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Maria Gaetana Agnesi: Female Mathematician and Brilliant Expositor of the Eighteenth Century

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Drawing by Benigno Bossi (1727-1792)
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Introduction

Maria Gaetana Agnesi is credited as being the first female mathematician of the Western world, which is quite an accomplishment considering that the time period in which she flourished was during the mid 1700s. The title is much deserved, as she is the author of the second Calculus textbook ever written. Yet much of her mathematical work is surrounded by conflicting opinions. An analysis of this text, *Analytical Institutions,* which includes authentic examples from which the reader may draw his or her own conclusions, will be presented later. Equally misunderstood are Agnesi's intentions for entering the field of mathematics. In fact, this career of Maria's came to a sudden halt in 1752. Clifford Truesdell perhaps says it best in that "the rule of Maria Gaetana's life,… was passionate obedience" [8, p. 141]. This so clearly encapsulates her life and her motives for all that she did because at no matter what age she is spoken of, it is never Maria who comes first.

Early Years

Born May 16, 1718, Agnesi grew up in Milan, Italy, in what would today be considered an extremely upper-middle class household. She was the oldest child of Pietro and Anna Fortunato Brivio Agnesi, and subsequently became one of twenty-one children by three wives of her father. The family money came from the Agnesi family work in the silk field, not from Pietro's alleged position as Professor of Mathematics at the University of Bologna, which did not exist. In fact, it was her father's quest for social status that shaped much of Maria's childhood. The Agnesi family was actually part of the bourgeois class, but Pietro had desperate desire for "a coat of arms and the title of Milanese patrician" [6, p. 666]. To help this cause, the education of his children, which will be covered in detail shortly, was of great importance.

However, before looking at the specifics of Agnesi's education, the general climate for education at the time, especially for that of females, must be examined. Italy stood apart from other countries in the eighteenth century with respect to the education of women, and the reason can be attributed to its leaders, both political and religious [4, p. 2]. Empress Maria Theresa of Austria (which controlled Italy then) apparently looked fondly upon female scholarly efforts as both Maria Gaetana and her younger sister, Maria Teresa, received accolades from her. In addition, Pope Benedict XIV, who was from Bologna [8, p. 127], a city which housed a university known for its acceptance even of women professors [3, p. 68], epitomized the "reformist Catholic tradition," which endorsed educational opportunities for females [6, p. 658]. Indeed, this time period, including the Renaissance and Catholic Enlightenment, has been called "the heyday of the intellectual woman throughout the Italian peninsula—a time when women enjoyed the same scholastic freedom as men" [7, p. 37]. Furthermore, men looked at a woman who was educated as "one who had but enhanced the graces and virtues of her sex by the added attractions of a cultivated mind and a developed intellect" [p. 38]. Thus, it was not uncommon for wealthy families to take on the cost of educating a child in hopes that he or she "might later find a prestigious academic position" [6, p. 666].

That was exactly what Pietro Agnesi did, except that due to his social motives, education was a focus for him with all of his children, male and female, not just Maria Gaetana. Regardless of the reasoning, Maria was provided with exceptional tutors in numerous subjects including "languages, philosophy, mathematics, natural sciences, and music" [8, p. 115]. Her most prominent tutors were Count Carlo Belloni of Pavia, and Ramiro Rampinelli, an Olivetan monk who later became a professor of mathematics at the University of Pavia. From very early on, it was evident that she had an intellectual gift, most apparent in her linguistic abilities. By the time she was five years old she had mastered her native Italian as well as French [7, p. 40]. In addition to those languages, by age eleven Agnesi had added Latin, Greek, German, Spanish, and Hebrew. In fact, by age nine she knew enough Latin to translate from Italian into Latin a discourse that encouraged women's higher education, written by one of her tutors.

