Estimations of π: The Kerala School of Astronomy and Mathematics, The Gregory-Liebniz Series, and the Eurocentrism of Math History

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Introduction

Long before it received its current name and value, the idea of pi was explored. Because of the common use of circles for tools and simple machines, it became important for ancient civilizations to be able to calculate the area and circumference of a circle relative to its radius. Before the current ratio of circumference to diameter was discovered, ancient peoples used approximations for what came close to finding the actual area of a circle. As time progressed, these approximations became more accurate. For example, records of the use of pi during the time of the Babylonians (1900 BCE), can be found as 3 or 3.125, while the ancient Egyptians used 3.1605 ("A Brief History" n.d.). As times changed and mathematical knowledge progressed, mathematicians were able to develop calculations for the approximation of pi. These improved calculations were accompanied by new mathematical proofs rather than half formed conjectures. Particular proofs of note, and the foci of this paper, are the works of Mādhava of Saṅgamagrāma and James Gregory and Gottfried Leibniz.

The Kerala School of Astronomy and Mathematics

When asked to think about the history of mathematics, a number of famous mathematicians may come to mind. For example, many may recall Pythagoras and his famous theorem for the hypotenuse of a triangle. Or one might think of the invention of calculus and the storied rivalry of Newton and Leibniz. For the more seasoned mathematician, Euler or Gauss may seem more impactful. In any case, Mādhava of Saṅgamagrāma is unlikely to be at the forefront of one's mind. And yet, Mādhava and his legacy of the Kerala School of Astronomy and Mathematics, not only made major contributions to both of these subjects but also made many of the aforementioned contributions before parallel discoveries were made in Europe.

Mādhava established the Kerala School in the 14th century in the city of Kerala, nestled between the Western Ghat mountains and the Arabian Sea at the southern tip of India. Mādhava was born in 1360 in the town of Bakulavihāra— modern-day Irinjalakuda— into the Brahmana caste— the most well-respected caste made up of those held in the highest esteem. He was specifically an Empranthiri— a subcaste unique to Kerala (Plofker 2009). During this age, Hinduism had a strong cultural influence and the caste system stood as the main social structure of the Indian people. In Hindu tradition, the titles of "priest" and "educator" were typically reserved for those of the Brahmana class, but the Kerala school was unique in that it did not strictly abide by this convention. This is partially because the region of Kerala had a far more complex caste system than the rest of India with around 420 castes (Hargrave 2021).

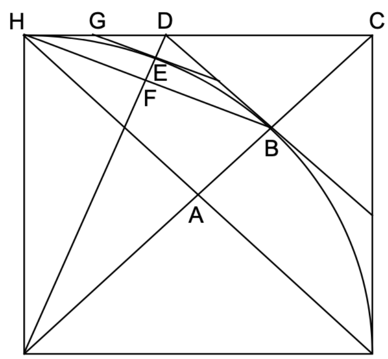
While Mādhava was indeed a Brahmana, many attendees of the Kerala school were members of lower castes or followers of a non-Hindu faith. For example, there are records of Muslim merchants, Nestorian Christians, and Jews in Kerala (Plofker 2009). Many of the students of the Kerala school were of the caste Ambalavasis. Similar to Mādhava's Empranthiri subcaste, Ambalavasi was a caste specific to Kerala, and Ambalavasis were traditionally allowed to join in intellectual studies, but lacked influence and status. Under the guidance of the Kerala school, several Ambalavasis were able to gain sway and receive recognition for their mathematical and astronomical success (Pingree 2014).

Among the Kerala school's students, there were several noted individuals whose works have survived. Of those directly influenced by Mādhava, Parameśvara is perhaps the most important. Living along the Nilā River, Parameśvara is the only known direct pupil of Mādhava. Although the exact timeline of his life and work is unknown, it is certain that Parameśvara had a long and successful academic career with twenty-five documented works on astronomy, astrology, and mathematics spread over a span of more than ninety years (Plofker 2009). Following Parameśvara was his son and student Dāmodara. While he is known to have been a member of the Kerala school, Dāmodara's work has unfortunately not survived to modern-day. Despite this fact, Dāmodara's influence has not been entirely lost, as works from his students Nīlakaṇṭha and Jyeṣṭhadeva remain. Nīlakaṇṭha (born 1443) (Pingree 2014) is best known for his work in astronomy in which he authored the Tantra-sangraha— a groundbreaking piece of astronomical literature. Jyeṣṭhadeva, the second student of Dāmodara, is best known for his work the Yukti-bhasa which provided proofs for the mathematical procedures in the Tantra-sangraha (Plofker 2009).

