

## Book 11

### Proposition 18

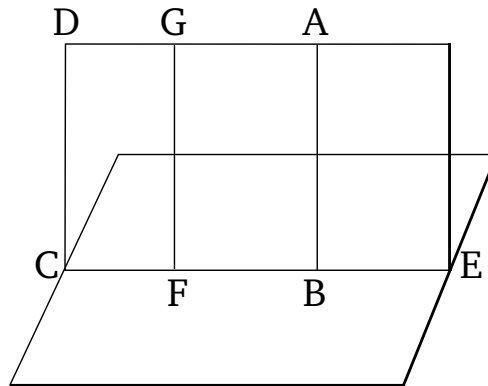
If a straight-line is at right-angles to some plane then all of the planes (passing) through it will also be at right-angles to the same plane.

For let some straight-line  $AB$  be at right-angles to a reference plane. I say that all of the planes (passing) through  $AB$  are also at right-angles to the reference plane.

For let the plane  $DE$  have been produced through  $AB$ . And let  $CE$  be the common section of the plane  $DE$  and the reference (plane). And let some random point  $F$  have been taken on  $CE$ . And let  $FG$  have been drawn from  $F$ , at right-angles to  $CE$ , in the plane  $DE$  [Prop. 1.11].

And since  $AB$  is at right-angles to the reference plane,  $AB$  is thus also at right-angles to all of the straight-lines joined to it which are also in the reference plane [Def. 11.3]. Hence, it is also at right-angles to  $CE$ . Thus, angle  $ABF$  is a right-angle. And  $GFB$  is also a right-angle. Thus,  $AB$  is parallel to  $FG$  [Prop. 1.28]. And  $AB$  is at right-angles to the reference plane. Thus,  $FG$  is also at right-angles to the reference plane [Prop. 11.8]. And a plane is at right-angles to a(nother) plane when the straight-lines drawn at right-angles to the common section of the planes, (and lying) in one of the planes, are at right-angles to the remaining plane [Def. 11.4]. And  $FG$ , (which was) drawn at right-angles to the common section of the planes,  $CE$ , in one of the planes,  $DE$ , was shown to be at right-angles to the reference plane. Thus, plane  $DE$  is at right-angles to the reference (plane). So,

similarly, it can be shown that all of the planes (passing) at random through  $AB$  (are) at right-angles to the reference plane.



Thus, if a straight-line is at right-angles to some plane then all of the planes (passing) through it will also be at right-angles to the same plane. (Which is) the very thing it was required to show.