Deconstructing Descartes: An Analysis of the Mathematical Influences on Descartes' Philosophy

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1. Introduction

During the winter of 1619, a young René Descartes weathered the frigid Danubian cold while fighting with the Bavarian army. During his brief military career, Descartes frequently stayed in an overheated room known as a "*poêle*," the French word for stove (Burton 363). In the "*poêle*," Descartes continued his academic studies in warmth while the rest of the army labored in the cold outdoors (Burton 363). On one particular evening, the heat lulled Descartes to sleep, and he experienced an event that would change the rest of his life. He dreamt three vivid dreams in which he learned "'the foundations of a marvelous science'" and discovered his calling to become a mathematician and philosopher (Burton 363). René Descartes devoted the remainder of his life to advancing the fields of mathematics and philosophy.

While most historians recognize René Descartes for his philosophical contributions like the creation of Rationalism that established modern philosophy, few realize the mathematical influences upon which Descartes' philosophy was based. Born to a French noble family in 1596, René Descartes spent most of his childhood and adolescence studying at the "Jesuit College of La Flèche" one of the most prestigious schools in France at the time (Burton 362). He found mathematics the most interesting subject "because of the certainty of its demonstrations" (Burton 362). Descartes later earned a degree in law before traveling throughout Europe as a volunteer in the armies of both Holland and Bavaria, where he had his life-altering, heat-induced dream (Burton 363). Descartes published many works during his lifetime covering academic topics like physics and astronomy, but his two most significant works: *The Geometry of René Descartes* and *Discourse on the Method* explored his two favorite subjects: mathematics and philosophy. Both works were monumental in scope in their respective fields. *The Geometry of René Descartes* combined geometry and algebra to create analytical geometry, while *Discourse on the Method* described Descartes' philosophy of Rationalism. What few have recognized, however, is how the mathematics of *The Geometry of René Descartes* impacted the philosophical development of *Discourse on the Method*. Descartes' analytical geometry strongly influenced his philosophical method of Rationalism as it offered a proof-like structure and established its goal of confidently seeking scientific knowledge for the benefit of humanity.

To analyze the influence mathematics had on Descartes' philosophy, one must first have at least a rudimentary understanding of his Rationalism. Descartes argued that humans have the capacity to understand all parts of the natural world by applying themselves to rational scientific study. Scientists must begin their inquiry by doubting all previous knowledge. For example, in order for mathematicians to prove a theorem, they first reject all previous knowledge about the problem, and only those theorems, postulates, and axioms that have already been proven are accepted. In this way, a mathematician's proof is shown absolutely true if it stems solely from previously proven, factual ideas. Rationalism then instructs scientists to carefully examine and collect information from the simplest to the most complex, only accepting information that is completely logical and without doubt as fact. This process references the methods of the great mathematician Euclid. Euclid began his *Elements* by defining and proving the most basic definitions, postulates, and common notions – such as the definition of a circle – before moving to more advanced proofs, like the Pythagorean Theorem, which utilized combinations of previously proven truths. Descartes believed that people have a duty to seek truth and knowledge in the sciences since the discoveries made in these fields can improve the lives of others (Descartes, *Discourse* 78). This paper will detail more of the specifics of Descartes' philosophy as it examines how mathematics impacted his theory.

2. Mathematical Inspirations for the Method

2.1 Descartes' Mathematical Method

René Descartes became interested in and studied mathematics long before diving into philosophy, so this paper will first examine the method behind Descartes' mathematics before connecting it to his Rationalism. *The Geometry of René Descartes*, like most other mathematical works, develops its claims and ideas through solving example problems. Descartes explains his method to solve mathematics problems in the first section of his work by first assuming that a solution exists for his given problem. Then, he assigns names to all the lines needed to construct the problem, both the known lines and the unknown lines (Descartes, *Geometry* 28). After assigning names, Descartes uses algebra to manipulate the quantities of each line so that each unknown quantity is expressed by an equation (31). Next, Descartes manipulates the equations so that there will be only one equation with an unknown quantity to a power equaling either a constant, linear equation, quadratic equation, cubic equation, etc. (31). Finally, Descartes solves this equation to find the unknown quantity (32).

