This paper studies the *Libra Astronómica* by Carlos de Sigüenza y Góngora, printed in Mexico in 1690, and in particular the mathematics found therein. It is the earliest example of the use of decimal notation and logarithms in print and in the New World. The paper begins with a summary of the interpretation of comets, followed by an introduction to the two main characters, Carlos de Sigüenza y Góngora and Eusebio Francisco Kino. Both authors wrote about comets, but their views differed considerably, so that a literary duel between them ensued. We will describe the events that led to this battle. Then, an important part of the paper focuses on the mathematics of this *Libra*, including a brief explanation of spherical geometry and trigonometry. Each spherical trigonometry rule that Sigüenza uses will be explained, and every spherical law will be derived. Translations of several excerpts from the *Libra Astronómica* are found in the Appendix. This will be the first time that any part of this book has been translated into English.

Logarithms were invented by John Napier for the purpose of making calculation easier. He lived in an age of significant innovation in astronomy. In 1563, Copernicus had published his theory of the solar system, and many mathematicians and astronomers began mathematical computations based on his theory.[1] Napier's invention made such calculations far less complicated. The breakthrough helped make dense calculation into mere additions and subtractions.

Sources say that manuscripts had been written on the subject of logarithms in the seventeenth century, but were never taken to print. Diego Rodríguez had written many manuscripts, including one named *De los Logaritmos y Aritmética*.[2] Apparently, he attempted to print the manuscript, but failed to do so. The subject was known and written about, but no such work was ever printed in the America's before Sigüenza's *Libra Astronómica y Philosóphica.* This is why this book is significant and deserves our attention.

At that time in history, the interpretation and theory of comets was not universally agreed upon. In the days before Halley and Newton, fear was the most common reaction to comets. Many regarded

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[1] Carslaw, H. S., *The discovery of logarithms by Napier,* Mathematical Gazette, Volume 8 (1915-1916).

[2] Bruce S. Burdick, *Mathematical Works Printed in the New World in the Sixteenth and Seventeenth Centuries*. Baltimore, MD: Johns Hopkins University Press, to appear, page 137.

them as heavenly signs of negative events to come, while others believed that to be merely superstition. The fear was associated with the tails of comets, many seeing them as portents stretching across the sky.[3] The ancient philosopher Aristotle influenced the general feelings about comets since he rejected them as planets. His proof rested on the fact that comets did not appear to travel the same way as planets. Furthermore, Aristotle argued that comets were completely atmospheric. He stated that they were the product of basic earthly gases eventually rising into the atmosphere. At a certain point these gases ignited, forming the comet. Aristotle also viewed other celestial objects as atmospheric, such as meteors, and even the Milky Way galaxy.

Over time, the path and behavior of comets began to be studied and observed. Many acknowledged that they went uninfluenced by earthly phenomena such as the wind. In addition, Tycho Brahe was able to use the method of measurable parallax to determine distances. A parallax is the apparent shift, or change in angular position, of an object due to the motion of the observer. The distance can be determined by viewing this parallax, measuring the successive angles and applying geometry. He used this exact process to conclude that comets must be outside the Earth's atmosphere as the parallax was minute. Still, others remained true to the Aristotelian view of an atmospheric position. At the time, the behavior of comets was viewed as unpredictable. Aristotle viewed comets as an indication of wind and drought and the fear of comets eventually spread. The controversy was common among not only the average people but also some of the most intelligent minds from antiquity to the seventeenth century.[4] A controversy occurred among Carlos de Sigüenza y Góngora, Francisco Kino, and others. However, both Carlos de Sigüenza y Góngora and Eusebio Francisco Kino stood out, as they used mathematics in their arguments regarding the significance of comets.[5]

Sigüenza was born in Mexico City in 1645. Early in his life he rigorously studied mathematics and astronomy. His education was influenced by his father, who was a tutor for the royal family in

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[3] James Howard Robinson states in his *The Great Comet of 1680: A Study in the History of Rationalism* (page 1), that the tail, or beard, of the comet is "the greatest cause of alarm."

[4] See Robinson's *The Great Comet of 1680.*

[5] Bruce S. Burdick, *Mathematical Works Printed in the New World in the Sixteenth and Seventeenth Centuries*. Baltimore, MD: Johns Hopkins University Press, to appear.

Spain. Sigüenza took as second name Góngora, from his mother's side. He received seven years of training in theology and humanistic studies from scholarly Jesuits, and was extremely strong willed. But, his strong will is not all that he is known for. He is renowned as an illustrious scholar, former Jesuit priest, poet, mathematician, historian, author, and geographer. All in all, he is one of the major intellects of Colonial Mexico.

In 1660, he joined the Society of Jesus but was dismissed within the decade as "He escaped from the dormitory to taste the forbidden fruit of nocturnal rambles about the city streets."[6] It is unknown exactly what he did to break his vows. He made several attempts to be reinstated, but to no avail. He was eventually named chair of mathematics and astrology at the University of México after winning a balloted voting for the position. However, he then failed to attend many classes, frequently asking for lengthy leaves. This may have been caused by his lack of love for astrology, which was favored at the university even more than mathematics. He stated that astrology had a weak foundation, "alien to science, method, principle, and truth."[7]

Then, in 1680, a series of events took place that ignited an intellectual feud regarding science and superstition. This paralleled the controversial struggle between religion and science that was to come shortly thereafter during the Enlightenment.