Noticing the precociousness of his daughter, Pietro began to use this to further his status-seeking cause by joining the salon culture popular with patrician families at the time [6, p. 667]. Mazzotti captures the grandeur of these gatherings well, saying, "magistrates, senators, Arcadian literati, university professors, ecclesiastics, and foreign travelers came together regularly in the domestic *accademie* at the Palazzo Agnesi, where they declaimed poetry and discussed scientific issues while sampling chocolate (in the winter) and sorbets (in the summer)" [p. 667]. It was at such gatherings that Pietro would have Maria Gaetana debate with the guests on any topic that they chose in their native languages. Typical areas of discussion included "logic, ontology, mechanics, hydromechanics, elasticity, celestial mechanics and universal gravitation, chemistry, botany, zoology, and mineralogy, among others" [5, p. 75]. And here, at the age of nine, she delivered the memorized Latin discourse that she had translated from Italian. Guests, in awe of her young talent, printed that discourse as a gesture of admiration [6, p. 667], making it Agnesi's first published work in 1729 [8, p. 116].

With the passing of time, the gatherings at the Palazzo Agnesi became famous across Europe. In 1738, Maria Gaetana had gathered enough theses from participating in so many of her father's assemblies, that she published 191 of them in her *Propositiones philosophicae*. This second publication of Maria's drew even more notable attendees and written acclaim. One such account came in a letter written by Charles De Brosses, "president of the parliament of Burgundy" [7, p. 40], who saw Agnesi on July 16, 1739. He even went so far as to say that he declined a visit to Countess Clelia Borromeo, "'who not only knows all the sciences and languages of Europe but also speaks Arabic like the Koran,'" in order to see Maria Gaetana, whom he said was "'a walking dictionary of all languages and who, not content with knowing all the oriental languages, gives out that she will defend a thesis against all comers about any science whatever'" [8, p. 116]. He arrived to find twenty-one year old Maria, seated in the center of a circle of approximately thirty people, "awaiting questions and challenges" [6, p. 670]. Upon hearing her speak, De Brosses commented that it was "something more stupendous than the cathedral of Milan" [8, p. 117]. Also noted in his letter on the evening was that Agnesi was "'much attached to the philosophy of Newton, and it [was] marvelous to see a person of her age so conversant with such abstract subjects'" [p. 118]. He said this because earlier she had defended Newtonian ideas when speaking about the causes of tides. Maria Gaetana's knowledge of mathematics and the sciences was clearly great even at her very young age. But ultimately De Brosses was most struck by her linguistic skill, saying, "'I was perhaps yet more amazed to hear her speak Latin…with such purity, ease, and accuracy that I do not recall having read any book in modern Latin in such a good style as her discourses'" [p. 118]. This was not the first time that others made comments about her linguistic talent and it would not be the last. Even her later mathematical critics still greatly praised her writing abilities.

Finally, two last great spectacles at the Palazzo Agnesi were the attendance on November 29, 1739 by the Prince of Wolfenbüttel and the later appearance of Augustus the Strong, Elector of Saxony and King of Poland, who came to witness Maria's intellect [8, p. 119]. This visit drew such attention that it was reported in the December 2, 1739 issue of the *Gazetta di Milano* [p. 119]. While the exhibition of his daughters was working as Pietro had wished, it was not in accordance with Maria Gaetana's best interests or desires since becoming such a spectacle was quite out of character for her shy and reserved personality.

Beginning in earlier adolescence but intensifying around this time, all of the social functions and rigorous academic work caused Maria to develop serious health problems. She would often break into "seizures of chorea, or St. Vitus's dance," for which physical activity was recommended, such as "dancing and horseback riding" [4, p. 3]. In addition to those problems, Maria simply became tired of her father's exploitation of her at the palazzo debates. This is evidenced by two specific things—her comment about one of the evenings and her desire to enter the convent. Maria's comment was described in De Brosses' account of the palazzo. She apparently said that "'she was very sorry that this visit had so taken the form of a thesis; that she did not like at all to speak of such things in company, where for one that was amused, twenty were probably bored to death…'" [8, p. 118]. This clearly shows that she was unhappy with the structure of the social evenings. In fact, she wished to withdraw from them and become a nun. However, this wish was met with great opposition by Pietro, and thus Maria agreed to stay in the home under the following conditions—"that she go to church whenever she wished, that she dress simply and humbly, [and] that she abandon altogether balls, theatres, and profane amusements" [p. 123]. With that, in early 1740, Maria retreated from public life to focus on her spirituality and her mathematical study.

