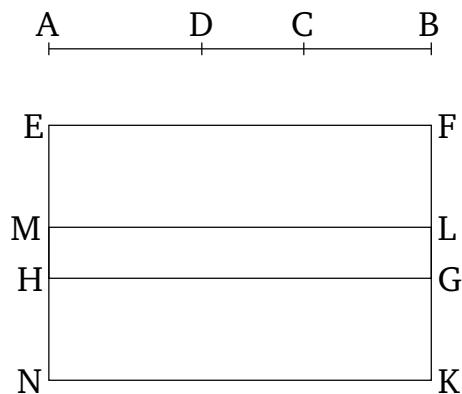


# Book 10

## Proposition 47

The square-root of (the sum of) two medial (areas) can be divided (into its component terms) at one point only.



Let  $AB$  be [the square-root of (the sum of) two medial (areas)] which has been divided at  $C$ , such that  $AC$  and  $CB$  are incommensurable in square, making the sum of the (squares) on  $AC$  and  $CB$  medial, and the (rectangle contained) by  $AC$  and  $CB$  medial, and, moreover, incommensurable with the sum of the (squares) on ( $AC$  and  $CB$ ) [Prop. 10.41]. I say that  $AB$  cannot be divided at another point fulfilling the prescribed (conditions).

For, if possible, let it have been divided at  $D$ , such that  $AC$  is again manifestly not the same as  $DB$ , but  $AC$  (is), by hypothesis, greater. And let the rational (straight-line)  $EF$  be laid down. And let  $EG$ , equal to (the sum of) the (squares) on  $AC$  and  $CB$ , and  $HK$ , equal to twice the (rectangle contained) by  $AC$  and  $CB$ , have been applied to  $EF$ . Thus, the whole of  $EK$  is equal to the square on  $AB$  [Prop. 2.4]. So, again, let  $EL$ , equal

to (the sum of) the (squares) on  $AD$  and  $DB$ , have been applied to  $EF$ . Thus, the remainder—twice the (rectangle contained) by  $AD$  and  $DB$ —is equal to the remainder,  $MK$ . And since the sum of the (squares) on  $AC$  and  $CB$  was assumed (to be) medial,  $EG$  is also medial. And it is applied to the rational (straight-line)  $EF$ .  $HE$  is thus rational, and incommensurable in length with  $EF$  [Prop. 10.22]. So, for the same (reasons),  $HN$  is also rational, and incommensurable in length with  $EF$ . And since the sum of the (squares) on  $AC$  and  $CB$  is incommensurable with twice the (rectangle contained) by  $AC$  and  $CB$ ,  $EG$  is thus also incommensurable with  $GN$ . Hence,  $EH$  is also incommensurable with  $HN$  [Props. 6.1, 10.11]. And they are (both) rational (straight-lines). Thus,  $EH$  and  $HN$  are rational (straight-lines which are) commensurable in square only. Thus,  $EN$  is a binomial (straight-line) which has been divided (into its component terms) at  $H$  [Prop. 10.36]. So, similarly, we can show that it has also been (so) divided at  $M$ . And  $EH$  is not the same as  $MN$ . Thus, a binomial (straight-line) has been divided (into its component terms) at different points. The very thing is absurd [Prop. 10.42]. Thus, the square-root of (the sum of) two medial (areas) cannot be divided (into its component terms) at different points. Thus, it can be (so) divided at one [point] only.