An Emanji Temple Tablet

Heath Yates

University of Missouri – Kansas City

Introduction

The mathematics unique to Japan has been called Wasan. The term translates to mean the great mathematics of the Yamamoto (*Japanese*) race. Japanese began using this term to describe old Japanese mathematics after they adopted western methods following the Meiji Restoration of 1867[1, p. 423]. The Meiji Restoration was a period when Japan came out of two hundred years of political isolation and began rapid modernization and as a result Japan abandoned many of its old traditions. One of these old and unique traditions to Wasan was the art of Sangaku.

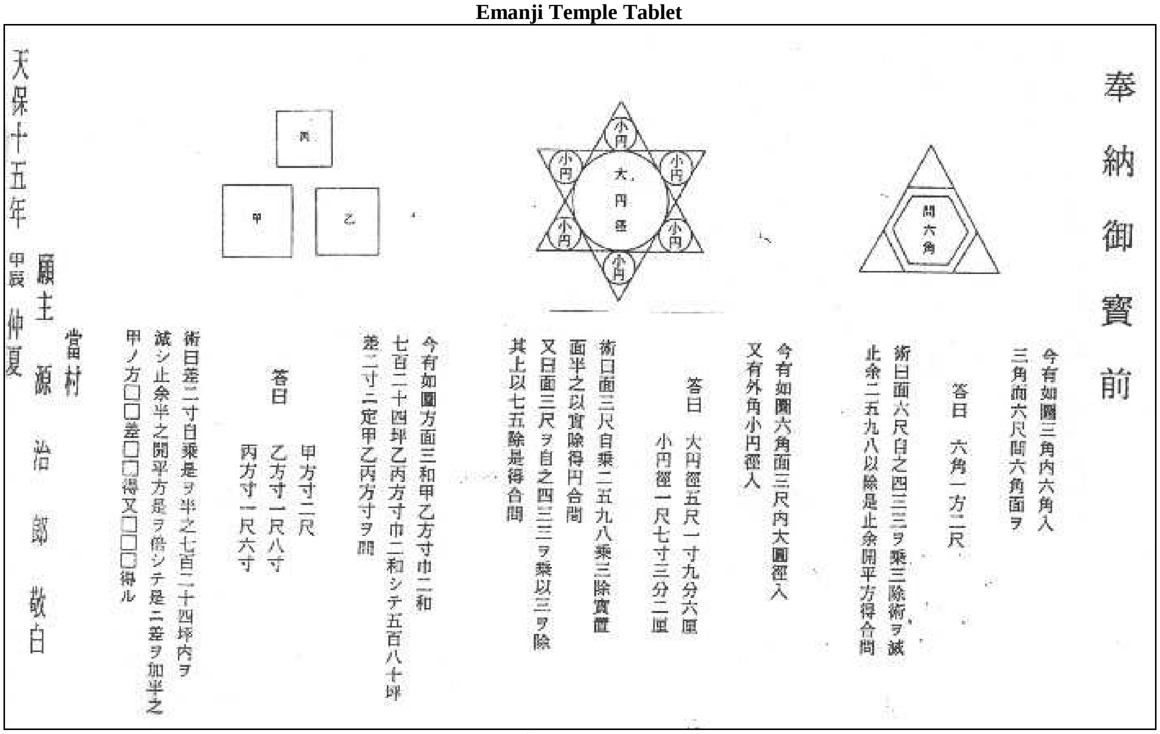
Sangaku is Japanese temple mathematics. The word Sangaku itself specifically means "mathematics tablet" in Japanese. The authors of these tablets vary from the nobleman, samurai, to the common farmer. Sangaku is mathematics artistically carved on wood. The construction of a tablet is extremely simple. The author would dedicate the tablet to a god and petition a temple to display his work where the grateful author would credit the gods for enlightening him. The basic tablet shows one or more Euclidean geometric problems and theorems. Many tablets have beautiful geometric figures carved into them and were meant to be as much a work of art as they were mathematics. The problems are verbal and have little in common with modern mathematical writing methods.

Given that the tablets are a relatively unknown part of Japanese cultural history a short historical summary on them will be provided here. Interestingly, the tradition of Japanese temple geometry correlates exactly to the most intense period of isolation in Japanese history, the Tokugawa Period (1603-1867). To this date only 850 Japanese Temple tablets are known to have survived. The period is named after Tokugawa Ieyasu (the name is given in Japanese style, family name first), a powerful shogun ruler who united Japan under his feudal law after the steady decline of the power of the emperor and state. The nation was closed off from the world and only limited trade was allowed. Japan during this period probably did not even have one western mathematical work within its borders [2, p.xi]. The next two hundred years Japanese mathematicians developed mathematics relatively independent from the rest of the world. There is even speculation that the Japanese predate Leibniz in work on determinants during this period [3, p. 95].