Analytical Institutions

For the task of learning technical, advanced mathematics, Ramiro Rampinelli became Maria's primary instructor in 1740 [4, p. 1]. She had previously become well versed in the mathematics of "Newton, Leibniz, Fermat, Descartes, Euler, and the Bernoulli brothers"

[7, p. 40] and had even written an analysis of a posthumous L'Hôpital work on conics, though it was never published [8, p. 123]. Now, however, she really seemed to thrive under Rampinelli's direction, saying that, without him she would "have become altogether tangled in the great labyrinth of insuperable difficulty, had not his secure guidance and wise direction led [her] forth from it" [p. 124]. The first large work that she analyzed in great detail appears to be Reyneau's *Analyse démontré*, which has been criticized for lacking application and for being written in such a way as to turn people off to further mathematical study [6, p. 678]. Despite this criticism, Agnesi's future mathematics largely followed Reyneau's structure. With Rampinelli's encouragement, she began work on what would be the second Calculus text ever written, called in full *Instituzioni Analitiche ad Uso della Gioventù Italiana* (*Textbook of Analysis for the Use of Young Italians*) [8, p. 124].

The text is often called *Analytical Institutions*, stemming from a later English translation of it, and that is how we will refer to it here. It was published by "the publishing house Richini" [4, p. 3] in 1748, which her father actually had installed within the home. Interestingly, the printers were extremely grateful to Maria for allowing them the opportunity to work with the mathematical symbols needed for Calculus, as it helped them with future work [p. 3]. The work appeared in two volumes, composed of four books, which total over one thousand pages. The original Italian version, made of handmade paper, also had additional fold out pages in the back that Marc' Antonio Dal Rè engraved with all of the mathematical figures referred to within the text [4, p. 3]. Additionally, within the Italian edition, the title page "[carried] also, in the style of

the period, an engraving of a handsome, half-naked, reclining woman sketching geometrical figures on a large board supported by an admiring little cupid…" [8, p. 125]. From an aesthetic point of view, the book made quite an impression and showed the same kind of great attention to detail as the mathematical text contained within it.

Agnesi's *Analytical Institutions* was actually a compendium of mathematics, beginning with the most basic arithmetic and advancing through the Calculus. The four books were: *The Analysis of Finite Quantities*, which would today be thought of as the basics of arithmetic, algebra, and analytical geometry; *The Analysis of Quantities Infinitely Small*, which is equivalent to Differential Calculus; *Of the Integral Calculus*; and *The Inverse Method of Tangents*, which is basically an introduction to differential equations. At the time of this book [1748], Calculus had two different styles of presentation, due to its co-creators Leibniz and Newton, who each used a different notation and focused on different pieces of the subject. Maria Gaetana, living in Italy, used Leibniz's differential notation. However, within *Analytical Institutions*, "Agnesi declared that the 'differential' was in fact equivalent to the Newtonian 'fluxion'…" [6, p. 679]. And, in fact, her work has been called "a valuable introduction to algebra and calculus in the Newtonian—'geometrical'—tradition" [6, p. 679]. This is so because Maria attempted to demonstrate "how the most recent developments in calculus could be understood in purely geometrical terms" [p. 675]. Thus, in this work she bridged the gap between both Leibnizian and Newtonian Calculus.

