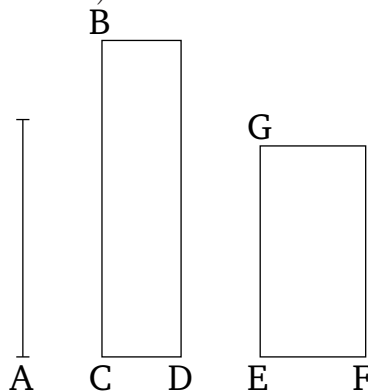


Book 10

Proposition 22

The square on a medial (straight-line), being applied to a rational (straight-line), produces as breadth a (straight-line which is) rational, and incommensurable in length with the (straight-line) to which it is applied.



Let A be a medial (straight-line), and CB a rational (straight-line), and let the rectangular area BD , equal to the (square) on A , have been applied to BC , producing CD as breadth. I say that CD is rational, and incommensurable in length with CB .

For since A is medial, the square on it is equal to a (rectangular) area contained by rational (straight-lines which are) commensurable in square only [Prop. 10.21]. Let the square on (A) be equal to GF . And the square on (A) is also equal to BD . Thus, BD is equal to GF . And (BD) is also equiangular with (GF) . And for equal and equiangular parallelograms, the sides about the equal angles are reciprocally proportional [Prop. 6.14]. Thus, proportionally, as BC is to EG , so EF (is) to CD . And, also, as the (square) on BC is to the (square) on

EG , so the (square) on EF (is) to the (square) on CD [Prop. 6.22]. And the (square) on CB is commensurable with the (square) on EG . For they are each rational. Thus, the (square) on EF is also commensurable with the (square) on CD [Prop. 10.11]. And the (square) on EF is rational. Thus, the (square) on CD is also rational [Def. 10.4]. Thus, CD is rational. And since EF is incommensurable in length with EG . For they are commensurable in square only. And as EF (is) to EG , so the (square) on EF (is) to the (rectangle contained) by FE and EG [see previous lemma]. The (square) on EF [is] thus incommensurable with the (rectangle contained) by FE and EG [Prop. 10.11]. But, the (square) on CD is commensurable with the (square) on EF . For they are rational in square. And the (rectangle contained) by DC and CB is commensurable with the (rectangle contained) by FE and EG . For they are (both) equal to the (square) on A . Thus, the (square) on CD is also incommensurable with the (rectangle contained) by DC and CB [Prop. 10.13]. And as the (square) on CD (is) to the (rectangle contained) by DC and CB , so DC is to CB [see previous lemma]. Thus, DC is incommensurable in length with CB [Prop. 10.11]. Thus, CD is rational, and incommensurable in length with CB . (Which is) the very thing it was required to show.