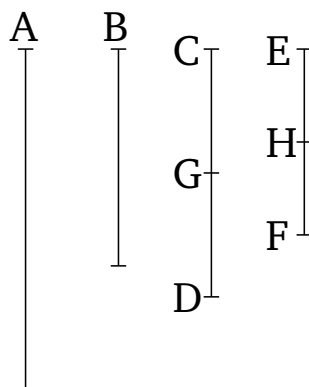


# Book 7

## Proposition 20

The least numbers of those (numbers) having the same ratio measure those (numbers) having the same ratio as them an equal number of times, the greater (measuring) the greater, and the lesser the lesser.

For let  $CD$  and  $EF$  be the least numbers having the same ratio as  $A$  and  $B$  (respectively). I say that  $CD$  measures  $A$  the same number of times as  $EF$  (measures)  $B$ .



For  $CD$  is not parts of  $A$ . For, if possible, let it be (parts of  $A$ ). Thus,  $EF$  is also the same parts of  $B$  that  $CD$  (is) of  $A$  [Def. 7.20, Prop. 7.13]. Thus, as many parts of  $A$  as are in  $CD$ , so many parts of  $B$  are also in  $EF$ . Let  $CD$  have been divided into the parts of  $A$ ,  $CG$  and  $GD$ , and  $EF$  into the parts of  $B$ ,  $EH$  and  $HF$ . So the multitude of (divisions)  $CG$ ,  $GD$  will be equal to the multitude of (divisions)  $EH$ ,  $HF$ . And since the numbers  $CG$  and  $GD$  are equal to one another, and the numbers  $EH$  and  $HF$  are also equal to one another, and the multitude of (divisions)  $CG$ ,  $GD$  is equal to

the multitude of (divisions)  $EH$ ,  $HF$ , thus as  $CG$  is to  $EH$ , so  $GD$  (is) to  $HF$ . Thus, as one of the leading (numbers is) to one of the following, so will (the sum of) all of the leading (numbers) be to (the sum of) all of the following [Prop. 7.12]. Thus, as  $CG$  is to  $EH$ , so  $CD$  (is) to  $EF$ . Thus,  $CG$  and  $EH$  are in the same ratio as  $CD$  and  $EF$ , being less than them. The very thing is impossible. For  $CD$  and  $EF$  were assumed (to be) the least of those (numbers) having the same ratio as them. Thus,  $CD$  is not parts of  $A$ . Thus, (it is) a part (of  $A$ ) [Prop. 7.4]. And  $EF$  is the same part of  $B$  that  $CD$  (is) of  $A$  [Def. 7.20, Prop 7.13]. Thus,  $CD$  measures  $A$  the same number of times that  $EF$  (measures)  $B$ . (Which is) the very thing it was required to show.