

Book 10

Proposition 18

If there are two unequal straight-lines, and a (rectangle) equal to the fourth part of the (square) on the lesser, falling short by a square figure, is applied to the greater, and divides it into (parts which are) incommensurable [in length], then the square on the greater will be larger than the (square on the) lesser by the (square) on (some straight-line) incommensurable (in length) with the greater. And if the square on the greater is larger than the (square on the) lesser by the (square) on (some straight-line) incommensurable (in length) with the greater, and a (rectangle) equal to the fourth (part) of the (square) on the lesser, falling short by a square figure, is applied to the greater, then it divides it into (parts which are) incommensurable [in length].

Let A and BC be two unequal straight-lines, of which (let) BC (be) the greater. And let a (rectangle) equal to the fourth [part] of the (square) on the lesser, A , falling short by a square figure, have been applied to BC . And let it be the (rectangle contained) by BDC . And let BD be incommensurable in length with DC . I say that that the square on BC is greater than the (square on) A by the (square) on (some straight-line) incommensurable (in length) with (BC).

$A \perp \text{—————} \perp$

$\begin{array}{ccccccc} & | & & | & & | & & | & \\ \text{B} & & \text{F} & & \text{E} & & \text{D} & & \text{C} \end{array}$

For, similarly, by the same construction as before, we

can show that the square on BC is greater than the (square on) A by the (square) on FD . [Therefore] it must be shown that BC is incommensurable in length with DF . For since BD is incommensurable in length with DC , BC is thus also incommensurable in length with CD [Prop. 10.16]. But, DC is commensurable (in length) with the sum of BF and DC [Prop. 10.6]. And, thus, BC is incommensurable (in length) with the sum of BF and DC [Prop. 10.13]. Hence, BC is also incommensurable in length with the sum of BF and DC . And the square on BC is greater than the (square on) A by the (square) on FD . Thus, the square on BC is greater than the (square on) A by the (square) on (some straight-line) incommensurable (in length) with (BC) .

So, again, let the square on BC be greater than the (square on) A by the (square) on (some straight-line) incommensurable (in length) with (BC) . And let a (rectangle) equal to the fourth [part] of the (square) on A , falling short by a square figure, have been applied to BC . And let it be the (rectangle contained) by BD and DC . It must be shown that BD is incommensurable in length with DC .

For, similarly, by the same construction, we can show that the square on BC is greater than the (square) on A by the (square) on FD . But, the square on BC is greater than the (square) on A by the (square) on (some straight-line) incommensurable (in length) with (BC) . Thus, BC is incommensurable in length with FD . Hence, BC is also incommensurable (in length) with the remaining sum of BF and DC [Prop. 10.16]. But, the sum of BF and DC is commensurable in length with

DC [Prop. 10.6]. Thus, BC is also incommensurable in length with DC [Prop. 10.13]. Hence, via separation, BD is also incommensurable in length with DC [Prop. 10.16].

Thus, if there are two ... straight-lines, and so on