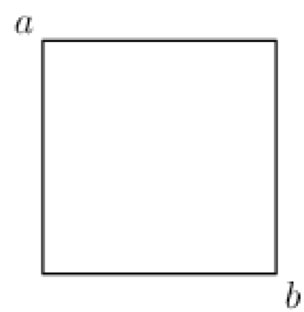
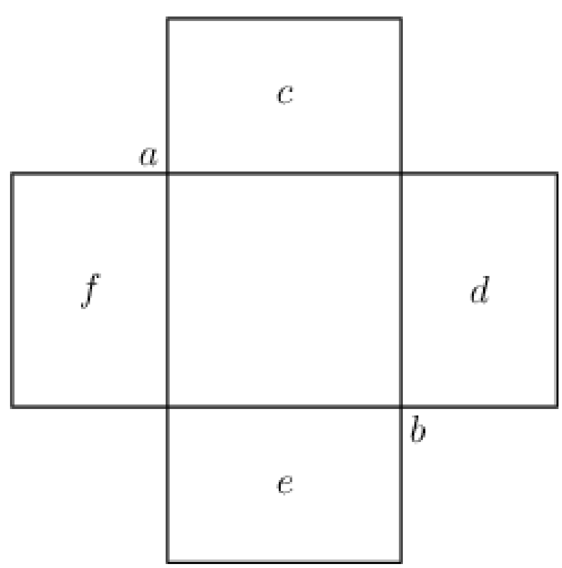
Source: Al-Khwarizmi, from *The Book of Algebra and Almucabola*, as translated in *The History of Mathematics, A Reader*, editors John Fauvel and Jeremy Gray, Open University, 1987, pp. 230-231.

Now, however, it is necessary that we should demonstrate geometrically the truth of the same problems which we have explained in numbers. Therefore our first proposition is this, that a square and 10 roots equal 39 units.

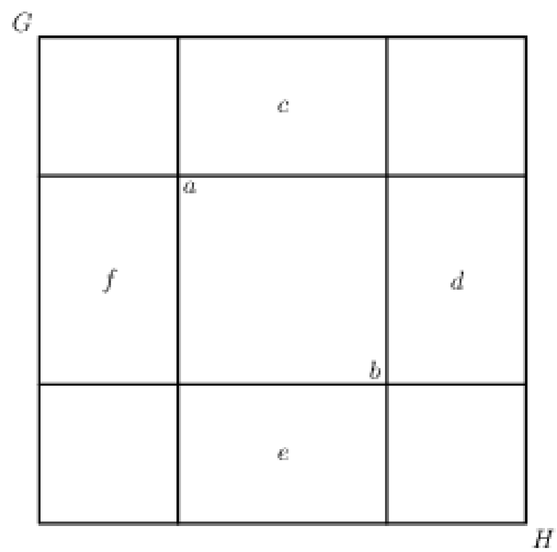
The proof is that we construct a square of unknown sides, and let this square figure represent the square (second power of the unknown) which together with its root you wish to find. Let the square, then, be ab, of which any side represents one root. When we multiply any side of this by a number (or numbers) it is evident that that which results from the multiplication will be a number of roots equal to the root of the same number (of the square). Since then ten roots were proposed with the square, we take a fourth part of the number ten and apply to each side of the square an area of equidistant sides, of which the length should be the same as the length of the square first described and the breadth $2\frac{1}{2}$, which is a fourth part of 10. Therefore four areas of equidistant sides are applied to the first square, ab. Of each of these the length is the length of one root of the square ab and also the breadth of each is $2\frac{1}{2}$, as we have just said. These now are the areas c, d, e, f. Therefore it follows from what we have said that there





there will be four areas having sides of unequal length which also are regarded as unknown. The size of the areas in each of the four corners, which is found by multiplying $2\frac{1}{2}$ by $2\frac{1}{2}$, completes that which is lacking in the larger or whole area. Whence it is we complete the drawing of the larger area by the addition of the four products, each $2\frac{1}{2}$ by $2\frac{1}{2}$; the whole of this multiplication gives 25.

And now it is evident that the first square figure, which represents the square of the unknown [x^2], and the four surrounding areas [$10x$] make 39. When we add 25 to this, that is, the four smaller squares which indeed are placed at the four angles of the square ab, the drawing of the larger square, called GH, is completed. Whence also



the sum total of this is 64, of which 8 is the root, and by this is designated one side of the completed figure. Therefore when we subtract from 8 twice the fourth part of 10, which is placed a the extremities of the larger square GH, there will remain but 3. Five being subtracted from 8, 3 necessarily remains, which is equal to one side of the first square ab. This 3 then expresses one root of the square figure, that is, one root of the proposed square of the unknown, and 9 the square itself. Hence we take half of 10 and multiply this by itself. We then add the whole product of the multiplication to 39, that the drawing of the larger square GH may be completed; for the lack of the four corners rendered incomplete the drawing of the whole of this square. Now it is evident that the fourth part of any number multiplied by itself and then multiplied by four gives the same number as half of the number multiplied by itself. Therefore if half of the root is multiplied by itself, the sum total of this multiplication will wipe out, equal, or cancel the multiplication of the fourth part by itself and then by 4.