2.2 Solving a Quadratic Geometrically using Descartes' Mathematical Method

To illustrate this point, Descartes solves the quadratic equation $z^2 = az + b^2$ geometrically with z representing an unknown quantity (35). To increase the clarity of the problem, the variable z representing the unknown quantity shall be represented as the letter x since modern mathematics primarily uses the variable x to represent unknown quantities. Descartes begins this problem by first "constructing right triangle NLM with one side LM, equal to b, … and the other side LN equal to ½ a" (35). Next, he extends the hypotenuse to O "so that NO is equal to NL, the whole line OM is the required line z" (35). Descartes arrives at these measurements for the sides of the triangle by manipulating the given quadratic equation to have it equal the constant b^2 term. Descartes constructs the following figure.

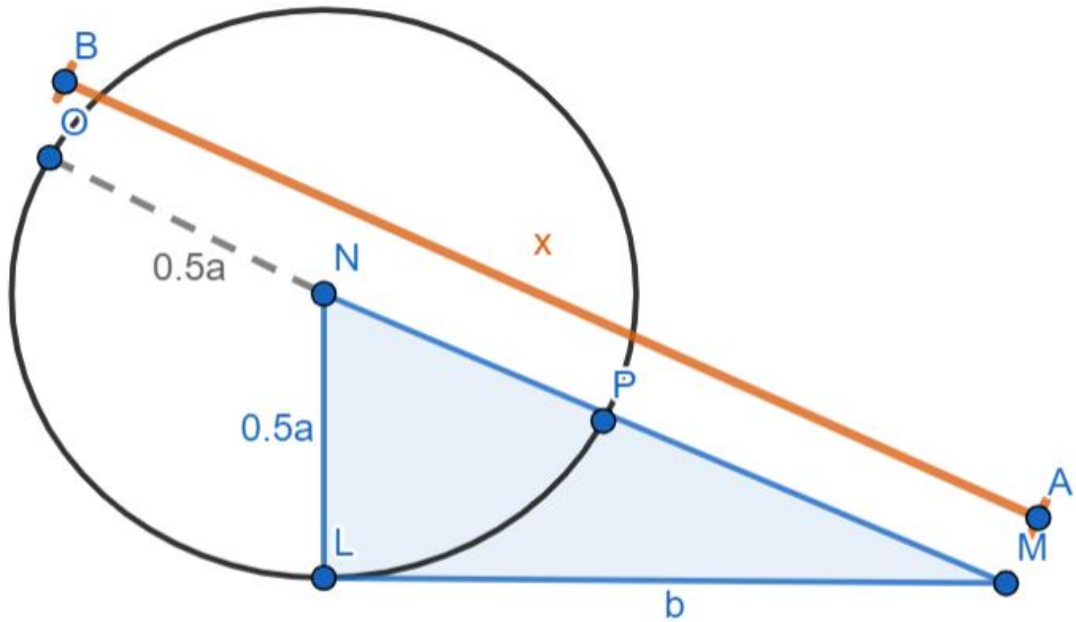

Figure 1: Geometric Figure to Solve $x^2 = ax + b^2$ Recreated in Geogebra (Descartes, *The Geometry of René Descartes* 34).

The following equations show the algebra Descartes used to make the variables equal b^2.

$$x^2 = ax + b^2$$

$$x^2 - ax = b^2$$

$$x(x - a) = b^2$$

The lines of the triangle then correspond to the variables of this equation.

$$OM = x$$

$$PM = x - a$$

$$LM = b$$

$$NL, NP, ON = \frac{1}{2}a$$

He then expresses the equation of the line:

$$x = \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 + b^2}\ .$$

Descartes continues describing how this method applies to all quadratics except for the following equation: $x^2 = ax - b^2$ (35). For this equation, Descartes constructs a different figure. He continues to draw lines $LM = b$ and $LN = \frac{1}{2}a$, but rather than forming a triangle by drawing a side MN he instead draws "line MQR parallel to LN, and with N as a center describe[s] a circle through L cutting MQR in the points Q and R" (36). The following is a drawing of the figure described above. (37).