In 1680, a comet was visible in Mexico. While others ran in fear, Sigüenza marveled in excitement. He looked for data in something that others deeply feared. Being rather confident in his work, Sigüenza knew that the comet was not a signal of terrible events to come. On January 13, 1681, Sigüenza published the *Manifiesto philosóphico contra los cometas despojados del imperio que tenían sobre los tímidos*, in an attempt to calm the general fears. Sigüenza probably knew his text would cause controversy, but that didn't stop him from publishing it. He genuinely wanted to remove this ridiculous fear from the hearts of the people.

This text caused a stir as Sigüenza had justly expected. Shortly after, the Flemish Martín de la

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[6] Irving Leonard, "Sigüenza y Góngora and the Chaplaincy of the Hospital del Amor de Dios," *The Hispanic American Historical Review*, XXXIV, no. 4, Nov., 1956, pp. 580-587.

[7] Leonard, Irving, A. *Baroque Times in Old Mexico, Seventeenth Century Persons, Places, and Practices*, p. 198.

Torre, from Campeche, wrote against Sigüenza in his *Manifiesto Cristiano en favor de los cometas mantenidos en su natural significación*. (This incident is related in paragraph 28, in the Appendix.) In his pamphlet, Martín de la Torre tried to prove, with astrological data, that comets were indications of calamitous events to come. This prompted Sigüenza to respond with the allegorically titled *Belerofonte Matemático contra la quimera astrológica de Don Martín de la Torre*. The title refers to the storied battle in Greek mythology between Bellerophon and the Chimera. The hero Bellerophon was sent to slay the three-headed monster Chimera with only the help of the winged Pegasus. The story represents the triumph of good over evil. It seems Sigüenza compared himself to the hero, and Martín de la Torre's perspective on comets to the chimera. Some controversy exists as to whether the pamphlet was ever printed.[8]

Seis Obras consisted of six works written by Sigüenza, including the *Libra Astronómica*. The prologue is done by Irving A. Leonhard, while the endnotes and introduction are written by a William G. Bryant. Bryant's seventyfourth endnote appears as follows: Compendia el contenido de la Manifiesto y la Belerofonte. This endnote implies that some part of the following paragraphs, 317 to 395, are taken directly from the *Belerofonte Matemático contra la quimera astrológica de Don Martín de la Torre*. Furthermore, paragraphs 320–328 appear to be quotes from Martín de la Torre himself, taken from his *Manifiesto Cristiano en favor de los cometas mantenidos en su natural significación.* All in all, paragraphs 317–395 appear to be taken directly from Sigüenza's *Belerofonte*. On the other hand, the prologue to this *Libra Astronómica* suggests indirectly that the work was never printed. Sebastián de Guzmán y Córdova proposes that it "perished on the reef of [Siguenza's] carelessness".

Sigüenza's list of critics continued to grow. Soon after, a Jesuit missionary by the name of Father Eusebio Kino entered the picture and strongly disagreed with Sigüenza's ideas. Mathematics and astronomy brought Sigüenza into contact with this great missionary and explorer of Mexico and the

[8] Bruce S. Burdick, *Mathematical Works Printed in the New World in the Sixteenth and Seventeenth Centuries*. Baltimore, MD: Johns Hopkins University Press, to appear.

southwest United States.

Father Kino was roughly the same age as Sigüenza, and like him, possessed a passion for mathematics and astronomy, as well as having entered the Society of Jesus. Kino intended to travel to Mexico City far earlier, but was delayed until 1681. But, during his wait in Cádiz he was able to observe the same comet Sigüenza had seen thousands of miles away. He was able to survey it while he traveled. Eventually, in 1681, Kino arrived at the port of Veracruz and went on to Mexico City. At the time, Sigüenza longed for an intellect with whom he could discuss his observations. Therefore, he eagerly sought out Kino when he reached Mexico. He welcomed the future Jesuit missionary, and even lent him maps and charts for his expeditions toward California. Sigüenza most likely hoped he had found an intellectual ally, someone who also understood that comets were no indication of misfortunes.

Kino visited Sigüenza just before leaving for Sinaloa, and handed him his newly printed *Exposición Astronomica*, suggesting to Sigüenza that, if he were not busy at the time, he would have plenty to read and write about. At that moment, Sigüenza felt challenged to a literary duel. Sigüenza probably suspected that Kino did not truly appreciate or respect him. Although his name did not literally appear in the text, Sigüenza felt the work was at least partially aimed at him. He declared, "No one knows better where the shoe pinches than the one who wears it and, since I assert that I was the object of his invective, everyone may believe that, without question, it was I".[9] Sigüenza felt singled out, as everyone else at the time had stated that comets were portents of misfortune.

Sigüenza's *Libra Astronómica* was written in 1681, but was not published until 1690 due to lack of funding. After nine years, his friend Don Sebastián de Guzmán y Córdova paid the cost. The introduction and prologue of the text are written by Don Sebastián. The book describes each of the works that came out in response to his *Manifiesto Filosófico*. Specifically, Sigüenza summarizes Kino's book by chapters.

In the *Libra*, Sigüenza often quotes authorities to support his ideas, however there is also a lot

[9] Sigüenza y Góngora, Carlos de, *Libra Astronómica y Philosóphica,* paragraph 315.

of mathematics, especially at the end. To solidify his arguments, the end of the *Libra Astronómica* includes mathematics. In paragraphs 388 through 395, there is extensive use of spherical trigonometry, which is trigonometry applied on the two-dimensional surface of a sphere. In this trigonometry, “straight lines” are great circles;[10] therefore, any two lines meet at two points causing intersections. This goes against Euclid’s 5th postulate in his *Elements* which is equivalent to the statement that for any given line ℓ and a point A, which is not on ℓ, there is exactly one line through A that does not intersect ℓ. This forces spherical geometry to be considered as a non-Euclidean geometry as it does not satisfy all of Euclid’s postulates.

