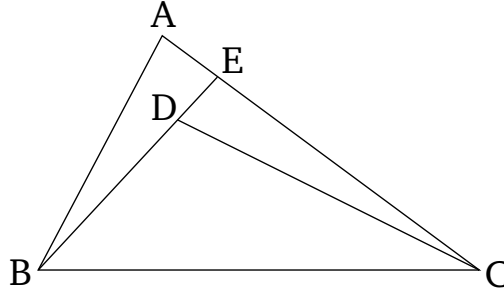


Book 1

Proposition 21

If two internal straight-lines are constructed on one of the sides of a triangle, from its ends, the constructed (straight-lines) will be less than the two remaining sides of the triangle, but will encompass a greater angle.



For let the two internal straight-lines BD and DC have been constructed on one of the sides BC of the triangle ABC , from its ends B and C (respectively). I say that BD and DC are less than the (sum of the) two remaining sides of the triangle BA and AC , but encompass an angle BDC greater than BAC .

For let BD have been drawn through to E . And since in any triangle (the sum of any) two sides is greater than the remaining (side) [Prop. 1.20], in triangle ABE the (sum of the) two sides AB and AE is thus greater than BE . Let EC have been added to both. Thus, (the sum of) BA and AC is greater than (the sum of) BE and EC . Again, since in triangle CED the (sum of the) two sides CE and ED is greater than CD , let DB have been added to both. Thus, (the sum of) CE and EB is greater than (the sum of) CD and DB . But, (the sum of) BA and AC was shown (to be) greater than (the sum of) BE and EC . Thus, (the sum of) BA and AC is much

greater than (the sum of) BD and DC .

Again, since in any triangle the external angle is greater than the internal and opposite (angles) [Prop. 1.16], in triangle CDE the external angle BDC is thus greater than CED . Accordingly, for the same (reason), the external angle CEB of the triangle ABE is also greater than BAC . But, BDC was shown (to be) greater than CEB . Thus, BDC is much greater than BAC .

Thus, if two internal straight-lines are constructed on one of the sides of a triangle, from its ends, the constructed (straight-lines) are less than the two remaining sides of the triangle, but encompass a greater angle. (Which is) the very thing it was required to show.