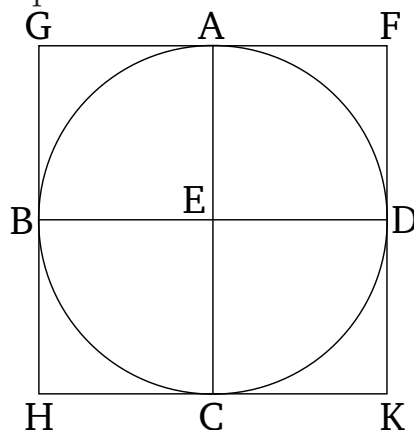


Book 4

Proposition 7

To circumscribe a square about a given circle.

Let $ABCD$ be the given circle. So it is required to circumscribe a square about circle $ABCD$.



Let two diameters of circle $ABCD$, AC and BD , have been drawn at right-angles to one another.[†] And let FG , GH , HK , and KF have been drawn through points A , B , C , and D (respectively), touching circle $ABCD$.[‡]

Therefore, since FG touches circle $ABCD$, and EA has been joined from the center E to the point of contact A , the angles at A are thus right-angles [Prop. 3.18]. So, for the same (reasons), the angles at points B , C , and D are also right-angles. And since angle AEB is a right-angle, and EBG is also a right-angle, GH is thus parallel to AC [Prop. 1.29]. So, for the same (reasons), AC is also parallel to FK . So that GH is also parallel to FK [Prop. 1.30]. So, similarly, we can show that GF and HK are each parallel to BED . Thus, GK , GC , AK , FB , and BK are (all) parallelograms. Thus, GF is equal to HK , and GH to FK [Prop. 1.34]. And since

AC is equal to BD , but AC (is) also (equal) to each of GH and FK , and BD is equal to each of GF and HK [Prop. 1.34] [and each of GH and FK is thus equal to each of GF and HK], the quadrilateral $FGHK$ is thus equilateral. So I say that (it is) also right-angled. For since $GBEA$ is a parallelogram, and AEB is a right-angle, AGB is thus also a right-angle [Prop. 1.34]. So, similarly, we can show that the angles at H , K , and F are also right-angles. Thus, $FGHK$ is right-angled. And it was also shown (to be) equilateral. Thus, it is a square [Def. 1.22]. And it has been circumscribed about circle $ABCD$.

Thus, a square has been circumscribed about the given circle. (Which is) the very thing it was required to do.