Spherical geometry differs significantly from normal plane geometry. In plane geometry, a straight line is the shortest path between any pair of points. But, in spherical geometry, the shortest path is actually a “great circle” . Also, the angles of a triangle do not sum to 180°, as they do for every triangle in plane geometry. This correlates with the fact that there is no concept of similar triangles that can be applied to a spherical plane. Furthermore, the sides of polygons in spherical geometry actually are arcs and have angle measures themselves (see Figure 1). Note that, in this geometry, it is possible to take trigonometric functions of the sides of figures. Here, sides are not specified by length but rather by the arc formed by the central angle.

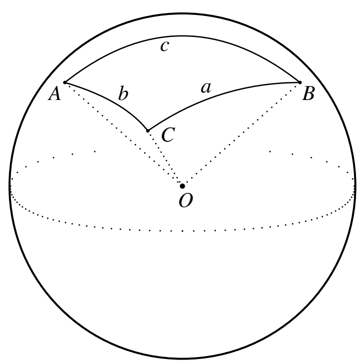


Figure 1: A spherical triangle

Multiplying large numbers is not always as easy as adding them. However, adding logarithms is essentially equivalent to direct multiplication. Observe the basic example of multiplying the

[10] A great circle is a circle on the sphere whose center is the same as the sphere’s.

substantially large numbers 725,392 and 649,218. Calculating the base-10 logarithm of each gives 5.860572762 and 5.812390552, respectively. Now, performing a simple addition of these two values is less complicated than multiplying the original numbers, and yields 11.67296331. Application of the inverse of the logarithmic function will produce the needed answer. Namely, since logarithms are base-10, we must raise the number 10 to this sum, $10^{11.67296331}$. This gives the product of our initial numbers, $4.709375435 \times 10^{11}$. At the time, tables for logarithms of trigonometric functions were easily available.

To avoid negative numbers, Sigüenza uses the logarithms plus ten. For example, take the numbers 0.753915 and 0.286428. Their logarithms are, respectively, -0.1226776158 and -0.5429845295. Here, Sigüenza adds 10 to each value, resulting in 9.877322384 and 9.457015471. To multiply the original numbers he adds these and subtracts 10, resulting in 9.334337855. Since he has a table of logarithms plus ten, he is able to do a reverse lookup to find the answer 0.215942366. In addition to using logarithms plus ten, when he has to divide, he uses the negative of the logarithm. This way he can combine multiplication and division into one calculation.

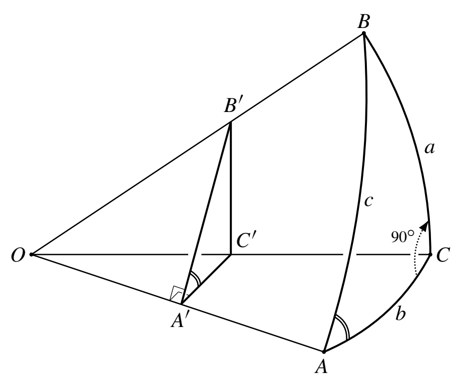


Figure 2: Right-angled spherical triangle

Throughout the *Libra Astronómica* Sigüenza uses trigonometric rules in his calculations for spherical triangles. First, he uses laws that apply to spherical triangles that have a right angle. The triangle ABC in Figure 2 is a right triangle on the surface of a sphere with C as its right angle. The picture has introduced a new plane, perpendicular to the line segment OA at the point A'. Here, the normal planar angle $C'A'B'$ is equal in measure to the angle A of the spherical triangle. We are now able to deduce information about the spherical triangle simply by using the usual planar rules for right triangles.

We will start with a derivation of the first rule for sin A. We are able to set this equal to the sine of the angle $C'A'B'$. Next, we can assume that the angle $A'C'B'$ is a right angle. We can use the fact that in planar right triangles, the sine of an angle is equal to the opposite side divided by the hypotenuse. Next, we divide both the top and bottom of the fraction by the segment OA'. By definition, these are equal to the sines of the sides a and c, respectively:

$$\begin{aligned}
\sin A &= \sin C'A'B' \\
&= \frac{C'B'}{A'B'} \qquad \text{as } A'C'B' \text{ is a right angle} \\
&= \frac{C'B'}{OB'} \Big/ \frac{A'B'}{OB'} \\
&= \frac{\sin a}{\sin c} \qquad \text{as } OB'C' \text{ and } OA'B' \text{ are right angles.} \qquad (1)
\end{aligned}$$

This is one of the basic trigonometric rules for spherical trigonometry. The others can be derived using formula (1). We start with a common identity.

$$\begin{aligned}
\cos A &= \cos C'A'B' \\
&= \frac{A'C'}{A'B'} \qquad \text{as } A'C'B' \text{ is a right angle} \\
&= \frac{A'C'}{OA'} \Big/ \frac{A'B'}{OA'} \\
&= \frac{\tan b}{\tan c} \qquad \text{as } OA'B' \text{ and } OA'C' \text{ are right angles} \qquad (2)
\end{aligned}$$

The third basic rule for planar right triangles can be derived from a combination of (1) and (2). By definition, the tangent of A is equal to the sine of A divided by the cosine of A.

