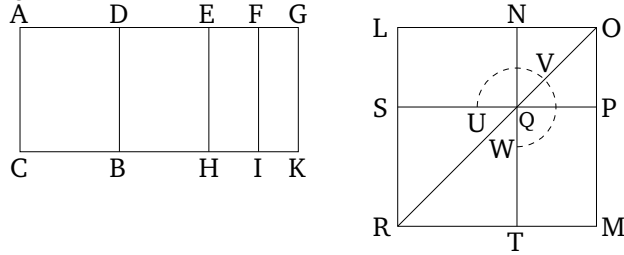


Book 10

Proposition 95

If an area is contained by a rational (straight-line) and a fifth apotome then the square-root of the area is that (straight-line) which with a rational (area) makes a medial whole.



For let the area AB have been contained by the rational (straight-line) AC and the fifth apotome AD . I say that the square-root of area AB is that (straight-line) which with a rational (area) makes a medial whole.

For let DG be an attachment to AD . Thus, AG and DG are rational (straight-lines which are) commensurable in square only [Prop. 10.73], and the attachment GD is commensurable in length the the (previously) laid down rational (straight-line) AC , and the square on the whole, AG , is greater than (the square on) the attachment, DG , by the (square) on (some straight-line) incommensurable (in length) with (AG) [Def. 10.15]. Thus, if (some area), equal to the fourth part of the (square) on DG , is applied to AG , falling short by a square figure, then it divides (AG) into (parts which are) incommensurable (in length) [Prop. 10.18]. Therefore, let DG have been divided in half at point E , and let (some area), equal to the (square) on EG , have been applied to AG ,

falling short by a square figure, and let it be the (rectangle contained) by AF and FG . Thus, AF is incommensurable in length with FG . And since AG is incommensurable in length with CA , and both are rational (straight-lines), AK is thus a medial (area) [Prop. 10.21]. Again, since DG is rational, and commensurable in length with AC , DK is a rational (area) [Prop. 10.19].

Therefore, let the square LM , equal to AI , have been constructed. And let the square NO , equal to FK , (and) about the same angle, LPN , have been subtracted (from NO). Thus, the squares LM and NO are about the same diagonal [Prop. 6.26]. Let PR be their (common) diagonal, and let (the rest of) the figure have been drawn. So, similarly (to the previous propositions), we can show that LN is the square-root of area AB . I say that LN is that (straight-line) which with a rational (area) makes a medial whole.

For since AK was shown (to be) a medial (area), and is equal to (the sum of) the squares on LP and PN , the sum of the (squares) on LP and PN is thus medial. Again, since DK is rational, and is equal to twice the (rectangle contained) by LP and PN , (the latter) is also rational. And since AI is incommensurable with FK , the (square) on LP is thus also incommensurable with the (square) on PN . Thus, LP and PN are (straight-lines which are) incommensurable in square, making the sum of the squares on them medial, and twice the (rectangle contained) by them rational. Thus, the remainder LN is the irrational (straight-line) called that which with a rational (area) makes a medial whole [Prop. 10.77]. And

it is the square-root of area AB .

Thus, the square-root of area AB is that (straight-line) which with a rational (area) makes a medial whole. (Which is) the very thing it was required to show.