COLORADO STATE UNIVERSITY-PUEBLO

JEAN LE ROND D'ALEMBERT
BIOGRAPHY OF A MATHEMATICIAN, *PHILOSOPHE*, AND MAN OF LETTERS

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Jean Le Rond d'Alembert's life can be construed as a life of fantastic variety. d'Alembert was fortunate enough to live in a time period, 1717-1783, when the Enlightenment was giving birth to an amazing diversity of new ideas, institutions, and even academic disciplines. However, one would be hard pressed to argue d'Alembert led a life characterized by luxurious discovery and indomitable success. From the very beginning of his existence d'Alembert was plagued by unfortunate episodes, and set amongst surroundings that would have stunted the intellectual growth of most individuals. He attended schools where his teachers endeavored to indoctrinate his developing mind with their own philosophical agendas. His real family abandoned him at birth while his foster family could offer little intellectual direction, and the traditional backing of numerous, permanent friendships was largely lacking. He never married and the only mistress he ever expressed interest in entertained other men and died early. Surprisingly, considering the near-celebrity renown many of his early accomplishments garnered, d'Alembert was promoted slowly and his fiscal position improved even more leisurely. Additionally, once d'Alembert had finally selected a field to pursue with all the vigor of his talented mind, he encountered other realms of thought which demanded his attention. Despite all these obstacles and the strain of his duties as one of the most determined *philosophes*[1], d'Alembert still strove to work on mathematics in the hectic, tumultuous circumstances he confronted in the 1750's and 1760's. Many times in the latter period of his life d'Alembert lamented not being more focused on mathematics, and

[1] The *philosophes* were both a philosophical and political entity among whose goals was the delivery of humanity from ignorance, fanaticism, and the tyranny of despotic governments.

no longer being able to produce tangible results in that field. Yet, his accomplishments in the arenas of philosophy, literature, and the general enlightenment of the human race strongly demonstrate that his work was of infinite value.

d'Alembert was the son of an eminent salon hostess and a cavalry officer. His mother, Mme. Tencin left him exposed in a wooden box in front of the Parisian church St. Jean Lerond while his father Louis-Camus Destouches was abroad.[2] His father supposedly suffered horribly on account of this action, but did ensure that the boy was placed with a good foster family; the family of Mme. Rousseau.[3] d'Alembert lived with his foster parents until he was 47 years old, demonstrating their generosity and the positive filial atmosphere that developed in the family.[4] He even refused to meet with his real parents for many years because of what he perceived as their abandonment, and an envisioned moral responsibility to his foster parents. Some evidence exists that later in his life "d'Alembert was accustomed to dine with the Destouches family;" suggesting relations improved.[5]

d'Alembert's foster father was only a glazier, and therefore had little to offer the boy intellectually. Luckily, when Destouches died he "left enough to provide the boy with a good education."[6] After attending a small private school d'Alembert enrolled in the Jansenist *Collége de Quatre-Nations* in Paris which was founded by the great

[2] Ronald Grimsley, *Jean D'Alembert 1717-83* (Oxford: Clarendon Press, 1963), 1.

[3] Ibid., 2.

[4] Ioan James, *Remarkable Mathematicians: from Euler to von Neumann* (New York: Cambridge University Press, 2002), 8.

[5] Grimsley, *Jean D'Alembert,* 2.

[6] Other sources debate just how much money was left to d'Alembert for education. The general consensus is that the amount was just enough. Howard Eves, *Mathematical Circles*, vol. I, (Washington D.C.: Mathematical Association of America, 2003), 218.

Cardinal Marazin.[7] While attending this college his teachers realized d'Alembert's many gifts in science, languages, literature, and other fields. Some of d'Alembert's Jansenist teachers tried to persuade him to enter their own vocation by introducing him into "theological controversy".[8] They hoped to make d'Alembert into a leader in their rivalry against the Jesuit order. While d'Alembert avoided confrontation with the Jesuits at this point in his life by rejecting a theology career, he would later be targeted by the Jesuits for his enlightened philosophy and projects.[9] Considering the derisive terms in which he later spoke of the only significant teaching institution he ever attended, one is able to understand why d'Alembert always proved reluctant to enter into theological philosophy while a metaphysical grounding of some sort would likely have enhanced the evolution of some his mechanical theories.

After finishing at the Jansenist College d'Alembert made brief forays into the disciplines of law and even medicine. In 1738 he even qualified as an advocate following examination and briefly practiced.[10] However, d'Alembert seems to have developed a serious interest in mathematical mechanics and other mathematics by this time, and therefore he never took a position in law. In 1739 d'Alembert submitted his first article to the *Académie des Sciences* and his ability as a "geometer" was immediately recognized by Clairaut and others who reviewed his work. ."[11]Almost immediately in 1739

[7] Morton J. Briggs, "Jean D'Alembert" in *Dictionary of Scientific Biography*, vol. I, ed. Charles Coulston Gillispie, 110-117, (New York: Charles Scribner's Sons, 1970), 110.