$$\tan A = \tan C'A'B'$$

$$
\begin{aligned}
&= \frac{C'B'}{A'C'} \qquad \text{as } A'C'B' \text{ is a right angle} \\
&= \frac{C'B'}{OC'} \bigg/ \frac{A'C'}{OC'} \\
&= \frac{\tan a}{\sin b} \qquad \text{as } OC'B' \text{ and } OA'C' \text{ are right angles}
\end{aligned} \tag{3}
$$

Another useful formula is the spherical form of the Pythagorean theorem:

$$
\begin{aligned}
\tan A &= \frac{\sin A}{\cos A} \\
&= \frac{\sin a}{\sin c} \times \frac{\tan c}{\tan b} \qquad \text{from formulas (1) and (2)} \\
&= \frac{\sin a}{\tan b} \times \frac{\tan c}{\sin c} \\
&= \frac{\sin a}{\tan b} \times \frac{1}{\cos c}
\end{aligned}
$$

But, from formula (3),

$$
\begin{aligned}
\tan A &= \frac{\tan a}{\sin b} \\
&= \frac{\sin a}{\tan b} \times \frac{1}{\cos a \ \cos b}
\end{aligned}
$$

By comparing the two formulas we get,

$$
\cos c = \cos a \ \cos b. \tag{4}
$$

Each of these rules is used by Sigüenza in his complex calculations. However, not all spherical triangles have a right angle. Therefore, a different equation is necessary for triangles without a 90° angle. This is where the spherical law of cosines is needed. However, before this, an introduction to the spherical law of sines is required.

To start, we must look at the angle B in Figure 3. Using formula (1), we can state that the sine of

B must be equal to the sine of the height divided by the sine of side a. Similarly, the sine of angle A is equal to the sine of the height divided by the sine of side b. Using a combination of the two, we can now derive the law of sines as follows:

$$\begin{aligned}
\sin B &= \frac{\sin p}{\sin a}, \quad \sin A = \frac{\sin p}{\sin b} \\
\frac{\sin B}{\sin A} &= \frac{\sin p}{\sin a} \Big/ \frac{\sin p}{\sin b} \\
&= \frac{\sin p}{\sin a} \times \frac{\sin b}{\sin p} \\
&= \frac{\sin b}{\sin a} \\
\sin B &= \frac{\sin b \ \sin A}{\sin a} \\
\frac{\sin B}{\sin b} &= \frac{\sin A}{\sin a}
\end{aligned}$$

And then, by symmetry,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

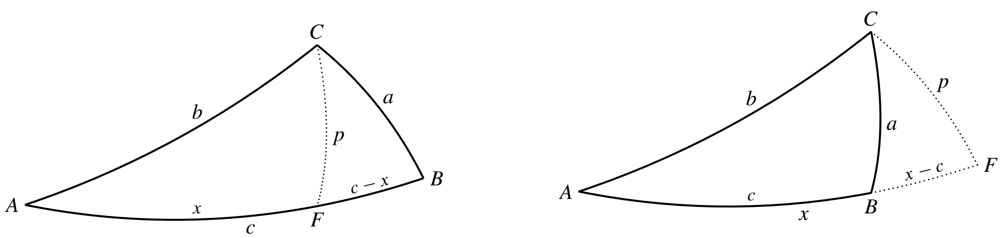


Figure 3: General spherical triangles

Sigüenza uses this law of sines in paragraph 388. His figure shows us a triangle CNO. To determine the measure of side CN, Sigüenza initially takes the logarithm of the sines of the side CO and of the angles CNO and CON. At first, the sine of angle CNO is equal to 0.0070006524. Next, he takes the logarithmic function of this side, resulting in $\log 0.0070006524 = -2.154861487$. Similarly, the base-10 logarithm of the sine of side CO is equal to -1.803897983, and the base-10 logarithm of the sine of CON is equal to -0.3958063875. And, as stated earlier, to avoid the use of negative numbers, Sigüenza adds 10 to each value, resulting in 8.196102017 and 9.604193612. Note that, in the chart below, the first value in the column of logarithms is a positive 2.1548687, because this corresponds to a factor by which he wants to divide. This is where the law of sines is applied. Using the law of sines to solve for side CN,

$$\frac{\sin CN}{\sin CON} = \frac{\sin CO}{\sin CNO},$$

yields

$$\sin CN = \frac{\sin CO \ \sin CON}{\sin CNO}.$$

Instead of multiplying and then dividing, Siguenza simply adds the logarithms of $\sin CO$ and $\sin CON$, and then only has to subtract $\log(\sin CNO)$:

$$\log(\sin CO) + \log(\sin CON) - \log(\sin CNO) = \log(\sin CN).$$

Therefore,

$$9.604193612 + 8.196102017 - (-2.154861487) = 19.95515712.$$

However, he subtracts 10 from this number (because there is an extra ten), resulting in 9.95515712. To evaluate the side CN, Sigüenza looks in his table and obtains $64°24'38''$. This is the way the calculation actually looks in Sigüenza's *Libra*:

Sine	CNO	$24'04''$	C.L. 2.1548687
Sine	CO	$54'00''$	8.1961020
Sine	CON	$23°42'05''$	9.6041935
Sine	CN	$64°24'38''$	9.9551642

"C.L." marks the places where Sigüenza uses the negative of the logarithm instead of ten plus the logarithm.

