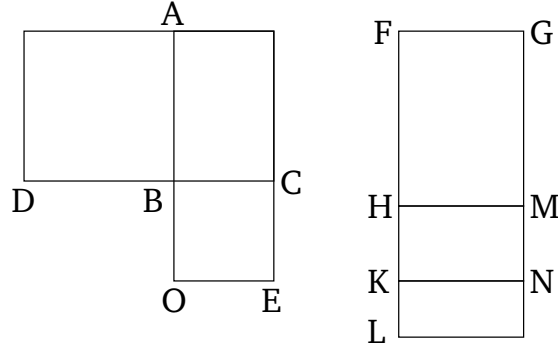


# Book 10

## Proposition 25

The rectangle contained by medial straight-lines (which are) commensurable in square only is either rational or medial.



For let the rectangle  $AC$  be contained by the medial straight-lines  $AB$  and  $BC$  (which are) commensurable in square only. I say that  $AC$  is either rational or medial.

For let the squares  $AD$  and  $BE$  have been described on (the straight-lines)  $AB$  and  $BC$  (respectively).  $AD$  and  $BE$  are thus each medial. And let the rational (straight-line)  $FG$  be laid out. And let the rectangular parallelogram  $GH$ , equal to  $AD$ , have been applied to  $FG$ , producing  $FH$  as breadth. And let the rectangular parallelogram  $MK$ , equal to  $AC$ , have been applied to  $HM$ , producing  $HK$  as breadth. And, finally, let  $NL$ , equal to  $BE$ , have similarly been applied to  $KN$ , producing  $KL$  as breadth. Thus,  $FH$ ,  $HK$ , and  $KL$  are in a straight-line. Therefore, since  $AD$  and  $BE$  are each medial, and  $AD$  is equal to  $GH$ , and  $BE$  to  $NL$ ,  $GH$  and  $NL$  (are) thus each also medial. And they are applied to the rational (straight-line)  $FG$ .  $FH$  and  $KL$  are thus

each rational, and incommensurable in length with  $FG$  [Prop. 10.22]. And since  $AD$  is commensurable with  $BE$ ,  $GH$  is thus also commensurable with  $NL$ . And as  $GH$  is to  $NL$ , so  $FH$  (is) to  $KL$  [Prop. 6.1]. Thus,  $FH$  is commensurable in length with  $KL$  [Prop. 10.11]. Thus,  $FH$  and  $KL$  are rational (straight-lines which are) commensurable in length. Thus, the (rectangle contained) by  $FH$  and  $KL$  is rational [Prop. 10.19]. And since  $DB$  is equal to  $BA$ , and  $OB$  to  $BC$ , thus as  $DB$  is to  $BC$ , so  $AB$  (is) to  $BO$ . But, as  $DB$  (is) to  $BC$ , so  $DA$  (is) to  $AC$  [Props. 6.1]. And as  $AB$  (is) to  $BO$ , so  $AC$  (is) to  $CO$  [Prop. 6.1]. Thus, as  $DA$  is to  $AC$ , so  $AC$  (is) to  $CO$ . And  $AD$  is equal to  $GH$ , and  $AC$  to  $MK$ , and  $CO$  to  $NL$ . Thus, as  $GH$  is to  $MK$ , so  $MK$  (is) to  $NL$ . Thus, also, as  $FH$  is to  $HK$ , so  $HK$  (is) to  $KL$  [Props. 6.1, 5.11]. Thus, the (rectangle contained) by  $FH$  and  $KL$  is equal to the (square) on  $HK$  [Prop. 6.17]. And the (rectangle contained) by  $FH$  and  $KL$  (is) rational. Thus, the (square) on  $HK$  is also rational. Thus,  $HK$  is rational. And if it is commensurable in length with  $FG$  then  $HN$  is rational [Prop. 10.19]. And if it is incommensurable in length with  $FG$  then  $KH$  and  $HM$  are rational (straight-lines which are) commensurable in square only: thus,  $HN$  is medial [Prop. 10.21]. Thus,  $HN$  is either rational or medial. And  $HN$  (is) equal to  $AC$ . Thus,  $AC$  is either rational or medial.

Thus, the ... by medial straight-lines (which are) commensurable in square only, and so on ....