

# Book 10

## Proposition 82

Only one straight-line, which is incommensurable in square with the whole, and (together) with the whole makes the (sum of the) squares on them rational, and twice the (rectangle contained) by them medial, can be attached to a minor (straight-line).



Let  $AB$  be a minor (straight-line), and let  $BC$  be (so) attached to  $AB$ . Thus,  $AC$  and  $CB$  are (straight-lines which are) incommensurable in square, making the sum of the squares on them rational, and twice the (rectangle contained) by them medial [Prop. 10.76]. I say that another another straight-line fulfilling the same (conditions) cannot be attached to  $AB$ .

For, if possible, let  $BD$  be (so) attached (to  $AB$ ). Thus,  $AD$  and  $DB$  are also (straight-lines which are) incommensurable in square, fulfilling the (other) aforementioned (conditions) [Prop. 10.76]. And since by whatever (area) the (sum of the squares) on  $AD$  and  $DB$  exceeds the (sum of the squares) on  $AC$  and  $CB$ , twice the (rectangle contained) by  $AD$  and  $DB$  also exceeds twice the (rectangle contained) by  $AC$  and  $CB$  by this (same area) [Prop. 2.7]. And the (sum of the) squares on  $AD$  and  $DB$  exceeds the (sum of the) squares on  $AC$  and  $CB$  by a rational (area). For both are rational (areas). Thus, twice the (rectangle contained) by  $AD$  and  $DB$  also exceeds twice the (rectangle contained) by  $AC$  and  $CB$  by a rational (area). The very thing is impossible.

For both are medial (areas) [Prop. 10.26].

Thus, only one straight-line, which is incommensurable in square with the whole, and (with the whole) makes the squares on them (added) together rational, and twice the (rectangle contained) by them medial, can be attached to a minor (straight-line). (Which is) the very thing it was required to show.