One of the final contributors out of the Kerala school was an Ambalavasi named of Śaṅkara. Śaṅkara was a student of both Nīlakaṇṭha and Jysethadeva. During the mid-16th century, Śaṅkara wrote commentary on both the Tantra-sangraha and the Yukti-bhasa (Plofker 2009). In addition to Śaṅkara's original works, he included extensive coverage of Mādhava's work in his publications— Yukti-dipika and Kriya-kramakari. Śaṅkara's works have provided much of the knowledge we have regarding Mādhava's proofs.

Mādhava and π

While Mādhava's work is extensive and a number of his proofs were groundbreaking for his time, the one that we will focus on in this paper is his approximation of pi. In his approximation of pi, Mādhava established two methods to find the circumference of a circle based on their radius. Both of these methods are outlined in Śaṅkara's writings. The first proof discussed by Śaṅkara relies on calculating the perimeter of many successive polygons. To do this, a circle of radius, r, is constructed and a regular square, octagon, and 16-gon are constructed around it. By definition, the square will have a side length of $2r$ as the sides are congruent to the full diameter of the circle. Then, two tangent lines DB and GE are constructed within the square. Point B bisects the first tangent line, and point E bisects the second. The construction of the first quadrant of the circle can be seen below (Plofker 2009):



The next step is to find the length of CD. This is done using the similarity of triangles HAC and DBC. This similarity can be seen as both are 45-45-90 triangles. Using the similarity of these two triangles, the following equation follows:

$$CD = BC \cdot \frac{CH}{AC}$$

In this equation, we let $s = 2r$ which is the length of the circumscribed square. The length of BC can then be found as the length from the center of the circle to point C (OC) minus the length from the center to point B (OB) giving $BC = OC - OB$. The length of OC can be found using the Pythagorean Theorem which gives us

$$OC = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2} = \sqrt{\frac{2s^2}{4}} = \sqrt{\frac{s^2}{2}} = \frac{s}{\sqrt{2}}$$

The length of OB is the radius of the circle, making $OB = \frac{s}{2}$, thus, $BC = \frac{s}{\sqrt{2}} - \frac{s}{2}$.For the second part of the proportion, lengths CH and AC must be identified. CH is one half of the side of the square giving us $CH = \frac{s}{2}$. The length of AC can be found by using the Pythagorean Theorem to find OC and dividing by 2. This gives us $AC = \frac{s}{2\sqrt{2}}$. This can then be substituted into the original equation to find CD as follows:

$$CD = \left(\frac{s}{\sqrt{2}} - \frac{s}{2}\right) \cdot \frac{s/2}{s/(2\sqrt{2})} = \left(\frac{s}{2\sqrt{2}} - \frac{s}{4}\right) \cdot \frac{s}{s/(s/\sqrt{2})}$$

Using the length of CD, the length DH is found which gives half of the length of the side of the octagon circumscribing the circle. Doubling this gives the length of the octagonal sides. Using the length of the octagonal sides, it is now possible to find the length of the sides of the 16-gon. The first vertex of the 16-gon is seen marked as point G on the construction. A similar process using similar triangles is constructed and the following equation is produced (Plofker 2009):

$$DG = DE \cdot \frac{DH}{DF} = \left(\sqrt{DH^2 + r^2} - r\right) \cdot \frac{DH}{DH^2/\sqrt{DH^2 + r^2}}$$

This can be used to find GH which is half of a 16-gon side. This same process can be repeated to find a 32-gon and so on and so forth to reach a circumference as precise as desired. While this construction can be refined indefinitely, each iteration requires a heavy amount of construction and increasingly complicated calculations.

