A TREATISE OF ALGEBRA,

In TWO BOOKS.

BOOK I.

CONTAINING,

The Fundamental Principles of this ART.

Together with

All the Practical Rules of OPERATION.

BOOK II.

CONTAINING,

A great VARIETY of PROBLEMS,

In the most important

BRANCHES of the MATHEMATICS.

William Emerson

Vix quicquam in universa Mathesi ita difficile aut arduum accurrere posse, quo non inoffenso pede per hanc methodum penetrare liceat.

SCHOOT. Pref. to DES CARTES.

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MDCCLXXX.

Ex. 5.

Extract the cube root of $-10+\sqrt{-243}$.

Here $A^2 - B^2 = 100 + 243 = 343 = D$, and $D^{\frac{1}{3}} = 7$. Therefore $x^3 + 3x \times xx - 7 = -10$, or $4x^3 - 21x = -10$, and the root is $x = 2$, whence v or $\sqrt{y} = \sqrt{4-7} = \sqrt{-3}$, and $x + \sqrt{y} = 2 + \sqrt{-3}$, as required.

In like manner the cube root of $-10 - \sqrt{-243}$ is $2 - \sqrt{-3}$.

Ex. 6.

Extract the 5th root of $843 - 589\sqrt{2}$.

Here $AA - BB = 16807$, and $D^{\frac{1}{5}} = 7$.

And $16x^5 - 140x^3 + 245x = 843$, and the root is $x = 3$; and $\sqrt{y} = -\sqrt{9-7} = -\sqrt{2}$, and $x + \sqrt{y} = 3 - \sqrt{2}$ the root required.

Ex. 7.

What is the 7th root of $568 + 328\sqrt{3}$.

Here $A^2 - B^2 = -128 = D$, and $\sqrt[7]{D} = -2$. Then $A + B = 1136.112$, $A - B = -.112$, and

$$x = \frac{\sqrt[7]{1136.112} - \sqrt[7]{-.112}}{2} = \frac{2.732 - .732}{2} = 1.$$

And $v = \dfrac{\sqrt[7]{1136.112} + \sqrt[7]{-.112}}{2} = \dfrac{2.732 + .732}{2} = 1.732 = \sqrt{3}$, and $x + v = 1 + \sqrt{3}$ = the root.

SCHOLIUM.

In the former method, if $\sqrt[n]{D}$ is not rational, neither member of the root will be rational, and in in the second, if neither the sum nor difference of $\sqrt[n]{A+B}$ and $\sqrt[n]{A-B}$, is rational; neither member of the root will be so: and in these cases the rules are of no use. Logarithms will be useful here in finding these roots, being exact enough in finding whether any of the quantities be rational or not. When none of these quantities are rational, multiply the given equation by some number, till $\sqrt[n]{D}$, or $\sqrt[n]{A+B} \pm \sqrt[n]{A-B}$, comes out rational; then extract the root as before. But remember to divide the values of x, v, at last, by the root of that number. Thus $22 + \sqrt{486}$ has not such a cube root; but multiply by 2, and then $\frac{44 + \sqrt{1944}}{2}$, will have a cube root, for the numerator.

PROBLEM LXXIII.

To explain the several properties of (o) nothing, and infinity.

It is plain, nothing added to, or subtracted from, any quantity, makes it neither bigger nor less.

Likewise, if any quantity is multiplied by o, that is, taken no times at all; the product will be nothing.

Let $\frac{b}{a} = q$; that is, let the quotient, of b divided by a, be q. Then if b remains the same, it is plain the less a is, the greater the quotient q will be. Let a be indefinitely small beyond all bounds, then q will be indefinitely great beyond all bounds. Therefore when a is nothing, the quotient q will be infinite. Whence

Alſo ſince $\frac{b}{0}$ = infinity, therefore b = nothing × infinity.

Let there be ſeveral geometrical proportionals, x, x^2, x^3, x^4, x^5, &c. If this ſeries be continued backwards, it will be x, 1, $\frac{1}{x}$, $\frac{1}{xx}$; that is, x^1, x^0, x^{-1}, x^{-2}, the indices continually decreaſing by 1. Then its plain x^0 is equal to 1, whatever x be; for it may ſtand univerſally for any thing. Therefore 0^0 is = 1.

Let x be an indefinitely ſmall quantity, beyond all conception; then in the ſeries x, x^2, x^3, &c. each term will be indefinitely greater than the following one. And when x is 0, then in the ſeries $\frac{1}{0}$, 0^0, 0^1, 0^2, *&c.* $\frac{1}{0}$ is infinite, and 0 is nothing, by what goes before. Therefore the mean 0^0 is a finite quantity. Suppoſe = b, whence $\frac{1}{0} \times 0 = bb$, that is $bb = \frac{1 \times 0}{0} = 1$, and $b = 1$, whence it is plain again, that (b) $0^0 = 1$.