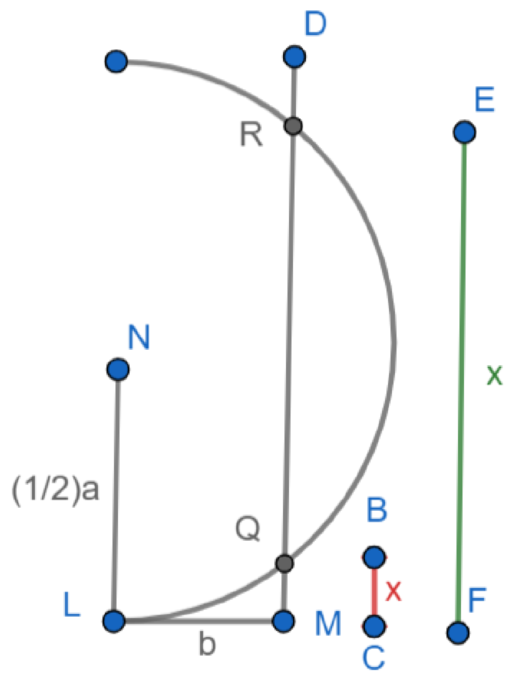

Figure 2: Geometric Figure to Solve $x^2 = ax - b^2$ Recreated in Geogebra (Descartes, *The Geometry of René Descartes* 37).

Descartes concludes that the desired line x is "either MQ or MR" which can be expressed by one of the equations below.

$$x = \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - b^2}$$

$$x = \frac{1}{2}a - \sqrt{\frac{1}{4}a^2 - b^2}$$

By using this method, Descartes described a way to solve quadratic equations geometrically. One can see how the semicircle LQR resembles a quadratic function with line MR and line LM serving as the x-axis and y-axis respectively. This example serves as a precursor to Descartes' development of the Cartesian plane which will be discussed later in this paper. Notice how Descartes arrived at the equations for both of these problems by adhering to his mathematical method. Descartes assumed that a solution existed for a geometric problem and reduced it to one quadratic equation with unknown quantities. He then constructed geometric figures that allowed him to solve for the last unknown quantity using deductive reasoning. Descartes was able to solve for this final unknown quantity by examining its relation to other known quantities and then manipulating those relations to discover a solution. This idea of utilizing previously established truths to find new knowledge in mathematics was a major influence on Descartes' intellectual philosophy.

2.3 Relating Mathematics to Philosophy

Descartes' Rationalism as described in his monumental *Discourse on the Method,* is organized in a manner similar to the mathematical demonstrations described in *The Geometry of René Descartes*. At the beginning of *Discourse on the Method*, Descartes expounds on the variety of topics he studied in order to create his philosophical method. Of all the subjects he studied, none appealed to him more than mathematics "on account of their certitude and evidence of their reasonings" (Descartes, *Discourse on the Method* 13). As a young student, Descartes was fascinated by mathematics due to its binding authority on the truth. In order for a new mathematical discovery to become accepted, it had to be proven through deductive reasoning. Descartes believed that the same criterion applied to general philosophy as he remarked how the variety of conflicting values, fashions, and customs of people from differing cultures at different times in history indicated the fallibility of relying on one's own cultural perceptions as criteria of truth (24). In crafting his philosophy, Descartes explicitly mentioned his desire to create a method that combined "logic, and among those of the mathematics geometrical analysis and algebra" (25). Descartes had attempted to base his philosophy purely on mathematics but discovered that the geometry of the ancients and the algebra of modern mathematics were too abstract to use (26). Despite this drawback, Descartes still incorporated many elements from mathematics in his method as the subject provided him with the tools to make new discoveries about the natural world. The next section will discuss how the advent of the Cartesian plane corresponds to the structure of Descartes' philosophical method in *Discourse on the Method*.