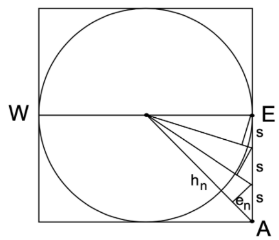
In his second method of approximating circumference, Mādhava established an infinite series with a correction term that is defined as follows (Plofker 2009)[1]:

$$C \approx \frac{4D}{1} - \frac{4D}{3} + \frac{4D}{5} - \cdots + (-1)^{n-1}\frac{4D}{2n-1} + (-1)^n \frac{4Dn}{(2n)^2 + 1}$$

While using this series to estimate circumference estimate is much easier than the first method shown, the proof is far more complicated. The general idea of the proof is to find incrementally

[1]The final term of the series is the correction term **not** the general term for the series

small arclengths of the circle using a series of constructed right triangles seen below (Plofker 2009):



Within this construction, a hypotenuse line, h_i, and a perpendicular edge e_i are identified in which $h_i = \sqrt{r^2 + (is)^2}$ and $e_i = \frac{sr}{h_i}$. Using these two values, the arc c_i can be found as $c_i \approx \frac{e_i r}{h_i - 1} = \frac{sr^2}{h_i h_{i-1}} \approx \frac{sr^2}{h_i^2}$. This only works under the assumption that sequential values of h_i are significantly close because we make the assumption that $h_{i-1} \approx h_i$. To continue the proof from here requires a method of finding c_i as a series and a way to combine these c_i together. These two factors can eventually be used to find a eighth of the circumference, the equation for which can be seen below (Plofker 2009):

$$\frac{C}{8} = \sum_{i=1}^{n} c_i \approx r - \frac{s}{r^2}\sum_{i=1}^{n}(is)^2 + \frac{s}{r^4}\sum_{i=1}^{n}(is)^4 - \frac{s}{r^6}\sum_{i=1}^{n}(is)^6 + \ldots$$

With more simplification, we arrive at:

$$\frac{C}{8} \approx r - \frac{1}{r^2}\frac{r^3}{3} + \frac{1}{r^4}\frac{r^5}{5} - \frac{1}{r^6}\frac{r^7}{7} + \ldots$$

which when D is substituted for $2r$, is the same as our starting equation.

Using his proof, Mādhava was able to reach an approximation of pi stating that, $\pi = \frac{2827433388233}{900000000000} \approx 3.14159265359$ which is accurate to the eleventh decimal place. This approximation was then expressed in verse using both the bhūtasaṅkhyā and the kaṭapayādi systems. The bhūtasaṅkhyā system is a method of numeric expression in which words are used to represent different numbers (Pingree 2014). Each word corresponds to generally accepted values of these words, mostly related to Hindu culture. For example, the word "Gods" is equal to the number thirty-three due to either the 330 million total Hindu gods or the thirty-three kinds of gods. We see thirty-three used here because unlike our modern base ten, during

Mādhava's time a sexagesmal system was the standard. In addition, these numbers are taken in reverse order— beginning with the ones and tens places and working their way up. In this case, Mādhava's approximation is written as, "Gods [33], eyes [2], elephants [8], snakes [8], fires [3], three [3], qualities [3], Vedas [4], nakṣatras [27], elephants [8], and arms [2]— the wise say that this is the measure of the circumference when the diameter of a circle is nine hundred billion" (Pingree 2014). The kaṭapayādi system of representation assigns consonants to each number zero through nine. Then by using these consonants, followed by a vowel, sentences can be formed as a representation of equations.

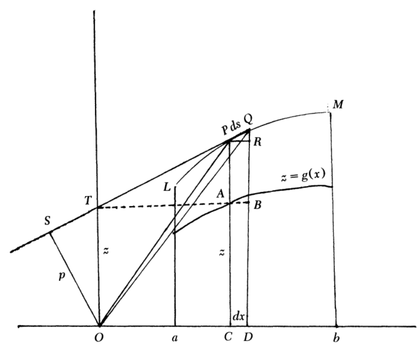
Mādhava's approximation of pi was not his only mathematical contribution. In fact, he had several, arguably better known, proofs including infinite series approximating the trigonometric functions. One of Mādhava's most well known verses is for his series for sine, written using the kaṭapayādi system. In it Mādhava writes, "The ruler whose army has been struck down gathers together the best of advisors and remains firm in his conduct in all matter; then he shatters the (rival) king whose army has not been destroyed" (Pingree 2014). This verse corresponds to the first six terms of the power series for sine using a sexagesimal translation. Similarly, series for cosine and tangent were also discovered and recorded by Mādhava. These discoveries along with the subsequent development of a series for arctangent, lead to another approximation of pi by the name of the Gregory-Leibniz series.