Maria herself may have best elucidated the full purpose of her *Analytical Institutions*. Since great controversy later arose surrounding what she included in the text, we should enter into the discussion armed with authentic knowledge of Agnesi's intentions for the book. In the Author's preface, she stated that "the necessity of this science appears so evident as to excite our youth to the earnest study of it; yet great are the difficulties to be overcome in the attainment of it" [1, p. XXI]. She further explained that without tutorial help, the subject would be nearly impossible and that she understood that not all people who had the desire to learn it were capable of having such help. She then commented that while the content had been dealt with by other authors, that "these pieces [were] scattered and dispersed in the works of various authors…so that it [was] impossible for a beginner to methodize the several parts, even though he were furnished with all the books necessary for his purpose" [p. XXII]. Thus, she felt that "a new Digest of Analytical Principles might be useful and acceptable" [p. XXII]. Accordingly, that is just what *Analytical Institutions* was—a digest or compendium of others' work. As a last note of interest, it was written in Italian, not the standard Latin. She acknowledged the reasoning behind the choice in her writing, saying that when she began work on the text, she hadn't intended to publish it. Therefore she wrote in her native language, and once it grew to such a large volume, it became too great a task to translate into Latin [p. XXIII]. She also revealed that another purpose of the book was to teach the content to her younger brothers, hence the mention of youth in the original title.

Upon its publication in 1748, *Analytical Institutions* received great acclaim from numerous sources. Granted, some comments were based on mere amazement that a person of her gender could produce something so technical, such as Montucla's comment in his history of mathematics text that "we cannot behold without the greatest astonishment a person of a sex that seems so little fitted to tread the thorny paths of these abstract sciences, penetrate so deeply as she has done into all the branches of Algebra, both the common and the transcendental, or infinitesimal" [1, p. XV]. Yet many others had no mention of her sex and offered praise just on the merit of the work itself. For instance, upon reviewing Agnesi's text, the French Academy of Sciences in 1749 remarked that a French translation of the second volume would be useful [8, p. 127], and said of the work [4, p. 3]:

> "It takes a good deal of knowledge and skill to reduce to an almost always uniform method, as indeed was done, the various discoveries in the works of modern geometers, where these are often explained by methods quite different one from another. Order, clarity, and precision reign in every part of this work. Up to now we have seen no work, in any language, which allows the student to penetrate so quickly and so far into mathematical analysis. We regard this treatise as the most complete and well written of its kind."

Comments such as these, especially ones about the lucidity of her arguments and writing style, were present in nearly everything written about the book.

Further honors for Maria Gaetana came from the Academy of Sciences of Bologna, Empress Maria Theresa of Austria (to whom she dedicated the book), and Pope Benedict XIV. As evidenced in the original Italian title page of *Analytical Institutions*, the Bolognese Academy of Sciences had already made her a member. Then, as a congratulatory gift, she received from the Empress a crystal container filled with a diamond ring and other loose diamonds and stones.

However, the most distinguishing honor was probably the one from Pope Benedict, given Maria's extreme religious faith. From him, she received not only "a gold wreath set with precious stones and a gold medal" [4, p. 2], but also a "personal letter of congratulation that showed some knowledge of the contents of her textbook" [6, p. 680]. In addition to that, she was named honorary lecturer at the University of Bologna in a diploma dated October 5, 1750 [4, p. 2]. Although Maria never accepted this position, her name was listed on the faculty roster until 1795-1796 [7, p. 46]. Thus, the girl famous throughout Europe in her younger years for her linguistic skills and scholarly aptitude eventually earned attention specifically for her mathematical capabilities.

Analysis of the Text

Now we proceed to an analysis of *Analytical Institutions*, as translated into English by John Colson, and printed under the direction of editor John Hellins in 1801. First, in addition to the division of the work into volumes and books already mentioned, we note that the books are further broken down into sections. There are between four and six sections per book, and each section is subsequently broken down into numbered articles, which are the individual detailed lessons on the topics. As a final structural note, sporadically throughout the text appear sections called Scholiums, which can be thought of as the teaching parts of the book. This is so because they follow specific examples and alert the reader to notice certain points, much like a teacher does when lecturing. Lastly, this textbook differs greatly from those of today in the sense that it is not a book filled with page after page of practice problems with answers in the back. It is most easily described as a detailed account of many mathematical processes and techniques, ranging from basic arithmetic to Calculus. Do not mistake this as a criticism for a lack of examples, however, because there are many examples given—they just differ from those of today because they are all worked out in detail for the reader to simply follow. With that, we may now begin to look at its mathematical contents.