There is no doubt that there are many examples of more advanced geometric mathematics from this period [2, p. xii]. However, for the most part, the Japanese desire for complete isolation left them in some ways mathematically in the time of the Greeks. The Japanese at this time were still doing their mathematics in the temples and had schools much like the Greeks did. To provide a broader perspective, note that at the beginning of the Tokugawa period (1603), Isaac Newton and Leibniz invented Calculus and by the end of the Tokugawa period (1867), western mathematics was to evolve into a sophisticated form of rigorous symbolic analysis.

The specific Japanese tablet [4] that will be translated and discussed in this paper is from the Emanji temple in Nara. The city of Nara was the first capitol of Japan in 710 C.E. The city is rich in history and has provided a good sample of Japanese Temple geometry. The Emanji tablet contains three problems each illustrated with its own unique illustration. It contains simple geometry and was translated to reflect the style in which it was written. The solutions provided are modern extrapolations. It is important to note again that Japanese mathematics is more verbal than symbolic.

Emanji Temple Tablet



(illustration and modifications used without permission)

This tablet is written in archaic Japanese and is comprised entirely of Chinese symbols called kanji. The tablet is read in the traditional Japanese manner, right to left, top to bottom. The numbered arrows in the picture below illustrate how to read the text.

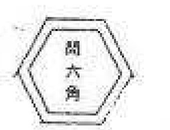
The first portion of the tablet states that this is a dedication in the presence of a god. The first portion can be seen in the picture on the top of the next page. It is important to reiterate that each problem is indicated by a separate illustration.

奉納御寳前

Problem One

The first problem sets the standard for the other two in the tablet. It contains one unique illustration that corresponds to three vertical paragraphs.

The reader should note that the illustration shown below is also not entirely practical as the hexagon contains an indication of thickness appearing as two nested hexagons that are not needed in the problem.

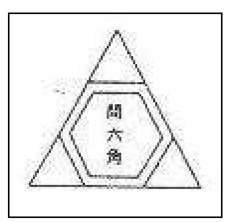


The illustrations are as much a work of art as they are illustrations of a mathematical problem. Nevertheless, most of the information provided is strictly useful and mathematical. For example, the characters 間六角 inside of the hexagon are labels: 間 means middle, 六 means six, and 角 means side which indicates that the figure being shown is a hexagon is in the "middle" of a triangle.

The problem is stated below. It translates as follows:

> "The above figure represents a triangle whose whole sides are 6 shaku that envelopes the hexagon. What are the sides of the hexagon?"


今有如圖三角内六角入
三角面六尺間六角面ヲ



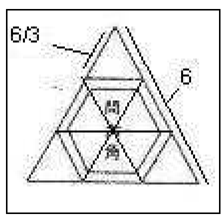
The solution to the problem is on the next illustration on the next page. The translation of the solution is the following:

"The hexagon side is 2 shaku."

答曰 六角一方二尺

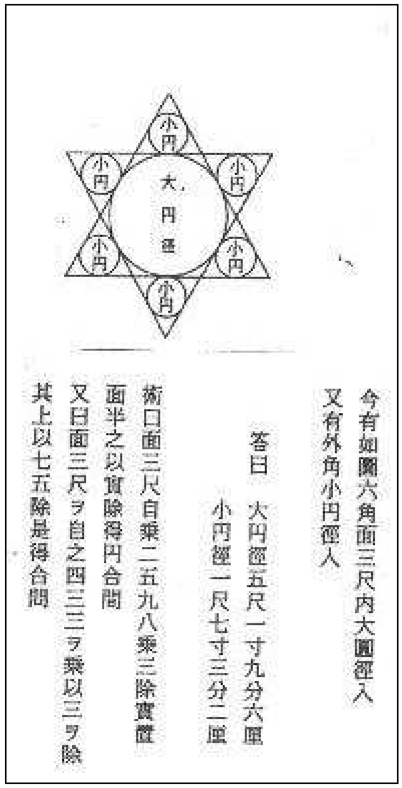
It is important to note to the reader that the Kanji 答日 are present in all three problems in the tablet. These two together mean “solution”.

The modern solution to the problem is relatively straightforward. See the figure below. The equilateral triangle can be divided into 9 small equilateral triangles. The side of 6 shaku is divided into 3 equal portions and from the picture it is evident that one side of the hexagon will be 2 shaku.



Problem Two

The problem is interesting that it involves a hexagram, a 12 sided geometric figure, also known famously as the Star of David. There are two labels provided in the illustration: 大円 means large circle (the third kanji after it means “full”), and 小円 means small circle.



