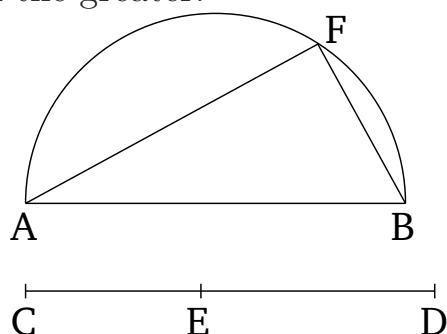


# Book 10

## Proposition 29

To find two rational (straight-lines which are) commensurable in square only, such that the square on the greater is larger than the (square on the) lesser by the (square) on (some straight-line which is) commensurable in length with the greater.



For let some rational (straight-line)  $AB$  be laid down, and two square numbers,  $CD$  and  $DE$ , such that the difference between them,  $CE$ , is not square [Prop. 10.28 lem. I]. And let the semi-circle  $AFB$  have been drawn on  $AB$ . And let it be contrived that as  $DC$  (is) to  $CE$ , so the square on  $BA$  (is) to the square on  $AF$  [Prop. 10.6 corr.]. And let  $FB$  have been joined.

[Therefore,] since as the (square) on  $BA$  is to the (square) on  $AF$ , so  $DC$  (is) to  $CE$ , the (square) on  $BA$  thus has to the (square) on  $AF$  the ratio which the number  $DC$  (has) to the number  $CE$ . Thus, the (square) on  $BA$  is commensurable with the (square) on  $AF$  [Prop. 10.6]. And the (square) on  $AB$  (is) rational [Def. 10.4]. Thus, the (square) on  $AF$  (is) also rational. Thus,  $AF$  (is) also rational. And since  $DC$  does

not have to  $CE$  the ratio which (some) square number (has) to (some) square number, the (square) on  $BA$  thus does not have to the (square) on  $AF$  the ratio which (some) square number has to (some) square number either. Thus,  $AB$  is incommensurable in length with  $AF$  [Prop. 10.9]. Thus, the rational (straight-lines)  $BA$  and  $AF$  are commensurable in square only. And since as  $DC$  [is] to  $CE$ , so the (square) on  $BA$  (is) to the (square) on  $AF$ , thus, via conversion, as  $CD$  (is) to  $DE$ , so the (square) on  $AB$  (is) to the (square) on  $BF$  [Props. 5.19 corr., 3.31, 1.47]. And  $CD$  has to  $DE$  the ratio which (some) square number (has) to (some) square number. Thus, the (square) on  $AB$  also has to the (square) on  $BF$  the ratio which (some) square number has to (some) square number.  $AB$  is thus commensurable in length with  $BF$  [Prop. 10.9]. And the (square) on  $AB$  is equal to the (sum of the squares) on  $AF$  and  $FB$  [Prop. 1.47]. Thus, the square on  $AB$  is greater than (the square on)  $AF$  by (the square on)  $BF$ , (which is) commensurable (in length) with  $(AB)$ .

Thus, two rational (straight-lines),  $BA$  and  $AF$ , commensurable in square only, have been found such that the square on the greater,  $AB$ , is larger than (the square on) the lesser,  $AF$ , by the (square) on  $BF$ , (which is) commensurable in length with  $(AB)$ . (Which is) the very thing it was required to show.