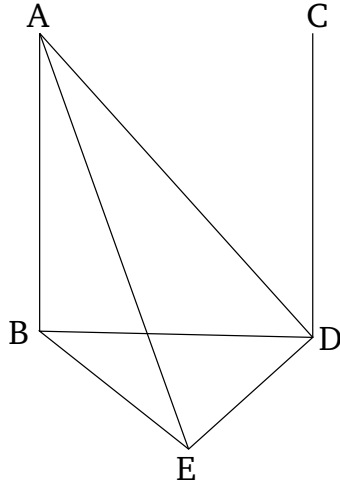


# Book 11

## Proposition 8

If two straight-lines are parallel, and one of them is at right-angles to some plane, then the remaining (one) will also be at right-angles to the same plane.



Let  $AB$  and  $CD$  be two parallel straight-lines, and let one of them,  $AB$ , be at right-angles to a reference plane. I say that the remaining (one),  $CD$ , will also be at right-angles to the same plane.

For let  $AB$  and  $CD$  meet the reference plane at points  $B$  and  $D$  (respectively). And let  $BD$  have been joined.  $AB$ ,  $CD$ , and  $BD$  are thus in one plane [Prop. 11.7]. Let  $DE$  have been drawn at right-angles to  $BD$  in the reference plane, and let  $DE$  be made equal to  $AB$ , and let  $BE$ ,  $AE$ , and  $AD$  have been joined.

And since  $AB$  is at right-angles to the reference plane,  $AB$  is thus also at right-angles to all of the straight-lines joined to it which are in the reference plane [Def. 11.3]. Thus, the angles  $ABD$  and  $ABE$  [are] each right-angles. And since the straight-line  $BD$  has met the parallel (straight-

lines)  $AB$  and  $CD$ , the (sum of the) angles  $ABD$  and  $CDB$  is thus equal to two right-angles [Prop. 1.29]. And  $ABD$  (is) a right-angle. Thus,  $CDB$  (is) also a right-angle.  $CD$  is thus at right-angles to  $BD$ . And since  $AB$  is equal to  $DE$ , and  $BD$  (is) common, the two (straight-lines)  $AB$  and  $BD$  are equal to the two (straight-lines)  $ED$  and  $DB$  (respectively). And angle  $ABD$  (is) equal to angle  $EDB$ . For each (is) a right-angle. Thus, the base  $AD$  (is) equal to the base  $BE$  [Prop. 1.4]. And since  $AB$  is equal to  $DE$ , and  $BE$  to  $AD$ , the two (sides)  $AB$ ,  $BE$  are equal to the two (sides)  $ED$ ,  $DA$ , respectively. And their base  $AE$  is common. Thus, angle  $ABE$  is equal to angle  $EDA$  [Prop. 1.8]. And  $ABE$  (is) a right-angle.  $EDA$  (is) thus also a right-angle. Thus,  $ED$  is at right-angles to  $AD$ . And it is also at right-angles to  $DB$ . Thus,  $ED$  is also at right-angles to the plane through  $BD$  and  $DA$  [Prop. 11.4]. And  $ED$  will thus make right-angles with all of the straight-lines joined to it which are also in the plane through  $BDA$ . And  $DC$  is in the plane through  $BDA$ , inasmuch as  $AB$  and  $BD$  are in the plane through  $BDA$  [Prop. 11.2], and in which (ever plane)  $AB$  and  $BD$  (are found),  $DC$  is also (found). Thus,  $ED$  is at right-angles to  $DC$ . Hence,  $CD$  is also at right-angles to  $DE$ . And  $CD$  is also at right-angles to  $BD$ . Thus,  $CD$  is standing at right-angles to two straight-lines,  $DE$  and  $DB$ , which meet one another, at the (point) of section,  $D$ . Hence,  $CD$  is also at right-angles to the plane through  $DE$  and  $DB$  [Prop. 11.4]. And the plane through  $DE$  and  $DB$  is the reference (plane).  $CD$  is thus at right-angles to

the reference plane.

Thus, if two straight-lines are parallel, and one of them is at right-angles to some plane, then the remaining (one) will also be at right-angles to the same plane. (Which is) the very thing it was required to show.