Gregory-Leibniz

The Gregory-Leibniz series utilizes the infinite series for arctangent to approximate pi. This series was independently discovered and proved by both Gregory and Leibniz in the mid-1600s and thus both receive the credit for this approximation (Roy 1990). There are however, many disputed claims on whether or not Gregory found the specific case of $\tan^{-1}(1)$ or $\frac{\pi}{4}$ which is what Leibniz used to find pi. This case was essential to the approximation at the time because it meant that the series was made up entirely of rational numbers. Regardless of this fact, both of these men's proofs are acknowledged as crucial for this approximation of pi.

Gottfried Leibniz was born 1646 in Leipzig, Germany where he spent his early years and received his undergraduate education (Look, Belaval 2021). After being denied his doctorate of law from the University of Leipzig, he moved to Nürnberg where he received his doctorate before eventually traveling to Paris (Look, Belaval 2021). It is in Paris that Leibniz met Christiaan Huygens who inspired him to begin his study of geometry and expand on his knowledge of mathematics (Roy 1990). Leibniz's studies brought him to the work of Blaise Pascal which in turn inspired his construction of a geometric proof for the infinite equation representing $\frac{\pi}{4}$. This was done by employing the idea of an infinitesimal triangle (Roy 1990).

To begin his construction, Leibniz constructed a curve, $y = f(x)$. He then took this curve and constructed a triangle between the origin and two arbitrarily close points, P and Q, on the curve. This can be seen in the diagram below (Roy 1990).



In addition to triangle PQO, there are several other features of note. Line PT is tangent to the curve, $f(x)$, at P and OS is perpendicular to the tangent line. In addition, the constructed rectangle $ABCD$ is exactly double the area of the triangle OPQ. Leibniz also designates p as the length of OS and z as the length of OT. Then using the fact that triangle PQR is similar to OST then the following ratio can be established:

$$\frac{dx}{p} = \frac{ds}{z}$$

Based on this ratio, we find that area$(OPQ) = \frac{1}{2}pds = \frac{1}{2}zdx$. Then by establishing a point A from every point P a second curve $z = g(x)$ can be created. Then allowing the sector OLM to be the region enclosed by f and lines OL and OM, then area$(OLM) = \frac{1}{2}\int_a^b g(x)dx$. From this, the area under the curve f can be calculated as

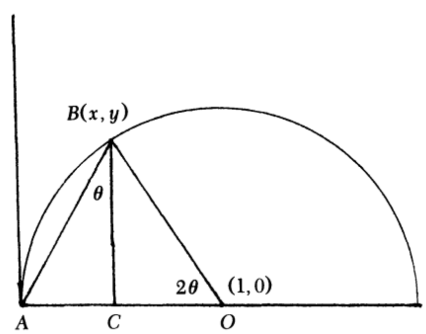
$$\begin{aligned}\int_a^b y dx &= \frac{b}{2}f(b) - \frac{a}{2}f(a) + \text{area (sector } OLM)\\ &= \frac{1}{2}\left(bf(b) - af(a) + \int_a^b g(x)dx\right)\\ &= \frac{1}{2}\left([xy]_a^b + \int_a^b zdx\right)\end{aligned}$$

From the diagram, z can be established as $y - x\frac{dy}{dx}$ and through substitution we arrive on

$$\int_a^b y dx = [xy]_a^b - \int_{f(a)}^{f(b)} xdy$$

which should be recognizable as integration by parts.

We now apply this to the particular case of a circle. With the designation that this circle is centered at $(1,0)$ and has radius 1, we arrive on the equation $y^2 = 2x - x^2$ and, using the previously established equation for z, find that $z = \frac{x}{y}$ and $x = \frac{2z^2}{1+z^2}$. An arbitrary θ is then created using the established circle as seen in the diagram below:



Using this, an equation for theta can be established as

$$\theta = z - \int_0^z \frac{t^2}{1+t^2}dt$$

From this, and using additional information from Nicolaus Mercator's work, we arrive on the equation

$$\theta = z - \frac{z^3}{3} + \frac{z^5}{5} - \dots$$

Since $\angle ABC = \theta$ and $z = \frac{x}{y} = \tan\theta$, this series becomes a definition for $\arctan z$

As Leibniz studied, a Scottish man by the name of James Gregory was completing much the same work. Gregory was born in 1638 Scotland (Malet 2021). He attended his initial schooling in Aberdeen, before traveling Europe and eventually settling in Italy (Malet 2021). In Italy, Gregory began his study of geometry and in turn discovered the work of many notable geometrists including Pierre de Fermat and Torricelli (Roy 1990). Following his studies in Italy, Gregory traveled to London before returning to his homeland of Scotland where he worked at the University of St. Andrews and then the University of Edinburgh (Malet 2021).