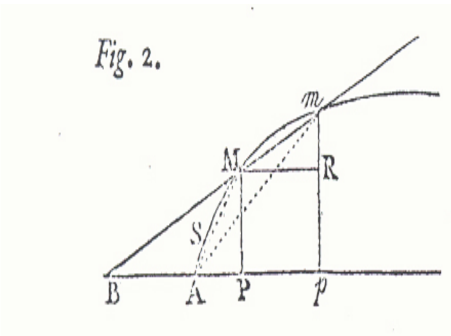
The first book, *The Analysis of Finite Quantities*, is most impressive because of the extremely precise detail in which she handles the most basic mathematics, such as addition, subtraction, and other arithmetical processes. It is important to note here that almost at the very beginning of Book I, Agnesi addresses the issue of infinity, saying, "the sign ∞ denotes infinite, and therefore $a = \infty$ signifies that a is equal to infinite, or is an infinite quantity" [1, p. 2]. This is definitely a precursor to the Calculus that follows in Books II and III. To illustrate her great depth in explanation, we look at the following passage dealing with why, when subtracting a negative number from another number, it is necessary to change the sign of the negative number to positive [p. 3-4]:

> "…to subtract one quantity from another is the same thing as to find the difference between those quantities. Now the difference between a and $-b$ is $a + b$, just in the same manner as the difference between a capital of 100 crowns and a debt of 50 is 150 crowns. For from having an hundred and having none, the difference is an hundred; and from having none to having a debt of fifty, the difference is fifty; therefore, from having an hundred to having a debt of fifty, the difference must be an hundred and fifty."

Her explanation even parallels the modern push within teaching to include as many real world situations as possible which follow from mathematics to best relate to students and connect to their prior knowledge. For that reason, this example highlights Agnesi's teaching abilities well.

In the second book, *The Analysis of Quantities Infinitely Small*, Agnesi deals with differentials and their applications as we do today, including tangents as well as maxima and minima. To begin, she first defines a *Difference* or *Fluxion* as "any infinitely little portion of a variable quantity… when it is so small, as that it has to the variable itself a less proportion than any that can be assigned…" [2, p. 2]. [Note that here and in all following mathematical work presented in this paper, author commentary will appear in square brackets.]

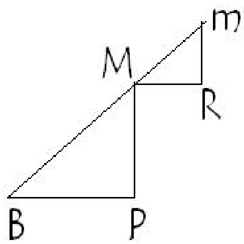
The following geometric illustration is taken from [2, p. 2-3].



"Let AM (Fig. 2) be a curve whose [horizontal] axis or diameter is AP;

[A]nd if, in AP produced [extended], we take an infinitely little portion P*p*, it will be the difference or fluxion of the absciss [abscissa] AP,

[A]nd therefore the two lines AP, A*p*, may still be considered as equal, there being no assignable proportion between the finite quantity AP, and the infinitely little portion P*p*.



From the points P, *p*, if we raise the two parallel [vertical] ordinates PM, *pm*, in any angle, and draw the chord *m*M produced [extended] to B, and the right line MR parallel to AP;

[T]hen, because the two triangles BPM, MR*m*, are similar [see figure on left], it will be BP . PM :: MR . R*m* [in modern notation, $\frac{BP}{PM} = \frac{MR}{Rm}$].

But the two quantities BP, PM, are finite, and MR is infinitely little; then also R*m* will be infinitely little, and is therefore the [difference or] fluxion of the ordinate PM."

