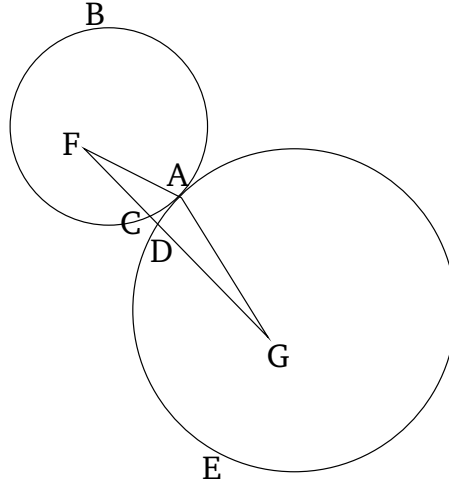


## Book 3

### Proposition 12

If two circles touch one another externally then the (straight-line) joining their centers will go through the point of union.



For let two circles,  $ABC$  and  $ADE$ , touch one another externally at point  $A$ , and let the center  $F$  of  $ABC$  have been found [Prop. 3.1], and (the center)  $G$  of  $ADE$  [Prop. 3.1]. I say that the straight-line joining  $F$  to  $G$  will go through the point of union at  $A$ .

For (if) not then, if possible, let it go like  $FCDG$  (in the figure), and let  $AF$  and  $AG$  have been joined.

Therefore, since point  $F$  is the center of circle  $ABC$ ,  $FA$  is equal to  $FC$ . Again, since point  $G$  is the center of circle  $ADE$ ,  $GA$  is equal to  $GD$ . And  $FA$  was also shown (to be) equal to  $FC$ . Thus, the (straight-lines)  $FA$  and  $AG$  are equal to the (straight-lines)  $FC$  and  $GD$ . So the whole of  $FG$  is greater than  $FA$  and  $AG$ . But, (it is) also less [Prop. 1.20]. The very thing is impossible. Thus,

the straight-line joining  $F$  to  $G$  cannot not go through the point of union at  $A$ . Thus, (it will go) through it.

Thus, if two circles touch one another externally then the [straight-line] joining their centers will go through the point of union. (Which is) the very thing it was required to show.