Before his time at Edinburgh, Gregory had already released several publications. The *Optica Promota* was his first book which focused on astronomy and described a reflecting telescope (Roy 1990). He published two more books during his time in Italy: *Vera Circuli et Hyperbolae Quadratura* and *Geometriae Pars Universalis*. Both of these books covered extensive geometric proofs— some more successfully than others— but one of the most important was in *Geometriae Pars Universalis* which provided a geometric proof to the fundamental theorem of calculus. Finally, Gregory published the *Exercitationes Geometricae* which discussed the logarithmic function as well as indefinite integrals for secant and tangent (Roy 1990).

During his time at Edinburgh, Gregory continued his mathematical explorations though he did not publish any of his works. Instead, records of Gregory's discoveries can be found in correspondence between Gregory and his friend John Collins who he had met during his time in London (Roy 1990). In these correspondences, Gregory described his binomial expansion for arbitrary exponents similar to the more generally accepted method by Newton. In addition Gregory used Newtons findings, sent to him by Collins, to derive equations for $\arctan x$, $\tan x$, $\sec x$, $\log \sec x$, $\log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, $\operatorname{arcsec}\left(\sqrt{2}e^{x}\right)$, and $2 \arctan \tanh\left(\frac{x}{2}\right)$ (Roy 1990). Gregory did nothing with these findings due to the fact that he believed himself to have found them using the method that Newton had already established. Further investigation into Greogry's work however, shows that he had actually done the opposite, instead outlining steps that would later become general rules for Taylor series. This was done by taking successive derivatives and operating them together with previous steps in the series (Roy 1990). While these were indeed groundbreaking discoveries, Gregory, believing his work to be unoriginal, never published, and passed away in 1675 before receiving any real credit for his work in calculus (Roy 1990).

From the work of both Gregory and Leibniz, a more modern proof for the approximation of pi can be established. This approximation uses the concept of Taylor series and integrals to create a series that, as the number of terms approaches infinity, the sum approaches the exact value of pi. We begin with a simple Taylor Series:

$$\frac{1}{1-y} = 1 + y + y^2 + \cdots + y^n + \ldots$$

This series is the base formula for a geometric series with common ratio: y, a first term of 1, and a domain limitation of $-1 < y < 1$ in order for the series to converge. While this is not extremely relevant in this stage of the process, it is important to keep in mind that a domain has been established and said domain will continue to need to be applied throughout the process. We then substitute the variable y for $-x^2$ and create the following series:

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 + \cdots + (-x^2)^n + \ldots$$

It is prudent to notice that this series is still a geometric series, now with a common ratio: $-x^2$ and there is still the domain restriction of $-1 < x < 1$.

For the next step of the process, we must recognize that the left side of the equation, $\frac{1}{1+x^2}$, as the derivative of $\arctan x$. Thus, to find $\arctan x$, we must take the integral of both sides of the equation as follows:

$$\tan^{-1} x = \int \frac{1}{1+x^2} dx = \int 1 - x^2 + x^4 + \cdots + (-x^2)^n + \ldots dx = \int \sum_{n=0}^{\infty} (-x^2)^n dx$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + \frac{(-1)^n}{2n+1} x^{2n+1} + C = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} + C$$

The C value can then be calculated used a scenario when $x = 0$. This process is as follows:

$$\tan^{-1} 0 = \sum_{n=0}^{\infty} \frac{(-1)^n (0)^{2n+1}}{2n+1} + C = 0 - \frac{0^3}{3} + 0^5 5 - \cdots + \frac{(-1)^n}{2n+1} 0^{2n+1} + C = 0$$

$$C + 0 = 0$$

$$C = 0$$

Having now obtained a complete Taylor series for arctangent, we can use the fact that

$\tan^{-1} 1 = \frac{\pi}{4}$ to find pi by setting x equal to 1 leaving:

$$\begin{aligned}\tan^{-1} 1 &= 1 - \frac{1^3}{3} + \frac{1^5}{5} - \cdots + \frac{(-1)^n}{2n+1} 1^{2n+1} + \ldots \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} 1^{2n+1} \\ &= \frac{\pi}{4}\end{aligned}$$

It is important to verify that this is a valid application of x to the equation. The original domain established in the first geometric series, states that $-1 < y < 1$, however, the while the center, 0, and radius, 1, are held constant, the changes made to the equation have altered the end behaviors— in this case both endpoints converge[2].

