David P. Robbins Prize

In 2005, the family of David P. Robbins gave the Mathematical Association of America funds sufficient to support a prize honoring the author or authors of a paper reporting on novel research in algebra, combinatorics, or discrete mathematics. The David P. Robbins Prize of $5,000 is awarded every third year at a national meeting of the Association. Papers are judged on quality of research, clarity of exposition, and accessibility to undergraduates. The paper must have been published within six years of the presentation of the prize, and must be written in English. In the event of joint authors, the prize shall be divided equally. The recipient need not be a member of the Association.

Samantha Dahlberg, Angèle Foley, and Stephanie van Willigenburg

"Resolving Stanley's e-positivity of claw-contractible-free graphs." *J. Eur. Math. Soc.* (JEMS) 22 (2020), no. 8, 2673–2696. doi.org/10.4171/JEMS/974

A coloring of the vertices of a graph is called *proper* if no two adjacent vertices have the same color. Proper colorings have been studied since the late 1800s and have a wide range of applications. The *chromatic polynomial* $\chi_G(t)$ counts the number of proper colorings of a graph G with t given colors. In the 1990s Stanley introduced a multivariate generalization of this polynomial: the chromatic symmetric function $X_G(x_1, x_2,\ldots)$, where the variables x_i represent the colors. One obtains χG(t) from $X_G(x_1, x_2,\ldots)$ by setting t of the variables x_i equal to 1 and the remaining ones to 0. Every symmetric function can be written as a linear combination of elementary symmetric functions, and a symmetric function is called *e-positive* if all the coefficients are non-negative when the function is written in the elementary basis. In his seminal 1995 paper, Stanley asked whether the chromatic symmetric function of graphs that were not contractible to a claw were *e*-positive. For twenty-five years this question remained unanswered, until Dahlberg, Foley, and van Willigenburg settled the question negatively by constructing several infinite families of graphs that show that claws and *e*-positivity are unrelated.

Many questions related to the chromatic symmetric function of a graph remain open and constitute very active research problems in algebraic combinatorics. In particular, conjectures related to the *e*-positivity of the chromatic symmetric function are still open and have applications to geometry and representation theory. The paper "Resolving Stanley's e-positivity of claw-contractible-free graphs" by Dahlberg, Foley, and van Willigenburg contains high quality results that are beautifully written and provide an engaging invitation for undergraduates to get involved in research.

Responses

Samantha Dahlberg: When I first received the email, I couldn't believe it. It was only after reading the excited replies from my co-authors, did I actually understand that this was real. I am extremely honored to be receiving this award with my two co-authors Dr. Angèle Foley and Dr. Stephanie van Willigenburg. The paper was the perfect collaborative storm formed from our individual talents and expertise. I am grateful to have met and worked with my coauthors. The entire journey of this paper has been nothing less than joyous, and I am humbled that the MAA has chosen to give this paper an award. I would also like to thank my thesis advisor, Dr. Bruce Sagan, whose patience and encouragement throughout my graduate school education and following mathematical career have been a cornerstone and an example I exemplify when mentoring. Also, a thank you to my other postdoctoral mentors, Dr. Susanna Fishel and Dr. Hemanshu Kaul, who have encouraged me and supported me throughout my journey. Some of the kindling that inspired this paper came from computer searches using Sage, a Python-based language. Computers have come so far since dial-up in the 1990's, the decade Richard Stanley wrote his unanswered question. While dial-up might have been able to do these calculations on the 21 connected graphs on five vertices, I would bet average computers in the 1990s couldn't have done these calculations on the 112 connected graphs on 6 vertices, where we found our first counterexamples to Stanley's question. We found even our modern computers had limits during our follow-up search for a more particular kind of graph. It took three days to search through graphs on eight vertices, with no avail. We predicted it would take three years to finish the search on nine vertices. Ridiculous! However, we decided a weekend worth of searching would give us more information. On that Monday I had a note on my desk from Dr. van Willigenburg saying "Please see me" (My word, I was so very worried. Was I in trouble?). Later that day I meekly entered her office to find a very excited Dr. van Willigenburg. Our computer search turned something out! (My word, I was so very relieved). We now had the makings for a very fun result and paper. Thank you, Steph and Angèle, for the fun times.