In paragraph 393, Sigüenza depicts a figure with vertices labeled M, S, P, and C for the comet itself; S represents the star Scheat and M represents the star Markab. We will focus on the triangle formed by the vertices S, M, and C. But, this spherical triangle does not possess a right angle. Therefore, he needs to apply a different formula. Sigüenza now has the values for the three sides, but no angles. Therefore, he needs a law of cosines.

To derive the law of cosines we must denote the sides of a spherical triangle as a, b, and c. We will again use Figure 3. Assume p to be the height of this spherical triangle. Dropping an altitude from angle C to side c, results in the two values, x and $c - x$, which together make up the side AB. Let ϑ_1 and ϑ_2 be the angles ACF and BCF. The following also uses properties of right triangles. The proof starts with

$$\begin{aligned}\cos C &= \cos(\vartheta_1 + \vartheta_2)\\ &= \cos\vartheta_1 \cos\vartheta_2 - \sin\vartheta_1 \sin\vartheta_2\end{aligned}$$

By using (1) and (2),

$$\cos C = \frac{\tan p}{\tan b}\cdot\frac{\tan p}{\tan a} - \frac{\sin x}{\sin b}\cdot\frac{\sin(c-x)}{\sin a}$$

After multiplying through by $\sin a\ \sin b$, we obtain

$$\begin{aligned}\sin a\ \sin b\ \cos C &= (\tan^2 p)\ \cos a\ \cos b - \sin x\ \sin(c-x)\\ &= (\sec^2 p - 1)\ \cos a\ \cos b - \sin x\ \sin(c-x)\end{aligned}$$

Using (4),

$$\begin{aligned}\sin a\ \sin b\ \cos C &= \frac{1}{\cos^2 p}\cos p\ \cos(c-x)\cos p\ \cos x - \cos a\ \cos b - \sin x\ \sin(c-x)\\ &= \cos x\ \cos(c-x) - \cos a\ \cos b - \sin x\ \sin(c-x),\end{aligned}$$

and thus

$$\cos x\ \cos(c-x) - \sin x\ \sin(c-x) = \cos a\ \cos b + \sin a\ \sin b\ \cos C.$$

We also have

$$\cos\bigl(x + (c-x)\bigr) = \cos a\ \cos b + \sin a\ \sin b\ \cos C,$$

and so

$$\cos c = \cos a\ \cos b + \sin a\ \sin b\ \cos C.$$

However, the law of cosines is not compatible with logarithms, because of the addition that is performed after trigonometric functions have been taken. Sigüenza actually uses formulas that can be derived from the spherical law of cosines. A derivation results in an equation that he uses in paragraph 393. The following equation does not include such an addition of trigonometric functions:

$$\sin^2\frac{C}{2} = \frac{\sin(s-a)\ \sin(s-b)}{\sin a\ \sin b}, \qquad \text{where} \quad s = \frac{a+b+c}{2}.$$

This can be derived from the law of cosines as follows:

$$\cos c = \cos a\ \cos b + \sin a\ \sin b\ \cos C,$$

$$\cos(a-b) = \cos a\ \cos b + \sin a\ \sin b \qquad \text{(identity)}.$$

Subtracting the two,

$$\cos(a-b) - \cos c = \sin a\ \sin b\,(1 - \cos C).$$

Using the identity

$$\sin\frac{C}{2} = \pm\frac{1-\cos C}{2},$$

we have

$$\sin^2\frac{C}{2} = \frac{1-\cos C}{2} = \frac{\cos(a-b) - \cos c}{2\sin a\ \sin b}.$$

Using the identity

$$\sin\alpha\,\sin\beta = \frac{1}{2}\big(\cos(\alpha-\beta) - \cos(\alpha+\beta)\big)$$

and letting $\alpha = s - b$ and $\beta = s - a$, we have

$$\begin{aligned}
\sin(s-b)\,\sin(s-a) &= \frac{1}{2}\big(\cos(a-b) - \cos c\big),\\
\sin^2\frac{C}{2} \;=\; \frac{2\sin(s-a)\,\sin(s-b)}{2\sin a\,\sin b} &\;=\; \frac{\sin(s-a)\,\sin(s-b)}{\sin a\,\sin b}.
\end{aligned}$$

Again, to avoid actual multiplication and division, Sigüenza uses addition and subtraction of logarithms for simpler calculations:

$$\begin{aligned}
2\log\sin\frac{C}{2} &= \log\frac{\sin(s-a)\,\sin(s-b)}{\sin a\,\sin b}\\
&= \log\sin(s-a) + \log\sin(s-b) - \log\sin a - \log\sin b.
\end{aligned}$$

His calculation begins with the summation of the three sides CS, CM, and SM. He then takes half of this sum. In order to get the value of $\frac{1}{2}(a-b+c)$, he takes the difference of this semiperimeter and the side SM. He then takes the difference between this semiperimeter and the side MC to get $\frac{1}{2}(a+b-c)$. After taking logarithms of each term, he finds the sum. He must take half of this sum to determine the arc of the half of this sum. He simply doubles this found angle, to obtain the measure of arc SMC. He is able to subtract this value from the arc SMP to establish that what is remaining must be the measure of PMC, namely $1'47''$. (See Table 1.)

The mathematics used by Sigüenza in the conclusion of the *Libra Astronómica* is aimed at tracking the position of the comet over time. The main point of the *Libra* is to calm the general fear of comets. Sigüenza does state his point throughout the *Libra*, but it is not evident that the mathematics in the book supports this. Does the mathematics actually prove that comets should not be feared? Sigüenza may have added the complexity of spherical geometry simply because he could, maybe even showing off. It is possible that he knew others at the time would not be able to grasp the complexity

Table 1:

The angle SMP and the arc SM were found in paragraph 389.