[8] Grimsley, *Jean D'Alembert*, 3.

[9] The Jesuits felt threatened by the *Encyclopédie* and d'Alembert. Ibid., 23, 34-35.

[10] J. J. O'Connor and E.F. Robertson, 1998, *Jean Le Rond D'Alembert*, in the St. Andrews database, http://turnbull.mcs.st-and.ac.uk/~history/Printonly/D'Alembert.html [accessed September 5,2007], 1.

[11] The *Académie des Sciences* was founded in 1666 while the French Academy was founded in 1635. During the time when d'Alembert became involved the Académie des Sciences nor the French

d'Alembert began work in earnest for the Académie and even read a minor paper revising some of Reyneau's ideas to the Académie's forum.[12] Although d'Alembert's initial interactions with Clairaut were friendly enough, their relationship had dissolved by 1742-1743.[13] This decomposition was evocative of d'Alembert's relations with many mathematicians.[14] Over the next couple of years d'Alembert proved how quickly he had been able to master many mathematical endeavors in his presentations to the Académie on fluids, and in 1741 he was appointed as an astronomy adjunct at the Académie with a small salary.[15] However, the salary provided by this appointment to the Académie des Sciences matured very slowly.[16]

Not long after being appointed to the Académie des Sciences in 1741 d'Alembert began to frequent many of Paris' noteworthy *salons*. Despite his somewhat withdrawn and introverted personality as a child and in school, d'Alembert achieved great renown as a conversational talent. He quickly acquired many friendships due to his rational, semi-Cartesian philosophical outlook, and the fact that "he was extremely well read and possessed an excellent memory."[17] Combined with his growing mathematical renown at

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Academy had many seats held by members of the philosophes and therefore were somewhat exclusive and reluctant to admit certain social elements. Thomas L. Hankins, *Jean D'Alembert: Science and the Enlightenment* (Oxford: Clarendon Press, 1970), 27,31.

[12] O'Connor and E.F. Robertson, 1998, *Jean Le Rond…,* 2.

[13] This rivalry resulted from the fact that the two choose to work on similar subjects and often obtained similar results despite the differences in their methodologies. Hankins, *Science and the Enlightenment…,* 31.

[14] He encouraged many rivalries besides with Clairaut, and his published articles were often sloppy and hurried in order "to forestall any other mathematician who might be working on the same subject."Ibid., 27, 31.

[15] Briggs, "Jean D'Alembert", 111.

[16] Particularly in the Académie des Sciences, promotion came very slowly. In fact despite his many awards and contributions d'Alembert endured a life-long struggle with personal finances. d'Alembert Hankins, *Science and the Enlightenment*…, 137.

[17] These cultivated qualities allowed d'Alembert to make clever connections between wide-ranging philosophical topics, and to impress figures in literature all over Paris and even around Europe. Ibid., 16.

the beginning of the 1740's d'Alembert became a social figurehead, and also a de facto member of the *philosophes* because of his philosophical outlook, reliance on logic, and espoused goal of delivering enlightenment and defeating fanaticism.[18]

Salon connections with Paris and European society proved extremely advantageous for d'Alembert in his professional advancement, and contributed greatly to his philosophical development. Immersed in the salon environment d'Alembert began to acquire interests in fields which he had not explored since his days at the Jansenist College. He began to read more and more works of literature and philosophy, and through his conversational ability, began to impress many of Europe's most famous literary satirists and philosophers. Nonetheless, the salon atmosphere provided more than a way for d'Alembert to mature intellectually. For the first time in his life d'Alembert was engaged in society, and developing friendships which, though troubled, would prove important to his personal satisfaction and his ability to lead the *philosophes* later on.

One of the most significant Parisian literary figures d'Alembert became friends with due to his attendance of salons was Denis Diderot. After the original editor of the Encylcopédie lost the confidence of the publishers d'Alembert became the scientific editor of the Encylcopédie; at the same time as Diderot received the head editorship.[19] His participation and guidance in the precarious work of creating the Encyclopédie came to take a preeminent place in his life for nearly two decades.[20] In the beginning d'Alembert's role in the project was immense as "he took a large share [also] in the

[18] The replacement of blind religious zeal and unreasonable faith by deduction was a primary goal of Enlightenment thinkers.

[19] Ibid., 10.

[20] The Encyclopédie project was one of immense proportions in which the philosophical beliefs of the philosophes were imbued. The project took over twenty years to publish in subsequent volumes, and was heavily persecuted by various enemies in France and farther abroad while the editors and contributors hurried to complete the vast academic encyclopedia.

general editorship."[21] In fact d'Alembert's role was of such importance that some historians argue that he was even more significant to the project than Diderot early on.[22] During the tremendous effort to complete the Encylcopédie d'Alembert was often able to rely on his salon contacts and ability to garner additional political support not only for the *philosophes* movement but for the completion and acceptance of the Encyclopédie.[23] Unlike some of the other literary figures involved in different stages of the Encyclopédie, d'Alembert could draw on a diverse arena of academic experience and contacts to produce support for this *philosophes* masterwork.