The final step in the estimation process is to multiply both sides by 4 in order to change $\frac{\pi}{4}$ into π.

$$\begin{aligned}4\left(\frac{\pi}{4}\right) &= 4\left[1 - \frac{1^3}{3} + \frac{1^5}{5} - \cdots + \frac{(-1)^n}{2n+1} 1^{2n+1} + \ldots\right] \\ \pi &= 4 - \frac{4}{3} + \frac{4}{5} - \cdots + \frac{4(-1)^n}{2n+1} + \ldots\end{aligned}$$

As the number of terms in the series approaches infinity, the value becomes increasingly closer to the true value of pi. While the initial equation is vastly inaccurate with $P_1 = 4$ and an error of over 1.33, as more terms are included, accuracy increases. For example, $P_{100} = 3.1316$ and has an error of less than 0.02.[3]

As more and more terms are added to the series, the rate at which the approximation becomes more accurate slows down. For example, $P_{1000} = 3.14059$, which is 900 more terms than P_{100}, is only more accurate by 0.01790 while P_{100}, which is only 99 more terms than P_1, is more accurate by 0.96970. A more complete list of approximations using this method can be seen below:

[2] This can be found through the alternating series test

[3] These errors can be calculated due to the alternating characteristics of the series for pi

Number of Terms	Value	Error
1	4	≤ 1.33333
5	3.33968	≤ 0.36364
10	3.04184	≤ 0.19048
20	3.09162	≤ 0.09756
50	3.12159	≤ 0.03960
100	3.13159	≤ 0.01990

The Gregory-Leibniz series at $x = 1$ converges extremely slowly. In order to get a more precise value more quickly, there are variations of this series that can be used. The simplest way to increase accuracy is to use a smaller value of x. For example, if we set x equal to $\frac{1}{\sqrt{3}}$ which is the tangent of $\frac{\pi}{6}$ the resulting series is (Lynn n.d.):

$$\tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\frac{1}{\sqrt{3}}^3}{3} + \frac{\frac{1}{\sqrt{3}}^5}{5} - \cdots + \frac{(-1)^n}{2n+1}\frac{1}{\sqrt{3}}^{2n+1} + \ldots$$

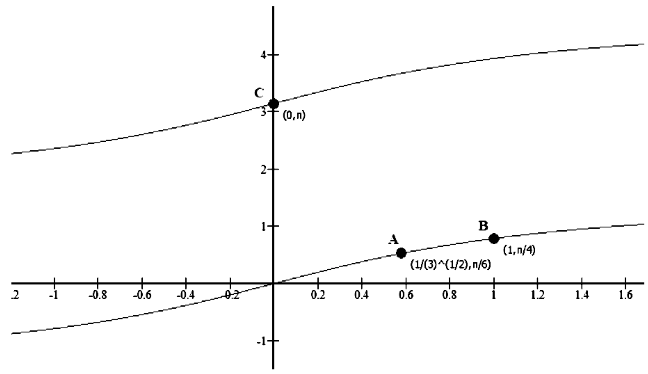
$$6\left(\frac{\pi}{6}\right) = 6\left[\frac{1}{\sqrt{3}} - \frac{\frac{1}{\sqrt{3}}^3}{3} + \frac{\frac{1}{\sqrt{3}}^5}{5} - \cdots + \frac{(-1)^n}{2n+1}\frac{1}{\sqrt{3}}^{2n+1} + \ldots\right]$$

$$\pi = \frac{6}{\sqrt{3}} - \frac{\frac{6}{\sqrt{3}^3}}{3} + \frac{\frac{6}{\sqrt{3}^5}}{5} - \cdots + \frac{6(-1)^n}{2n+1}\frac{1}{\sqrt{3}}^{2n+1} + \cdots = \sum_{n=0}^{\infty}\frac{6(-1)^n}{2n+1}(\frac{1}{\sqrt{3}})^{2n+1}$$