3. Organizing Rationalism like a Proof

3.1 Creating the Cartesian Plane

Mathematics had a clear impact on René Descartes when he formulated his philosophical method of Rationalism. One of Rationalism's most obvious traces of mathematical influence is its proof-like organization. One can compare the reasons Descartes developed his geometric coordinate system to the methods he prescribed for gaining scientific knowledge through Rationalism. Descartes developed his Cartesian plane out of a need for a system to solve for the lengths of all lines in a figure without knowing the lengths of any of the lines. His solution involved measuring line segments of unknown quantities in terms of two designated segments. These two designated reference segments gradually evolved into the x-axis and y-axis mathematicians use today. Descartes named these two referential line segments x and y in the following problem.

Descartes solved for the coordinate location of the point C in the figure below.

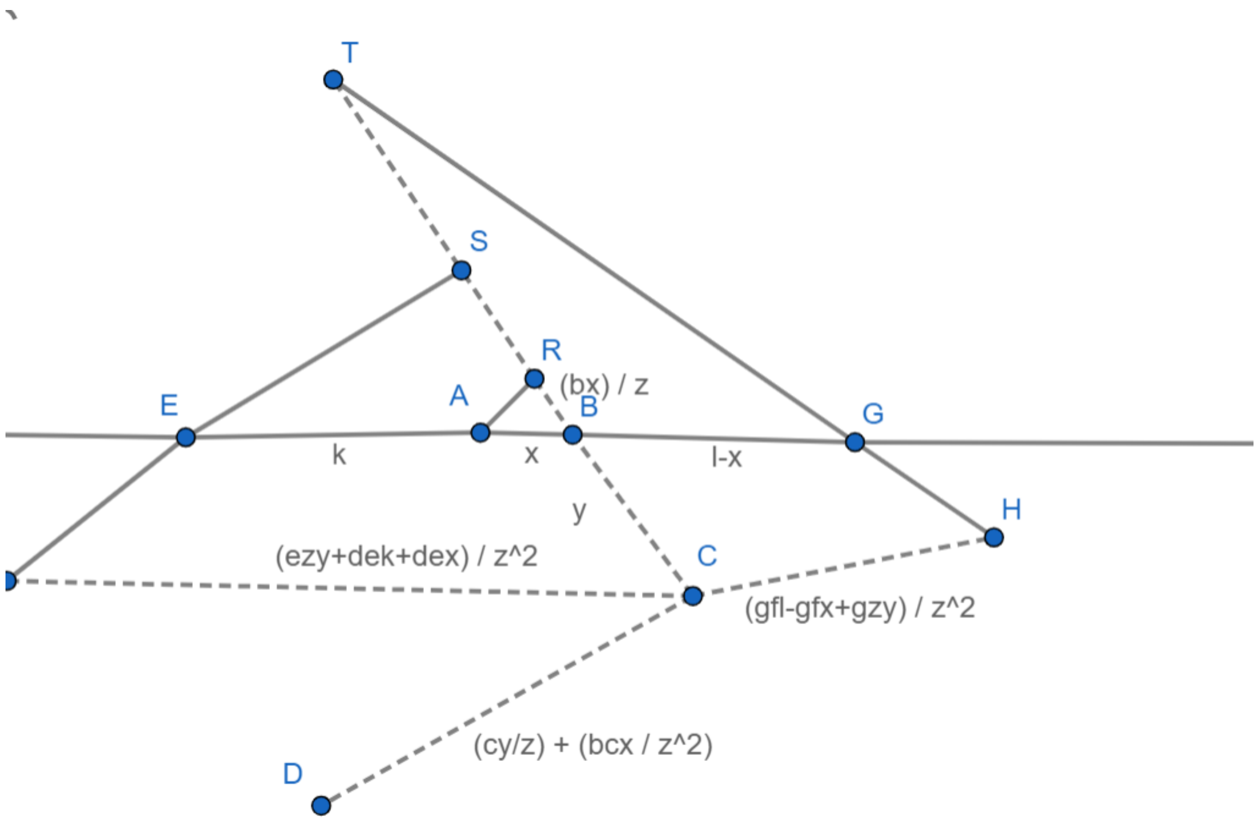

Figure 3: Geometric Figure Solving for Coordinate Location of Point C Recreated in Geogebra (Descartes, *The Geometry of René Descartes* 53).