At first his work on the Encyclopédie only proved advantageous for d'Alembert. His *Preliminary Discourse* was widely accepted as a philosophical masterpiece, and is still regarded as a landmark piece of literature from the Enlightenment. In the *Preliminary Discourse* he proceeded to "articulate modern knowledge solely in light of its empirical origins."[24] This philosophical position seems odd considering the lack of empirical rigor which many of d'Alembert's contemporaries criticized in his work, but one must realize that d'Alembert always strove to base his work on a firm experiential foundation even he often experimented only in the context of his own mind.

By the time the Encyclopédie had printed more than a couple of its multiple volumes, the problems with the Encyclopédie became more difficult for d'Alembert to juggle. Censorship was attempted by many societies and institutions, and became very aggressive during certain stages. Then d'Alembert wrote an article entitled *Genéva* in which he grossly misapprehended the character of the protestant religion of the Geneva

[21] Ibid., 11
[22] Ibid., 11-15.
[23] Ibid., 14-15.
[24] Grimsley, *Jean D'Alembert,* 20.

after visiting Voltaire at his home in that city.[25] Owing to the political fallout from this article, d'Alembert resigned as editor of the Encyclopédie and his relations with Diderot "became strained" due to his unwillingness to retract the article.[26] By 1760 his involvement with the Encyclopédie was strictly confined to scientific and math writings.[27]

The enormous amount of work d'Alembert spent in organizing, promoting, and writing for the Encyclopédie consumed the middle portion of his life. This period of his life was incredibly prolific, as he had not only evolved into one of the leaders of the philosophes, but kept busy maintaining relationships with literary notables like Rousseau, Voltaire, David Hume, and many of the leading monarchs in Europe. The strain on him was evident in the many serious bouts of illness he dealt with,[28] and the fact that he fell into states of cynicism. Nevertheless, d'Alembert's periodic disgusts with humanity were, "rather the reaction of a man appalled by the magnitude of his task."[29] A particular indication that d'Alembert cannot be described as a pessimist is his disagreement with "Rousseau's deeply felt animadversions against civilization."[30] d'Alembert was optimistic about many aspects of the future, as demonstrated by his encouragement of the genius of the younger generation.

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[25] Voltaire was partially responsible for this misinterpretation because he didn't impart to d'Alembert that this sacrilegious conception of the Genevan religion was part of Voltaire's own idealism. Ibid., 52.

[26] Briggs, "Jean D'Alembert", 115.

[27] Grimsley, *Jean D'Alembert*, 70.

[28] d'Alembert blamed his bad health on many sources ranging from physical abnormalities to exhaustion. E.T. Bell, *Men of Mathematics*, (New York: Simon and Schuster, 1937), 156-157. The strain on him seems to elect the latter cause as the most likely culprit for his periodic illness. But near the end of his life he writes Lagrange and "councils work as the only remedy for Lagrange's psychic ills," implying he believes exhaustion is not the cause. Ibid., 157.

[29] Ibid., 119.

[30] Ibid., 136.

d'Alembert's work on the Encyclopédie combined with his stunning mathematical achievements gained him entrance into political circles that many academics could only dream about. In 1746 a paper submitted by d'Alembert, which involved planetary wind and the attempt to explain atmospheric tides won the Berlin Academy prize, and he was recognized by Frederick II, 'The Great', as one of Europe's leading mathematicians.[31] For the rest of his life d'Alembert would remain one of the Prussian King's favorite correspondents, and "d'Alembert's admiration for the philosopher-king is[was] as sincere as it is[was] unbounded."[32]

Frederick relied on d'Alembert's opinions concerning the proper operation of his Berlin Academy for decades, and nominated him to be the President. d'Alembert refused the nomination likely because of feelings of loyalty to France, but Frederick II continued trying to compel him to accept.[33] Instead, d'Alembert recommended that Frederick place Euler in the position of president.[34] This recommendation did much to improve relations between the two mathematicians despite Frederick's espoused mistrust of Euler and reluctance to install the genius as President of his Academy. d'Alembert may have been somewhat egotistical concerning his claims to many of "his" inventions, but a constant in

[31] Although academic institutions did circulate prizes on a grand scale, during this period developments in the sciences were characterized by the patronage of monarchs and universities in the form of academic contests in many cases.

[32] Ibid., 166. The King was one of the greatest admirers of the *Preliminary Discourse* and strove to support d'Alembert on many occasions. \ The only time d'Alembert ever left France besides his visit to Voltaire in Geneva, a trip which Frederick paid for, was to visit the King's court in Wessel in 1755. Ibid., 159. Frederick also advanced other assistance to d'Alembert in terms of money on other occasions after observing his ill position. Ibid., 158. \

[33] O'Connor and E.F. Robertson, 1998, *Jean Le Rond…,* 4.

