ESSAYS,

Mathematical and Physical

CONTAINING

NEW

THEORIES AND ILLUSTRATIONS

OF SOME VERY

Important and Difficult Subjects

OF THE

SCIENCES.

NEVER BEFORE PUBLISHED.

EST ANIMORUM INGENIORUMQUE NOSTRORUM NATURALE QUODDAM QUASI PABULUM, CONSIDERATIO CONTEMPLATIOQUE NATURÆ: ET INDAGATIO IPSA RERUM MAGNARUM OCCULTARUMQUE HABET OBLECTATIONEM.

Cicero.

IN GEOMETRIA PARTEM FATENTUR ESSE UTILEM TENERIS ÆTATIBUS; AGITARI NAMQUE ANIMOS ET ACUI INGENIA AD CELERITATEM PERCIPIENDI VENIRE INDE CONCEDUNT; SED PRODESSE EAM NON UT CETERAS ARTES CUM PERCEPTÆ SINT, SED CUM DISCATUR, EXISTIMANT. EA VULGARIS OPINIO EST; NEC SINE CAUSA SUMMI VIRI ETIAM IMPENSAM HUIC SCIENTIÆ OPERAM DEDERUNT.

Quintilian.

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Of NOTHING *and* INFINITY.

CONNECTED in a mannner with the foregoing, and ſubject to ſimilar animadverſions and cavils, is that part of the mathematics which relates to the properties of algebraical expreſſions when they are conſidered as nothing or infinite. If the ſtrict ſenſe of the words only be regarded, there appears to be juſt ground of cenſure towards thoſe who have vitiated a ſcience renowned for the clearneſs and preciſion of its reaſoning, with an unintelligible jargon of nothings and infinites. The fault, however, is ſimply in the terms adopted, and by no means extends to the ideas and truths of the mathematics, which are every where clear and undeniable and ſtand in no need of myſtery to veil them. Let metaphyſicians diſpute and wrangle about infinity and other matters equally *metaphyſical* or beyond nature, it is certain that the true ſubject on which the genius of mathematics delights to dwell is *phyſics*, or thoſe objects which are within the bounds of nature and intelligible to every diligent *reſearcher* of her works. If improper terms have been introduced into this ſcience it is to be regretted; but as names are arbitrary, this will not be thought important, if by a definition, preciſe ideas be aſſociated to them.

It is obvious, that magnitude may be conſidered, as ſuſceptible of increaſe or diminution beyond any aſſignable limit.—— Whenever it is thus conſidered, it is denominated infinite, indefinite, or unlimited; which terms are often uſed in their ſtrict ſenſe, ſo that the lexicographic definition of them becomes that of the *thing* to which they are applied; but actual *infinity* producing on the one hand the limit of the ratios of quantities indefinitely great, and *nothing* that of quantities indefinitely ſmall, theſe terms for the ſake of conciſeneſs have been ſometimes uſed inſtead of the others; thus in the expreſſion $\frac{x}{y}$ if x be increaſed to infinity and y be a finite quantity the value of $\frac{x}{y}$ is infinite; if x be a finite quantity and y infinite, the value of $\frac{x}{y}$, is nothing; by which we are to underſtand that if x be increaſed beyond any aſſignable quantity, the ratio of x to y is great beyond any aſſignable ratio and if y be increaſed be-

$\sqrt{\frac{a^2}{4}-b^2}=Mn$; and the roots are either $AC-Mn$ or $AC+Mn$, or PM and PM, which ſhews that the circle may be cut in two places and the two roots diſcover the correſponding values of the line PM. But if we ſuppoſe AP to vary, then when $-b^2$ under the radical ſign $=\frac{a^2}{4}$; then $\sqrt{\frac{a^2}{4}-b^2}=0$, and $x=\frac{a}{4}$; that is the two roots PM, PM become equal to one another and to PN the radius of the circle. If b^2 be greater, than $\frac{a^2}{4}$ or b be greater than $\frac{a}{2}$, then the expreſſion $\sqrt{\frac{a^2}{4}-b^2}$ is impoſſible, becauſe the quantity under the radical ſign is negative, and the ſquare root of a negative quantity is impoſſible; which alſo exactly correſponds with the figure, for when AP (b) is greater than CN ($\frac{a}{2}$) then the problem is impoſſible, for the line no where interſects the circle.