Descartes began solving the problem by setting $AB = x$ and $BC = y$ (Descartes, *The Geometry of René Descartes* 51). He denoted these two line segments as the unknown quantities of x and y so he had quantities to which he could refer lines in relation to the rest of the terms in the figure. Next, he drew lines from the remaining points to meet AB and BC as long as they were not parallel to either x or y, because drawing a parallel line would negate the benefits of defining AB and BC as referential axes. Since BC cuts AB forming the triangle ARB, and AB equals x one can have:

$$AB: BR = z: b$$

Following this understanding, one can solve for BR.

$$\frac{x}{BR} = \frac{z}{b}$$

$$\frac{bx}{z} = BR$$

This is equal to $\frac{bx}{z} = BR$. Since the point B lies between CR, Descartes wrote that $CR = CB + RB$ which amounted to $CR = y + \frac{bx}{z}$. This formed the triangle DRC where Descartes solved for line CD.

$$\frac{z}{c} = \frac{y + \frac{bx}{z}}{CD}$$

$$CD = \frac{cy}{z} + \frac{bcx}{z^2}$$

After solving for these sides, Descartes called $AE = k$ which allowed him to state $EB = k + x$. This created the triangle ESB where the ratio of BE to BS was equal to the ratio of z to d. This allowed Descartes to solve for the sides BS and CS.

$$\frac{k + x}{BS} = \frac{z}{d}$$

$$\frac{dk + dx}{z} = BS$$

The line CS is composed of lines BC and BS which equates to $CS = \frac{zy+dk+dx}{z}$. This calculation makes the triangle FCS known and allows us to solve for side CF using the ratio $CS{:}CF$ equalling $z{:}e$. This makes $CF = \frac{ezy+dek+dex}{z^2}$.

$$\frac{\frac{cy + dk + dx}{z}}{CF} = \frac{z}{e}$$

$$\frac{ecy + edk + edx}{z^2} = CF$$

Descartes then assigned the line $AG = l$ and from that line $BG = l - x$. Descartes used this new line to make the triangle BGT with the lines BG and BT equalling the ratio of $z\!:\!f$. The following solves for BT

$$\frac{l-x}{BT} = \frac{z}{f}$$

$$\frac{fl - fx}{z} = BT$$

This allows us to calculate the length CT which is the sum of BT and BC equalling $CT = \frac{fl-fx+zy}{z}$. Descartes repeated this process one last time creating a triangle TCH using the ratio of the lines $TC\!:\!CH$ and $c\!:\!g$. This resulted in

$$\frac{\frac{fl - fx + zy}{z}}{CH} = \frac{z}{g}$$

$$\frac{gfl - gfx + gzy}{z^2} = CH$$

René Descartes finished this problem by explaining how this method can be applied no matter how many given lines pass through the point C (55). All these lines will equal "the unknown quantity y multiplied or divided by some known quantity; another consisting of the unknown quantity x multiplied or divided by some other known quantity; and the third consisting of a known quantity" (55). Descartes was able to solve this problem due to his decision to assume that lines AB and BC were equal to two unknown quantities he could use to describe the position of every line on the plane. By making one small logical assumption, Descartes was able to deduce the length of every single line of the plane without knowing any of the measurements beforehand.

Through this demonstration, Descartes opened an entirely new field within mathematics, analytical geometry, that moved beyond the geometry one could perform using only a compass and a straight edge. The creation of the coordinate plane provided mathematicians with a canvas on which to experiment with new curves. By utilizing Descartes' x/y coordinate system, mathematicians had the ability to create and solve plane figures with any number of unknown lengths. The ideas of analytical geometry set the foundations for modern mathematics and directly contributed to the formation of calculus by Isaac Newton and Gottfried Leibniz in the decades after Descartes' death. Both the way in which Descartes solved the previous problem and the significance of his finding this problem's solution heavily influenced the method and purpose of Rationalism. The next section will compare how Descartes' philosophical method resembled his approach to solving mathematics problems.