[34] Earlier in his career d'Alembert had been angry with Euler for criticizing some of his work, and revising some of his ideas. The two mathematicians became more and more estranged as Euler made vast improvements on the basic ideas d'Alembert had proposed and reformed them to the general satisfaction of the academic community. James, *Remarkable Mathematicians*, 11.

his career was his ability to recognize talent in others.[35] d'Alembert also maintained a committed correspondence with Catherine 'The Great' of Russia, and was offered the opportunity to tutor her son. As part of his efforts to raise the prestige of the French Academy d'Alembert also entertained foreign dukes and other politicians; often taking precious time away from his work on the Encyclopédie and mathematics.

After d'Alembert gained the influential position of secretary of the French Academy in1766,[36] he began to devote time to, "enhance the Academy's prestige both at home and abroad."[37] d'Alembert vastly improved the waning reputation of the French Academy with the public and also internationally. Part of the new direction he emphasized for the French Academy was the maintenance of all humanity. Of course, this and other agendas were consequences of his fervent allegiance to the philosophes.[38] The French academy enjoyed this popularity d'Alembert had helped create until the 1770's.[39]

The convergence between d'Alembert's political/philosophical ideas and his mathematical ideas is somewhat difficult to describe. Part of his philosophical beliefs involved the preeminence of sense-perception and empirically derived knowledge.[40] He adhered to the idea of *The Great Chain* of knowledge, and it was his goal as part of the Encyclopédie to "trace out this Chain as much as possible" by illuminating all of the

[35] Especially when he was older d'Alembert encouraged younger mathematicians and can be considered a significant patron to many including Lagrange and Laplace. Grimsley, *Jean D'Alembert*.

[36] Briggs, "Jean D'Alembert", 117.

[37] Grimsley, *Jean D'Alembert*, 88.

[38] D'Alembert even struggled bitterly to include more enlightened individuals in the French Academy's ranks Grimsley, *Jean D'Alembert*, 95.

[39] Ibid., 100-101.

[40] Briggs, "Jean D'Alembert", 111.

sources and tenets of human knowledge.[41] For example, in the development of his mathematics, especially his *Principle,* one observes a rigorous search for assumptions which cannot be debated. Although he realized that mathematics was imperfect in its complete description of the world, he strove to base his mathematics on ground as free from abstract and artificial contrivances as possible. d'Alembert said:

> Geometrical truths are in a way asymptotes to physical truths; that is to say, the latter approach the former indefinitely near without ever reaching them exactly.[42]

Other mathematicians like Daniel Bernoulli and Clairaut were often correct when they described the inadequacies of D'Alembert's evidence. But if one then recognizes the fact that, for d'Alembert theory was much superior to numerical calculation,[43] it is possible to understand why his numerical justifications were juvenile in comparison to those of Clairaut and other contemporaries. The fact that in many circumstances, "There is no question d'Alembert was guilty of neglecting observation," might seem to undermine his philosophical method of empiricism.[44] However, for d'Alembert there was no contradiction. He worked to ensure that his theorems and ideas were derived from a firm, empirical footing, such as the foundation he felt his *Principle* encapsulated. He did not often explicitly check to ensure his results agreed with physical expectations, even

[41] Hankins, *Science and the* Enlightenment…, 104
[42] Jean Le Rond d'Alembert, unknown publication. Quoted in Eves, *Mathematical Circles*, vol. II, 129.
[43] Hankins, *Science and the* Enlightenment…, 41.
[44] Ibid., 37.

though he agreed this was overwhelmingly important, and therefore did not often have recourse to alter his ideas. This lack of physical justifications in much of his work can be explained partially by his preoccupation with so many other extensive duties.

d'Alembert's rapid elevation in scientific ability before 1739 is mind-boggling. He received little formal mathematical or scientific training during his time at the Jansenist College, where he was primarily focused on Latin, philosophy, and literature studies. After deciding the careers of advocate and doctor would not satisfy his inquisitive mind, d'Alembert began an intensive program of mathematics. During the course of his twenty years study of the subject, d'Alembert would manage to gain recognition as one of the era's most original, though stubborn, mathematicians.

In only a year or two around 1738 d'Alembert was able to acquire knowledge of contemporary mathematical advancements and begin to develop his own ideas. On his own d'Alembert read works of Newton, L'Hospital, and the Bernoullis. Early on d'Alembert was so interested in mathematics that he worked feverishly through English works, despite a language handicap. He read most of Newton's *Principia* and Maclaurin's *Treatise of Fluxions*, which proved instrumental to his mechanical ideas.[45] By the time d'Alembert published his famous article *Traité de dynamique* in 1743 he was already endeavoring to develop a clear logical principle on which to base all of his future mathematical exploits. The attempt to generate a principle that would provide a logical, yet non-metaphysical[46] groundwork for all of mechanics would plague d'Alembert throughout his career. Even at this early stage his struggle with the idea of *force* as

[45] Hankins, *Science and the* Enlightenment…, 22.