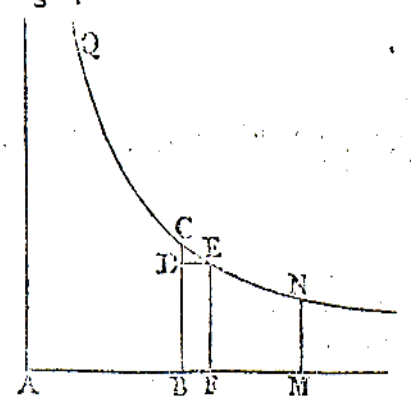
Now that Agnesi has showed us how differences or fluxions are geometrically created, she moves on to introduce the notation for them*: "the mark or characteristic by which Fluxions are…expressed, is by putting a point over the quantity of which it is the fluxion" [2, p. 3]. She

*It is important to understand that in the translation from Italian to English, Newtonian notation for fluxions [the point over the quantity] was adopted as opposed to the Leibnizian notation [a "d" before the quantity] Agnesi used in the original.

then relates this notation to the example, saying: "thus, if the absciss [abscissa] AP = x, then will it be Pp or MR = $\dot{x}$. And, in the like manner, if the ordinate PM = y, then it will be Rm = $\dot{y}$" [2, p. 3]. Additionally, she mentions that these fluxions are First Fluxions or Differences of the first Order [we would say first derivatives]. Finally, Maria concludes the notation section by pointing out that if the flowing quantity [what we would call a "function" today] generating the fluxion is decreasing, the sign of that fluxion will be negative [p. 3].

It is common throughout *Analytical Institutions* to see Maria supplement mathematical examples with verbal explanations as well. And while one might think this would just complicate matters, she writes so clearly that such paragraphs really do make the topics more meaningful. For instance, in a very clearly worded explanation, Agnesi reinforces the ideas of the existence of differentials for any readers who might still be doubtful [2, p. 3].

Fig. 4.



"That these differential quantities are real things, and not merely creatures of the imagination,…, may be clearly perceived from only considering that the [vertical] ordinate MN (Fig. 4) moves continually approaching towards [the vertical ordinate] BC, and finally coincides with it.

But it is plain, that, before these two lines coincide, they will have a distance between them, or a difference, which is altogether inassignable, that is, less that any given quantity whatever.

In such a position let the [vertical] lines BC, FE, be supposed to be, and then [the horizontal distance] BF, [and the vertical distance] CD, will be quantities less than any that can be given, and therefore will be *inassignable*, or *differentials*, or *infinitesimals*, or, finally, *fluxions*."

Thus, both differences [or fluxions] and infinitesimals are now deeply rooted within the reader's understanding.

With the basic definitions of Differential Calculus covered, we may now move on to see how Agnesi handles a commonly known rule of differentiation—the Product Rule. First note that within this text, reflecting vocabulary of the time period, "to take the derivative of" a quantity is referred to as "to difference" a quantity. For example, to "difference" the quantity x in Agnesi's way, we would add its fluxion, $\dot{x}$, to it to get $x + \dot{x}$. Then, we would subtract x from this, giving us $(x + \dot{x}) - x = \dot{x}$, as desired.

In modern language, the Product Rule is often stated as: *the derivative of a product is the first quantity times the derivative of the second quantity plus the second quantity times the derivative of the first quantity*. Here, we will present this rule in Agnesi's own words [2, p. 18]:

> "But if the quantity proposed to be differenced [meaning take the derivative of] shall be the product of several variables, as xy;
>
> [B]ecause x becomes $x + \dot{x}$, and y becomes $y + \dot{y}$; and xy becomes $xy + y\dot{x} + x\dot{y} + \dot{x}\dot{y}$, which is the product of $x + \dot{x}$ into $y + \dot{y}$
> [$(x + \dot{x})(y + \dot{y}) = xy + x\dot{y} + y\dot{x} + \dot{x}\dot{y}$];
>
> [F]rom this product subtracting, therefore, the proposed quantity xy, there will remain $y\dot{x} + x\dot{y} + \dot{x}\dot{y}$
> [since $(xy + x\dot{y} + y\dot{x} + \dot{x}\dot{y}) - xy = y\dot{x} + x\dot{y} + \dot{x}\dot{y}$].
>
> But $\dot{x}\dot{y}$ is a quantity infinitely less than either of the other two, which are the rectangle of a finite quantity into an infinitesimal.
> [Here, both $x\dot{y}$ and $y\dot{x}$ are thought of as actual geometrical rectangles whose sides are of length $x, \dot{y}$ and $y, \dot{x}$ respectively.]
>
> But $\dot{x}\dot{y}$ is the rectangle of two infinitesimals [meaning its sides are of length $\dot{x}$ and $\dot{y}$], and therefore is infinitely less, and must be supposed entirely to vanish
>
> The fluxion, therefore, of xy will be $x\dot{y} + y\dot{x}$."

