebra book begins with the problem which we would write $x^2=5x$. Al-Khowarizmi wrote, "The following is an example of squares equal to roots: a square is equal to 5 roots. The root of the square then is 5, and 25 forms its square, which, of course, equals five of its roots."[3] Thus we see that a square was apparently thought of as having grown out of or having been generated by its side. When this Arabic was translated into Latin, *roots* naturally became *radices*, the same word from which we derive *radical, radix, radish.* The Latin verb *extrahere* (*ex*+*trahere*), meaning to pull or draw out, was then as appropriately used for finding (extracting) the root of a given square, as for pulling a radish or a tooth! Thus tractors and square root are brothers under the skin etymologically!!

Two of the most useful word origins are those of *numerator* and *denominator* which are obviously, via Latin, "the numberer" and "the namer" (compare "denomination" as used for bills and religious groups). The essential notion that one can combine only like quantities in addition together with the idea that the denominator tells the kind of thing with which one is dealing (thirds, fifths, etc.) should help one teach with meaning and understanding the need for and nature of common denominators. Incidentally, the fact that denominators are today written below a line or to the right of a solidus is strictly an historical, not a logical, fact. In fact some early Greeks did write the numerator below and the denominator above. In such a case and in our symbols $\frac{2}{3}+\frac{1}{2}=\frac{4}{6}+\frac{3}{6}=\frac{7}{6}$ would have been written $\frac{3}{2}+\frac{2}{1}=\frac{6}{4}+\frac{6}{3}=\frac{6}{7}$, but the process would have still involved finding a common denominator!

Some further accounts of word origins may be found in all the standard histories of mathematics, in particular: D. E. Smith and Jekuthiel Ginsburg, *Numbers and Numerals*, now published by the National Council of Teachers of Mathematics, Chapter VIII, and L. C. Karpinski, *The History of Arithmetic*, Chapter VII. (This unfortunately is now out of print). An article by L. C. Karpinski and Adelaid M. Fiedler, "The Terminology of Elementary Geometry," *School Science and Mathematics*, Vol. xxiv (1924), pp. 162–67, is of some interest too.

Word studies may make interesting class or individual projects. Any good dictionary will offer some information, but one compiled on historical principles such as the *New English Dictionary* will be better.

Perhaps some readers will send their word findings or questions in to this Department?

[2] Howard Eves, *An Introduction to the History of Mathematics.* (New York: Rinehart and Co., Inc., 1953), 422 pp., $6.00.

[3] *Robert of Chester's Latin Translation of the Algebra of Al-Khowarizmi,* L. C. Karpinski (New York: Macmillan, 1915).