今有如圖六角面三尺内大圓徑入
又有外角小円徑入
答曰　大円徑五尺一寸九分六厘
小円徑一尺七寸三分二厘
術曰面三尺自乘二五九八乘三除實體
面半之以實除得円合問
又曰面三尺ヲ自之四三三ヲ乘以三ヲ除
其上以七五除是得合問

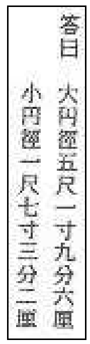
The problem can be seen in the illustration below and it translates as follows:

> "The hexagon side is 3 shaku. Inside of it is a large circle. Outside of the hexagon there are triangles that accommodate small circles. What are the diameters of the circles in the diagram?"

今有如圖六角面三尺内大圓徑入
又有外角小円徑入

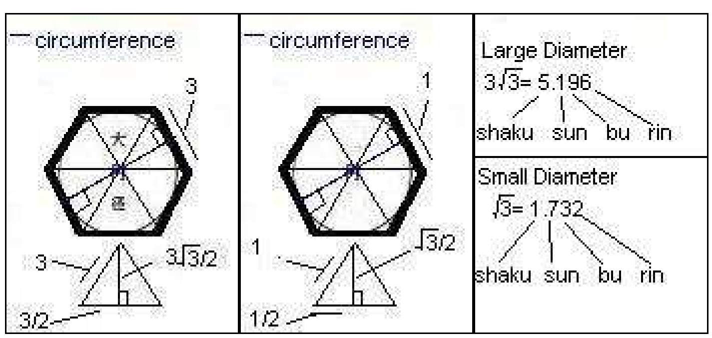
The solution to the problem can be seen in the illustration below[1]. The translation of the solution is the following:

> "The large circle is [has diameter] five shaku, one sun, nine bu, and six rin. The small circle is [has diameter] one shaku, seven sun, three bu, and two rin ."


答曰　大円徑五尺一寸九分六厘
小円徑一尺七寸三分二厘

The diagram below helps explain how the diameter could have been found using mathematics available at the time. Each circle can be shown to be inscribed in a hexagon divided into six equal parts. The length of the side of the hexagon for the small circle can be found using the methods from problem one. The diameter of either circle can be found by finding the radius of the circle and applying the Pythagorean Theorem. The numbers for the calculations are provided in the illustration below in shaku units[6].

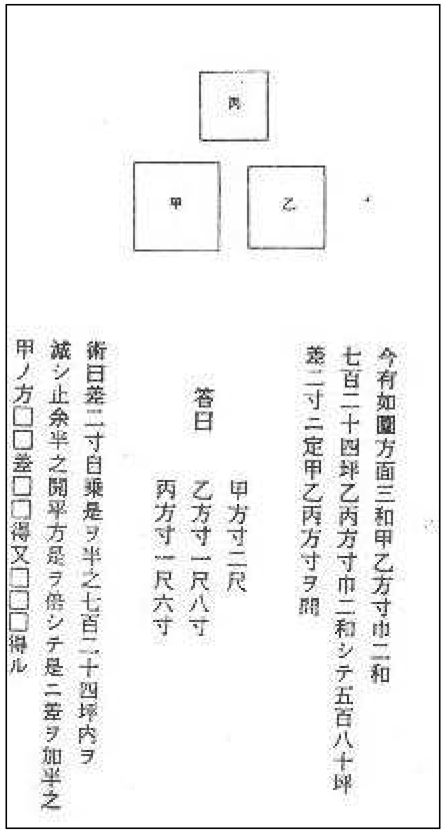
Large Circle　　Small Circle　　Solution



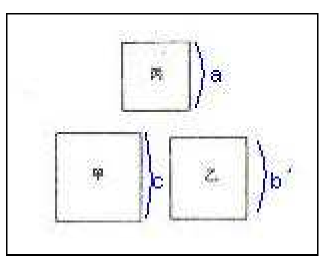
[1] 1 sun=.1 shaku , 1bu=.01 shaku, 1 rin=.001 shaku

Problem Three

The illustration provided with this problem indicates it is not related to the previous two given on the tablet.



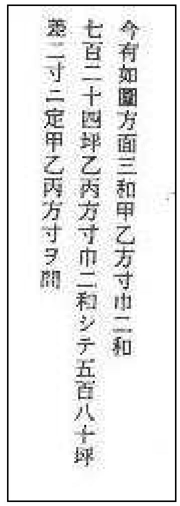
The kanji in the squares provided by the author are labels which can be better understood with the following figure, where each kanji represents a variable, a, b, c, as shown.