[46] d'Alembert's definition of "metaphysical" involves grand, abstract, and unwieldy systems. His irritation with metaphysics is not based on a nihilist approach.

conceived by Isaac Newton is apparent, as he strives to set mechanics more firmly within the realm of.[47]

d'Alembert's mathematical ideas were extremely dependent on inquisitive thinking about a subject and the consequent attempt to find basic principles. d'Alembert declared that these principles were usually beyond reach, but he struggled to discover them.[48] In mechanics d'Alembert rejected all obscure and metaphysical entities and strove to base that science, then considered a field of mathematics, on a secure axiomatic basis like to that of geometry.[49] *d'Alembert's Principle* included three laws of motion which were very similar to those of Newton, except for the notion of forces.[50] His third law in particular was a divergence from the ideas of Newton, and expressed unclear ideas. Much of the difficulty in understanding *d'Alembert's Principle* of motion results from not recognizing that he always thought in terms of 'original impact'.[51] Although his claims that he could reduce all of dynamics to the statics of collisions as a result of his principle are certainly presumptuous, the thinking established in d'Alembert's principle was an important development in the theory of mechanics. In the end d'Alembert's principle was successful because it went beyond particles and attempted to deal with constraints inherent in continuous physical systems.

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[47] Here we observe again the apparent contradiction in his philosophical demeanor. But d'Alembert believed more in the power of mathematics than he did on human perception in some instances. He thought mathematics should adhere to reality by becoming more perfect, rather than believing reality should adhere to mathematics.

[48] As one author has stated, "the only method was to accept the fact of observation and experiment, decompose them into their simplest elements and then by careful comparison of these simples, work back to more general principles." [48] Hankins, *Science and the Enlightenment*…, 114.

[49] Ibid., 152.

[50] d'Alembert always recognized the genius in the work of Isaac Newton. But he was determined to show that the concept of the force, which was gaining widespread acceptance, was flawed in many essential aspects. For instance, d'Alembert claimed that a force could be known only through its effects (Ibid., 207), and since the causes of a force remained mysterious, mathematicians should be reluctant to utilize forces as basic components in their theory when those forces remained unknowable.

[51] Ibid., 192.

In order to work in the advanced mechanical concepts which d'Alembert confronted, he had to be extremely competent in the mathematical tools behind the physical ideas. Resulting from this necessity, d'Alembert is also known for his work in perfecting certain elements of the calculus, and inventing new techniques in differential equations, partial differential equations, and even some techniques which are reminiscent of matrix theory. The fundamental search for simplicity characterizes d'Alembert's efforts to expound the theory of the differential calculus.[52] In the evolution of that discipline d'Alembert work must be considered the next stage after Newton.[53]

d'Alembert proposed his own theory of differentiation as:

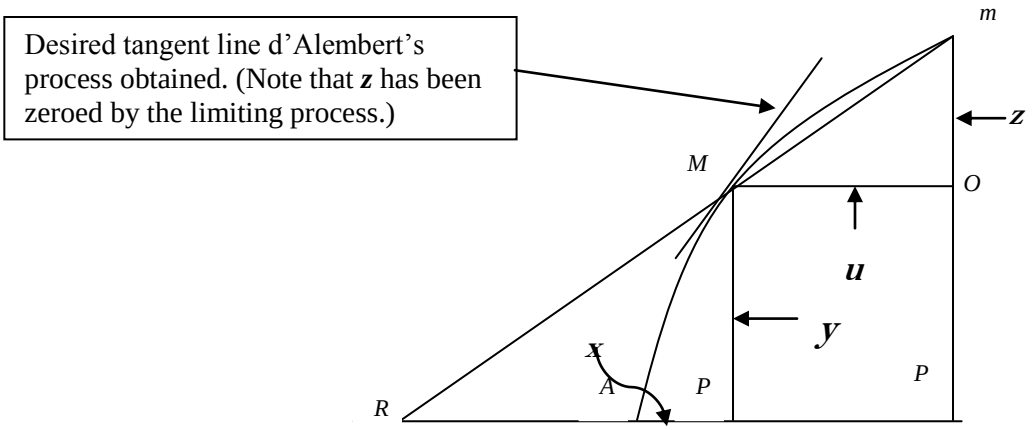
> Every equation involves a relation between two variables and the differentiation of equations consists merely in finding the limit of the ratio of the finite differences of the two quantities contained in the equation.[54]

In other words the derivative *dy/dx* is the limit (independent variable approaching 0) of a ratio of quantities. In the same article d'Alembert went on to demonstrate a geometric proof of this idea as applied to a traditional conic section; the parabola $y^2=ax$. In this example d'Alembert constructed the geometric version of part of this parabola:

[52] In his approach to calculus we can observe the characteristic attitude of d'Alembert. At the beginning of the article he wrote for the Encyclopédie entitled *Differentiel* d'Alembert declared, "since this calculus yields reliable, simple, and exact methods, the principles on which it depends must also be simple and exact." d'Alembert, "Différentiel," *Encyclopédie*, ed. Calinger, 482.