A list of approximations using $\frac{\pi}{6}$ and the corresponding error using alternating series properties can be seen below:

Number of Terms Added	Value	Error
1	3.46410	≤ 0.38490
5	3.14260	≤ 0.00130
10	3.1415905109	≤ 0.00000279
20	3.1415926535	$\leq 2.42316 \times 10^{-11}$

This simple change of the value for x that is being substituted has large effects on the convergence of the equation because it affects the constant that each term is multiplied (4 vs 6) which affects the size of the value of each term. This change is also important because it affects how close the starting value is to the true value of π. The $\frac{\pi}{6}$ multiplied by 6, which is what is used when $\frac{1}{\sqrt{3}}$ is used as the x value, is equal to 3.464 which is a starting value much closer to pi than 4 — the starting value when 1 is used as the value of x. This is because the value of $\frac{\pi}{6}$ is closer to having a matching value with pi on a regular tangent curve. The tangent of pi is 0 and thus the closer the starting tangent is to 0, the more rapidly the system converges. This can be easily seen on an arctangent graph.



As seen on the graph, point A has a closer x-value to that of point C than point B. This greater similarity to pi leads to a quicker rate of convergence.

While these series certainly converge more quickly, they require the ability to calculate irrational square roots which was not a highly developed method at the time. Isaac Newton would later publish his binomial theorem which could be used to approximate the values of any rational exponent but this would not appear in correspondence until 1676, after both Leibniz and Gregory had already made their respective discoveries.

Recognition

Credit for the creation of this series and approximation is extremely muddled. In addition to both Gregory and Leibniz working during the same time period but separately, Newton's development of calculus certainly plays a role in the modernized version of this proof. Not only that, Mādhava is generally acknowledged as having discovered the series representations of these trigonometric functions several centuries before the Europeans were even born, but is not given credit for this approximation of pi. In fact even Mādhava's credit is jumbled as his work has also been credited to Nīlakaṇṭha. This, of course, begs the question—why did Mādhava, or Nīlakaṇṭha, perhaps not receive sole credit for this work? And why does the historical lens of mathematics remain so firmly in Europe?

The answer largely comes down to an issue of isolation. While major ideas were explored and well documented within the Kerala school, dispersion of these discoveries was highly limited. Due to this seclusion, very few publications made it out of the Kerala school, and those that did only made it to surrounding areas of India. In addition to its physical separation, there were major issues of cultural isolation. While the Kerala school was somewhat progressive for its time, the rest of the nation remained heavily divided with only Brahmins being involved in education and information exchange (Dani 2012). This greatly restricted the passage of knowledge among scholars and kept publications from being shared to a wider audience.

Secondly, a major language barrier further limited the transfer of knowledge. Nearly all of the publications that came out of the Kerala school were published in either Sanskrit or Malayālam. Even if all of Mādhava's publications had reached Europe by the time Gregory and Leibniz had begun their work it is unlikely anyone would have been able to understand the information being conveyed. Because of this, when Gregory and Leibniz published their work, this Taylor series approximation had never been seen by the majority of the mathematical world. Naturally, the credit for this "discovery" went to these two men.

In addition, there is the fact that the Kerala School was far before its time and thus there were few people who would've been able to understand the concepts being put on by these mathematicians even without these other obstacles. The Kerala School rose and fell before European mathematics had reached its level of understanding and thus there was no one to give proper recognition to these brilliant minds. Although Vasco da Gama did make contact with the Kerala coast in the 16th century, it was not until the early 17th century, when the British East India company began to establish trade, that European contact with India truly began. However this trade was limited to east coast cities such as Calcutta (Plofker 2009). Exchange of mathematical knowledge between Europe and India did not begin until the late 1700s (Plofker 2009)— over a century after the work of Gregory and Leibniz. Just as Gregory did not receive credit for his works on calculus due to his untimely death, so did the Kerala School's praise remain unsung.

Although the Kerala school has not received recognition and credit for its mathematical and astronomical discoveries, its contributions should not be understated. This school stands as a phenomenal representation of academic excellence beyond a Eurocentric scope.

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