Let $\frac{a}{1-1}$ or its equal $\frac{a}{-1+1}$ be an infinite quantity, then by actually dividing, $\frac{a}{1-1} = a + a + a + \frac{a}{1-1}$, and $\frac{a}{-1+1} = -a - a - a + \frac{a}{-1+1}$. Therefore $\frac{a}{1-1} + a + a + a$ &c. $= \frac{a}{1-1} - a - a - a$ &c. that is, an infinite quantity is neither increaſed nor decreaſed by finite quantities.

Cor. 1. *If* 0 *multiply any finite quantity, the product will be nothing.*

Cor.

Cor. 2. *If o multiply an infinite quantity, the product is a finite quantity. Or a finite quantity is a mean proportional between nothing and infinity.*

For $0 \times \text{infinity} = b$.

Cor. 3. *If a finite quantity is divided by o, the quotient is infinite* ($\frac{b}{0} = \text{inf.}$).

Cor. 4. *If o be divided by o, the quotient is a finite quantity of some sort.*

For (Co. 1.) $b \times 0 = 0$, and therefore $\frac{0}{0} = b$, a finite quantity, or nothing.

Cor. 5. *Hence also* $0^0 = 1$, *or the infinitely small power, of an infinitely small quantity, is infinitely near 1.*

Cor. 6. *Adding or subtracting any finite quantities to or from an infinite quantity, makes no alteration.*

Cor. 7. *Therefore in any equation, where are some quantities infinitely less than others; they may be thrown out of the equation.*

Cor. 8. *An infinite quantity may be considered either as affirmative or negative.*

For infinity $= \frac{b}{+0}$ or $\frac{b}{-0}$.

SCHOLIUM.

There is something extremely subtle, and hard to conceive, in the doctrine of *infinites* and *nothings*. Yet although the objects themselves are beyond our comprehension; yet we cannot resist the force of demonstration, concerning their powers, properties, and effects; which properties, under such and such conditions, I think, I have truly explained in this proposition. Any metaphysical notions, that go beyond these mathematical operations, are

 not

not the business of a mathematician. But thus much may be observed, that o, in a mathematical sense, never signifies absolute nothing; but always nothing in relation to the object under consideration. For illustration thereof, suppose we are considering the area contained between the base of a parallelogram and a line drawn parallel to the base. As this line draws nearer the base, the area diminishes; till at last, when the line coincides with the base, the area becomes nothing. So the area here degenerates into a line; which is nothing, or no part of the area. But it is a line still, and may be compared with other lines.

PROBLEM LXXIV.

To find the value of a fraction, when the numerator and denominator, is each of them nothing.

1 RULE.

Consider, from the nature of the question proposed, what quantities are infinitely greater than others, when they are all taken infinitely small. Then throw out of the equation, all those terms that are infinitely less than others; retaining only those that are infinitely greater than the rest; by which expunge one of the unknown quantities, and the value of the fraction will be known.

Ex. 1.

Let $x^3 + y^3 = axy$, *and y infinitely greater than x, when they vanish; to find the value of* $\frac{yy}{x}$, *when x and y are* $= 0$.

Here x^3 is infinitely less than axy or y^3, whence $y^3 = axy$, or $yy = ax$. Then $\frac{yy}{x} = \frac{ax}{x} = a$, the value of the fraction proposed.

Ex.

Ex. 2.

If $2ax + xx = yy$, *what is the value of* $\frac{x}{yy}$, *when* x *and* $y = 0$, and y *infinitely greater than* x.

Here reject xx being infinitely less than the rest; then $yy = 2ax$, and $\frac{x}{yy} = \frac{1}{2a}$.

Ex. 3.

What is the value of $\frac{y}{x}$, *when* $2ay + yy = rx$; y, x *being* $= 0$.

Here yy is infinitely less than $2ay$. Whence $2ay = rx$, and $\frac{y}{x} = \frac{r}{2a} = \frac{0}{0}$.

2 RULE.

Observe what the unknown quantity is equal to, when the numerator, &c. vanishes; put the unknown quantity = that value $\pm e$, where e is supposed infinitely small. Which being substituted for that unknown quantity, and the roots of all surds, extracted to a sufficient number of places of e; at last you will have some terms in both the numerator and denominator, which will determine the value of the fraction.

Ex. 4.

What is the value of $\frac{a\sqrt{ax} - xx}{a - \sqrt{ax}}$, *when* $x = a$.

Put $x = a + e$, then expunging x; $\frac{a\sqrt{ax} - xx}{a - \sqrt{ax}}$

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