After the diſcrimination which we have made of quantities which may and which cannot admit of the negative ſign without abſurdity, there will be no need of an explanation of negative roots in numbers. They muſt either repreſent thoſe quantities which have been ſhewn to be ſuſceptible of negative values or they are to be rejected as wholly impoſſible and abſurd.

the whole expreſſion or that of the ratio of $1-x^2$ to $1-x$, in their evaneſcent ſtate, will differ from that of 2 to 1, by a ratio leſs than any aſſignable one and conſequently 2 is the only definite expreſſion for the value of $\frac{1-x^2}{1-x}$ in the caſe where it is at the limit of *nothing*.

In the ſame manner $\frac{1-x^3}{1-x}$ when $x=1$ is 3 and $\frac{1-x^4}{1-x}$ is 4, &c. from which it is evident that the value of ſuch expreſſions when they are abſolutely nothing is not pretended to be known in mathematical inquiries, and is only a figment of thoſe ſcioliſts who know ſcarcely enough of the ſciences to point out their real faults, much leſs to amend them.

After the definitions we have given of *nothing* and *infinity*, the following rules taken from the writings of an eminent mathematician, it is preſumed, will not be thought prepoſterous; and in the higher branches of the mathematics they will be found of conſiderable utility to ſtudents—*

1. If *nothing* multiply any finite quantity, the product will be nothing.

2. If *nothing* multiply an infinite quantity, the product is a finite quantity. Or a finite quantity is a mean proportional between nothing and infinity.

3. If a finite quantity be divided by nothing, the quotient is infinite,

4. If nothing be divided by nothing, the quotient is a finite quantity.

5. If a quantity be nothing and the index of its power nothing, that quantity is equal to unity; or the infinitely ſmall power of an infinitely ſmall quantity is infinitely near 1.

6. Adding or ſubtracting any finite quantities to or from an infinite quantity, makes no alteration, therefore,

* *Emerson's Algebra.*

yond any assignable quantity, the value of $\frac{x}{y}$ will be less than any assignable quantity: Moreover if x be nothing, $\frac{x}{y}$ is nothing, and if y become nothing, $\frac{x}{y}$ is infinite.

By nothing, is to be understood an indefinitely small quantity, and by no means a total privation of magnitude or absolute nothing, this will be manifest from the following, let $\frac{1-x^2}{1-x}$ be another algebraical expression, wherein x is a variable quantity, which may be taken at pleasure. If x be taken $=1$, then $1-x^2=0$ and $1-x=0$; from which an unskilful algebraist would assign 0, as the value of this expression, whereas its real value is 2; for when $x=0$, then $\frac{1-x^2}{1-x}=1$. Suppose x to have some value less than 1: the greater this value is, the more will $\frac{1-x^2}{1-x}$ exceed 1, and when $x=1$ then $\frac{1-x^2}{1-x}$, or $1+x\times\frac{1-x}{1-x}=1+x=2$. In the same manner $\frac{1-x}{1-x}$, when $x=1$, becomes equal to 1, which must be considered as its true value and not 0, as the learner might imagine. The reason of which is, that absolute nothing is wholly incapable of mathematical ratio and comparison, and must be considered only as the limit of indefinitely small decreasing quantities, at which, when they have arrived, they are said to become nothing, that is, when they are in their evanescent state, or less than any assignable quantity. Now the ratio of quantities one to another, in this state, may be determined, though their values be incapable of assignation, by means of a comparison of the ultimate tendency of this ratio; thus $\frac{1-x^2}{1-x}$ becomes nearer 2, as x tends to an equality with 1 or the numerator and denominater approach to 0, and if we suppose x susceptible of a negative value, when it has passed the limit of 0, or when x exceeds 1, then $\frac{1-x^2}{1-x}$ or $1+x\times\frac{1-x}{1-x}$ will exceed 2; hence the value of