CS	6°00′00″	
CM	8°37′00″	C.L. 0.8244216
SM	12°52′41″	C.L. 0.6519291
Sum of the three sides	27°29′41″	
Semi-sum	13°44′50″	
Difference of the semi-sum and MS	52′09″	8.1809595
Difference of the semi-sum and MC	5°07′50″	8.9514609
Sum of the logarithms		18.6087711
Arc of the half of this sum	11°37′39″	9.3043855
The double of the angle SMC	23°15′19″	
Reduced SMP	23°17′06″	
And it will remain PMC	1′47″	

of the mathematics. Overall, the mathematics may not have supported his case.

This literary battle was unique because it involved an intellectual from the Old World and one from the New World. Sigüenza genuinely attempted to free the people from anxious fear and superstition. He did this without ever actually traveling to Europe. In many respects, he kept up with and was ahead of his times. He made use of decimal fractions and logarithms. His thoroughness and brilliance were profound. There is no doubt he enhanced the scientific tradition in Mexico while keeping up with the advances of Europe.

The mathematics from Sigüenza's *Libra Astronómica y Philosóphico* appears to be a process of calculating the longitudes of the comet and stars measured by the ecliptic, or the sun's path. All such computations involve spherical geometry and trigonometry. These calculations were made easier by the use of logarithms and decimal notation. Both of these concepts had appeared in Europe before the publication of the *Libra Astronómica y Philosóphico.* John Napier invented logarithms in 1614 when he published his *Mirifici logarithmorum canonis descriptio.* A manuscript (by Diego Rodríguez) existed in Mexico, but was probably never printed. Therefore, suprisingly, logarithms did not appear in print in Mexico until 1690. This paper includes the first ever English translation of excerpts from this important book by Sigüenza.

Appendix

The following passages are the first-ever English translation of any part of Sigüenza's *Libra*. I had difficulty with the translation. First, Sigüenza wrote in the style of Gongorism. This is a Baroque literary style that involves flowery, elaborate puns and metaphors. This style, linked directly to the Spanish poet Luis Góngora y Argote, often uses run-on sentences. Difficulties in translation arose owing to the nature of this metaphoric style. Prime examples exist throughout the text. A look into paragraph 314 shows some of the complexity faced by the translation. The paragraph consists of two complicated sentences:

> "Protesto, delante de Dios, haberme precisado y aun compelido el reverendo padre a tomar la pluma en la mano para escribir este libro, diciéndome, cuando se dignó de regalarme con su Exposición astronómica, no me faltaría qué escribir y en qué ocupar el tiempo si lo leyese, como en el número 4 quedó apuntado. Así lo he hecho por parecerme el que no sólo a mí, sino a mi patria y a mi nación, desacreditaría con el silencio, si — calificándome por de trabajoso juicio y objecionándome el que sólo estando enamorado de las astrosas lagañas y oponiéndome al universal sentir de altos y bajos, nobles y plebeyos, doctos e indoctos, pude decir lo que de los cometas en mi Manifiesto filosófico se contenía — disimulase con tan no esperada censura, supuesto que dirían, y con razón, cuantos leyesen su escrito, tenían los españoles en la Universidad Mexicana por profesor público de las matemáticas a un hombre loco y que tenía por opinión lo que nadie dijo".

First, observe the infrequency of the use of periods; this entire excerpt is made up of two sentences. Second, the frequency of commas is far greater; there are sixteen. The initial sentence is complicated, so that identifying the subject is not straightforward. The translation that follows shows that the subject of the first sentence is actually the action of the Reverend Father. Later in the sentence, Sigüenza states that something obliged and even compelled him to take the pen in hand and write this book. However, it is not immediately clear what exactly compelled Sigüenza to write this book. There are many ways to interpret this passage, but the way I have chosen is to make "the Reverend Father's telling me" the subject of "obliged and compelled". In this case, the order of the verb and subject are not as in English.

Translation often involves expressions that cannot be translated literally. In this paragraph, two stand out. The phrases "trabajoso juicio" and "astrosas lagañas" are translated as "dull wit" and "contemptible bleariness", respectively. In the translation, I attempted to maintain the structure of each sentence, although it was problematic.

The *Libra* consists of 395 numbered paragraphs. We have used these numbers here to show where paragraphs have been skipped.

The Translation

4. The days ran until eventually the Reverend Father brought to the public light his EXPOSICION ASTRONOMICA which came to my hands by those of the Reverend Father who gave it to me with total liberality one day when (like many other times he had) he visited me in my house, and, while saying goodbye in departing that same afternoon for the Provinces of Sinaloa, he asked me in what was I then occupied? And responding to him that I did not have anything in particular that required me to study, he urged me that, in reading his book, I would not lack something to write about and occupy my time; with this I confirmed the truth of what they had warned me about, and I considered myself challenged to a literary duel [. . .]