[53] He considered the derivative as a ratio defined at the ultimate limit.

[54] Ibid.



Next d'Alembert observed the secant line through *RMm,* in modern notation through points *(x,y)* and *(x+u,y+z),* and proceeded to show the ratio *z:u* is equal to the ratio *a:2y+z*. He then explained how the geometer may choose the value of *Om=z* as small, or as close to zero, as desired. Therefore when *Om=z* is selected to be extremely small the limit is observed and the desired derivative expression becomes merely *a:2y*.[55]

In another article written for the Encyclopédie d'Alembert contributed further to the development of calculus by providing an early definition of the limiting process.[56] d'Alembert's conception and definition of the limit was predominantly geometric, and unlike the arithmetic versions of his contemporaries.[57] It was not until Augustine Cauchy in the early 19th Century that d'Alembert's definition reemerged into the main stream of mathematics and became widely accepted.

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[55] Ibid., 483.

[56] Victor J. Katz, *A History of Mathematics: Brief Edition*, (Boston: Pearson, Addison Wesley, 2004), 356-357. d'Alembert expressed his idea: "One magnitude is said to be the limit of another magnitude when the second may approach the first within any given magnitude, however small, though the second magnitude may never exceed the magnitude it approaches." This definition is evocative of d'Alembert's emphasis on maintaining simplicity by observing a simple geometric situation involving two magnitudes. Jean Le Rond d'Alembert, *Encyclopédie.* Quoted in Katz, *A History of Mathematics*, 356-357.

[57] Ibid., 357.

Despite the important contributions d'Alembert made in the solutions to ordinary differential equations, his work in partial differential equations generated the esteem for his name in future generations of mathematicians. d'Alembert positively can be given credit for developing and solving the first partial differential equation.[58] The capacity of mathematics to involve such entities was made possible in d'Alembert's time because of the advancing theory of functions and their application to situations where more than one variable was essential to describe the phenomena.

The first appearance of d'Alembert's work on partial differential equations appears publicly in two articles he wrote in 1747. In one of those articles d'Alembert introduced the form, and parts of the derivation behind that form, of a particular partial differential equation, and in another demonstrated a limited solution to this 'wave equation'. [59] d'Alembert's work on the wave equation originated as an attempt to solve the puzzling matter of quantifying the physical properties of the vibrating string. This problem involved a partial differential equation because the shape of the string varies with two variables: position and time. In order to derive the wave equation d'Alembert primarily used Newton's Laws[60], at this time in his career he had yet to develop his

[58] This fact is widely acknowledged by historians of d'Alembert and has been well documented in the history of that field. Current texts in partial differential equations to this day have sections such as "The D'Alembert Solution of the Wave Equation", in Stanley J. Farlow, *Partial Differential Equations for Scientists and Engineers*, (New York: Dover Publications Inc., 1993), 129. This text follows a solution procedure similar to the one d'Alembert employed even though it does not explicitly detail how the procedure relates to that of d'Alembert.

[59] Briggs, "Jean D'Alembert", 113.

[60] Although the direct derivation is somewhat incomplete and fuzzy in this source it clearly illustrates d'Alembert was considering tension and the force of gravity when he derived this equation. Such use of Newton's forces is surprising considering the fact that later in his career, d'Alembert decried the use of force by mathematicians or other scientists to any purpose. Jean Le Rond d'Alembert, "Vibrating String: D'Alembert, Euler, D. Bernoulli," in *A Source Book in Mathematics 1200-1800* ed. D.J. Struik (Cambridge: Harvard University Press, 1969), 352.

famous principle. He thought about the string as being composed of an infinite number of tiny masses to obtain:[61]

$$\frac{d^2y}{\partial t^2} = c^2 \frac{d^2y}{\partial x^2} \quad \text{or rewritten:} \quad (\delta_{tt} - c^2\delta_{xx})y = 0$$