Angèle Foley: I am honoured and grateful to receive the Robbins Prize, especially as Robbins' work has inspired much of my own research on alternating sign matrices. I am especially delighted to share this award with co-authors. In graduate school, I was surprised to learn that mathematicians were among the most widely-travelled researchers in the university. Why? Because mathematics runs on the exchange of ideas, at conferences or together at a chalkboard. Such collaboration is the lifeblood of mathematical research, and good research partners are as valuable as they are rare. Good mentors are also rare,

and I appreciate all those who have helped form me into the mathematician I am today. And although I have had many fine mentors, the first and best was my father, Professor Michael Foley, who taught me to love words and who awakened in me the beauty of discovering new things.

Stephanie van Willigenburg: I am extremely grateful and truly elated to receive the 2023 MAA David P. Robbins Prize, which I dedicate to my parents, Judy and Evert van Willigenburg; my sister, Michelle van Willigenburg; and my husband, Niall Christie. Your love and support have propelled me further than I thought possible, and words cannot express my love and gratitude. I am as proud of you as you are of me. Heartfelt thanks also go to my postdoc mentor at Cornell, Lou Billera, for being the most superlative mentor and role-model I could wish for, and for convincing me to become a mathematician. Many thanks are also owed to Richard Stanley for posing the problem and for being so enthusiastic when we shared our solution with you—25 years later. I am indebted to my fantastic research group, my students, and my dear friends at UBC and in algebraic combinatorics for making work feel like play. Equal thanks go to my cherished non-mathematical friends who bring balance to the equation that is my life. Amongst my dear (mathematical) friends are my brilliant coauthors Samantha and Angèle, whom I thank heartily for a most thrilling and delightful collaboration. I will always treasure our epic adventure together and remember how it took our unique combination of skills to crack the problem. Once we had, I truly felt that we were onto something special, and my final earnest thanks go to the MAA and the Robbins Prize Committee for deciding this too.

Biographical Sketches

Samantha Dahlberg is a native Michigander who received her undergraduate degree from Grand Valley State University in 2010. She earned her PhD in mathematics from Michigan State University in 2016. Under the advising of Dr. Bruce Sagan she got to delve into the wonderful world of combinatorics with the irresistible gateway topic of pattern avoidance in permutations. After graduating, Samantha had several postdoctoral positions. The first was at the University of British Columbia, with Dr. Stephanie van Willigenburg, where Samantha explored chromatic symmetric functions as well as snowboarded down the beautiful mountains in Vancouver. The second was at Arizona State University, with Dr. Susanna Fishel, where Samantha dived into the combinatorial and geometric world of reduced words of permutations as well as hiked up desert mountains. The third was at Illinois Institute of Technology, where she is currently employed and is working with Dr. Hemanshu Kaul. There she is surveying various new probabilistic and algebraic techniques to apply to

variations on graph colorings as well as enjoying walking on the other side of Lake Michigan that you can see in Chicago.

Angèle Foley is a professor at Wilfrid Laurier University in Waterloo, Ontario, Canada. She graduated with a BSc (mathematics) from the University of Prince Edward Island, and an MMath (pure mathematics) and a PhD (combinatorics and optimization) from the University of Waterloo. After completing her PhD, she kept moving west, first to Christchurch, New Zealand, and then to Southampton, United Kingdom, before completing the circle by returning to Waterloo. She has worked at Laurier for over 20 years as a professor in the Department of Physics and Computer Science, and she is cross appointed to the Department of Mathematics. Her research interests are in the area of algebraic combinatorics, particularly symmetric functions, tableaux, and all things enumerative. Mathematics is one of her favourite ways of making art, and she devotes herself to creating theorems and proofs that exemplify Keats' aphorism: "Beauty is truth, truth Beauty." A mathematician and a writer, she has published in numerous mathematics and computer science journals and conferences, in economics journals, in a physics journal, and in a literary quarterly.

Stephanie van Willigenburg is a professor of mathematics at the University of British Columbia (UBC) in Vancouver, Canada. Before this, she obtained her PhD from St Andrews University in Scotland and was a visiting assistant professor at Cornell University.

Her many awards for research include winning the Krieger-Nelson Prize from the Canadian Mathematical Society for outstanding research in 2017, and she was inducted as a Fellow of the American Mathematical Society in 2023. She is a dedicated mentor to her research group, has won a Killam Award for teaching, and has given numerous plenary lectures including an Invited Address at the Joint Mathematics Meeting.

Stephanie is also currently the associate dean for equity, diversity and inclusion in science at UBC. She is one of the co-founders of the Algebraic Combinatorics Research Community that fosters mentoring, collaborations and networking for minorities in her area of algebraic combinatorics and related areas.