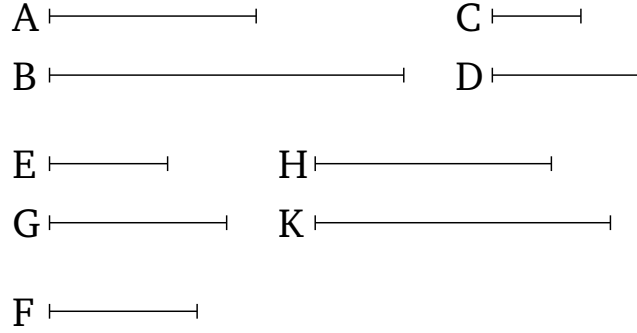


## Book 8

### Proposition 15

If a cube number measures a(nother) cube number then the side (of the former) will also measure the side (of the latter). And if the side (of a cube number) measures the side (of another cube number) then the (former) cube (number) will also measure the (latter) cube (number).

For let the cube number  $A$  measure the cube (number)  $B$ , and let  $C$  be the side of  $A$ , and  $D$  (the side) of  $B$ . I say that  $C$  measures  $D$ .



For let  $C$  make  $E$  (by) multiplying itself. And let  $D$  make  $G$  (by) multiplying itself. And, further, [let]  $C$  [make]  $F$  (by) multiplying  $D$ , and let  $C, D$  make  $H, K$ , respectively, (by) multiplying  $F$ . So it is clear that  $E, F, G$  and  $A, H, K, B$  are continuously proportional in the ratio of  $C$  to  $D$  [Prop. 8.12]. And since  $A, H, K, B$  are continuously proportional, and  $A$  measures  $B$ , ( $A$ ) thus also measures  $H$  [Prop. 8.7]. And as  $A$  is to  $H$ , so  $C$  (is) to  $D$ . Thus,  $C$  also measures  $D$  [Def. 7.20].

And so let  $C$  measure  $D$ . I say that  $A$  will also measure  $B$ .

For similarly, with the same construction, we can show that  $A, H, K, B$  are continuously proportional in the

ratio of  $C$  to  $D$ . And since  $C$  measures  $D$ , and as  $C$  is to  $D$ , so  $A$  (is) to  $H$ ,  $A$  thus also measures  $H$  [Def. 7.20]. Hence,  $A$  also measures  $B$ . (Which is) the very thing it was required to show.