Allowing doubly infinitesimal quantities [like $\dot{x}\dot{y}$] to "vanish" whenever needed in calculations reflects a common practice of mathematicians doing Calculus at this time. Aside from that minor detail in Maria's argument, we see that while the phrasing is different from the Product

Rule of today, the result is still the same:

the derivative of a product [xy] is

the first quantity times the derivative of the second quantity [$x\dot{y}$ *]*

plus

the second quantity times the derivative of the first quantity [$y\dot{x}$ *],*

achieving the desired outcome, $x\dot{y} + y\dot{x}$.

Now that we have sampled some sections of Book II dealing with Differential Calculus, we will move to a brief overview of Book III, which speaks of the Integral Calculus. Maria is once again clear [2, p.109]:

> "The *Integral Calculus*, which …also used to be called the *Summatory Calculus*, is the method of reducing a differential or fluxional quantity, to that quantity of which it is the difference or fluxion.
>
> Whence the operations of the Integral Calculus are just the contrary of those of the Differential; and therefore it is also called *The Inverse Method of Fluxions*, or of *Differences*. Thus, for example, the fluxion or differential of y is $\dot{y}$, and consequently the *fluent* or *integral* of $\dot{y}$ is y.
>
> Hence it will be a sure proof that any integral is just and true, if, being differenced [differentiated] again, it shall restore the given fluxion, or the quantity whose integral was to be found."

Readers can now easily see the relationship between the differential and the integral. Note that the last sentence is the statement of what we call today the First Fundamental Theorem of Calculus [in modern notation: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$]. It may be of interest to know that the modern sign for an integral [$\int$] was used within this text. Agnesi then gives the most basic rule for integration—that the integral of a variable taken to a power "is the variable raised to a power the exponent of which is increased by unity, divided by the same exponent so increased" [2, p. 110]. [For an example, take $\int x^2 dx.$ According to her rule, the same one we follow today,

$$\int x^2 dx = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C.$$ The "*C*" is explained below.]

Completing a basic but thorough introduction to integration, Maria covers what could be called today the "Plus *C* Rule," meaning that for an integral to be completely general, one must always add an arbitrary constant *C* to the result [2, p. 111]:

> "But here we are to observe, that, in order to have the integrals complete, we ought always to add to them, or to subtract from them, some constant quantity at pleasure, which, in particular cases, is afterwards to be determined as occasion may require.
>
> Thus the complete integral of $\dot{x}$, for example, will be $x \pm a$, where a signifies some constant quantity."

While the notation for the constant differs from our modern one, we see that the process is still the same. Furthermore, Agnesi does not simply leave the reader with this rule, but rather goes on to clearly explain in words why it is needed: "The reason of which is, that, as constant quantities have no differentials [the derivative of a constant is zero], but $\dot{x}$ may as well be the differential of $x + a$, or $x - b$, &c. as of x" [p. 111]. Furthermore, if one was in the least bit doubtful why such a rule would be needed, Maria meticulously details the reason for it through multiple examples. Never in this text is the reader left wondering why something must be done the way that she says; Agnesi always supplements rules and procedures with not only their derivations but also an explanation for their uses in words.

With that, we have now seen several illustrations of Maria Agnesi's amazing mathematical writing talents, as she clarifies the most basic arithmetic through more advanced Integral Calculus. Thus, our brief analysis of her *Analytical Institutions* is complete.

Criticism of the Text

Most criticism of Agnesi's *Analytical Institutions* comes from a 1989 article by Clifford Truesdell of Johns Hopkins University, who completed a substantial body of research on the subject for the "University and Research" symposium commemorating the 900th anniversary of the University of Bologna [8, p. 141, 142]. While his comments are shared by other reviewers, it is his that are the most supported with authentic, primary sources and his that will be looked at here.

