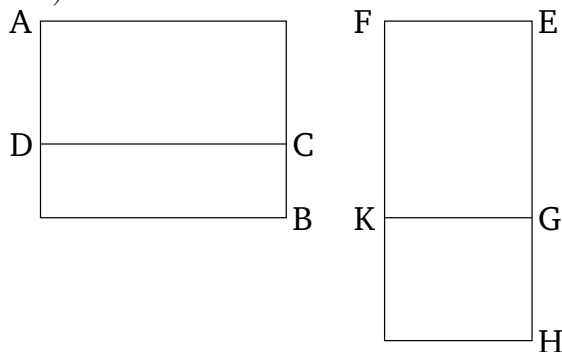


Book 10

Proposition 26

A medial (area) does not exceed a medial (area) by a rational (area).



For, if possible, let the medial (area) AB exceed the medial (area) AC by the rational (area) DB . And let the rational (straight-line) EF be laid down. And let the rectangular parallelogram FH , equal to AB , have been applied to to EF , producing EH as breadth. And let FG , equal to AC , have been cut off (from FH). Thus, the remainder BD is equal to the remainder KH . And DB is rational. Thus, KH is also rational. Therefore, since AB and AC are each medial, and AB is equal to FH , and AC to FG , FH and FG are thus each also medial. And they are applied to the rational (straight-line) EF . Thus, HE and EG are each rational, and incommensurable in length with EF [Prop. 10.22]. And since DB is rational, and is equal to KH , KH is thus also rational. And (KH) is applied to the rational (straight-line) EF . GH is thus rational, and commensurable in length with EF [Prop. 10.20]. But, EG is also rational, and incommensurable in length with

EF . Thus, EG is incommensurable in length with GH [Prop. 10.13]. And as EG is to GH , so the (square) on EG (is) to the (rectangle contained) by EG and GH [Prop. 10.13 lem.]. Thus, the (square) on EG is incommensurable with the (rectangle contained) by EG and GH [Prop. 10.11]. But, the (sum of the) squares on EG and GH is commensurable with the (square) on EG . For (EG and GH are) both rational. And twice the (rectangle contained) by EG and GH is commensurable with the (rectangle contained) by EG and GH [Prop. 10.6]. For (the former) is double the latter. Thus, the (sum of the squares) on EG and GH is incommensurable with twice the (rectangle contained) by EG and GH [Prop. 10.13]. And thus the sum of the (squares) on EG and GH plus twice the (rectangle contained) by EG and GH , that is the (square) on EH [Prop. 2.4], is incommensurable with the (sum of the squares) on EG and GH [Prop. 10.16]. And the (sum of the squares) on EG and GH (is) rational. Thus, the (square) on EH is irrational [Def. 10.4]. Thus, EH is irrational [Def. 10.4]. But, (it is) also rational. The very thing is impossible.

Thus, a medial (area) does not exceed a medial (area) by a rational (area). (Which is) the very thing it was required to show.