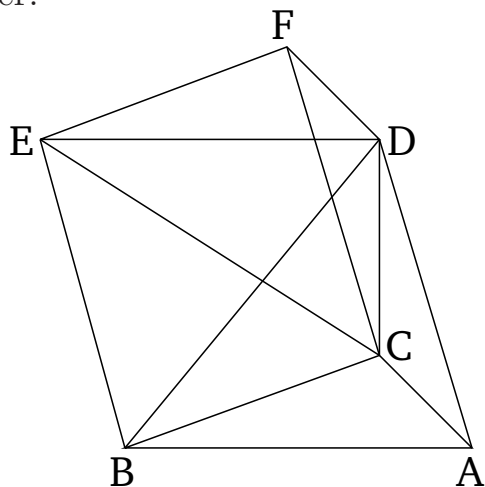


## Book 12

### Proposition 7

Any prism having a triangular base is divided into three pyramids having triangular bases (which are) equal to one another.



Let there be a prism whose base (is) triangle  $ABC$ , and opposite (plane)  $DEF$ . I say that prism  $ABCDEF$  is divided into three pyramids having triangular bases (which are) equal to one another.

For let  $BD$ ,  $EC$ , and  $CD$  have been joined. Since  $ABED$  is a parallelogram, and  $BD$  is its diagonal, triangle  $ABD$  is thus equal to triangle  $EBD$  [Prop. 1.34]. And, thus, the pyramid whose base (is) triangle  $ABD$ , and apex the point  $C$ , is equal to the pyramid whose base is triangle  $DEB$ , and apex the point  $C$  [Prop. 12.5]. But, the pyramid whose base is triangle  $DEB$ , and apex the point  $C$ , is the same as the pyramid whose base is triangle  $EBC$ , and apex the point  $D$ . For they are contained by the same planes. And, thus, the pyramid whose base is  $ABD$ , and apex the point  $C$ , is equal to

the pyramid whose base is  $EBC$  and apex the point  $D$ . Again, since  $FCBE$  is a parallelogram, and  $CE$  is its diagonal, triangle  $CEF$  is equal to triangle  $CBE$  [Prop. 1.34]. And, thus, the pyramid whose base is triangle  $BCE$ , and apex the point  $D$ , is equal to the pyramid whose base is triangle  $ECF$ , and apex the point  $D$  [Prop. 12.5]. And the pyramid whose base is triangle  $BCE$ , and apex the point  $D$ , was shown (to be) equal to the pyramid whose base is triangle  $ABD$ , and apex the point  $C$ . Thus, the pyramid whose base is triangle  $CEF$ , and apex the point  $D$ , is also equal to the pyramid whose base [is] triangle  $ABD$ , and apex the point  $C$ . Thus, the prism  $ABCDEF$  has been divided into three pyramids having triangular bases (which are) equal to one another.

And since the pyramid whose base is triangle  $ABD$ , and apex the point  $C$ , is the same as the pyramid whose base is triangle  $CAB$ , and apex the point  $D$ . For they are contained by the same planes. And the pyramid whose base (is) triangle  $ABD$ , and apex the point  $C$ , was shown (to be) a third of the prism whose base is triangle  $ABC$ , and opposite (plane)  $DEF$ , thus the pyramid whose base is triangle  $ABC$ , and apex the point  $D$ , is also a third of the pyramid having the same base, triangle  $ABC$ , and opposite (plane)  $DEF$ .

Corollary

And, from this, (it is) clear that any pyramid is the third part of the prism having the same base as it, and an equal height. (Which is) the very thing it was required to show.