The procedure employed to solve this equation essentially employs the same ideas and steps today as it did when d'Alembert first published. The only real difference is that some of d'Alembert's confusing geometric schemes are not included.[62] First d'Alembert let the ordinate in a diagram (y-coordinate algebraically), be a function $\varphi(t, s)$. Next d'Alembert noted the derivative $d[\varphi(t, s)] = pdt + qds$ as his characteristic equation.[63] In the modern solution $p = \frac{dt}{dw}$ and $q = \frac{dx}{dw}$ are the derivatives of the characteristic curves $x(w) and\ t(w)$ and dt represents the partial y_t and dx the partial y_x. One next proceeds to factor the original equation into: $(\partial_t + c\partial_x)(d_t - c\partial_x)y$. When either of these factors is 0 the half of the solution to the entire expression$(\partial_{tt} - c^2\partial_{xx})y = 0$. In modern terms this is justified by simple linear operator theory. Each of the factors above is in the form of d'Alembert's characteristic equation[64] and can be solved, in a modern sense, by formulating characteristic curves $x(w) and\ t(w)$ starting at some initial condition like: $w = 0, t = 0, x = \tau$. In the modern process one finishes by relating ($x\ and\ t$) to ($w\ and\ \tau$); d'Alembert clearly used a little different notation in his characteristic equation above. By adding the contribution of each factor d'Alembert received, in his notation, the solution of the equation in the form of: $y = \Psi(t + s) + \Gamma(t - s)$.[65]

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[61] Katz, *A History of Mathematics…,* 343.

[62] Even Euler found some of d'Alembert's geometric methodology puzzling, and unnecessary.

[63] d'Alembert, "Vibrating String…," 353.

[64] Take for example the factor $(\partial_t + c\partial_x)$. This can be equated to a characteristic equation in the form $pdt + qds$, where $p = \frac{dt}{dw} = 1$, and $q = \frac{dx}{dw} = c$. (Also note that the s in d'Alembert's notation is the x in the modern notation).

[65] Ibid., 355.

d'Alembert's solution to this equation was originally published in the absence of initial data; *f(x)* and *g(x),* representing the initial position and initial velocity of the string. In a later article he generalized his solution where the initial position was given by $y(x,0) = f(x)$ and initial velocity by $y_t(x,0) = g(x)$. However, d'Alembert tried to prove these initial conditions had to be functions with a period of exactly $2l$; l being the length of the string. He also demanded these functions had to be expressible as single analytic expression which was twice differentiable as was common at the time.[66] This limitation on d'Alembert's solution is evocative of his hesitation to deal with discontinuous functions which characterized all of his mathematics.[67]

d'Alembert was never able to give a complete solution to the wave equation and it was left to his contemporaries to expand and more carefully explain the details. For instance, two years later in 1749 Leonard Euler confirmed that the initial shape of the string could be arbitrary and discarded the periodicity limitations d'Alembert had placed on the initial conditions.[68] But regardless of the criticisms of Euler, Daniel Bernuoulli and some of his other contemporary mathematicians, d'Alembert's consideration of, and work on, this first equation in the field certainly demand that he be given tremendous credit. Historians all agree that d'Alembert did not have the same ability in comparison to Euler, however, his originality provided results which even that great mathematician respected and honored by attempting to improve and expand himself.

Another realm of mathematics which d'Alembert's name is often connected with is early probability theory. During d'Alembert's period, probability was only just

[66] Ibid., 344.
[67] Although it must be noted this hesitation was shared by most mathematicians of the time.
[68] Euler, "Vibrating String: D'Alembert, Euler, D. Bernoulli," in *A Source Book in Mathematics 1200-1800* ed. D.J. Struik (Cambridge: Harvard University Press, 1969), 357.

becoming legitimate. However, d'Alembert had grave misgivings about the field because of the inadequate caution exercised by those who practiced it. d'Alembert believed probability concepts should delicately embrace not only the objective facts of infinite chance, but also address the moral complexities of the problem, and that it incorporate "standards of physical plausibility"[69]. Some historians have characterized his thoughts and work on probability as "aberrations of an otherwise outstanding mathematician" because of such flagrant disregard for the standards involved in this subject.[70] d'Alembert was known to have to rejected some of the basic foundations of much of probability which other mathematicians took for granted. For instance, d'Alembert attempted to describe examples of situations in which equi-probability was a false assumption.[71] Independence of events was another basic issue which d'Alembert rejected in almost every instance because he believed multiple variables were involved in almost any conceivable situation.

The ultimate hope d'Alembert had for probability, however, was not the dissolution of the subject.[72] Rather he hoped to increase the level of rigor inherent in that subject by careful revision based on intense attention to physical experience.[73] In his own articles on probability d'Alembert maintained that the simplest branches of probability be "perfected before progressing to more complex topics."[74] Hence he divided the subject into three subcategories of increasing complexity: games of chance, "common life" or issue like insurance and life expectancy, and "true conjecture" or abstract application to

[69] L. J. Daston, "D'Alembert's Critique of Probability Theory," *Historia Mathematica 6* (1979): 260.
[70] Ibid., 259.
[71] Ibid., 267.
[72] Ibid., 260.
[73] Ibid., 264.
[74] Ibid.

topics like history, politics, and medicine.[75] Adhering to his own criterion, d'Alembert began his critical articles on probability usually by referencing the difficulties posed by even the lowest category of probability; games of chance.

