

# Paul R. Halmos - Lester R. Ford Awards

## Paul Thomas Young

“From Madhava-Leibniz to Lehmer’s Limit,” *The American Mathematical Monthly*, 129:6, 524–538. doi.org/10.1080/00029890.2022.2051405

The starting point of this article is a paper by D.H. Lehmer. In this paper, Lehmer considered a family of infinite series revolving around the central binomial coefficient. Some ideas of this paper came from Apéry’s “famous paper” about the irrationality of  $\zeta(3)$ .

The term “Lehmer limit” is coined by Dyson et al. in their 2012 paper, which was extended in a 2012 paper of Glasser. Their proof used the ideas from Gauss’ hypergeometric functions using beta, gamma, and Bessel functions, and Fourier and Hankel transforms.

In the present paper, a short history of Gregory-Leibniz series is given and mentions the credits should also be extended to Madhava for the same series, hence the part of the paper’s title. The background of the article is very nice and elementary, incorporating Taylor series, Stirling numbers of the second kind, and the Lehmer polynomials to mention a few. All of these are used in palatable doses, thus making the article accessible to anybody! The author also extended Lehmer’s idea to the field of 2-adic numbers in a 2016 talk. Using this extension, his proof of the (complex) Lehmer limit is elementary in the sense that it does not employ the fancy tools mentioned in the previous paragraph. He emphasizes the connection between the Lehmer limit and Madhava-Leibniz formula for  $\pi$ .

It is nice to see how Lehmer’s interesting series  $S_k(z)$  transforms a power series whose partial sums converge only on a disk into a sequence of rational functions that converge on almost the entire complex plane.

## Response

I am grateful to the Halmos-Ford committee and to the MAA for this award celebrating the art of mathematical exposition. In light of the long tradition of excellent articles in the *Monthly*, I am honored and humbled as a recipient. This article was among the most magical journeys on which mathematics has guided me. It certainly owes a great deal to the inspirational articles of Lehmer and of Dyson, Frankel, and Glasser. I am very grateful for the opportunity to share this thrill of discovery with the readership of the *Monthly*. I thank the College of Charleston for its support, and M. Lawrence Glasser for helpful, encouraging and stimulating exchanges.

## Biographical Sketch

Paul Thomas Young received his PhD in mathematics from Oklahoma State University in 1988 and is a professor of mathematics at the College of Charleston, where he has taught for the last thirty-two years. The longest day of his life was Pi Day, 3/14/16, a thirty-six-hour day which he began playing bass in a club in Wuhan, and ended, after crossing the International Date Line, doing mathematics at his home in Charleston.