Truesdell's most detrimental remarks about Maria's efforts are that her text lacked mechanical application and that it did not contain enough innovative mathematics to be of any significance to later readers. Because of this, he even goes so far as to say that due to her documented inquiries and examples, "while writing an exposition of differential calculus, she is a beginner, just learning the subject" [8, p. 132]. Yet it is a bit puzzling that a "beginner" could write a textbook consisting of more than one thousand pages full of detailed mathematical examples and explications of famous rules if she did not herself know the content. Moreover, much of this criticism can be handled by simply returning to Agnesi's stated purposes for writing the book: to serve as a compendium for young students of mathematics from basic arithmetic through Calculus.

First, consider Truesdell's complaint about the failure to include mechanical examples [8, p. 133]: "Though Agnesi wrote an exposition by examples, for the eighteenth century the most telling applications of the calculus beyond those in geometry were drawn from rational mechanics [physics], yet of these she gives none." In a letter to Count Jacopo Riccati, a famous mathematician of the sixteenth century, Agnesi herself divulges why she *chose* not to include any such applications: "I did not wish to get involved in physics, and I left aside all those problems that depend upon it so as not to spread out beyond pure analysis and its applications to geometry" [8, p. 133]. Mazzotti, another critic, also refutes Truesdell, saying that "these distinctive features of the textbook should be understood as deliberate choices made by Agnesi rather than as consequences of inadequate information or understanding" [6, p. 678]. Given a first hand account by Maria herself, it is hard to understand the basis of Truesdell's criticism.

Next, consider Truesdell's comments that Agnesi presents no original mathematical ideas within her text. As a result, he states that her book was not nearly as influential as all the acclaim for it would indicate. His sentiments stem mostly from the presence of Euler's texts on similar subjects published at nearly the same time as Agnesi's *Analytical Institutions.* Truesdell writes that Euler's text, *Introductio*, is "far from being like Agnesi's textbook, which is a compendium…it provided a true introduction at the highest level, to two particular branches of analysis…" [8, p. 136]. This comment, however, is almost contradictory in its mere phrasing; for, Maria's text was purposefully a compendium of current mathematical knowledge at the time it was written to serve those trying to learn the subject, not a book showcasing her ingenious mathematical abilities. Therefore, it should not be looked at in competition with Euler's text in any way. Maria's biographer, Luisa Anzoletti, echoes a similar opinion, saying, "How can there be any comparison of her analytic talent, which she applied only to things already known, putting them in order and giving them demonstrations, rules, and formulae, with the synthetic genius of the discoverers…?" [p. 137]. Thus, since Agnesi's intention was to create a "Digest of

Analytical Principles" [1, p. XXII], *Analytical Institutions* cannot be fairly criticized for lacking invention.

While Truesdell makes other criticisms, most can easily be refuted if one carefully compares them to Maria's objectives for writing the book. The fact that *Analytical Institutions* was fully translated into English and partially into French, and that copies of it still exist today, signifies that it had sufficient influence. Maria's clarity and brilliant writing abilities should be commended, and without question, *Analytical Institutions* should be regarded with the utmost respect.

Life after *Analytical Institutions*

Following all of the acclaim for *Analytical Institutions*, lasting for several years after its publication in 1748, Agnesi's life took a dramatic turn. Her father passed away on March 19, 1752 [4, p. 2], and from that point forward, Maria abandoned mathematical study completely. In 1762, she even rejected a request to review some of Lagrange's work at the University of Turin on the "calculus of variations" [7, p. 47], signifying that her decision to leave mathematics really was permanent: "Man always acts to achieve goals; the goal of the Christian is the glory of God. I hope my studies have brought glory to God, as they were useful to others, and derived from obedience, because that was my father's will. Now I have found better ways and means to serve God, and to be useful to others" [6, p. 682]. And with that, the story of the second chapter of Maria Gaetana Agnesi's life and the motivations for it must be left entirely to another essay.

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The title page picture is from Clifford Truesdell's article [8].