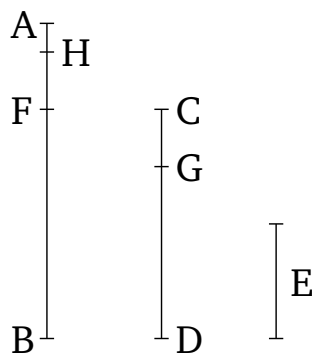


# Book 7

## Proposition 1

Two unequal numbers (being) laid down, and the lesser being continually subtracted, in turn, from the greater, if the remainder never measures the (number) preceding it, until a unit remains, then the original numbers will be prime to one another.



For two [unequal] numbers,  $AB$  and  $CD$ , the lesser being continually subtracted, in turn, from the greater, let the remainder never measure the (number) preceding it, until a unit remains. I say that  $AB$  and  $CD$  are prime to one another—that is to say, that a unit alone measures (both)  $AB$  and  $CD$ .

For if  $AB$  and  $CD$  are not prime to one another then some number will measure them. Let (some number) measure them, and let it be  $E$ . And let  $CD$  measuring  $BF$  leave  $FA$  less than itself, and let  $AF$  measuring  $DG$  leave  $GC$  less than itself, and let  $GC$  measuring  $FH$  leave a unit,  $HA$ .

In fact, since  $E$  measures  $CD$ , and  $CD$  measures  $BF$ ,  $E$  thus also measures  $BF$ . And  $(E)$  also measures the whole of  $BA$ . Thus,  $(E)$  will also measure the remainder

$AF$ . And  $AF$  measures  $DG$ . Thus,  $E$  also measures  $DG$ .  
 And  $(E)$  also measures the whole of  $DC$ . Thus,  $(E)$  will  
 also measure the remainder  $CG$ . And  $CG$  measures  $FH$ .  
 Thus,  $E$  also measures  $FH$ . And  $(E)$  also measures the  
 whole of  $FA$ . Thus,  $(E)$  will also measure the remaining  
 unit  $AH$ , (despite) being a number. The very thing is  
 impossible. Thus, some number does not measure (both)  
 the numbers  $AB$  and  $CD$ . Thus,  $AB$  and  $CD$  are prime  
 to one another. (Which is) the very thing it was required  
 to show.