28. This is the context of my published writing of January 13, 1681, whose brief clauses motivated, in the esteemed scholars, ignorant laughter and conceited objections; and, as the first does not make me proud, because it was not fair, neither does the second make me strong, because I had always had in my memory "I never tried to please the common people", which Seneca said, if the authors did not see that they passed to the printing form the manuscripts with which they provoked me to the arena, among all of the first that took to arms, Don Martín de la Torre, the Flemish gentleman who (persecuted by adverse fortune, and not being in the sphere that he had perhaps occupied and in which he should maintain himself for his nobility and stature) finds himself today in the port of San Francisco of Campeche, he who wrote a brief treatise entitled *Manifiesto cristiano en favor de los cometas mantenidos en su natural significación*, to which, if my self esteem did not mislead me, I fairly responded in another entitled *Belerofonte Matemático contra la quimera astrológica de, etc.* The second was the educated Josef de Escobar Salmerón y Castro, doctor and professor of anatomy and surgery in the Royal University, printing a *Discurso cometológico y relación del Nuevo cometa, etc.* to which I never thought to respond, as not being worthy of [response, due to] the extraordinary writing and the dreadful proposition that the comet had been formed from the exhalations of dead bodies and human sweat. The third is the very reverend Father Eusebio Francisco Kino, of the company of Jesus, whom gladly I try to satisfy and I have intent to examine his assertions in the present *Libra*; and it appears to me to give some news of his *Exposición Astronómica del cometa que el año 1680, se ha visto en todo el mundo y le ha observado en la ciudad de Cadíz el padre Eusebio Kino, de la Compañia de Jesús.* Licensed. In Mexico by Francisco Rodríguez Luperzio in 1681.

314. I protest, before God, that the Reverend Father's telling me, as I related in paragraph 4, when he deigned to gift me with his Astrological Exposition, that I would not lack, if I were to read it, for something to occupy my time and to write about, has obliged me and even compelled me to take pen in hand and write this book. Thus, I have made it, by being like the one who, with my silence, would discredit not only me, but my country and nation, if — qualifying me as a dull wit[11] and impugning me as one who is only enamored of the contemptible bleariness[12] and opposed to the universal feelings of highs and lows, the noble and plebeian, the educated and the uneducated,

[11] Leonhard's translation in *Baroque Times in Old Mexico, Seventeenth Century Persons, Places, and Practices.*

[12] Both trabajosos juicios and astrosas lagañas are direct quotes from Kino's Exposición Astronómica. I doubt that astrosas lagañas actually means contemptible bleariness, but I have not been able to find out an appropriate translation.

so could he relate what was included in my *Manifiesto Philosóphico* — I forgave him, in spite of so unexpected a condemnation, since they said, and with reason, as many who have read his writing, "Do the Spanish of the Mexican University have a crazy man as public teacher of mathematics?" who had for an opinion what no one had said.

315. If someone scrupulous will object that I tried to make mine the pain that was common, being that the reverend Father hadn't expressed my name in the *Exposición Astrónomica*, I do not have anything more adequate to respond with than, no one knows better where the shoe pinches than the one who wears it; and thus, I assure that I was the object of ridicule, they all can believe me that without a doubt it was I. It is not important that my name is not mentioned, therefore, like what happened there in Rome to Horatio in the book Carmina: "Therefore I am indicated by the pointing fingers of strangers", as many in Mexico have read the work of the reverend father this happens to me.

316. For us to remain reconciled and friends, and to end this controversy once and for all, I want to conclude it with the same words with which the eminent philosopher Pedro Gassendo responded to the no less educated Monsieur Descartes, and one reads in his works (tome 3, page 410):

> "I hoped that you would take the things in such form that, something too bitter being said by me, you attribute it to the ingeniousness with which I believed I was able to continue the norm imposed by you. And if, by chance, something got ignored that you consider unacceptable, there is reason for you to forgive me for it, as something approved first by your own attitude. I am of such character that for nature and application I inclined toward smoothness of habit; it seemed more to me to see, nevertheless, that you wanted to irritate my patience, believing to provoke with impunity as a bull that doesn't have hay in the horns from there resulted in it truly my judging that is not my concern to advise that you should not have so treated a man that did not deserve anything bad from you. In which concerns me, I assure one thing: that it will never happen that it should depend on

me that you should not house me, if I am considered worthy, as your very attentive and observant friend. Goodbye."

THE FILOSÓFICA LIBRA EXAMINES INCIDENTALLY THE BASES ON WHICH, THEY SAY, ASTROLOGY RESTS

Preamble: link with the preceding

317. The reverend father said in his letter, that remains inserted in paragraph 221, that the effects of this comet would last so many more years rather than days or months, it was obvious to us that this is what the astrologers thought in the judgement of what eclipses do; and to be read, in the beginning of its dedication, I am obligated to tell of what it persuades, without any doubt, being unfailing and certain what this teaches. And his forecast that is in his letter, does the same and with the same words as anyone of the many that are found in the manuals of astrology, would have had for certain (as if seen) while alone as much as in similar obligations as those who preceded him in these judgments inscribed and put in their books.

318. To think of it the field offered me enough to examine the correspondence between the years of comets and the days and months of the comet's duration; but being already encouraged to write against astrology not only in the Lunario del año 1675 but also the present of 1681 in my *Belerofonte Matemático contra la quimera astrológica de don Martín de la Torre*, *Matemático Campechano*, I do not find reason of motive for what is done here. This nevertheless, this approved astrology has led my friend Eusebio Francisco Kino to examine the comet, which was much more exquisite and fundamental than that in the books, judging that the curious reader will not find it unpleasant, I will set down, with your permission, some of what was in that manuscript as background for the material here.