The problem can be seen in the illustration below and it translates as follows[2]:

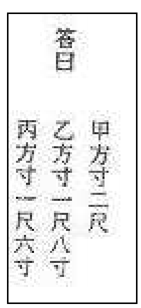
[2] 1 bo = 1 sun squared

> “There are three squares such that: $b^2 + a^2 = 580$ *bo and* $c^2 + b^2 = 724$ *bo.* Also, $c - b = 2$ *sun* and $b - a = 2$ *sun.* What are a^2, b^2, and c^2?”


今有如圖方面三和甲乙方寸巾二和
七百二十四坪乙丙方寸巾二和シテ五百八十坪
差二寸ニ定甲乙丙方寸ヲ問

The solution to the problem is shown in the next illustration. The translation of the solution is the following:

> “The square sun of *c* is two shaku. The square sun of *b* is one shaku and eight sun. The square sun of *a* is one shaku and six sun.”


答曰
甲方寸二尺
乙方寸一尺八寸
丙方寸一尺六寸

The author of this paper has no clear indication on how the Japanese tablet author solved this problem. The answer is found using modern methods. It is important to state that 1 bo indicates one sun squared and 1 sun is 1/10 of a shaku.

$c^2 + b^2 = 724$ $-(b^2 + a^2) = 580$	$c - b = 2$ $+ (b - a = 2)$	$c - b = 2$ $- (b - a = 2)$
$c^2 - a^2 = 144$ *bo*	$c - a = 4$ *sun*	$c + a = 2b$ *sun*

We now solve for *b*, *c*, and *a*

$(c - a)(c + a) = 144$ *bo*
$(4)(2b) = 144$ *bo*
$b = 18$ *sun*

$c - b = 2$
$c = 2 + b$
$c = 20$ *sun*

$b - a = 2$
$a = b - 2$
$a = 16$ *sun*

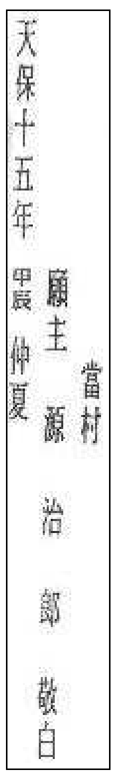
So, $a^2 = (16\ sun)^2$ [The square sun of *a* is 1 shaku + 6 sun]

$b^2 = (18\ sun)^2$ [The square sun of *b* is 1 shaku + 8 sun]

$c^2 = (20\ sun)^2$ [The square sun of *c* is 2 shaku + 0 sun]

Which is the solution desired.

Finally, the biographical portion of the tablet states when the work was carved. The biographical portion of the tablet can be seen in the illustration below.



This translates as follows:

> "Tenpou 15 year, mid-summer, temple petitioner: Minamoto Jiro."

The date corresponds to the last year of the Tenpou Era (1830-1844), and thus we know the work was carved in the summer of 1844. While educated speculation is useful in eliminating possibilities, detailed knowledge of this author is not known. The name Minamoto Jiro does not seem to be associated with any well known Japanese mathematicians of the time. It is possible the author could be a common farmer dedicating his work to a god and petitioning his local Shinto shrine. However, considering the noble and samurai class were relatively literate, it is most probable that the author was from one of these classes instead.

Conclusion

This tablet is no doubt a sample of the more simple mathematics that existed at the time of extreme isolation in Japan. The most charming aspect of this particular tablet is that the author might not necessarily be a mathematician. This is an indication of a rich culture where everyone actively participated in mathematics. The late date of the tablet compared to others also provides a clue to the art of Japanese at its artistic height, before the introduction of rigorous western mathematics. The Japanese were very precise in their verbal mathematics, much like how western mathematics is with symbolic theorems. While the temple geometry tablet themselves may have not influenced the development of mathematics in a significant way, they should be considered for what they are: beautiful and esthetic works of art that provide evidence of the great achievements Japanese would make using the ideas of western mathematics.

References

1. Selin, H (ed). Mathematics Across Cultures: The History of Non-Western Mathematics London, Kluwer Academic Publishers. 2000: pg 423

2. Fukagawa, H. Pedoe, D. Japanese Temple Problems Winnipeg, Charles Babbage Research Centre. 1989: pg. xi-xii

3. Mikami, Yoshio. Smith, David Eugene. A History of Japanese Mathematics Chicago, Open Court Publishing. 1914: pg. 95

4. Kotera, Hiroshi. "Sangaku: Japanese Temple Problems" 4 Jan, 2004 http://www.wasan.jp/nara/enman2.html

5. Green, Jim. "WWWJDIC Word Search 24 Feb", 2004. http://www.csse.monash.edu.au/~jwb/wwwjdic.html

6. Gershteins, Sergey. Gershteins, Anna. "Interactive Units Converter." 29 April, 2004 http://www.convert-me.com/en/convert/length.html

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