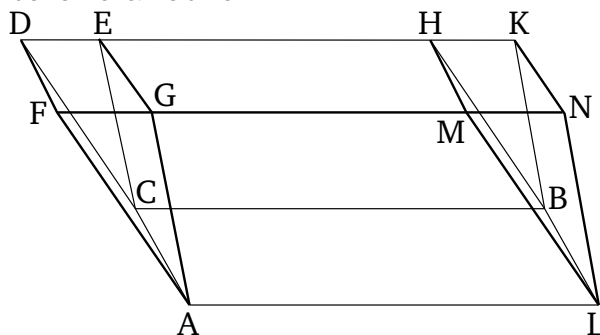


Book 11

Proposition 29

Parallelepiped solids which are on the same base, and (have) the same height, and in which the (ends of the straight-lines) standing up are on the same straight-lines, are equal to one another.



For let the parallelepiped solids CM and CN be on the same base AB , and (have) the same height, and let the (ends of the straight-lines) standing up in them, AG , AF , LM , LN , CD , CE , BH , and BK , be on the same straight-lines, FN and DK . I say that solid CM is equal to solid CN .

For since CH and CK are each parallelograms, CB is equal to each of DH and EK [Prop. 1.34]. Hence, DH is also equal to EK . Let EH have been subtracted from both. Thus, the remainder DE is equal to the remainder HK . Hence, triangle DCE is also equal to triangle HBK [Props. 1.4, 1.8], and parallelogram DG to parallelogram HN [Prop. 1.36]. So, for the same (reasons), triangle AFG is also equal to triangle MLN . And parallelogram CF is also equal to parallelogram BM , and CG to BN [Prop. 11.24]. For they are opposite.

Thus, the prism contained by the two triangles AFG and DCE , and the three parallelograms AD , DG , and CG , is equal to the prism contained by the two triangles MLN and HBK , and the three parallelograms BM , HN , and BN . Let the solid whose base (is) parallelogram AB , and (whose) opposite (face is) $GEHM$, have been added to both (prisms). Thus, the whole parallelepiped solid CM is equal to the whole parallelepiped solid CN .

Thus, parallelepiped solids which are on the same base, and (have) the same height, and in which the (ends of the straight-lines) standing up (are) on the same straight-lines, are equal to one another. (Which is) the very thing it was required to show.