3.2 Connecting Rationalism to Geometry

Descartes' primary goal in writing *Discourse on the Method* was to find a method one could utilize to deduce truths about the world. Descartes took heavy inspiration from mathematics when he described the four rules that allow any person to find verifiable truths about the universe. He said the first rule was to "never accept anything for true which I did not clearly know to be such" and to accept only "what was presented to my mind so clearly and distinctly as to exclude all ground of doubt" (Descartes, *Discourse on the Method* 27). Descartes explained that he would at first doubt everything and accept knowledge only as it was proven. This first rule clearly stemmed from his experience with mathematical proofs as mathematicians accept new theorems only if they clearly proceed from conclusions free from errors and doubts. Descartes advises scholars in his second rule "to divide each of the difficulties under examination into as many parts as possible, and as might be necessary for its adequate solution" (27). The second rule again followed the methods mathematicians use to solve complex mathematical problems. In the previous mathematical example described in this paper, Descartes found a solution to the whole problem by breaking up the given figure into parts and then solving for the lengths in question. Descartes' third rule described how he ascertained new information. Descartes gradually expanded his knowledge by connecting already-proven, simpler truths to new, more complex conclusions (27). He argued that by starting with simple truths, one could make logical conclusions that verify information of increasing complexity. This bears a striking resemblance to the way mathematicians produce new theorems and postulates. A mathematician begins proving a theorem by first assuming a set of previously proven statements is true. Then, using the already-proven statements, the mathematician applies those previously established truths to verify his/her new proof. The mathematics historian R. H. Moorman noticed this pattern in Descartes' writing. Moorman explained that "just as mathematicians deduce theorems from an array of postulates and assumptions, Descartes used deduction from a set of first principles, proceeding always from the simplest to the most complicated" (Moorman 299). Moorman also argued that Descartes' first and most important common notion was his realization of "*cogito ergo sum*" or "I think therefore I am" (301). Descartes' method for gaining knowledge emerged from this single axiomatic statement. After Descartes became assured of his existence, he had a principle upon which he could assimilate other truths. His fourth and final instruction for scholars was to rigorously and carefully verify discoveries to make sure they left no room for error (Descartes, *Discourse on the Method* 27). This rule proceeded from the field of mathematics as mathematicians must review the work of their peers to ensure that new proofs are accurate and logical before they are accepted as fact.

One can see Descartes' philosophical method at work in the third problem that was solved earlier in this paper. He removed any assumptions previous mathematicians held about the problem as those assumptions could not be verified. He did this in part because no one before Descartes solved the problem. He followed his first law by doubting everything about the problem until it could be proven. Next, Descartes found the solution to the problem by utilizing the factual assumption that he could measure all of the lines in the figure in terms of two referential lines with unknown quantities of x and y respectively. This foundational assumption correlated to his assertion "I think therefore I am" which allowed him to accumulate more proven knowledge (44). Following this assumption, Descartes divided each section of the figure into smaller parts, most of them being triangles and line segments. By dividing the problem into smaller sections, Descartes attended to the second rule of his method. He solved the figure, proceeding from simple triangles and lines to more complex triangles and lines composed of two or more composite lines. This action corresponds to his third rule, whereby one gains knowledge by beginning with the simple and ascending to the complex. After solving the problem, Descartes presented solutions for additional problems similar to the original only with slight variance. He adhered to the final rule of his philosophical method as he repeated the same mathematical process to solve similar figures to check for accuracy. Each figure differed only slightly from the original. Clearly, Descartes organized his philosophy of Rationalism in a manner very similar to his method of solving and proving mathematical problems. In the next section, we will examine how Descartes' analytical geometry aided him in establishing Rationalism's goal to confidently seek scientific knowledge for the benefit of humanity.