319. Don Martín de la Torre felt that what it said in my Manifiesto, did not ignore the authority of poets, philosophers, astrologers, and saints who were able to be open themselves up to what was written against the comets, and doing justice of its organization, I assured him that the astrology didn't have other things to tell rather that I am an astrologer and that I know very well which the foot that astrology limps on and on what weak bases the material is raised; seeming to be a sacrilege of in what I said, in which he shamefully incurs if, as the great astrologer he is, I did not punish it, taking in the hand the scourge of his elegant words and reasons, began to correct me brilliantly, saying thus: [...][13]

386. Thus it reflected I know that the Reverend Father Fray Diego Rodriguez, of the order of Our Mother of Mercy, the excellent mathematician and very the same to as many as they have been great in this century and my predecessor in the leadership of the Royal Department of mathematics, and Gabriel López de Bonilla, Mexican astrologer, have used (not by means of the observations) by the Tychonic Tables of *Supplement* of Juan Antonio Magino (that, he accordingly affirms, reduced to the meridian which is 11 minutes of time more eastern than Uraniburgo[14]) with a 7 hour 39 minute difference of the suitable I have always used a good management of events. Next, if of Mexico to Uraniburgo, according to these two authors there are 7 hours 28 minutes difference, or 112 degrees and in this 35 degrees and 54 minutes, it will be in Mexico on $283°54''$, that differs from what follows from the observations of Enrico Martinez by $31'$ from the Equator or in $2'$ of time, which at so great a distance is a rather stupendous concordance.

Observations of January 3, 1681

388. Friday, January 3 of 1681, at seven o'clock at night: the comet, the precedent and the subsequent of the mouth of the Small Horse formed a right triangle, being the northern comet and something more eastern than the stars mentioned above. Between the comet and the subsequent, by the reticule of silver threads (according to the honorable Cornelio Malavasia, like he said in his Observations) I accommodate, when I need to, in the focus of the ocular lens of my telescope,

[13] The next few paragraphs are a quote from Martín de la Torre's Manifiesto Christiano, now lost.
[14] Uraniburgo is the observatory created by Tycho Brahe in Denmark.

there was more than nine parts, of those between the subsequent and precedent there are ten, and I exquisitely observed a right angle at seven p.m. The longitudes and latitudes of the stars here and in front served me to the day January 1 of this year 1681, according to the hypothesis and corrections of the father Juan Bautista Ricciolo in his *Astronomica reformada*; and to know what of this the comet had when I observed it: there is, in the present delineation, the pole of ecliptic N, the place of the precedent P, that of the subsequent O, that of comet C. These extremes are joined with the arcs of maximum circles and will result in two triangles, the first ONP and the second CNO, for whose easy solution joining CP with the line CP, by exempting lines that will serve (although it is not) the mutual perpendicular at one and another triangle, it will arrange this way:

[...]

Look at this second: C N, complement of latitude of the comet

Sine	CNO	24′04″	C.L. 2.1548687
Sine	CO	54′00″	8.1961020
Sine	CON	23°42′05″	9.6041935
Sine	OP	64°24′38″	9.9551642

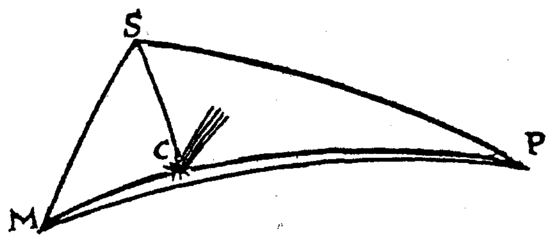
[...]

Being the complement of the arc CN 25°35′22″, rather another was the northern latitude of the comet at the same time. This observation, being made with great diligence and with an instrument that could not mislead me, I take as very good. The logarithms of the small tangents and sines were taken from the Tables of Cavalieri,[15] which are very precise, by being divided by seconds at the beginning and end of the quadrant. And if these same calculations were made by common tables,

[15] Bonaventura Cavalieri (1598–1647). See Katz *A History of Mathematics*.

there will be some difference, because the sines and tangents do not grow regularly at the sixtieth number.[16]

393. Thursday, January 9, at 7:54 p.m., the comet was at precisely a 6° distance of Scheat, 8°37′ from Markab, and the calculation inferred that it was in conjunction with Markab, because the angle SMC differs by SMP 1′47″; that is the disposable difference and of no consideration y CPM will be much less.



El ángulo SMP y el arco SM se hallaron en el número 389.

CS	6°00′00″	
CM	8°37′00″	C.L. 0.8244216
SM	12°52′41″	$C.L.$0.6519291
Sum of the three sides	27°29′41″	
Semi-sum	13°44′50″	
Difference of the semi-sum and MS	52′09″	8.1809595
Difference of the semi-sum and MC	5°07′50″	8.9514609
Sum of the logarithms		18.6087711
Arc of the half of this sum	11°37′39″	9.3043855
Its double angle SMC	23°15′19″	
Reduced SMP	23°17′06″	
And will remain PMC	1′47″	

[16] We think that what Sigüenza means is that the sine and tangent function grow regularly at the beginning, but by the time that you reach the sixtieth number in the table, the growth is not regular anymore.

Therefore, if the longitude of Markab today were 19°02′44″ of Pisces, likewise would it be of the Comet at this time. If the latitude of that star, 19°24′50″, is increased 8°37′ so there was between her and the comet, will be the latitude of this 28°01′50″.

Look the same latitude by means of the angle that was found in the number 390

Maximum Sine		90°00′00″	C.L. 0.0000000
Sine	EN	87°28′54″	9.9995803
Tangent	ENC	28°03′40″	9.7267910
Tangent	EC	28°02′17″	9.7263713

A latitude of the other only differs 27″; then, it has been observed well.

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