7. In any equation where are fome quantities infinitely lefs than others; they may be thrown out of the equation.

8. An infinite quantity may be confidered either as affirmative or negative.

Note.—As we can conceive of nothing but what is fufceptible of augmentation and diminution, actual infinity, or that which embraces the whole of an infinite quantity, is to the human mind totally incomprehenfible, and yet no one can deny the exiftence of this actual infinity in extenfion, fpace and time: but though our minds are incapable of forming any ideas of infinity in this fenfe of the word, we are able neverthelefs to conceive of the fucceffive augmentation or diminution of objects without end; and alfo of a limit to their ratios which they cannot pafs though actually infinite.

Thus the ratio of the tangent and fecant of an arch is always diminifhing in all finite degrees of their magnitude as the correfponding arch approaches to a quadrant; though the tangent is always lefs than the fecant by fome finite quantity nor is there any limit to this ratio of minority on the fide of the tangent, unlefs we fuppofe thofe quantities continued beyond all poffible finite magnitude or actually to become infinite; in which cafe their ratio is that of equality, and is the limit of the ratio to which the quantities themfelves increafing without any limit continually approach; and to which the ratio can come within any difference, which may be given, but can never pafs, nor even reach this limit before the quantities become actually infinite. Of this infinity of magnitudes, though we can have no conception, we may notwithftanding have very clear ideas of the quantity of their ratio—for the ratio of majority of the fecant to the tangent decreafing as the quantities themfelves increafe; it is manifeft that when thefe become great beyond any finite quantity, the excefs of the ratio of the fecant to the tangent above that of equality, becomes lefs than any finite quantity and when actually infinite, it will be infinitely little or nothing, confequently the ratio of fecant to tangent when thefe quantities become infinite, is that of equality.

In the fame manner, fuppofing two lines indefinitely continued to form an angle, and another line perpendicular or forming any given angle with either of the firft, to move from the angular point forming a triangle continually increafing. Now the ratio of the sides of this triangle, even when actually infinite, must be allowed to be in the same known or given ratio as when in a finite state; for the ratio continues invariably the same, whatever the magnitude of the sides may be.

Moreover the sides of the triangle, when actually infinite, form infinities of different magnitudes, and by varying the proportion of the sides, which may be done to an indefinite degree, different infinities will arise in indefinitely varying ratios, whence if one quantity be supposed *infinite*, others may be supposed infinite, yet differing from the former in an indefinite ratio of *majority* or *minority*.

These instances were adduced, with a view of exhibiting the futility and arrogance of those sciolists, who affectedly disclaim any knowledge of infinites, because magnitude, when actually infinite, is wholly incomprehensible to the human mind. We on the other hand maintain, that the *relation* and *ratios* of such quantities can be clearly ascertained in certain cases, though the quantities themselves be wholly beyond the reach of the human intellect.

But the mathematics has no occasion ever of considering quantities as infinite. It is only necessary that the varying magnitude be supposed susceptible of increase or decrease beyond any assignable quantity. Then that limit of the varying magnitude or ratio, which it can approach within any degree of nearness, or from which it differs by a quantity or ratio less than any assignable one, yet can never pass, is the magnitude or ratio of a quantity thus continued beyond any assignable quantity.

Since writing the above, which was merely the result of the authors reflection and experience in the mathematics; it was with great satisfaction that he found the principal ideas concerning infinity, confirmed by the greatest *genius* that ever adorned human nature. These are his words, *" Ultimæ rationes illæ, quibuscum quantitates evanescunt, revera non sunt rationes quantitatum ultimarum, sed limites ad quos quantitatum sine limite decrescentium rationes semper appropinquant; et quas proprius assequi possunt quam pro data quavis differentia, nunquam vero transgredi, neque prius at-

* *Newtoni princip. philos. lib.* 1. *lem.* 11 *in schol.*