3.3 The Mathematical Goal of Rationalism

The goal of any philosophy is to provide a method with which people can apply to improve their lives. To facilitate others living their best lives, philosophers offer an *epistemology*, or a method describing how one can find, verify, and classify knowledge. Throughout history, philosophers have centered their epistemologies around finding and verifying spiritual or ethical truths. Until the time of Descartes, few philosophers had offered a complete epistemology to explain how one might best gain knowledge in the natural sciences (including mathematics). The historian A. J. Snow in his article "Descartes' Method and the Revival of Interest in Mathematics" recognized how scientists of the 17th century needed "a new method which would put philosophy on a firm foundation" (Snow 612). In the centuries prior, scholars had followed the philosophy of the medieval "Scholastics" which emphasized explaining *metaphysics*, the philosophy concerned with reality itself, the soul, and the existence of God (612). Descartes trained under "Scholastic" teachers in school but rejected their philosophy because he felt it focused too much on metaphysics, a field "whose results were always being disputed" and therefore was "useless for the discovery of truth" (612). The scientific need for a strong method to discover and document new information about the natural world prompted Descartes to write *Discourse on the Method*. This work provided both a method scientists could use to apprehend new truths about the universe and a mission for scientists to purposefully pursue their studies.

In the final section of *Discourse on the Method*, Descartes gave an explanation for why he wrote this work. He believed it was of public interest for him to publish *Discourse on the Method* as he deemed it important that others acquire an understanding of the methodology he proposed for the advancement of knowledge (78). Descartes declared that he had made discoveries in physics that he "could not keep them concealed without sinning grievously against the law by which we are bound to promote, as far as in us lies, the general good of mankind" (78). From these scientific revelations, Descartes professed that people could "arrive at knowledge highly useful in life" which rendered humans "the lords and possessors of nature" (78). He firmly believed that scientific discovery was entirely beneficial for society. He desired that science be used to improve the human condition through "the invention of an infinity of arts, by which we might be enabled to enjoy without any trouble the fruits of the earth" as well as improving the health of people through advances in medicine (78). Descartes believed that his method provided scientists (including mathematicians) with the tools and confidence needed to continue the quest for truth in their chosen fields of study (79). Rather than merely observing the natural world as scholars had done in the past, Descartes encouraged scientists to enthusiastically seek out the truth about our world in an effort to improve the circumstances of humanity. Scientists inherited from Descartes a verified method with which to gain new knowledge, and he hoped they would use his work to bring about great benefits for humankind.

Descartes designed his philosophy of Rationalism with the intent to provide a method and a purpose to investigate science (including mathematics). His development of analytical geometry had an immense influence on his philosophy in *Discourse on the Method*. The historian R. H. Moorman noted that Descartes "wanted to expand the realm of pure geometry to the external world" (*Moorman* 304). He added that "the extension of the mathematical method to the universality of cosmological problems, was an innovation in the seventeenth century" (304). Moorman believed Descartes wanted to universally apply the method he used in mathematics to all sciences. Descartes was confident that his method would permit any person to make new discoveries about the natural world and use that new information to improve society.

4. Conclusion

Descartes lived at a time in which scientific advancements occurred rapidly. He noticed that the scientific world was expanding quickly, but it was using an antiquated method of learning, verifying, and classifying knowledge. He himself experienced these difficulties as he struggled to prove the validity of his mathematical achievements using the medieval "Scholastic" philosophy that was prevalent in his time. Through his efforts in creating the field of analytical geometry, Descartes formulated his own epistemology which allowed him to find truths about the natural world. Both the structure and goal of Descartes' method in his *Discourse on the Method* reflect his mathematical expertise. The organization of Rationalism resembles a mathematical proof. Descartes aspired to equip scientists with a method to expand their fields of study just as he had done in creating the mathematical field of analytical geometry. One can hardly understate the impact Rationalism had on the history of science and mathematics. In the decades following the publication of *Discourse on the Method*, mathematicians and scientists made great advances in their fields in what historians deem the Scientific Revolution. Later scientists developed their own philosophies in reaction to Rationalism like Francis Bacon's Empiricism. Each of these philosophies shared a common goal with Descartes' Rationalism: they all sought to find the best way for scientists (including mathematicians) to make discoveries that would improve the world. Had Descartes failed to experience his fateful fever dream, Western science may not have experienced the rapid advancements it needed to bring about positive change in the lives of millions of people.

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