An example situation d'Alembert often employed was the game involved in the St. Petersburg problem.[76] Two of d'Alembert's major criticisms of plausible solutions to the problem were that the moral expectation of the player "had been arbitrarily simplified and arbitrarily applied."[77] Any possible solution to this problem involved the notion of the possibility of a long sequence of identical events, and d'Alembert was simply unwilling to recognize the physical possibility of the probability $p = \frac{1}{2^n}$. existing for some large n of all tails. Likewise d'Alembert disagreed with contemporary use of probability to justify conclusions about the physical universe. He protested against Daniel Bernoulli's probability justification for uniform causality in the solar system. Although he agreed with Bernoulli's conclusion in this case, he claimed that true application of probability to this situation would require a result in which the uniform cause of the solar system was enormously improbable.

The most infamous work d'Alembert did on probability concerned the inoculation of French citizens against small pox. This inoculation had a 1 in 200 chance of causing death in the recipient, but as a philosophe d'Alembert supported the inoculations as a part of the scientific evolution of society. Here again d'Alembert clashed with Bernoulli, because he thought important moral and practical implications were being ignored.

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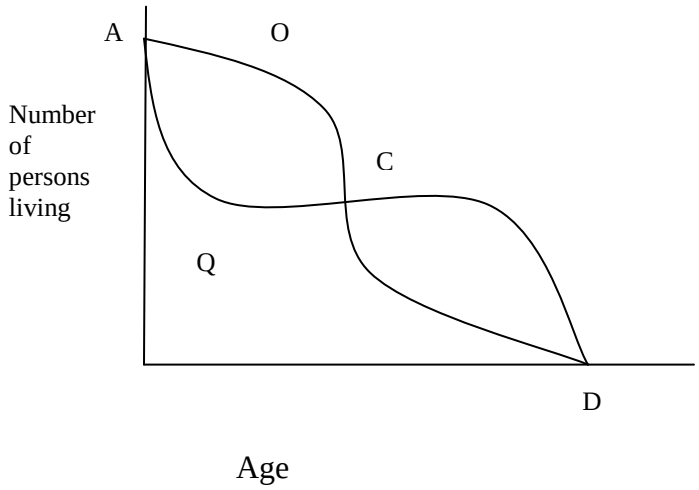
[75] Ibid.

[76] This problem involved a coin toss in which one player (A) would pay a stake to play. The second player (B) would then pay A according by an increasing scale the longer it took A to toss a heads. Ibid, 265.

[77] Ibid., 266.

d'Alembert judged the advantages of the inoculation program on a threefold basis.[78] Whereas Bernoulli and other traditionalists argued mortality could be based on a single average life expectancy, d'Alembert observed two distinct mortality curves with identical integrals.



As stated by Bernoulli the life expectancy is the same for both curves, yet d'Alembert pointed out that AOCD is a preferable course since less people die at an early age. Therefore d'Alembert argued that these integrals demonstrate how traditional probability could not be applied in this case. In any case d'Alembert was focused on reforming probability until it embraced the proper physical, moral, and other variables which he thought to be lacking. Even though his criticisms were and are still not considered valid, d'Alembert's questions and challenge to future mathematicians should still be honored, as did Condorcet in his eulogy of d'Alembert:[79]

[78] Consequences were considered in application to "physical life", "real life"—the part of life lived fully, without suffering, and "civic life"—the part of life in which one was useful to the purposes of the state. Ibid., 274.

[79] Condorcet's extensive work in probability refuted d'Alembert intensively, and hereby acknowledged the necessity of addressing d'Alembert's ideas. Ibid., 275.

> …if this calculus of probabilities one day rests on more certain foundations, it will be to M. d'Alembert that we will be obliged.[80]

Ultimately, d'Alembert focused the majority of his energies on the advancement of the *philosophes* in the French Academy, and on the Encyclopédie from the 1750's forward. By this time he relegated his work on mathematics further down on the list of his priorities, though he did continue to publish the occasional article in mathematics. He also collected all of his mathematical exploited into a series of volumes published between 1761 and 1780 titled *Opuscules Mathématiques*.[81] However, these volumes lacked sufficient organization, and the last five "contain a jumble of notes, comments on earlier work … but no new solid research—nothing well organized or complete."[82] By the time of his impending death d'Alembert had outlived the majority of his philosophes comrades and in his final correspondences expressed a disappointment about no longer being capable of working on mathematics due to illness and lack of the necessary concentration. Despite his divergence from mathematics following the 1750's d'Alembert always maintained that exact science was superior to art. One anecdote characterizes his standard of valuable academic pursuit particularly well:

[80] Condorcet *1847*, 92. Quoted in Daston, "D'Alembert's Critique on Probability Theory," 275.
[81] Hankins, *Science and the* Enlightenment…, 139.
[82] Ibid., 139.

In a desert island … I should think that a poet could scarcely be vain, whereas a mathematician might still enjoy the pride of discovery. [83]

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[83] Jean Le Rond d'Alembert, unknown publication. Quoted in Eves, *Mathematical Circles*, vol. II, 214.

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